Model Question Paper-II with effect from 2022 (CBCS Scheme)

USN

First Semester B.E Degree Examination

Mathematics-I for Electrical & Electronics Engineering Stream (22MATE11)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE.**

		Module -1	Marks
Q.01	a	With usual notations prove that $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$	06
	b	Find the angle between the curves $r = alog\theta$ and $r = \frac{a}{log\theta}$	07
	c	Find the radius of curvature for the cardioids $r = a(1+\cos\theta)$	07
		OR	
Q.02	a	Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ cut each other orthogonally.	06
	b	Find the pedal equation of the curve $r^n = a^n \cos n\theta$	07
	c	Show that the radius of curvature at $(a, 0)$ on the curve $y^2 = \frac{a^2(a-x)}{x}$ is $\frac{a}{2}$	07
		Module-2	
Q. 03	a	Expand $\sqrt{1+sin2x}$ by Maclaurin's series upto the term containing x^5	06
	b	If $u = tan^{-1}\left(\frac{y}{x}\right)$, where $x = e^t - e^{-t}$ $and\ y = e^t + e^{-t}$, find the total derivative $\frac{du}{dt}$ using partial differentiation	07
	c	If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$	07
		OR	
Q.04	a	Evaluate (i) $\lim_{x\to 0}(a^x + x)^{\frac{1}{x}}$ (ii) $\lim_{x\to\frac{\pi}{2}}(\tan x)^{\tan 2x}$	06
	b	If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$	07
	c	Find the maximum and minimum value of $x^3 + y^3 - 3axy$	07
		Module-3	
Q. 05	a	Solve $\frac{dy}{dx} + ytanx = y^3secx$	06
	b	When a resistance R Ohms connected in series with an inductance L henries with an emf of E volts, the current i amperes at time t is given by $L\frac{di}{dt} + Ri = E$. If $E = 100\ sint$ volts and $i = 0$ when $t = 0$, find i as a function of t.	07
	c	Solve $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$	07
		OR	

Q. 06	a	Solve $(x^2 + y^3 + 6x)dx + y^2xdy = 0$	06
	b	Prove that the system of parabolas $y^2 = 4a(x + a)$ is self orthogonal	07
	c	Find the general and singular solution of $$xp^2 + xp - yp + 1 - y = 0$$	07
		Module-4	
Q. 07	a	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdzdydx$	06
	b	Evaluate $\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} (2 - x)dydx$	07
	c	Define beta and gamma functions and show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$	07
		OR	
Q. 08	a	Evaluate by changing the order of integration $\int_0^a \int_0^{2\sqrt{ax}} x^2dydx, a > 0$	06
	b	Evaluate $\int_0^1 x^{\frac{3}{2}}(1 - x)^{\frac{1}{2}}dx$, by expressing in terms of beta and gamma functions	07
	c	Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.	07
		Module-5	
Q. 09	a	Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$	06
	b	Solve the system of equations by Gauss elimination method $$3x + y + 2z = 3,$$ $$2x - 3y - z = -3,$$ $$x + 2y + z = 4$$	07
	c	Using the Gauss-Seidel iteration method, solve the equations $$83x + 11y - 4z = 95$$ $$3x + 8y + 29z = 71$$ $$7x + 52y + 13z = 104$$ Carry out four iterations, starting with the initial approximations (0, 0, 0)	07
		OR	
Q. 10	a	Test for consistency and solve $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$	06
	b	Using Gauss Jordan method, solve $$x + y + z = 11$$ $$3x - y + 2z = 12$$ $$2x + y - z = 3$$	07

	c	Find the largest eigenvalue and the corresponding eigenvector of $$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$$ with the initial approximate eigenvector $[1 \quad 1 \quad 1]^T$	07

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome

Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 01
	(c)	L3	CO 01	PO 02
Q.2	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 01
	(c)	L3	CO 01	PO 02
Q.3	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 02
Q.4	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 03
Q.5	(a)	L2	CO 03	PO 02
	(b)	L3	CO 03	PO 03
	(c)	L2	CO 03	PO 02
Q.6	(a)	L2	CO 03	PO 02
	(b)	L3	CO 03	PO 03
	(c)	L2	CO 03	PO 02
Q.7	(a)	L2	CO 04	PO 02
	(b)	L2	CO 04	PO 02
	(c)	L2	CO 04	PO 01
Q.8	(a)	L2	CO 04	PO 01
	(b)	L2	CO 04	PO 02
	(c)	L2	CO 04	PO 02
Q.9	(a)	L2	CO 05	PO 01
	(b)	L3	CO 05	PO 01
	(c)	L3	CO 05	PO 02
Q.10	(a)	L2	CO 05	PO 01
	(b)	L3	CO 05	PO 01
	(c)	L3	CO 05	PO 02

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering(knowledge): L_1	Understanding Comprehension): L_2	Applying (Application): L3
	Higher order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6

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	Solution and Scheme for award of marks	

AY: 2022-23

Department: Mathematics **Model Question Paper – II**

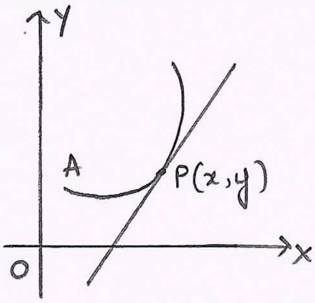
Branch: EC/EE **Semester: I**

Subject with Sub. Code: Mathematics-I for Electrical and Electronics Engineering Stream (22MATE11)

Name of the Faculty: Dr. Lata Lamani

Q.No.	Solution and Scheme	Marks

1. (a)



Let $y=f(x)$ be the equation of the Cartesian Curve and 'A' be a fixed point on it. Let $P(x,y)$ be a point on the Curve such that $\overrightarrow{AP}=s$. Let ψ be the angle made by the tangent at 'P' with the X-axis. Then we know that $\tan\psi=\frac{dy}{dx}$. (2M)

Differentiating w.r.t 's', we have

$$\frac{d}{ds}(\tan\psi)=\frac{d}{ds}\left(\frac{dy}{dx}\right)$$

ie $$\sec^2\psi\cdot\frac{d\psi}{ds}=\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{ds}$$ (1M)

But $\frac{dx}{ds}=\cos\psi$ and by definition $\frac{d\psi}{ds}=\frac{1}{\rho}$.

$$\therefore\ \sec^2\psi\cdot\frac{1}{\rho}=\frac{d^2y}{dx^2}\cdot\cos\psi$$

or $$\sec^3\psi=\rho\cdot\frac{d^2y}{dx^2}$$ (1M)

or $$\rho=\frac{\sec^3\psi}{\left(\frac{d^2y}{dx^2}\right)}=\frac{(\sec^2\psi)^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}=\frac{(1+\tan^2\psi)^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$$ (1M)

Denoting $y_1=\frac{dy}{dx}$ and $y_2=\frac{d^2y}{dx^2}$, we get

$$\boxed{\rho=\frac{(1+y_1^2)^{3/2}}{y_2}}$$ (1M)

(6M)

Q.No.	Solution and Scheme	Marks
1. (b)	Given Curves are $r = a\log\theta$ and $r = \frac{a}{\log\theta}$.	
	Now, $r = a\log\theta$	
	Taking log on both sides, we get	
	$\log r = \log a + \log(\log\theta)$.	
	Differentiating w.r.t 'θ', we get	
	$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{1}{\log\theta}\cdot\frac{1}{\theta}$	
	or $r\cdot\frac{d\theta}{dr} = \theta\log\theta$	(1M)
	ie $\tan\phi_1 = \theta\log\theta$	
	Now, $r = \frac{a}{\log\theta}$	
	Taking log on both sides, we get	
	$\log r = \log a - \log(\log\theta)$	
	Differentiating w.r.t 'θ', we get	
	$\frac{1}{r}\cdot\frac{dr}{d\theta} = \frac{-1}{\log\theta}\cdot\frac{1}{\theta}$.	
	or $r\cdot\frac{d\theta}{dr} = -\theta\log\theta$	(1M)
	ie $\tan\phi_2 = -\theta\log\theta$	
	WKT, $\tan(\phi_1-\phi_2) = \frac{\tan\phi_1 - \tan\phi_2}{1+\tan\phi_1\tan\phi_2}$	
	$= \frac{\theta\log\theta - (-\theta\log\theta)}{1+(\theta\log\theta)(-\theta\log\theta)}$	
	$\therefore \tan(\phi_1-\phi_2) = \frac{2\theta\log\theta}{1-(\theta\log\theta)^2}$ ——— (1)	(1M)

