Model Question Paper-I with effect from 2022 (CBCS Scheme)

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First Semester B.E Degree Examination
Mathematics-I for Civil Engineering Stream (22MATC11)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE.**

		Module -1	Marks
Q.01	a	With usual notations prove that $\tan\varphi = r\frac{d\theta}{dr}$	06
	b	Find the angle between the curves $r = a(1-\cos\theta)$ *and* $r = 2a\cos\theta$	07
	c	Show that the radius of curvature for the curve $r^n = a^n\cos n\theta$ varies inversely as r^{n-1}.	07
		OR	
Q.02	a	Derive an expression for the radius of curvature for a Cartesian curve.	06
	b	Find the pedal equation of the curve $r = 2(1+\cos\theta)$	07
	c	Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point where it cuts the line $y = x$.	07
		Module-2	
Q. 03	a	Expand $\log(1+e^x)$ by Maclaurin's series up to the term containing x^4.	06
	b	If $u = f(x-y, y-z, z-x)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$.	07
	c	Examine the function $f(x,y) = xy(a-x-y)$ for extreme values.	07
		OR	
Q.04	a	Evaluate (i) $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$ (ii) $\lim_{x\to 0}(\cot x)^{\tan x}$	06
	b	If $z = f(x+ay)+g(x-ay)$ prove that $\frac{\partial^2 z}{\partial y^2} = a^2\frac{\partial^2 z}{\partial x^2}$.	07
	c	If $u = x+3y^2-z^3$, $v = 4x^2yz$, $w = 2z^2-xy$ *find* $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ *at* $(1,-1,0)$.	07
		Module-3	
Q. 05	a	Solve $\frac{dy}{dx}+xy = xy^3$.	06
	b	Find the orthogonal trajectories of the cardioids $r = a(1-\cos\theta)$.	07
	c	Solve $p^2+2py\cot x = y^2$.	07
		OR	

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Q. 06	a	Solve $(4xy+3y^2-x)dx+x(x+2y)dy=0$	06
	b	A body originally at 80° C cools down to $60°C$ in 20 minutes; the temperature of the air being $40°c$. What will be the temperature of the body after 40 minutes from the original?	07
	c	Find the general and singular solution of the equation $x^2(y-px)=p^2y$ by reducing into Clairaut's form, using the substitution $X=x^2$, $Y=y^2$.	07
		Module-4	
Q. 07	a	Solve $(4D^4-4D^3-23D^2+12D+36)y=0$.	06
	b	Solve $(D-2)^2y=8(e^{2x}+sin2x+x^2)$.	07
	c	Solve by the method of variation of parameter $y''-6y'+9y=\frac{e^{3x}}{x^2}$.	07
		OR	
Q. 08	a	Solve $y''+3y'+2y=12x^2$.	06
	b	Solve $\frac{d^2y}{dx^2}-4y=\cosh(2x-1)+3^x$.	07
	c	Solve $(2x-1)^2\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}-2y=8x^2-2x+3$.	07
		Module-5	
Q. 09	a	Find the rank of the matrix $\begin{bmatrix}4&0&2&1\\2&1&3&4\\2&3&4&7\\2&3&1&4\end{bmatrix}$	06
	b	Solve the system of equations by Gauss-Jordan method $x+y+z=9$, $x-2y+3z=8$, $2x+y-z=3$	07
	c	Using Rayleigh's power method find the dominant eigenvalue and the corresponding eigenvector of $\begin{bmatrix}25&1&2\\1&3&0\\2&0&-4\end{bmatrix}$ by taking $[1\quad 0\quad 0]^T$ as the initial eigenvector [carry out 6 iterations].	07
		OR	
Q. 10	a	Find the rank of the matrix $\begin{bmatrix}11&12&13&14\\12&13&14&15\\13&14&15&16\\14&15&16&17\end{bmatrix}$	06

	b	For what values λ and μ the system of equations $2x + 3y + 5z = 9\,,\ \ 7x + 3y - 2z = 8\ \ ,\ \ 2x + 3y + \lambda z = \mu$, has (i) no solution (ii) a unique solution and (iii) infinite number of solutions	07
	c	Solve the system of equations $5x + 2y + z = 12;\ \ x + 4y + 2z = 15; x + 2y + 5z = 20$ Using Gauss-Seidel method, taking (0, 0, 0) as an initial approximation. (Carry out 4 iterations).	07

Table showing the Bloom's Taxonomy Level, Course outcome and Program outcome

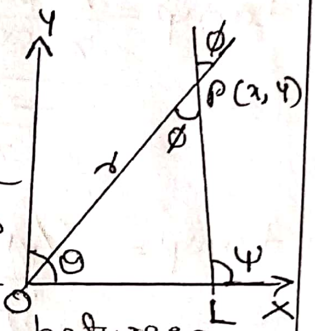
Question		Bloom's taxonomy level attached	Course outcome	Program outcome
Q.1	a)	L1	CO 01	PO 01
	b)	L2	CO 01	PO 01
	c)	L3	CO 01	PO 02
Q. 2	a)	L1	CO 01	PO 01
	b)	L2	CO 01	PO 01
	c)	L3	CO 01	PO 02
Q. 3	a)	L2	CO 02	PO 01
	b)	L2	CO 02	PO 01
	c)	L3	CO 02	PO 03
Q. 4	a)	L2	CO 02	PO 01
	b)	L2	CO 02	PO 01
	c)	L3	CO 02	PO 02
Q. 5	a)	L2	CO 03	PO 02
	b)	L3	CO 03	PO 03
	c)	L2	CO 03	PO 01
Q. 6	a)	L2	CO 03	PO 02
	b)	L3	CO 03	PO 03
	c)	L2	CO 03	PO 01
Q. 7	a)	L2	CO 04	PO 01
	b)	L2	CO 04	PO 01
	c)	L2	CO 04	PO 02
Q. 8	a)	L2	CO 04	PO 01
	b)	L2	CO 04	PO 01
	c)	L2	CO 04	PO 02
Q. 9	a)	L2	CO 05	PO 01
	b)	L3	CO 05	PO 01
	c)	L3	CO 05	PO 02
Q. 10	a)	L2	CO 05	PO 01
	b)	L3	CO 05	PO 02
	c)	L3	CO 05	PO 01

Department: Mathematics **AY: 2022-23**
Subject with Sub. Code: Mathematics-I for Civil and Mechanical Engineering stream (22MATC11&22MATM11)
Name of Faculty: Prof. Akshata Patil **Semester: I**

Q.No.	Solution and Scheme	Marks

MODULE - 1.

Q.01
a] Let $P(r,\theta)$ be any point on the curve $r = f(\theta)$
$X\hat{O}P = \theta$ and $OP = r$
Let PL be the tangent to the curve at P substending an angle ψ with the positive direction of the initial line (x-axis) and ϕ be the angle between the radius vector OP and the tangent PL.



That is,
$O\hat{P}L = \phi$
From the Figure
$\psi = \phi + \theta$
$\tan\psi = \tan(\phi + \theta)$ (2)
$$\tan\psi = \frac{\tan\phi + \tan\theta}{1 - \tan\phi\tan\theta} \quad \text{-----} (1)$$
Let (x,y) be the Cartesian co-ordinates of P
So that,
$x = r\cos\theta$, $y = r\sin\theta$
Since, r is a function of θ. we also know from the geometrical meaning of the derivative that,
$$\tan\psi = \frac{dy}{dx} = \text{slope of the tangent PL.}$$
$$\tan\psi = \frac{dy/d\theta}{dx/d\theta} \quad \text{since } x \,\&\, y \text{ are function of } \theta$$

Q.No.	Solution and Scheme	Marks
	$\tan\psi = \dfrac{\frac{d}{d\theta}(r\sin\theta)}{d/d\theta\,(r\cos\theta)} = \dfrac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$	(2)
	where, $r' = dr/d\theta$	
	Dividing both the Nr & Dr by $r'\cos\theta$ we have,	
	$\tan\psi = \dfrac{\frac{r\cos\theta}{r'\cos\theta} + \frac{r'\sin\theta}{r'\cos\theta}}{\frac{-r\sin\theta}{r'\cos\theta} + \frac{r'\cos\theta}{r'\cos\theta}}$	
	$\tan\psi = \dfrac{r/r' + \tan\theta}{1 - r/r'\tan\theta}$ ----- (2)	
	comparing eqn (1) and (2)	
	$\tan\phi = r/r' = r/(dr/d\theta)$	
	$\tan\phi = r(d\theta/dr)$	
	Equivalently we can write it in the form	
	$\dfrac{1}{\tan\phi} = 1/r\,(dr/d\theta)$ or $\cot\phi = 1/r\,(dr/d\theta)$	(2)
	$\boxed{\tan\phi = r \cdot d\theta/dr}$	6M
1 b	$r = a(1-\cos\theta)$: $r = 2a\cos\theta$	
	Taking log on both sides	
	$\log r = \log a + \log(1-\cos\theta)$: $\log r = \log 2a + \log(\cos\theta)$	
	Diff w.r.t 'θ'	
	$\dfrac{1}{r}\cdot\dfrac{dr}{d\theta} = \dfrac{\sin\theta}{1-\cos\theta}$: $\dfrac{1}{r}\dfrac{dr}{d\theta} = \dfrac{-\sin\theta}{\cos\theta}$	2
	$\cot\phi_1 = \dfrac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$: $\cot\phi_2 = -\tan\theta$	1

Q.No.	Solution and Scheme	Marks

$\cot\phi_1 = \cot(\theta/2)$; $\cot\phi_2 = \cot(\pi/2+\theta)$

$\phi_1 = \theta/2$; $\phi_2 = \pi/2+\theta$ (2)

$|\phi_1-\phi_2| = |\theta/2-\pi/2-\theta| = \pi/2+\theta/2$ ------ (1)

Now consider,

$r = a(1-\cos\theta)$ and $r = 2a\cos\theta$

$\therefore a(1-\cos\theta) = 2a\cos\theta$

$3\cos\theta = 1$ or $\theta = \cos^{-1}(1/3)$ (2)

$\frac{\theta}{2} = 1/2 \cdot \cos^{-1}(1/3)$

we shall substitute this value in (1)

Thus the angle of intersection $= \pi/2 + 1/2\cos^{-1}(1/3)$

7 M

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1 c. $r^n = a^n \cos n\theta$

Taking log on both sides.

$n\log r = n\log a + \log(\cos n\theta)$

$$\frac{n}{r}\frac{dr}{d\theta} = \frac{-n\sin n\theta}{\cos n\theta} = -\tan n\theta$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\tan n\theta$$ (2)

$r_1 = -r\tan n\theta$.

$r_2 = -[r_1\tan n\theta + r\sec^2 n\theta \cdot n]$

$r_2 = -r_1\tan n\theta - nr\sec^2 n\theta$

we have,

$$\rho = \frac{[r^2+r_1^2]^{3/2}}{r^2+2r_1^2-rr_2}$$ (1)

$$= \frac{[r^2+r^2\tan^2 n\theta]^{3/2}}{r^2+2r^2\tan^2 n\theta - r[-r_1\tan n\theta - nr\sec^2 n\theta]}$$

$$= \frac{r^3\sec^3 n\theta}{r^2+2r^2\tan^2 n\theta + rr_1\tan n\theta + nr^2\sec^2 n\theta}$$ (2)

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Q.No.	Solution and Scheme	Marks

$$\rho = \frac{r^3 \sec^3 n\theta}{r^2 + 2r^2 \tan^2 n\theta - r^2 \tan^2 n\theta + n r^2 \sec^2 n\theta}$$

$$= \frac{r^3 \sec^3 n\theta}{r^2[1+\tan^2 n\theta] + n r^2 \sec^2 n\theta}$$

$$= \frac{r^3 \sec^3 n\theta}{\sec^2 n\theta (1+n)}$$

$$= \frac{r \sec n\theta}{(n+1)} = \frac{r\left(a^n/r^n\right)}{(n+1)}$$ (2)

$$\rho = \frac{a^n}{(n+1)} \cdot \frac{1}{r^{n-1}}$$

7M

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2 a] Consider a cartesian curve $y = f(x)$

$$y_1' = \frac{dy}{dx} = \tan\psi$$

$$y_2'' = \frac{d^2y}{dx^2} = \sec^2\psi \cdot \frac{d\psi}{dx}$$ (2)

$$= (1+\tan^2\psi)\frac{d\psi}{ds} \cdot \frac{ds}{dx}$$

$$y'' = [1+(y')^2] \cdot \frac{d\psi}{ds} \cdot \frac{ds}{dx}$$ (1)