Q.No.	Solution and Scheme	Marks
	Given: $r = a\log\theta$ and $r = \dfrac{a}{\log\theta}$	
	$\therefore a\log\theta = \dfrac{a}{\log\theta} \Rightarrow (\log\theta)^2 = 1 \Rightarrow \log\theta = 1 \Rightarrow \theta = e.$	(1M)
	From (1), $\tan(\phi_1 - \phi_2) = \dfrac{2\theta\log\theta}{1-(\theta\log\theta)^2}$	
	$= \dfrac{2e\log e}{1-(e\log e)^2}$	
	$\therefore \tan(\phi_1-\phi_2) = \dfrac{2e}{1-e^2}$	
	ie $\phi_1 - \phi_2 = \tan^{-1}\left[\dfrac{2e}{1-e^2}\right]$ — (2)	(1M)
	Put $e = \tan\theta \Rightarrow \theta = \tan^{-1}e$.	(1M)
	From (2), $\phi_1-\phi_2 = \tan^{-1}\left[\dfrac{2\tan\theta}{1-\tan^2\theta}\right] = \tan^{-1}[\tan 2\theta] = 2\theta = 2\tan^{-1}e$	
	$\therefore \underline{\underline{\phi_1 - \phi_2 = 2\tan^{-1}e}}$.	(1M)
		(7M)
1. (c)	Given Curve is $r = a(1+\cos\theta)$.	
	Taking log on both sides, we get	
	$\log r = \log a + \log(1+\cos\theta)$.	
	Differentiating w.r.t 'θ', we have	
	$\dfrac{1}{r}\cdot\dfrac{dr}{d\theta} = \dfrac{-\sin\theta}{1+\cos\theta} = \dfrac{-\cancel{2}\sin(\theta/2)\cancel{\cos(\theta/2)}}{\cancel{2}\cos^{\cancel{2}}(\theta/2)} = -\tan\left(\dfrac{\theta}{2}\right)$	
	ie $\dfrac{dr}{d\theta} = -\tan\left(\dfrac{\theta}{2}\right)\cdot r$	
	Or $r_1 = -r\tan\left(\dfrac{\theta}{2}\right)$	(1M)

Q.No.	Solution and Scheme	Marks
	Differentiating w.r.t 'θ', we get	
	$r_2 = -\frac{r}{2}\sec^2\left(\frac{\theta}{2}\right) - r_1\tan\left(\frac{\theta}{2}\right)$	
	ie $r_2 = -\frac{r}{2}\sec^2\left(\frac{\theta}{2}\right) + r\tan^2\left(\frac{\theta}{2}\right)$	(1M)
	WKT, $\rho = \frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$	(1M)
	$\therefore \rho = \frac{\left[r^2+r^2\tan^2(\theta/2)\right]^{3/2}}{r^2+2r^2\tan^2(\theta/2)+\frac{r^2}{2}\sec^2\left(\frac{\theta}{2}\right)-r^2\tan^2\left(\frac{\theta}{2}\right)}$	
	$= \frac{r^3\left(\sec^2(\theta/2)\right)^{3/2}}{r^2\left[1+\tan^2(\theta/2)+\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\right]}$	
	$= \frac{r\sec^3(\theta/2)}{(3/2)\cancel{\sec^2}(\theta/2)}$	
	$\therefore \rho = \frac{2r}{3}\sec\left(\frac{\theta}{2}\right)$ —— (1)	(2M)
	We have, $r = a(1+\cos\theta) = 2a\cos^2(\theta/2)$	
	$\therefore \sec^2\left(\frac{\theta}{2}\right) = \frac{2a}{r}$ or $\sec\left(\frac{\theta}{2}\right) = \frac{\sqrt{2a}}{\sqrt{r}}$	
	From (1), $\rho = \frac{2r}{3}\cdot\frac{\sqrt{2a}}{\sqrt{r}} = \frac{2}{3}\sqrt{2ar}$	(1M)
	$\therefore \rho = \frac{2}{3}\sqrt{2ar}$	(1M)
		(7M)

Q.No.	Solution and Scheme	Marks
2.(a)	Given Curves are $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$.	
	Now, $r^n = a^n \cos n\theta$	
	Taking log on both sides, we get	
	$n \log r = n \log a + \log(\cos n\theta)$	
	Differentiating w.r.t 'θ', we get	
	$\frac{\cancel{n}}{r} \cdot \frac{dr}{d\theta} = \frac{-\cancel{n} \sin n\theta}{\cos n\theta} \Rightarrow \frac{1}{r} \cdot \frac{dr}{d\theta} = -\tan n\theta \Rightarrow \tan\phi_1 = -\cot n\theta$	
	$\therefore \tan\phi_1 = \tan\left(\frac{\pi}{2} + n\theta\right)$	(1M)
	or $\phi_1 = \frac{\pi}{2} + n\theta$	(1M)
	Now, $r^n = b^n \sin n\theta$	
	Taking log on both sides, we get	
	$n \log r = n \log b + \log(\sin n\theta)$	
	Differentiating w.r.t 'θ', we get	
	$\frac{\cancel{n}}{r} \cdot \frac{dr}{d\theta} = \frac{\cancel{n} \cos n\theta}{\sin n\theta} \Rightarrow r \cdot \frac{d\theta}{dr} = \tan n\theta \Rightarrow \tan\phi_2 = \tan n\theta$	(1M)
	$\therefore \phi_2 = n\theta$	(1M)
	Now, $\lvert\phi_1 - \phi_2\rvert = \left\lvert \frac{\pi}{2} + n\theta - n\theta \right\rvert = \frac{\pi}{2}$	
	$\therefore \phi_1 - \phi_2 = \frac{\pi}{2}$.	(1M)
	$\therefore$ The given curves cut each other orthogonally.	(1M)
		(6M)

Q.No.	Solution and Scheme	Marks
2. (b)	Given equation is $r^n = a^n \cos n\theta$ —— (1)	(1M)
	Taking log on both sides, we get	
	$n \log r = n \log a + \log(\cos n\theta)$	
	Differentiating w.r.t 'θ', we get	
	$\frac{\cancel{n}}{r} \cdot \frac{dr}{d\theta} = \frac{-\cancel{n} \sin n\theta}{\cos n\theta}$	
	or $r \cdot \frac{d\theta}{dr} = -\cot n\theta = \tan\left(\frac{\pi}{2} + n\theta\right)$	(1M)
	$\therefore \tan\phi = \tan\left(\frac{\pi}{2} + n\theta\right)$	(1M)
	$\therefore \phi = \frac{\pi}{2} + n\theta$.	(1M)
	WKT, $p = r\sin\phi$	(1M)
	$\therefore p = r\sin\left(\frac{\pi}{2} + n\theta\right) = r\cos n\theta$	
	ie $p = r\cos n\theta$	
	$\therefore p = r \cdot \frac{r^n}{a^n}$ [$\because$ From (1)]	(1M)
	$\therefore r^{n+1} = pa^n$ is the required pedal curve.	(1M)
		(7M)
2. (c)	Given Curve is : $y^2 = \frac{a^2(a-x)}{x}$	
	$\therefore y^2 = \frac{a^3 - a^2x}{x}$	
	$\therefore y^2 = \frac{a^3}{x} - a^2$ —— (1)	(1M)
	Differentiating (1) w.r.t 'x', we get	