Since,
W.K.T

$$\frac{d\psi}{ds} = 1/\rho, \quad \frac{ds}{dx} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/2} = \left\{1+(y')^2\right\}^{1/2}$$ (2)

$$y'' = \frac{\left\{1+(y')^2\right\}^{3/2}}{\rho}$$ (1)

OR

$$\boxed{\rho = \frac{\left\{1+(y')^2\right\}^{3/2}}{y''}}$$

6M

Q.No.	Solution and Scheme	Marks
2b.	$r = 2(1+\cos\theta)$ Taking log on both sides. $\log r = \log 2 + \log(1+\cos\theta)$ Diff w.r.t 'θ' $\frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin\theta}{1+\cos\theta} = \frac{-2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)} = -\tan(\theta/2)$	(2)
	$\therefore \cot\phi = \cot(\pi/2+\theta/2)$ Consider,	(1)
	$p = r\sin\phi$ $p = r\sin(\pi/2+\theta/2) = r\cos(\theta/2)$ Now we have, $r = 2(1+\cos\theta)$ ----- (1)	(1)
	$p = r\cos(\theta/2)$ ----- (2) (1) can be put in the form $r = 2 \cdot 2\cos^2(\theta/2)$ ie $r = 4\cos^2(\theta/2)$ From eqn (2)	(2)
	$p/r = \cos(\theta/2)$ and hence (1) becomes $r = 4(p^2/r^2) \Rightarrow r^3 = 4p^2$	(1)
	Thus, $r^3 = 4p^2$ is the required pedal equation.	7 M
2c.	The equation of the line is $y=x$ and we shall find the point of intersection of this line with the curve. $\sqrt{x}+\sqrt{y} = \sqrt{a}$ This equation when $y=x$ becomes $\sqrt{x}+\sqrt{x} = \sqrt{a}$ or $2\sqrt{x} = \sqrt{a}$. Squaring	(2)
	$4x = a \Rightarrow x = a/4$ $\therefore$ The point of intersection is $\left(\frac{a}{4}, \frac{a}{4}\right)$	(1)

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Q.No.	Solution and Scheme	Marks
	Consider,	
	$\sqrt{x} + \sqrt{y} = \sqrt{a}$	
	Diff wrt 'x'	
	$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot y_1 = 0$ or $\frac{y_1}{\sqrt{y}} = -\frac{1}{\sqrt{x}}$	
	$y_1 = -\frac{\sqrt{y}}{\sqrt{x}}$ At $(a/4, a/4)$	
	we get,	
	$\boxed{y_1 = -1}$	(2)
	Now,	
	$y_2 = \frac{d^2y}{dx^2} = \frac{\sqrt{x}\left(-\frac{1}{2}\sqrt{y}\, y_1\right) - (-\sqrt{y})\left(\frac{1}{2}\sqrt{x}\right)}{x}$	
	at $(a/4, a/4)$	
	$y_2 = \frac{\sqrt{a/4}\left(-\frac{1}{2}\sqrt{a/4} \cdot -1\right) - \left(-\sqrt{a/4}\right)\left(\frac{1}{2}\sqrt{a/4}\right)}{a/4}$.	(2)
	$y_2 = \frac{\frac{a}{2} + \frac{a}{2}}{a/4} = \frac{a}{a/4} = 4$.	
	$\rho = \frac{[1 + y_1^2]^{3/2}}{y_2} = \frac{[1+1]^{3/2}}{4} = \frac{(2)^{3/2}}{4}$	(1)
	$\rho = \frac{1}{4} \times 2\sqrt{2} = \frac{\sqrt{2}}{2} = \frac{\cancel{\sqrt{2}}}{\sqrt{2}\cancel{\sqrt{2}}} = 1/\sqrt{2}$	
	Thus,	
	$\boxed{\rho = 1/\sqrt{2}}$	7M.

Q.No.	Solution and Scheme	Marks
	-: MODULE - 2 :-	
3a]	$y = \log(1+e^x)$ we have maclaurin's Series expansion $y(x) = y(0) + x y_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0) + \frac{x^4}{4!} y_4(0)$ Let, $y = \log(1+e^x)$; $y(0) = \log_e(1+1) = \log 2$ $y_1 = \frac{e^x}{1+e^x}$; $y_1(0) = 1/2$	(2)
	$(1+e^x) y_1 = e^x$ ------ (1) Diff wrt 'x' $(1+e^x) y_2 + e^x y_1 = e^x$ ---- (2) At $x=0$, $2y_2(0) + 1/2 = 1$ $\therefore y_2(0) = 1/4$	(2)
	Diff (2) wrt 'x' $(1+e^x) y_3 + 2e^x y_2 + e^x y_1 = e^x$ ---- (3) At $x=0$ $2 y_3(0) + \frac{1}{2} + \frac{1}{2} = 1$; $y_3(0) = 0$ Diff eq^n (3) wrt 'x' $(1+e^x) y_4 + 3e^x y_3 + 3e^x y_2 + e^x y_1 = e^x$ ---- (4) At $x=0$ $2y_4(0) + 3/4 + 1/2 = 1$; $y_4(0) = -1/8$	(2)
	Thus by substituting these values in the expansion of $y(x)$ $\log(1+e^x) = \log_e 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192}$.	6M.
3b]	Let, $u = f(p, q, r)$ where $p = x-y$, $q = y-z$, $r = z-x$. $\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial x} + \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x}$	2M.

Q.No.	Solution and Scheme	Marks
	$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\cdot 1 + \frac{\partial u}{\partial q}\cdot 0 + \frac{\partial u}{\partial r}(-1)$$	(1)
	Hence, $$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} - \frac{\partial u}{\partial r} \quad \text{------ (1)}$$	
	Similarly, we have by symmetry $$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial q} - \frac{\partial u}{\partial p} \quad \text{------ (2)}$$ $$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r} - \frac{\partial u}{\partial q} \quad \text{------ (3)}$$	(3)
	Thus by adding (1), (2) and (3) we get, $$\boxed{\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0}$$	(1)
		7M.
3c.	Let, $f(x,y) = axy - x^2y - xy^2$	
	we have, $f_x = ay - 2xy - y^2$, $f_y = ax - x^2 - 2xy$	(1)
	we shall find points such that $f_x = 0$ and $f_y = 0$	
	$y(a-2x-y) = 0$ ------ (1) $x(a-x-2y) = 0$ ------ (2)	(2)
	we now form the following pair of eq^n	
	$\left.\begin{matrix} y=0 \\ x=0 \end{matrix}\right\}$ $\left.\begin{matrix} y=0 \\ a-x-2y=0 \end{matrix}\right\}$ $\left.\begin{matrix} a-2x-y=0 \\ x=0 \end{matrix}\right\}$ $\left.\begin{matrix} a-2x-y=0 \\ a-x-2y=0 \end{matrix}\right\}$	(2)
	The stationary points (x,y) from the first three pair of equations are $(0,0)$ $(a,0)$ $(0,a)$	

Q.No.	Solution and Scheme	Marks
	By solving the last pair of equations	
	$2x+y=a$ and $x+2y=a$	
	we obtain,	
	$(x,y)=(a/3, a/3)$	
	The four stationary points are $(0,0)$ $(a,0)$ $(0,a)$ and $(a/3, a/3)$	(1)
	we shall examine these points for maxima & minima.	
	$A=f_{xx}$, $B=f_{xy}$, $C=f_{yy}$.	

	$(0,0)$	$(a,0)$	$(0,a)$	$(a/3, a/3)$
$A=-2y$	0	0	$-2a$	$-2a/3$
$B=a-2x-2y$	a	$-a$	$-a$	$-a/3$
$C=-2x$	0	$-2a$	0	$-2a/3$
$AC-B^2$	$-a^2<0$	$-a^2<0$	$-a^2<0$	$a^2/3>0$
conclusion	saddle pt	saddle pt	saddle pt	Depends on a

(2)

Here, $AC-B^2>0$. Also A is negative if $a>0$ and A is positive if $a<0$ (1)

Thus the given function is maximum or minimum according as $a>0$ or $a<0$ — 7 M

4 a) i) Let,

$$K=\lim_{x\to\pi/2}(\sin x)^{\tan x} \text{ ----- } (1^\infty)$$

$$\log K=\lim_{x\to\pi/2}\tan x\log(\sin x) \text{ ---- } (\infty\times 0)$$

$$=\lim_{x\to\pi/2}\frac{\log(\sin x)}{\cot x} \text{ ----- } (0/0)$$

(1)

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Q.No.	Solution and Scheme	Marks
	Applying L' Hospital rule.	
	$\log_e k = \lim_{x\to\pi/2} \frac{\cos x/\sin x}{-\operatorname{cosec}^2 x} = \lim_{x\to\pi/2} -\sin x\cos x = 0$	(2)
	$\log_e k = 0$ Thus $\boxed{k = e^0 = 1.}$	
	[ii] $k = \lim_{x\to 0} (\cot x)^{\tan x}$. - - - - $(\infty)^0$	
	Taking log on both sides.	(1)
	$\log k = \lim_{x\to 0} \tan x \log(\cot x)$ - - - - $(0\times\infty)$	
	$\log k = \lim_{x\to 0} \frac{\log \cot x}{\cot x}$ - - - - (∞/∞)	(2)
	$\log k = \lim_{x\to 0} \frac{\frac{1}{\cot x}\times -\cancel{\operatorname{cosec}^2 x}}{-\cancel{\operatorname{cosec}^2 x}}$	
	$\log k = \lim_{x\to 0} \tan x. = 0$	
	$\log k = 0 \implies \boxed{k = e^0 = 1}.$	6M
4b.	we have,	
	$z = f(x+ay) + g(x-ay)$	(2)
	$z_x = f'(x+ay) + g'(x-ay)(1)$	
	$z_{xx} = f''(x+ay) + g'(x-ay)$	
	$z_y = f'(x+ay)a + g'(x-ay)(-a)$	(2)
	$z_{yy} = a^2 f''(x+ay) + a^2 g''(x-ay)$	
	$z_{yy} = a^2[f''(x+ay) + g''(x-ay)]$	(2)
	$z_{yy} = a^2 z_{xx}$	
	Thus we get,	1
	$\boxed{\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}}.$	7M

Q.No.	Solution and Scheme	Marks

4c. $u = x+3y^2-z^3$, $v = 4x^2yz$, $w = 2z^2-xy$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \partial v/\partial z \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$ (2)

(2)

$$= \begin{vmatrix} 1 & 6y & -3z^2 \\ 8xyz & 4x^2z & 4x^2y \\ -y & -x & 4z \end{vmatrix}$$

It will be easier if the elements of the determinant are evaluated (1,−1,0) (2)

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} \text{ at } (1,-1,0) = \begin{vmatrix} 1 & -6 & 0 \\ 0 & 0 & -4 \\ 1 & -1 & 0 \end{vmatrix}$$ (1)

On expanding $1(0-4)+6(0+4)+0 = 20$

Thus,

$$[J]_{(1,-1,0)} = 20.$$ 7M

MODULE-03

5a. we have,

$$\frac{dy}{dx} + xy = xy^3$$

dividing y^3 on both sides.

$$y^{-3}\frac{dy}{dx} + xy^{-2} = x \text{ ------ } (1)$$ (2M)

Put,

$$y^{-2} = t$$

$$-2y^{-3}\frac{dy}{dx} = \frac{dt}{dx}$$ (1)

$$y^{-3}\frac{dy}{dx} = -\frac{1}{2}\,dt/dx.$$

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Q.No.	Solution and Scheme	Marks
	eqⁿ (1) becomes.	
	$-\frac{1}{2}\frac{dt}{dx}+xt=x$	
	$\frac{dt}{dx}-2xt=-2x$	
	where $P=-2x$ & $Q=-2x$	(2)
	$I.F=e^{\int P dx}=e^{\int -2x\,dx}=e^{-x^2}$	
	$\therefore t(IF)=\int Q.IF\,dx+c$	(1)
	$te^{-x^2}=\int(-2x)e^{-x^2}dx+c$	
	Put, $x^2=v \Rightarrow 2x\,dx=dv$	
	$te^{-x^2}=-\int e^{-v}dv+c$	
	$te^{-x^2}=-(-e^{-v})+c$	
	$te^{-x^2}=e^{-x^2}+c$.	
	$t=1+ce^{x^2} \Rightarrow y^{-2}=1+ce^{x^2}$ or $\boxed{1/y^2=1+ce^{x^2}}$.	6M.
5b]	$r=a(1-\cos\theta)$	
	Taking log on both sides.	
	$\log r=\log a+\log(1-\cos\theta)$	
	Diff w.r.t 'θ'	(1)
	$\frac{1}{r}\frac{dr}{d\theta}=0+\frac{\sin\theta}{1-\cos\theta}$	
	Replacing $\frac{dr}{d\theta}$ by $-r^2\,d\theta/dr$,	
	$\frac{1}{r}(-r^2\,d\theta/dr)=\frac{\sin\theta}{1-\cos\theta}$	
	$-r\frac{d\theta}{dr}=\frac{\sin\theta}{1-\cos\theta}=\frac{\cancel{2}\sin\theta/2\cos\theta/2}{\cancel{2}\sin^2\theta/2}$	(2)
	$-r\frac{d\theta}{dr}=\cot\theta/2 \Rightarrow \frac{d\theta}{\cot\theta/2}=-dr/r$	(1)
	$\int\tan\theta/2\,d\theta=-\int 1/r\,dr$	
	$\log(\sec\theta/2)=-\log r+\log c$	(2)
	$\log(\sec\theta/2)+\log r=\log c$	
	$\log[r\sec\theta/2]=\log c \Rightarrow r\sec\theta/2=C$.	1
		7M.