Q.No.	Solution and Scheme	Marks
	$2yy_1 = \frac{-a^3}{x^2}$ $\therefore y_1 = \frac{-a^3}{2yx^2}$ At $(a,0)$, y_1 becomes infinity and hence we have to Consider $\frac{dx}{dy}$.	(1M)
	Let $x_1 = \frac{dx}{dy} = \frac{-2yx^2}{a^3}$ At $(a,0)$, $x_1 = 0$	(1M)
	Now, $x_2 = \frac{-2}{a^3}\left[2xy \cdot x_1 + x^2\right]$ At $(a,0)$, $x_2 = \frac{-2}{a^3}\left[a^2\right] = \frac{-2}{a}$	(1M)
	WKT, $\rho = \frac{(1+x_1^2)^{3/2}}{x_2}$	(1M)
	$\therefore \rho = \frac{(1+0^2)^{3/2}}{\left(\frac{-2}{a}\right)} = \frac{-a}{2}$	(1M)
	Since 'ρ' cannot be negative, $\rho = \frac{a}{2}$	(1M)
		(7M)
3.(a)	We have Maclaurin's series as: $y(x) = y(0) + xy_1(0) + \frac{x^2}{2!}y_2(0) + \frac{x^3}{3!}y_3(0) + \frac{x^4}{4!}y_4(0) + \frac{x^5}{5!}y_5(0) + \cdots$	(1M)

Q.No.	Solution and Scheme	Marks
	Let $y=\sqrt{1+\sin 2x}=\sqrt{\cos^2 x+\sin^2 x+2\sin x\cos x}=\sqrt{(\cos x+\sin x)^2}$	(1M)
	$\therefore y=\cos x+\sin x \Rightarrow y(0)=1$	
	$y_1=-\sin x+\cos x \Rightarrow y_1(0)=1$	
	$y_2=-\cos x-\sin x \Rightarrow y_2(0)=-1$	
	$y_3=\sin x-\cos x \Rightarrow y_3(0)=-1$	(3M)
	$y_4=\cos x+\sin x \Rightarrow y_4(0)=1$	
	$y_5=-\sin x+\cos x \Rightarrow y_5(0)=1$	
	Thus by substituting these values in the expansion of $y(x)$, we get	
	$\sqrt{1+\sin 2x}=1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}-\cdots$	(1M)
		(6M)
3. (b)	The total derivative rule is:	
	$\frac{du}{dt}=\frac{\partial u}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial u}{\partial y}\cdot\frac{dy}{dt}$ — (1)	(1M)
	Given: $u=\tan^{-1}(y/x)$ where $x=e^t-e^{-t}$ and $y=e^t+e^{-t}$.	
	Now, $\frac{\partial u}{\partial x}=\frac{1}{1+\frac{y^2}{x^2}}\left(\frac{-y}{x^2}\right)=\left(\frac{\cancel{x^2}}{x^2+y^2}\right)\left(\frac{-y}{\cancel{x^2}}\right)=\frac{-y}{x^2+y^2}$	
	$\therefore \frac{\partial u}{\partial x}=\frac{-y}{x^2+y^2}$ — (2)	(1M)
	Now, $\frac{\partial u}{\partial y}=\frac{1}{1+\frac{y^2}{x^2}}\left(\frac{1}{x}\right)=\left(\frac{x^{\cancel{2}}}{x^2+y^2}\right)\left(\frac{1}{\cancel{x}}\right)=\frac{x}{x^2+y^2}$	
	$\therefore \frac{\partial u}{\partial y}=\frac{x}{x^2+y^2}$ — (3)	(1M)

Q.No.	Solution and Scheme	Marks
	Now, $\frac{dx}{dt} = e^{t} + e^{-t}$ —— (4)	(1M)
	and $\frac{dy}{dt} = e^{t} - e^{-t}$ —— (5)	(1M)
	Substituting (2), (3), (4) and (5) in (1), we get	
	$\frac{du}{dt} = \left[\left(\frac{-y}{x^2+y^2}\right)(e^{t}+e^{-t})\right] + \left[\left(\frac{x}{x^2+y^2}\right)(e^{t}-e^{-t})\right]$	
	$= \frac{-(e^{t}+e^{-t})(e^{t}+e^{-t})}{(e^{t}-e^{-t})^2+(e^{t}+e^{-t})^2} + \frac{(e^{t}-e^{-t})(e^{t}-e^{-t})}{(e^{t}-e^{-t})^2+(e^{t}+e^{-t})^2}$	
	$= \frac{(e^{t}-e^{-t})^2-(e^{t}+e^{-t})^2}{(e^{t}-e^{-t})^2+(e^{t}+e^{-t})^2} = \frac{-4}{2e^{2t}+2e^{-2t}}$	
	$\therefore \frac{du}{dt} = \frac{-2}{e^{2t}+e^{-2t}}$	(2M)
		(7M)
3. (c)	Given: $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$	
	Now, $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$	(1M)

Q.No.	Solution and Scheme	Marks
	$\therefore \dfrac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} -\frac{yz}{x^2} & \frac{z}{x} & \frac{y}{x} \\ \frac{z}{y} & -\frac{zx}{y^2} & \frac{x}{y} \\ \frac{y}{z} & \frac{x}{z} & -\frac{xy}{z^2} \end{vmatrix}$	(1M)
	ie $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = \left(-\frac{yz}{x^2}\right)\left[\frac{x^2\cancel{y}\cancel{z}}{y^{\cancel{2}}z^{\cancel{2}}} - \frac{x^2}{yz}\right] - \left(\frac{z}{x}\right)\left[\frac{-x\cancel{y}\cancel{z}}{\cancel{y}z^{\cancel{2}}} - \frac{x\cancel{y}}{\cancel{y}z}\right]$ $+\left(\frac{y}{x}\right)\left[\frac{x\cancel{z}}{y\cancel{z}} + \frac{x\cancel{y}\cancel{z}}{y^{\cancel{2}}\cancel{z}}\right]$	(2M)
	$= -\frac{x^2\cancel{yz}}{\cancel{x^2}yz} + \frac{x^2\cancel{yz}}{\cancel{x^2}yz} + \frac{\cancel{x}z}{\cancel{x}z} + \frac{\cancel{xz}}{\cancel{xz}} + \frac{\cancel{xy}}{\cancel{xy}} + \frac{\cancel{xy}}{\cancel{xy}}$	
	$= 1+1+1+1$	(2M)
	$\therefore \dfrac{\partial(u,v,w)}{\partial(x,y,z)} = 4$	(1M)
		(7M)
4.(a)	(i) Let $k = \lim_{x\to 0}(a^x + x)^{1/x}$ $[1^\infty \text{ form}]$ Taking log on both sides, we get $\log_e k = \lim_{x\to 0} \dfrac{\log(a^x+x)}{x}$ $\left[\frac{0}{0} \text{ form}\right]$	(1M)
	Applying L'Hospital's rule,	

Q.No.	Solution and Scheme	Marks
	$\log_e k = \lim_{x\to 0}\left[\dfrac{\frac{1}{a^x+x}\cdot(a^x\log a+1)}{1}\right]$	
	$=\lim_{x\to 0}\left[\dfrac{a^x\log a+1}{a^x+x}\right]$	
	$=\dfrac{\log a+1}{1}$	(1M)
	$\therefore \log_e k=\log a+1$	
	ie $k=e^{\log a+1}=e^{\log a}\cdot e^1=a\cdot e$	
	$\therefore \lim_{x\to 0}(a^x+x)^{1/x}=ae$	(1M)
		(3M)
	(ii) Let $k=\lim_{x\to\pi/2}(\tan x)^{\tan 2x}$ $[\infty^0$ form$]$	
	Taking log on both sides, we get	
	$\log_e k=\lim_{x\to\pi/2}[\tan 2x\log(\tan x)]$ $[0\times\infty$ form$]$	(1M)
	$\therefore \log_e k=\lim_{x\to\pi/2}\left[\dfrac{\log(\tan x)}{\cot 2x}\right]$ $\left[\dfrac{\infty}{\infty}\text{ form}\right]$	(1M)
	Applying L'Hospital's rule,	
	$\log_e k=\lim_{x\to\pi/2}\left[\dfrac{\frac{1}{\tan x}\cdot\sec^2 x}{-2\operatorname{cosec}^2 2x}\right]$	(1M)
	$=\lim_{x\to\pi/2}\dfrac{-\cancel{\cos x}}{2\sin x}\cdot\dfrac{1}{\operatorname{cosec}^2 2x}\cdot\dfrac{1}{\cos^{\cancel{2}} x}$	
	$=\lim_{x\to\pi/2}\dfrac{-1}{2\sin x}\cdot\sin^2 2x\cdot\dfrac{1}{\cos x}$	
	$=\lim_{x\to\pi/2}\left[\dfrac{-\sin^{\cancel{2}} 2x}{\cancel{\sin 2x}}\right]$	

Q.No.	Solution and Scheme	Marks
	$\therefore \log_e k = \lim_{x\to\pi/2}[-\sin 2x] = 0$	
	$\therefore k = e^0 = 1$	
	ie $\lim_{x\to\pi/2}(\tan x)^{\tan 2x} = 1$	(1M)
		(4M)
4.(b)	Given: $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$	
	Here we need to Convert the given function 'u' into a Composite function.	(1M)
	Let $u = f(p,q)$ where $p = \frac{y-x}{xy}$, $q = \frac{z-x}{xz}$	(1M)
	Now, $p = \frac{1}{x} - \frac{1}{y}$ and $q = \frac{1}{x} - \frac{1}{z}$	(1M)
	[ie $u \to (p,q) \to (x,y,z)$] $\Rightarrow u \to (x,y,z)$	
	$\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial x} + \frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial x} = \frac{\partial u}{\partial p}\left\{\frac{-1}{x^2}\right\} + \frac{\partial u}{\partial q}\left\{\frac{-1}{x^2}\right\}$	
	$\therefore \frac{\partial u}{\partial x} = -\frac{1}{x^2}\left[\frac{\partial u}{\partial p} + \frac{\partial u}{\partial q}\right]$	
	Multiplying both sides by x^2, we get	
	$x^2\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial p} - \frac{\partial u}{\partial q}$ —— (1)	(1M)
	Now, $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial y} + \frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial y} = \frac{\partial u}{\partial p}\left\{\frac{1}{y^2}\right\} + \frac{\partial u}{\partial q}(0)$	

Q.No.	Solution and Scheme	Marks
	$\therefore \frac{\partial u}{\partial y} = y^2 \cdot \frac{\partial u}{\partial p}$	
	Multiplying both sides by y^2, we get	
	$y^2 \cdot \frac{\partial u}{\partial y} = \frac{\partial u}{\partial p}$ —— (2)	(1M)
	Now, $\frac{\partial u}{\partial z} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial z} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial z} = \frac{\partial u}{\partial p}(0) + \frac{\partial u}{\partial q}\left\{\frac{1}{z^2}\right\}$	
	$\therefore \frac{\partial u}{\partial z} = \frac{1}{z^2}\frac{\partial u}{\partial q}$	
	Multiplying both sides by z^2, we get	
	$z^2 \cdot \frac{\partial u}{\partial z} = \frac{\partial u}{\partial q}$ —— (3).	(1M)
	Adding (1), (2) and (3), we get	
	$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = -\cancel{\frac{\partial u}{\partial p}} - \cancel{\frac{\partial u}{\partial q}} + \cancel{\frac{\partial u}{\partial p}} + \cancel{\frac{\partial u}{\partial q}} = 0.$	
	$\therefore x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$	(1M)
		(7M)
4.(c)	Let $u = x^3 + y^3 - 3axy$ —— (1)	
	Differentiating equation (1), we get	
	$p = u_x = \frac{\partial u}{\partial x} = 3x^2 - 3ay$; $q = u_y = \frac{\partial u}{\partial y} = 3y^2 - 3ax$	(1M)
	$A = u_{xx} = \frac{\partial^2 u}{\partial x^2} = 6x$; $B = u_{xy} = \frac{\partial^2 u}{\partial x \partial y} = -3a$; $C = \frac{\partial^2 u}{\partial y^2} = 6y$	
	Now, for maximum or minimum of 'u', we must have	
	$u_x = 0$ and $u_y = 0$.	(1M)

Q.No.	Solution and Scheme	Marks
	So from $u_x=0$, we get $x^2-ay=0$ —— (2)	
	and from $u_y=0$, we get $y^2-ax=0 \Rightarrow x=\frac{y^2}{a}$ —— (3)	(1M)
	Substituting (3) in (2), we get	
	$\left(\frac{y^2}{a}\right)^2-ay=0 \Rightarrow y^4-a^3y=0 \Rightarrow y(y^3-a^3)=0 \Rightarrow y=0,a$	(1M)
	Now, from equation (2), we have	
	When $y=0$, $x=0$ and When $y=a$, $x=\pm a$.	
	But $x=-a, y=a$ donot satisfy equation (3). Hence	(1M)
	are not solutions.	
	Hence the solutions are $x=0, y=0$ and $x=a, y=a$.	
	At $x=0, y=0$, we have $A=0, B=-3a, C=0$	
	$\therefore AC-B^2=0-(-3a)^2=-9a^2<0$.	
	and hence there is neither maximum nor minimum at $x=0, y=0$.	(1M)
	At $x=a, y=a$, we have $A=6a, B=-3a, C=6a$	
	$\therefore AC-B^2=(6a)(6a)-(-3a)^2=36a^2-9a^2=27a^2>0$.	
	Also, $A=6a>0$ if $a>0$ and $A=6a<0$ if $a<0$.	
	Hence, there is maximum or minimum according to $a>0$ or $a<0$ at $x=a, y=a$.	(1M)
		(7M)
5.(a)	Given: $\frac{dy}{dx}+y\tan x=y^3\sec x$ [Bernoulli's Equation]	
	Dividing throughout by y^3, we get	
	$\frac{1}{y^3}\frac{dy}{dx}+\frac{1}{y^2}\tan x=\sec x$ —— (1)	(1M)

Q.No.	Solution and Scheme	Marks
	Let $\frac{1}{y^2}=t$ ie $y^{-2}=t \Rightarrow -2y^{-3}\frac{dy}{dx}=\frac{dt}{dx} \Rightarrow \frac{-2}{y^3}\frac{dy}{dx}=\frac{dt}{dx}$	(1M)
	$\therefore$ Equation (1) becomes:	
	$-\frac{1}{2}\frac{dt}{dx}+t\cdot\tan x=\sec x$	
	or $\frac{dt}{dx}-(2\tan x)t=-2\sec x$ —— (2)	(1M)
	This is a linear equation in 't' with $P=-2\tan x$ and $Q=-2\sec x$.	
	$\therefore$ Integrating factor $(I.F)=e^{\int P\cdot dx}$	(1M)
	ie $I.F=e^{\int -2\tan x\, dx}=e^{-2\int \tan x\, dx}=e^{2\log\cos x}=e^{\log\cos^2 x}$	
	$\therefore I.F=\cos^2 x$.	(1M)
	Therefore, the general solution of (2) is:	
	$t\cdot\cos^2 x=\int(-2\sec x)\cos^2 x\, dx+c$	
	$=-2\int\cos x\, dx+c$	
	$\therefore \frac{1}{y^2}\cos^2 x=2\sin x+c$	
	ie $\cos^2 x=y^2(2\sin x+c)$	(1M)
	is the general solution of the given differential equation.	(6M)
5.(b)	Given differential equation is:	
	$L\frac{di}{dt}+Ri=E$	