Q.No.	Solution and Scheme	Marks
5c	$p^2 + 2py\cot x - y^2 = 0$	
	$p = \dfrac{-2y\cot x \pm \sqrt{4y^2\cot^2 x + 4y^2}}{2}$	
	$p = \dfrac{-2y\cot x \pm 2y\,\mathrm{cosec}\,x}{2} = y[-\cot x \pm \mathrm{cosec}\,x]$	
	$p = y[-\cot x + \mathrm{cosec}\,x]$ or $p = y[-\cot x - \mathrm{cosec}\,x]$	(2)
	we have,	
	$\dfrac{dy}{dx} = y[-\cot x + \mathrm{cosec}\,x]$	
	$\dfrac{dy}{y} = [\mathrm{cosec}\,x - \cot x]\,dx$	
	$\int \dfrac{dy}{y} = \int [\mathrm{cosec}\,x - \cot x]\,dx + k.$	
	$\log y = \log\tan(x/2) - \log\sin x + k.$	(2)
	$\log y = \log\left[\dfrac{c\tan(x/2)}{\sin x}\right]$ where $\log c = k.$	
	$y = \dfrac{c\tan(x/2)}{\sin x} = \dfrac{c\tan(x/2)}{2\sin(x/2)\cos(x/2)} = \dfrac{c}{2\cos^2(x/2)}$	(1)
	$y = \dfrac{c}{1+\cos x}$	
	Also,	
	$\dfrac{dy}{dx} = y[-\cot x - \mathrm{cosec}\,x] = -y[\cot x + \mathrm{cosec}\,x]$	
	$\dfrac{dy}{-y} = [\cot x + \mathrm{cosec}\,x]\,dx$ Integrating	
	$\int \dfrac{dy}{y} + \int [\cot x + \mathrm{cosec}\,x]\,dx = k.$	(2)
	$\log y + \log\sin x + \log\tan(x/2) = k.$	
	$\log[y\sin x\tan(x/2)] = \log c.$	
	$y \cdot 2\sin(x/2)\cos(x/2) \cdot \dfrac{\sin(x/2)}{\cos(x/2)} = c$	
	$y \cdot 2\sin^2(x/2) = c$ or $y(1-\cos x) = c.$	7M.
	The G. S is $\{y(1+\cos x) - c\}\{y(1-\cos x)\}= 0$	

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Q.No.	Solution and Scheme	Marks

6a. Let,

$M = 4xy + 3y^2 - x$ and $N = x(x+2y) = x^2 + 2xy$

$\frac{\partial M}{\partial y} = 4x + 6y$ and $\frac{\partial N}{\partial x} = 2x + 2y$ (1)

The equation is not exact.

consider,

$\frac{\partial N}{\partial y} - \frac{\partial N}{\partial x} = 2x + 4y = 2(x+2y)$ ---- close to N.

Now,

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2(x+2y)}{x(x+2y)} = \frac{2}{x} = f(x)$ (2)

Hence,

$e^{\int f(x)dx}$ is an integrating factor

ie $e^{\int f(x)\,dx} = e^{\int 2/x\,dx} = e^{2\log x} = e^{\log(x^2)} = x^2$

Multiplying the given equation by x^2 (1)

$M = 4x^3y + 3x^2y^2 - x^3$ and $N = x^4 + 2x^3y$

$\frac{\partial M}{\partial y} = 4x^3 + 6x^2y$ and $\frac{\partial N}{\partial x} = 4x^3 + 6x^2y$

The solution of the exact equation is (2)

$\int M\,dx + \int N(y)\,dy = C$

$\int (4x^3y + 3x^2y^2 - x^3)\,dx + \int 0\,dy = C$

Thus, $\boxed{x^4y + x^3y^2 - \frac{x^4}{4} = C}$ is the required solution.

6M

6b By the data,

$t_1 = 80°C$, $t_2 = 40°C$, $T = 60°C$ when $t = 20$ min

we have,

$T = t_2 + (t_1 - t_2)e^{-kt}$

$T = 40 + (80 - 40)e^{-kt}$ (2)

$T = 40 + 40e^{-kt}$

By applying the initial condition

$60 = 40 + 40e^{-20k}$

$20 = 40e^{-20k} \Rightarrow e^{-20k} = 1/2$ (2)

$e^{20k} = 2 \Rightarrow 20k = \log 2 = 0.6931$.

Q.No.	Solution and Scheme	Marks
	$K = \frac{0.6931}{20} = 0.034657$.	(1)
	we have find T when t = 40 mins.	
	Using the value of K.	
	we have,	
	$T = 40 + 40\, e^{-0.034657t}$	
	$(T)_{t=40} = 40 + 40\, e^{-0.034657\times 40}$	(2)
	$T = 40 + 40\, e^{-1.38628}$	
	$T = 50.00\,^\circ C$.	
	Thus the temp of the body at the end of 40 minutes is $50^\circ C$.	7M.
6C.	$X = x^2 \Rightarrow \frac{dX}{dx} = 2x$	
	$Y = y^2 \Rightarrow \frac{dY}{dy} = 2y$	
	Now,	
	$p = \frac{dy}{dx} = \frac{dy}{dY}\cdot\frac{dY}{dX}\cdot\frac{dX}{dx}$ and let $P = \frac{dY}{dX}$	
	$p = \frac{1}{2y}\cdot P\cdot 2x$ or $p = \frac{x}{y}P$.	(2)
	That is $p = \frac{\sqrt{X}}{\sqrt{Y}}P$	
	The given equation $x^2(y - px) = p^2 y$ becomes,	
	$X\left(\sqrt{Y} - \frac{\sqrt{X}}{\sqrt{Y}}P\sqrt{X}\right) = \frac{X}{Y}P^2\sqrt{Y}$	
	$\frac{X(Y - PX)}{\sqrt{Y}} = \frac{XP^2}{\sqrt{Y}}$ or $Y - PX = P^2$	(2)
	$Y = PX + P^2$ is in the clairauts form.	
	The associated general solution is	
	$Y = CX + c^2$	(1)
	Thus the required general solution of the given equation is $y^2 = cx^2 + c^2$	
	Now diff w.r.t 'c' partially	
	$0 = x^2 + 2c$ or $c = -x^2/2$.	(1)