Q.No.	Solution and Scheme	Marks
	Dividing throughout by 'L', we get	
	$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$ —— (1)	(1M)
	This is of the form $\frac{dy}{dx} + Py = Q$ whose solution is given by $ye^{\int P dx} = \int Q e^{\int P dx} + c$	(1M)
	From (1), $P = \frac{R}{L}$ and $Q = \frac{E}{L}$	
	Now, $\int P dt = \int \frac{R}{L} dt = \frac{R}{L} \int dt = \frac{Rt}{L}$	
	$\therefore e^{\int P dx} = e^{\frac{Rt}{L}}$	(1M)
	$\therefore$ Solution of (1) is:	
	$ie^{\frac{Rt}{L}} = \int \frac{E}{L} e^{\frac{Rt}{L}} dt + c = \frac{E}{L} \int e^{\frac{Rt}{L}} dt + c$	
	$\therefore ie^{\frac{Rt}{L}} = \frac{E}{L} \cdot e^{Rt/L} \cdot \frac{L}{RT} = \frac{E}{R} e^{\frac{RT}{L}} + c$	
	$\therefore i = \frac{E}{R} + ce^{\frac{-Rt}{L}}$ —— (2)	(1M)
	This is the general solution of the equation and we shall apply the condition that $i=0$ when $t=0$.	
	ie $0 = \frac{E}{R} + ce^{0} \Rightarrow c = \frac{-E}{R}$	(1M)
	$\therefore$ Equation (2) becomes: $i = \frac{E}{R} - \frac{E}{R} e^{\frac{-Rt}{L}} = \frac{E}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$	(1M)
	When $E = 100 \sin t$ volts,	
	$i = \frac{100 \sin t}{R}\left[1 - e^{\frac{-Rt}{L}}\right]$	(1M)
		(7M)

Q.No.	Solution and Scheme	Marks
5.(c)	Given: $p^3+2xp^2-y^2p^2-2xy^2p=0$.	
	ie $p(p^2+2xp-y^2p-2xy^2)=0$.	(1M)
	or $p[p(p+2x)-y^2(p+2x)]=0$	(1M)
	ie $p(p+2x)(p-y^2)=0$	
	ie $p=0$ or $p+2x=0$ or $p-y^2=0$.	(1M)
	Integrating, we get	
	$p=0 \Rightarrow \frac{dy}{dx}=0 \Rightarrow y=c$ — (1)	(1M)
	$p-y^2=0 \Rightarrow \frac{dy}{dx}-y^2=0 \Rightarrow \frac{dy}{dx}=y^2 \Rightarrow \frac{dy}{y^2}=dx \Rightarrow \frac{dy}{y^2}-dx=0$	
	$\therefore \int\frac{dy}{y^2}-\int dx=c_1 \Rightarrow -\frac{1}{y}-x=c_1 \Rightarrow x+\frac{1}{y}=c$ — (2)	(1M)
	$p+2x=0 \Rightarrow \frac{dy}{dx}+2x=0 \Rightarrow \frac{dy}{dx}=-2x \Rightarrow dy=-2x\,dx$	
	$\therefore dy+2x\,dx=0 \Rightarrow \int dy+2\int x\,dx=0 \Rightarrow y+x^2=c$ — (3)	(1M)
	Hence the general solution is:	
	$(y-c)\left(x+\frac{1}{y}-c\right)(y+x^2-c)=0$.	(1M)
		(7M)
6.(a)	Given: $(x^2+y^3+6x)dx+y^2x\,dy=0$ — (1)	
	The equation is of the form $Mdx+Ndy=0$.	
	Here, $M=x^2+y^3+6x$ and $N=y^2x$.	
	$\frac{\partial M}{\partial y}=3y^2$ $\qquad$ $\frac{\partial N}{\partial x}=y^2$	(1M)

Q.No.	Solution and Scheme	Marks
	ie $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ The given equation is not exact.	
	Now, $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 3y^2 - y^2 = 2y^2$, which is close to 'N'.	(1M)
	$\therefore \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{y^2 x}(2y^2) = \frac{2}{x} = f(x)$.	
	Integrating factor (I.F) $= e^{\int f(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\log x}$	
	$\therefore$ I.F $= e^{\log x^2} = x^2$	(1M)
	Multiplying equation (1) by x^2, we get	
	$(x^4 + x^2y^3 + 6x^3)dx + y^2x^3dy = 0$, which is exact.	(1M)
	The general solution of this exact equation is:	
	$\int x^4 dx + y^3\int x^2 dx + 6\int x^3 dx = c$	(1M)
	ie $\frac{x^5}{5} + \frac{x^3y^3}{3} + \frac{6x^4}{4} = c$	(1M)
	ie $6x^5 + 10x^3y^3 + 45x^4 = 30c$.	(6M)
6.(b)	The given family is:	
	$y^2 = 4a(x+a)$ —— (1)	
	Differentiating equation (1) w.r.t 'x', we get	
	$2yy_1 = 4a$, $y_1 = \frac{dy}{dx}$.	
	$\Rightarrow a = \frac{1}{2}yy_1$ —— (2)	(1M)
	Substituting the value of 'a' from (2) in (1),	
	$y^2 = 4\left[\frac{1}{2}yy_1\right]\left(x + \frac{1}{2}yy_1\right)$	(1M)

Q.No.	Solution and Scheme	Marks
	ie $y^2 = 2yy_1\left(x + \frac{y}{2}y_1\right)$	
	or $y = 2xy_1 + yy_1^2$ —— (3)	(1M)
	This is the differential equation of the given family.	
	Now replacing y_1 by $-1/y_1$, (3) becomes:	
	$y = 2x\left(\frac{-1}{y_1}\right) + y\left(\frac{-1}{y_1}\right)^2$	(1M)
	or $y = \frac{-2x}{y_1} + \frac{y}{y_1^2}$	
	$\therefore yy_1^2 + 2xy_1 = y$ —— (4)	(1M)
	Equation (4) is the differential equation of the orthogonal family which is same as (3) being the differential equation of the given family.	(1M)
	Thus the family of parabolas $y^2 = 4a(x+a)$ is self orthogonal.	(1M)
		(7M)
6.(c)	Given: $xp^2 + xp - yp + 1 - y = 0$.	
	ie $xp^2 + xp + 1 = y(p+1)$.	
	ie $y = \frac{xp^2 + xp + 1}{p+1}$	
	or $y = px + \frac{1}{p+1}$ —— (1)	(1M)
	Equation (1) is in Clairaut's form $y = px + f(p)$ whose general solution is $y = cx + f(c)$.	(1M)

Q.No.	Solution and Scheme	Marks
	Thus the general solution is $y = cx + \frac{1}{c+1}$	(1M)
	Now, differentiating partially w.r.t 'c', we have	
	$0 = x - \frac{1}{(c+1)^2}$	
	or $(c+1)^2 = \frac{1}{x} \Rightarrow c+1 = \frac{1}{\sqrt{x}}$ or $c = \frac{1}{\sqrt{x}} - 1$	(1M)
	Hence the general solution becomes:	(1M)
	$y = \left(\frac{1}{\sqrt{x}} - 1\right)x + \sqrt{x}$	
	ie $y = \sqrt{x} - x + \sqrt{x}$ or $x+y = 2\sqrt{x} \Rightarrow (x+y)^2 = 4x$	(1M)
	Thus the singular solution is $\underline{(x+y)^2 = 4x}$.	(1M)
		(7M)
7.(a)	Let $I = \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} \int_{z=0}^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$	
	$= \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} xy \left[\frac{z^2}{2}\right]_{z=0}^{\sqrt{1-x^2-y^2}} dy\,dx$	(1M)
	$= \frac{1}{2} \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} xy(1-x^2-y^2)\,dy\,dx$	(1M)
	$= \frac{1}{2} \int_{x=0}^{1} \left[\frac{xy^2}{2} - \frac{x^3y^2}{2} - \frac{xy^4}{4}\right]_{y=0}^{\sqrt{1-x^2}} dx$	
	$= \frac{1}{2} \int_{x=0}^{1} \left[\frac{x(1-x^2)}{2} - \frac{x^3(1-x^2)}{2} - \frac{x(1-x^2)^2}{4}\right] dx$	(1M)

Q.No.	Solution and Scheme	Marks

$$\therefore I = \frac{1}{8}\int_{x=0}^{1}\left[2x(1-x^2) - 2x^3(1-x^2) - x(1+x^4-2x^2)\right]dx$$

$$= \frac{1}{8}\left[x^2 - \frac{2x^4}{4} - \frac{2x^4}{4} + \frac{2x^6}{6} - \frac{x^2}{2} - \frac{x^6}{6} + \frac{2x^4}{4}\right]_{x=0}^{1}$$ (1M)