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Q.No.	Solution and Scheme	Marks
	Hence the general solution becomes,	
	$y^2 = -\frac{x^4}{2} + \frac{x^4}{4}$ or $4y^2 = -x^4$	(1)
	Thus the required singular solution is	
	$\boxed{x^4 + 4y^2 = 0}$	7M

MODULE - 04

7a] Let,

$[4D^4 - 4D^3 - 23D^2 + 12D + 36]\,y = 0$.

AE is,

$4m^4 - 4m^3 - 23m^2 + 12m + 36 = 0$

If,

$m = 2$; $64 - 32 - 92 + 24 + 36 = 124 - 124 = 0$

$m = 2$ is a root by inspection. Now by synthetic division. (2)

2	4	−4	−23	12	36
	0	8	8	−30	−36
	4	4	−15	−18	0

Now,

$4m^3 + 4m^2 - 15m - 18 = 0$

If

$m = 2$, $32 + 16 - 30 - 18 = 48 - 48 = 0$

Again $m = 2$ is a root. By insynthetic division (2)

2	4	4	−15	−18
	0	8	24	18
	4	12	9	0

Now, $4m^2 + 12m + 9 = 0$ (2)

or $(2m+3)^2 = 0 \Longrightarrow m = -3/2, -3/2$

Hence roots are $2, 2$; $-3/2, -3/2$

Thus $y = (c_1 + c_2 x)\,e^{2x} + (c_3 + c_4 x)\,e^{-3x/2}$ is the G.S. 6M

Q.No.	Solution and Scheme	Marks
7b.	$(D-2)^2 y = 8[e^{2x} + \sin 2x + x^2]$	
	A.E is $(m-2)^2 = 0 \Rightarrow m = 2, 2$	
	C.F $= (c_1 + c_2 x)e^{2x}$	
	P.I $= \frac{1}{f(D)} x = \frac{1}{(D^2-4D+4)} 8[e^{2x} + \sin 2x + x^2]$	
	Consider, $8\Big[\underbrace{\frac{1}{f(D)}e^{2x}}_{P_1} + \underbrace{\frac{1}{f(D)}\sin 2x}_{P_2} + \underbrace{\frac{1}{f(D)}x^2}_{P_3}\Big]$	(2)
	$P_1 = \frac{1}{D^2-4D+4} \cdot e^{2x} = \frac{1}{2}x^2 e^{2x}$	
	$P_2 = \frac{1}{f(D)}\sin 2x = \frac{1}{D^2-4D+4} \times \sin 2x \quad D^2 \rightarrow (-4)$	(2)
	$P_2 = \frac{1}{-4D}\sin 2x = -\frac{1}{4}\left[\frac{-\cos 2x}{2}\right] = \frac{1}{8}\cos 2x$	
	$P_3 = \frac{1}{D^2-4D+4} x^2$	
	$4-4D+D^2 \,\big)\, x^2 \,\big(\, x^2/4 + x/2 + 3/8$; $x^2 - \frac{4x}{2} + \frac{1}{2}$; $0 + 2x - 1/2$; $2x - 2$; $0 + 3/2$; $3/2$; 0	
	$P_3 = \frac{x^2}{4} + \frac{x}{2} + \frac{3}{8}$	(2)
	P.I $= 8\left[\frac{1}{2}x^2 e^{2x} + \frac{1}{8}\cos 2x + \frac{x^2}{4} + \frac{x}{2} + \frac{3}{8}\right]$	
	$= 8\left[\frac{1}{2}x^2 e^{2x} + \frac{1}{8}\cos 2x + \frac{2x^2+4x+3}{8}\right]$	1
	P.I $= 4x^2 e^{2x} + \cos 2x + 2x^2 + 4x + 3$	
	$y =$ C.F $+$ P.I.	
	$y = (c_1 + c_2 x)e^{2x} + 4x^2 e^{2x} + \cos 2x + 2x^2 + 4x + 3$.	7M
7c.	We have,	
	$(D^2 - 6D + 9)y = e^{3x}/x^2$	
	AE is	
	$m^2 - 6m + 9 = 0$ or $(m-3)^2 = 0 \Rightarrow m = 3, 3$	
	$\therefore y_c = (c_1 + c_2 x)e^{3x}$	(2)
	Let, $y = Ae^{3x} + Bxe^{3x}$ be the complete solution of the given equation where A and B are functions of x to be found.	
	We have,	
	$y_1 = e^{3x} \qquad y_2 = xe^{3x}$	(2)
	$y_1' = 3e^{3x} \qquad y_2' = 3xe^{3x} + e^{3x}$	