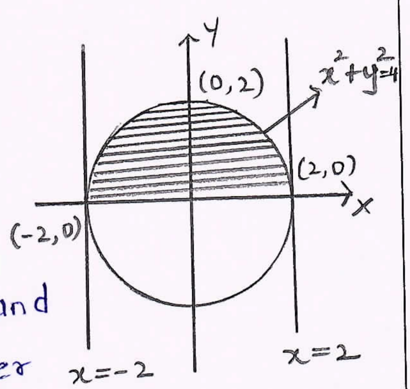
$$= \frac{1}{8}\left[1 - \cancel{\frac{1}{2}} - \frac{1}{2} + \frac{1}{3} - \frac{1}{2} - \frac{1}{6} + \cancel{\frac{1}{2}}\right]$$

$$\therefore I = \frac{1}{8}\cdot\frac{1}{6} = \frac{1}{48}$$ (1M)

ie $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx = \frac{1}{48}$. (1M)

(6M)

7.(b) Let $\displaystyle I = \int_{x=-2}^{2}\int_{y=0}^{\sqrt{4-x^2}} (2-x)\,dy\,dx$. (1M)



Here, $y=\sqrt{4-x^2}$ or $x^2+y^2=4$

This is a circle with centre origin and radius 2. $y=0$ to $\sqrt{4-x^2}$ is the upper half of the circle being bounded by the lines $x=-2$ and $x=2$. On changing the order we must have 'y' varying from 0 to 2 and 'x' from $-\sqrt{4-y^2}$ to $\sqrt{4-y^2}$. (3M)

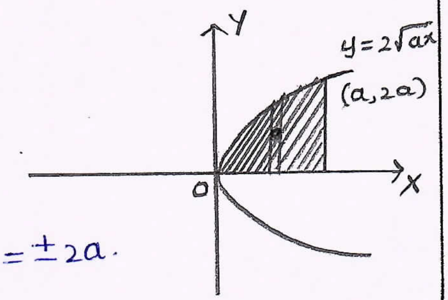
$$\therefore I = \int_{y=0}^{2}\int_{x=-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (2-x)\,dx\,dy = \int_{y=0}^{2}\left[2x - \frac{x^2}{2}\right]_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dy$$ (1M)

$$\text{ie } I = \int_{y=0}^{2}\left[2\cdot 2\sqrt{4-y^2} - 0\right]dy = 4\int_{y=0}^{2}\sqrt{2^2-y^2}\,dy = 4\left[\frac{y\sqrt{4-y^2}}{2} + \frac{2^2}{2}\sin^{-1}\frac{y}{2}\right]_0^2$$ (1M)

$$\therefore I = 4(0 + 2\sin^{-1}1) = 8\cdot\frac{\pi}{2} = 4\pi$$ (1M)

(7M)

Q.No.	Solution and Scheme	Marks
7.(c)	$\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx,\ (m,n>0)$ is called the Beta function	(1M)
	$\Gamma n=\int_0^\infty e^{-x}x^{n-1}dx,\ (n>0)$ is called the Gamma function.	(1M)
	To show: $\Gamma\frac{1}{2}=\sqrt{\pi}$.	
	By the definition of Beta function,	
	$\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx,\ (m,n>0).$	(1M)
	Let $x=\sin^2\theta \Rightarrow dx=2\sin\theta\cos\theta\, d\theta$. When $x=0, \theta=0$ and When $x=1,\ \theta=\pi/2$.	
	$\therefore \beta(m,n)=\int_0^{\pi/2}(\sin^2\theta)^{m-1}(1-\sin^2\theta)^{n-1}\cdot 2\sin\theta\cos\theta\cdot d\theta.$	
	$\therefore \beta(m,n)=2\int_0^{\pi/2}\sin^{2m-2+1}\theta\cos^{2n-2+1}\theta\, d\theta=2\int_0^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta\, d\theta$	(1M)
	For $m=n=1/2$, $\beta(1/2,1/2)=2\int_0^{\pi/2}\sin^0\theta\cos^0\theta\, d\theta=2\int_0^{\pi/2}1\cdot d\theta=\pi$	(1M)
	WKT, $\beta(m,n)=\frac{\Gamma m\cdot\Gamma n}{\Gamma m+n}$	(1M)
	For $m=n=\frac{1}{2}$, $\beta(1/2,1/2)=\frac{\Gamma 1/2\cdot\Gamma 1/2}{\Gamma 1/2+1/2}$	
	ie $\pi=\frac{\left(\Gamma\frac{1}{2}\right)^2}{\Gamma 1}$	
	$\therefore \Gamma\frac{1}{2}=\sqrt{\pi}\quad [\because \Gamma 1=1]$	(1M)
		(7M)

Q.No.	Solution and Scheme	Marks
8.(a)	Let $I=\int_{x=0}^{a}\int_{y=0}^{2\sqrt{ax}} x^2\,dy\,dx$	(1M)
		
	Here $y=2\sqrt{ax}$ or $y^2=4ax$	
	When $x=a$ on $y^2=4ax$, $y=4a^2 \Rightarrow y=\pm 2a$.	(2M)
	So, on $y=\sqrt{2ax}$, $y=2a$ when $x=a$.	
	The integral is over the shaded region. On changing the order of integrating,	
	$I=\int_{y=0}^{2a}\int_{x=\frac{y^2}{4a}}^{a} x^2\,dx\,dy=\int_{y=0}^{2a}\left[\frac{x^3}{3}\right]_{x=\frac{y^2}{4a}}^{a} dy=\int_{y=0}^{2a}\left[\frac{a^3}{3}-\frac{y^6}{192a^3}\right]dy$	(1M)
	$\therefore I=\left[\frac{a^3}{3}y-\frac{y^7}{192a^3\times 7}\right]_{x=0}^{2a}=\frac{2a^4}{3}-\frac{2^7a^4}{192\times 7}=a^4\left[\frac{2}{3}-\frac{2}{21}\right]$	(1M)
	$\therefore I=\frac{4a^4}{7}$	(1M)
		(6M)
8.(b)	Let $I=\int_0^1 x^{3/2}(1-x)^{1/2}\,dx$	
	$\therefore I=\int_0^1 x^{5/2-1}(1-x)^{3/2-1}\,dx=\beta\left(\frac{5}{2},\frac{3}{2}\right)$ $\left[\because \beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx\right]$	(1M)
	ie $I=\frac{\Gamma\frac{5}{2}\cdot\Gamma\frac{3}{2}}{\Gamma\left(\frac{5}{2}+\frac{3}{2}\right)}$ $\left[\because \beta(m,n)=\frac{\Gamma m\cdot\Gamma n}{\Gamma(m+n)}\right]$.	(2M)
	$\therefore I=\frac{\frac{3}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2}}{\Gamma 4}$ $\left[\because \Gamma(n+1)=n\Gamma n,\ \Gamma n=(n-1)!\right]$	(2M)
	ie $I=\frac{1}{\cancel{6}_2}\cdot\frac{\cancel{3}}{8}\cdot\pi$ $\left[\because \Gamma\frac{1}{2}=\sqrt{\pi}\right]$	(1M)
	ie $\int_0^1 x^{3/2}(1-x)^{1/2}\,dx=\frac{\pi}{16}$.	(1M)
		(7M)