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Q.No.	Solution and Scheme	Marks
	$W = y_1 y_2' - y_2 y_1' = e^{6x}$. Also $\phi(x) = e^{3x}/x^2$	
	$A' = \dfrac{-y_2 \phi(x)}{W}$; $B' = \dfrac{y_1 \phi(x)}{W}$	
	$A' = \dfrac{-x e^{3x} \cdot e^{3x}/x^2}{e^{6x}}$; $B' = \dfrac{e^{3x} \cdot e^{3x}/x^2}{e^{6x}}$	(2)
	$A' = -1/x$; $B' = 1/x^2$	
	$A = \int -1/x \, dx + k_1$; $B = \int 1/x^2 \, dx + k_2$	
	$A = -\log x + k_1$; $B = -1/x + k_2$	(1)
	Substituting these in $y = Ae^{3x} + Bxe^{3x}$	
	$y = (-\log x + k_1)e^{3x} + (-1/x + k_2)xe^{3x}$	
	$y = (k_1 + k_2 x)e^{3x} - e^{3x}\log x - e^{3x}$ since the	
	The term $-e^{3x}$ can be neglected in the solution.	
	term $k_1 e^{3x}$ is present	7M
	Thus, $\boxed{y = (k_1 + k_2 x)e^{3x} - e^{3x}\log x}$	
8a.	We have,	
	$(D^2 + 3D + 2)y = 12x^2$	
	AE is, $m^2 + 3m + 2 = 0$ or $(m+1)(m+2) = 0$	
	$\boxed{m = -1, -2}$	(2)
	$\therefore y_c = c_1 e^{-x} + c_2 e^{-2x}$	
	$y_p = \dfrac{12x^2}{D^2 + 3D + 2}$	
	We need to divide for obtaining the PI.	
	$$\begin{array}{r\|l} & 6x^2 - 18x + 21 \\ \hline 2 + 3D + D^2 & 12x^2 \\ & 12x^2 + 36x + 12 \\ \hline & -36x - 12 \\ & -36x - 54 \\ \hline & 42 \\ & 42 \\ \hline & 0 \quad 0 \end{array}$$	(2)

Q.No.	Solution and Scheme	Marks

Hence,

$y_p = 6x^2 - 18x + 21$

Complete solution $y = y_c + y_p$ (2)

Thus,

$$\boxed{y = c_1 e^{-x} + c_2 e^{-2x} + 6x^2 - 18x + 21.}$$

6M.

8b. We have,

$(D^2-4)y = \cosh(2x-1) + 3^x$

AE is, $m^2 - 4 = 0$ or $(m-2)(m+2) = 0 \Rightarrow m = 2, -2$ (1)

$\therefore y_c = c_1 e^{2x} + c_2 e^{-2x}$

$$y_p = \frac{\cosh(2x-1) + 3^x}{D^2-4} = \frac{1}{2}\left[\frac{e^{2x-1}}{D^2-4} + \frac{e^{-(2x-1)}}{D^2-4}\right] + \frac{3^x}{D^2-4}$$ (1)

$y_p = P_1 + P_2 + P_3$.

$$P_1 = \frac{1}{2} \cdot \frac{e^{2x-1}}{D^2-4} = \frac{1}{2} \frac{e^{2x-1}}{2^2-4} \quad [D_r = 0]$$

$$= \frac{1}{2} x \frac{e^{2x-1}}{2D} = \frac{1}{2} \cdot x \cdot \frac{e^{2x-1}}{4} = \frac{x}{8} e^{2x-1}$$

$$P_2 = \frac{1}{2} \cdot \frac{e^{-(2x-1)}}{D^2-4} = \frac{1}{2} \cdot \frac{e^{-(2x-1)}}{(-2)^2-4} \quad [D_r = 0]$$ (2)

$$= \frac{1}{2} \cdot x \cdot \frac{e^{-(2x-1)}}{2D} = \frac{1}{2} \cdot x \cdot \frac{e^{-(2x-1)}}{-4} = \frac{-x}{8} e^{-(2x-1)}$$

$$P_3 = \frac{3^x}{D^2-4} = \frac{(e^{\log 3})^x}{D^2-4} = \frac{e^{\log 3 \cdot x}}{D^2-4} = \frac{e^{\log 3 \cdot x}}{(\log 3)^2 - 4} = \frac{3^x}{(\log 3)^2 - 4}$$

Complete solution:-

$y = y_c + y_p$ (2)

where,

$y_p = P_1 + P_2 + P_3$

$$y = c_1 e^{2x} + c_2 e^{-2x} + \frac{x}{8} e^{2x-1} - \frac{x}{8} e^{-(2x-1)} + \frac{3^x}{(\log 3)^2 - 4}$$ (1)

Thus,

$$y = c_1 e^{2x} + c_2 e^{-2x} + \frac{x}{4} \sinh(2x-1) + \frac{3^x}{(\log 3)^2 - 4}.$$

7M.

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Q.No.	Solution and Scheme	Marks
36.	$(2x-1)^2\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}-2y=8x^2-2x+3.$ Put, $\log(2x-1)=t$ or $e^t=2x-1 \Rightarrow 2x=e^t+1 \Rightarrow x=\frac{e^t+1}{2}$ $2^2.D(D-1)y+2Dy-2y=8\left[\left(\frac{e^t+1}{2}\right)^2-2\left(\frac{e^t+1}{2}\right)+3\right]$ $4D(D-1)y+2Dy-2y=8\left[\left(\frac{e^t+1}{2}\right)^2-2\left(\frac{e^t+1}{2}\right)+3\right]$	(2)
	$[2D(D-1)+D-1]y=4\left[\left(\frac{e^t+1}{2}\right)^2-2\left(\frac{e^t+1}{2}\right)+3\right]$ $[2D^2-2D+D-1]y=4\left[\left(\frac{e^t+1}{2}\right)^2-2\left(\frac{e^t+1}{2}\right)+3\right]$ $[2D^2-D-1]y=4\left[\left(\frac{e^t+1}{2}\right)^2-2\left(\frac{e^t+1}{2}\right)+3\right]$ A.E is, $2m^2-m-1=0$	(2)
	$2m^2+m-2m-1=0$ $m(2m+1)-1(2m+1)=0$ $(2m+1)(m-1)=0$ $m=1,\ m=-1/2$ $C.F=c_1e^t+c_2e^{-t/2}=c_1(2x-1)+c_2(2x-1)^{-1/2}$	(1)
	$P.I=\frac{1}{f(D)}4\left[\frac{1}{4}(e^{2t}+1+2e^t)-e^t-1+3\right]$ $=\frac{1}{f(D)}\left[e^{2t}+1+2e^t-e^t-1+3\right]$ $=\frac{1}{f(D)}\left[e^{2t}+e^t+3\right]$	(2)
	$=\frac{1}{2D^2-D-1}e^{2t}+\frac{1}{2D^2-D-1}e^t+\frac{1}{2D^2-D-1}3e^{0.t}$ $=\frac{1}{5}e^{2t}+\frac{1}{3}e^t+\frac{1}{-1}3.$ $P.I=\frac{1}{5}e^{2t}+\frac{1}{3}e^t-3=\frac{1}{5}\left[(2x-1)^2+\frac{1}{3}(2x-1)-3\right]$ //	7M