Q.No.	Solution and Scheme	Marks
8.(c)	Let $V = \iiint dx\,dy\,dz$	(1M)
	$y+z=4$ gives $z=4-y$. Hence 'z' varies from 0 to $(4-y)$	
	Also $x^2+y^2=4$ gives $y^2=4-x^2$.	
	Hence 'y' varies from $-\sqrt{4-x^2}$ to $\sqrt{4-x^2}$	(1M)
	Further when $y=0$, $x^2=4$ and hence 'x' varies from -2 to 2	
	$\therefore V = \int_{x=-2}^{2}\int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{z=0}^{(4-y)} dz\,dy\,dx$	(1M)
	$= \int_{x=-2}^{2}\int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left[z\right]_{z=0}^{4-y} dy\,dx$	
	$= \int_{x=-2}^{2}\int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4-y)\,dy\,dx = \int_{x=-2}^{2}\left[4y-\frac{y^2}{2}\right]_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx$	(1M)
	$= \int_{x=-2}^{2}\left[4\left(\sqrt{4-x^2}+\sqrt{4-x^2}\right)-\frac{1}{2}\left\{(4-x^2)-(4-x^2)\right\}\right]dx$	
	$= \int_{x=-2}^{2} 8\sqrt{4-x^2}\,dx = 8\times 2\int_{x=0}^{2}\sqrt{4-x^2}\,dx$	(1M)
	$= 16\left[\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{x=0}^{2}$ $\left\{\because \int\sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right\}$	(1M)
	$V = 16\left[0+2\left(\sin^{-1}1-\sin^{-1}0\right)\right] = 32\left[\frac{\pi}{2}-0\right] = 16\pi$	
	Thus the required volume is 16π Cubic units.	(1M)
		(7M)

Q.No.	Solution and Scheme	Marks
9.(a)	Let $A=\begin{bmatrix}0&1&-3&-1\\1&0&1&1\\3&1&0&2\\1&1&-2&0\end{bmatrix}$	
	$R_1 \leftrightarrow R_2$	
	$A \sim \begin{bmatrix}1&0&1&1\\0&1&-3&-1\\3&1&0&2\\1&1&-2&0\end{bmatrix}$	(1M)
	$R_3 \to -3R_1+R_3,\ R_4 \to -R_1+R_4$	
	$A \sim \begin{bmatrix}1&0&1&1\\0&1&-3&-1\\0&1&-3&-1\\0&1&-3&-1\end{bmatrix}$	(1M)
	$R_3 \to -R_2+R_3,\ R_4 \to -R_2+R_4$	
	$A \sim \begin{bmatrix}1&0&1&1\\0&1&-3&-1\\0&0&0&0\\0&0&0&0\end{bmatrix}$	(1M)
	$C_3 \to -C_1+C_3,\ C_4 \to -C_1+C_4$	
	$A \sim \begin{bmatrix}1&0&0&0\\0&1&-3&-1\\0&0&0&0\\0&0&0&0\end{bmatrix}$	(1M)
	$C_3 \to 3C_2+C_3,\ C_4 \to C_2+C_4$	
	$A \sim \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} = \begin{bmatrix}I_2&0\\0&0\end{bmatrix}$	(1M)
	Thus $\rho(A)=2$.	(1M)
		(6M)

Q.No.	Solution and Scheme	Marks
9.(b)	Given: $3x+y+2z=3$ $2x-3y-z=-3$ $x+2y+z=4$ The augmented matrix of the system is $[A:B] = \left[\begin{array}{ccc:c} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{array}\right]$	(1M)
	$R_1 \to R_2,\ R_2 \to R_3;\ R_3 \to R_1$ $[A:B] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \end{array}\right]$	(1M)
	$R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - 2R_1$ $[A:B] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 0 & -5 & -1 & -9 \\ 0 & -7 & -3 & -11 \end{array}\right]$	(1M)
	$R_3 \to R_3 - \frac{7}{5}R_2$ $[A:B] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 4 \\ 0 & -5 & -1 & -9 \\ 0 & 0 & -8/5 & 8/5 \end{array}\right]$	(1M)
	Therefore, $x+2y+z=4$ —— (1) $-5y-z=-9$ —— (2) $-\frac{8}{5}z = \frac{8}{5} \Rightarrow \boxed{z=-1}$	(1M)
	$\therefore$ From (2), $-5y+1=-9 \Rightarrow 5y=10 \Rightarrow \boxed{y=2}$	(1M)
	From (1), $x+4-1=4 \Rightarrow x+3=4 \Rightarrow \boxed{x=1}$	(1M)
	$\therefore x=1,\ y=2$ and $z=-1$.	(7M)

Q.No.	Solution and Scheme	Marks
9. (c)	The given system of equations is not diagonally dominant and hence we rearrange the given system of equations as follows:	
	$83x+11y-4z=95$	
	$7x+52y+13z=104$	(1M)
	$3x+8y+29z=71$	
	Hence, $x=\frac{1}{83}\left[95-11y+4z\right]$	
	$y=\frac{1}{52}\left[104-7x-13z\right]$	(1M)
	$z=\frac{1}{29}\left[71-3x-8y\right]$	
	<u>I Iteration</u>: $x=0,\ y=0,\ z=0$.	
	$x^{(1)}=\frac{1}{83}\left[95-(11\cdot 0)+(4\cdot 0)\right]=1.4457$	
	$y^{(1)}=\frac{1}{52}\left[104-(7\cdot 0)-(13\cdot 0)\right]=1.84592$	(1M)
	$z^{(1)}=\frac{1}{29}\left[71-(3\cdot 0)-(8\cdot 0)\right]=1.82065$	
	<u>II Iteration</u>: $x=1.4457,\ y=1.84592,\ z=1.82065$	
	$x^{(2)}=\frac{1}{83}\left[95-11(1.84592)+4(1.82065)\right]=0.98768$	(1M)
	$y^{(2)}=\frac{1}{52}\left[104-7(0.98768)-13(1.82065)\right]=1.44118$	
	$z^{(3)}=\frac{1}{29}\left[71-3(0.98768)-8(1.44118)\right]=1.95662$	
	<u>III Iteration</u>: $x=0.98768,\ y=1.41188,\ z=1.95662$.	
	$x^{(3)}=\frac{1}{83}\left[95-11(1.41188)+4(1.95662)\right]=1.05176$.	
	$y^{(3)}=\frac{1}{52}\left[104-7(1.05176)-13(1.95662)\right]=1.36926$.	(1M)
	$z^{(3)}=\frac{1}{29}\left[71-3(1.05176)-8(1.36926)\right]=1.96175$.	

Q.No.	Solution and Scheme	Marks
	IV Iteration: $x=1.05176$, $y=1.36926$, $z=1.96175$.	
	$x^{(4)}=\frac{1}{83}[95-11(1.36926)+4(1.96175)]=1.05765$.	
	$y^{(4)}=\frac{1}{52}[104-7(1.05176)-13(1.96175)]=1.36719$.	(1M)
	$z^{(4)}=\frac{1}{29}[71-3(1.05765)-8(1.36719)]=1.96171$	
	$\therefore x=1.05765$, $y=1.36719$, $z=1.96171$	(1M)
		(7M)
10.(a)	Given: $5x+3y+7z=4$ $3x+26y+2z=9$ $7x+2y+10z=5$	
	$[A:B]=\left[\begin{array}{ccc:c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$ is the augmented matrix.	(1M)
	$R_2\to -3R_1+5R_2$, $R_3\to -7R_1+5R_3$	
	$\therefore [A:B]\sim\left[\begin{array}{ccc:c} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{array}\right]$	(1M)
	$R_2\to R_2/11$.	
	$\therefore [A:B]\sim\left[\begin{array}{ccc:c} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & -11 & 1 & -3 \end{array}\right]$	(1M)
	$R_3\to R_2+R_3$	
	$\therefore [A:B]\sim\left[\begin{array}{ccc:c} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$	
	We have $\rho[A]=2$, $\rho[A:B]=2$ Since $\rho[A]=\rho[A:B]=2<3$ we Conclude that the system is Consistent and will have infinite solutions.	(1M)

Q.No.	Solution and Scheme	Marks
	We now have, $5x+3y+7z=4$ — (1) $11y-z=3$ — (2) Let $z=k$ be arbitrary and from (2) $y=\frac{1}{11}(k+3)$	(1M)
	Also from (1), $5x+\frac{3}{11}(k+3)+7k=4 \Rightarrow x=\frac{1}{11}(7-16k)$ Thus, $x=\frac{1}{11}(7-16k), y=\frac{1}{11}(k+3), z=k$ is the required solution	(1M) (6M)
10.(b)	Given: $x+y+z=11$ $3x-y+2z=12$ $2x+y-z=3$ $[A:B]=\left[\begin{array}{ccc:c} 1 & 1 & 1 & 11 \\ 3 & -1 & 2 & 12 \\ 2 & 1 & -1 & 3 \end{array}\right]$	(1M)
	$R_2\to R_2-3R_1$; $R_3\to R_3-2R_1$ $[A:B]\sim\left[\begin{array}{ccc:c} 1 & 1 & 1 & 11 \\ 0 & -4 & -1 & -21 \\ 0 & -1 & -3 & -19 \end{array}\right]$	(1M)
	$R_1\to R_2+4R_1$, $R_3\to -R_2+4R_3$ $[A:B]\sim\left[\begin{array}{ccc:c} 4 & 0 & 3 & 23 \\ 0 & -4 & -1 & -21 \\ 0 & 0 & -11 & -55 \end{array}\right]$	(1M)
	$R_3\to \frac{-R_3}{11}$ $[A:B]\sim\left[\begin{array}{ccc:c} 4 & 0 & 3 & 23 \\ 0 & -4 & -1 & -21 \\ 0 & 0 & 1 & 5 \end{array}\right]$	(1M)
	$R_1\to R_1-3R_3$; $R_2\to R_2+R_3$ $[A:B]\sim\left[\begin{array}{ccc:c} 4 & 0 & 0 & 8 \\ 0 & -4 & 0 & -16 \\ 0 & 0 & 1 & 5 \end{array}\right]$	(1M)

Q.No.	Solution and Scheme	Marks

By back substitution, $4x = 8 \Rightarrow x = 2$

$-4y = -16 \Rightarrow y = 4$ (1M)

$z = 5$

Thus, $x = 2, y = 4, z = 5$ (1M)

(7M)

10.(c) Let $A = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$

By data, $x^{(0)} = [1 \;\; 1 \;\; 1]^T$.

$$Ax^{(0)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix} = 4\begin{bmatrix} -0.75 \\ 0.25 \\ 1 \end{bmatrix} = \lambda^{(1)} x^{(1)}$$

$$Ax^{(1)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} -0.75 \\ 0.25 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ -1 \end{bmatrix} = 1\begin{bmatrix} 0.5 \\ 0 \\ -1 \end{bmatrix} = \lambda^{(2)} x^{(2)}$$

$$Ax^{(2)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} 0.5 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \\ 1 \end{bmatrix} = 1\begin{bmatrix} 0 \\ -0.5 \\ 1 \end{bmatrix} = \lambda^{(3)} x^{(3)}$$

$$Ax^{(3)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ -0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1.5 \\ -1 \end{bmatrix} = 1.5\begin{bmatrix} -0.6667 \\ 1 \\ -0.6667 \end{bmatrix} = \lambda^{(4)} x^{(4)}$$ (1M)

$$Ax^{(4)} = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} -0.6667 \\ 1 \\ -0.6667 \end{bmatrix} = \begin{bmatrix} 2 \\ -2.3333 \\ 0.6667 \end{bmatrix} = 2.3333\begin{bmatrix} 0.8571 \\ -1 \\ 0.2857 \end{bmatrix} = \lambda^{(5)} x^{(5)}$$

$$Ax^{(5)} = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} 0.8571 \\ -1 \\ 0.2857 \end{bmatrix} = \begin{bmatrix} -2 \\ 2.1429 \\ -0.2857 \end{bmatrix} = 2.1429\begin{bmatrix} -0.9333 \\ 1 \\ -0.1333 \end{bmatrix} = \lambda^{(6)} x^{(6)}$$

$$Ax^{(6)} = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} -0.9333 \\ 1 \\ -0.1333 \end{bmatrix} = \begin{bmatrix} 2 \\ -2.0667 \\ 0.1333 \end{bmatrix} = 2.0667\begin{bmatrix} 0.9677 \\ -1 \\ 0.0645 \end{bmatrix} = \lambda^{(7)} x^{(7)}$$ (1M)

$$Ax^{(7)} = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} 0.9677 \\ -1 \\ 0.0645 \end{bmatrix} = \begin{bmatrix} -2 \\ 2.0323 \\ -0.0645 \end{bmatrix} = 2.0323\begin{bmatrix} -0.9841 \\ 1 \\ -0.0317 \end{bmatrix} = \lambda^{(8)} x^{(8)}$$

Q.No.	Solution and Scheme	Marks

$$Ax^{(8)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -0.9841 \\ 1 \\ -0.0317 \end{bmatrix} = \begin{bmatrix} 2 \\ -2.0159 \\ 0.0317 \end{bmatrix} = 2.0159 \begin{bmatrix} 0.9921 \\ -1 \\ 0.0157 \end{bmatrix} = \lambda^{(9)} X^{(9)}$$

$$Ax^{(9)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0.9921 \\ -1 \\ 0.0157 \end{bmatrix} = \begin{bmatrix} -2 \\ 2.0079 \\ -0.0157 \end{bmatrix} = 2.0079 \begin{bmatrix} -0.9961 \\ 1 \\ -0.0078 \end{bmatrix} = \lambda^{(10)} X^{(10)}$$ (1M)

$$Ax^{(10)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -0.9961 \\ 1 \\ -0.0078 \end{bmatrix} = \begin{bmatrix} 2 \\ -2.0039 \\ 0.0078 \end{bmatrix} = 2.0039 \begin{bmatrix} 0.998 \\ 1 \\ 0.0039 \end{bmatrix} = \lambda^{(11)} X^{(11)}$$

$$Ax^{(11)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0.998 \\ 1 \\ 0.0039 \end{bmatrix} = \begin{bmatrix} -2 \\ 2.002 \\ -0.0039 \end{bmatrix} = 2.002 \begin{bmatrix} -0.999 \\ 1 \\ -0.002 \end{bmatrix} = \lambda^{(12)} X^{(12)}$$

$$Ax^{(12)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -0.999 \\ 1 \\ -0.002 \end{bmatrix} = \begin{bmatrix} 2 \\ -2.001 \\ 0.002 \end{bmatrix} = 2.001 \begin{bmatrix} 0.9995 \\ -1 \\ 0.001 \end{bmatrix} = \lambda^{(13)} X^{(13)}$$ (1M)

$$Ax^{(13)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0.9995 \\ -1 \\ 0.001 \end{bmatrix} = \begin{bmatrix} -2 \\ 2.0005 \\ -0.001 \end{bmatrix} = 2.0005 \begin{bmatrix} -0.9998 \\ 1 \\ -0.0005 \end{bmatrix} = \lambda^{(14)} X^{(14)}$$

$$Ax^{(14)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -0.9998 \\ 1 \\ -0.0005 \end{bmatrix} = \begin{bmatrix} 2 \\ -2.0002 \\ 0.0005 \end{bmatrix} = 2.0002 \begin{bmatrix} 0.9999 \\ -1 \\ 0.0002 \end{bmatrix} = \lambda^{(15)} X^{(15)}$$

$$Ax^{(15)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0.9999 \\ -1 \\ 0.0002 \end{bmatrix} = \begin{bmatrix} -2 \\ 2.0001 \\ -0.0002 \end{bmatrix} = 2.0001 \begin{bmatrix} -0.9999 \\ 1 \\ -0.0001 \end{bmatrix} = \lambda^{(16)} X^{(16)}$$ (1M)

$$Ax^{(16)} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -0.9999 \\ 1 \\ -0.0001 \end{bmatrix} = \begin{bmatrix} 2 \\ -2.0001 \\ 0.0001 \end{bmatrix} = 2.0001 \begin{bmatrix} 1 \\ -1 \\ 0.0001 \end{bmatrix} = \lambda^{(17)} X^{(17)}$$

(1M)

∴ The largest eigenvalue $\lambda = 2.0001 \cong 2$

The Corresponding eigenvector is $\begin{bmatrix} 1 \\ -1 \\ 0.0001 \end{bmatrix} \cong \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ (1M)

(7M)

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