Q.No.	Solution and Scheme	Marks
	MODULE-05	
9a.	$A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$	
	$R_2 \longleftrightarrow R_1$	
	$A \sim \begin{bmatrix} 2 & 1 & 3 & 4 \\ 4 & 0 & 2 & 1 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$	(1)
	$R_2 \to -2R_1 + R_2,\ R_3 \to -R_1 + R_3,\ R_4 \to -R_1 + R_4$	
	$A \sim \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & -2 & -4 & -7 \\ 0 & 2 & 1 & 3 \\ 0 & 2 & -2 & 0 \end{bmatrix}$	(2)
	$R_3 \to R_2 + R_3,\ R_4 \to R_2 + R_4$	
	$A \sim \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & -2 & -4 & -7 \\ 0 & 0 & -3 & -4 \\ 0 & 0 & -6 & -7 \end{bmatrix}$	
	$R_4 \to -2R_3 + R_4$	
	$A \sim \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & -2 & -4 & -7 \\ 0 & 0 & -3 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	(2)
	$\frac{1}{2} \cdot R_1,\ -\frac{1}{2}R_2,\ -\frac{1}{3}R_3.$	
	$A \sim \begin{bmatrix} 1 & 1/2 & 3/2 & 2 \\ 0 & 1 & 2 & 7/2 \\ 0 & 0 & 1 & 4/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	
	All the 4 rows are non-zero in the row echelon form A.	(1)
	Thus,	
	$\boxed{\rho(A) = 4}$	
		6M.
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Q.No.	Solution and Scheme	Marks

9b. The augmented matrix of the system is.

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_2 \to -R_1 + R_2$, $R_3 \to -2R_1 + R_2$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$ (2)

We use the leading non zero entry in second row (−3) to make the element above 1 and below, 1 and −1 respectively. zero. (1)

$R_1 \to R_2 + 3R_1$, $R_3 \to R_2 - 3R_3$.

$$[A:B] \sim \left[\begin{array}{ccc:c} 3 & 0 & 5 & 26 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$$

$R_3 \to 1/11 \cdot R_3$. (2)

$$\sim \left[\begin{array}{ccc:c} 3 & 0 & 5 & 26 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

we use the element 1 in the third row to make the two elements above 2 and 5 zero.

$R_1 \to -5R_3 + R_1$, $R_2 \to -2R_3 + R_2$

$$\sim \left[\begin{array}{ccc:c} 3 & 0 & 0 & 6 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$ (2)

Hence, we have $3x = 6$, $-3y = -9$, $z = 4$

Thus,

$\boxed{x = 2,\ y = 3,\ z = 4}$ is the required Solution. 7M

Q.No.	Solution and Scheme	Marks
9c	$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$	
	Let, $X^{(0)} = [1, 0, 0]'$ be the initial eigen vector.	(1)
	$AX^{(0)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 25 \\ 1 \\ 2 \end{bmatrix} = 25 \begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} = \lambda^{(1)} X^{(1)}$	
	$AX^{(1)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.04 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 25.2 \\ 1.12 \\ 1.68 \end{bmatrix} = 25.2 \begin{bmatrix} 1 \\ 0.04 \\ 0.07 \end{bmatrix} = \lambda^{(2)} X^{(2)}$	(4)
	$AX^{(2)} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0.04 \\ 0.07 \end{bmatrix} = \begin{bmatrix} 25.18 \\ 1.12 \\ 1.72 \end{bmatrix} = 25.18 \begin{bmatrix} 1 \\ 0.04 \\ 0.07 \end{bmatrix} = \lambda^{(3)} X^{(3)}$	
	We observe that, $X^{(2)} = X^{(3)}$	(2)
	Thus the numerically largest eigen value of A is 25.18 and the corresponding eigen vector is $[1, 0.04, 0.07]'$.	7M
10a	$A = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 12 & 13 & 14 & 15 \\ 13 & 14 & 15 & 16 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	
	$R_1 \to R_1 - R_2$	
	$A \sim \begin{bmatrix} -1 & -1 & -1 & -1 \\ 12 & 13 & 14 & 15 \\ 13 & 14 & 15 & 16 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	(2)
	$R_1 \to (-) R_1$ $R_2 \to R_2 - R_3$	
	$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 13 & 14 & 15 & 16 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	(1)
	$R_2 \to (-) R_2$ $R_3 \to R_3 - R_4$.	

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Q.No.	Solution and Scheme	Marks
	$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	(2)
	$R_3 \to (-)R_3 \; ; \; R_2 \to R_2 - R_1$	
	$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	
	$R_3 \to R_3 - R_1$	(2)
	$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 14 & 15 & 16 & 17 \end{bmatrix}$	
	$\boxed{\rho(A) = 2}$	6 M
10 b.	$2x + 3y + 5z = 9$ $7x + 3y - 2z = 8$ $2x + 3y + \lambda z = \mu$	
	$A \sim \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$	
	$R_2 \to 2R_2 - 7R_1$ $R_3 \to R_3 - R_1$	(2)
	$\sim \begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{bmatrix}$	
	Case i :- No solution	(2)
	$\lambda - 5 \neq 0$ and $\mu - 9 = 0$	
	ie $\lambda \neq 5$ & $\mu = 9$	
	Case ii :- Unique solution	(1)
	$\lambda - 5 \neq 0$ $\lambda \neq 5$	

Q.No.	Solution and Scheme	Marks
	iii) Infinitely many solutions. $\lambda - 5 = 0$ and $\mu - 9 = 0$ ie $\lambda = 5$ and $\mu = 9$.	(2) 7M.
10C	The given system of equation are diagonally dominant and we put them in the following form. $x = \frac{1}{5}[12 - 2y - z]$ $y = \frac{1}{4}[15 - x - 2z]$ $z = \frac{1}{5}[20 - x - 2y]$ By the data, $x^{(0)} = 1, \; y^{(0)} = 0, \; z^{(0)} = 3$ First iteration: $x^{(1)} = \frac{1}{5}[12 - 2(0) - 3] = 1.8$ $y^{(1)} = \frac{1}{4}[15 - 1.8 - 2(3)] = 1.8$ $z^{(1)} = \frac{1}{5}[20 - 1.8 - 2(1.8)] = 2.92$.	(1) (2)

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Q.No.	Solution and Scheme	Marks

Second iteration:

$x^{(2)} = \frac{1}{5}[12 - 2(1.8) - 2.92] = 1.096$

$y^{(2)} = \frac{1}{4}[15 - 1.096 - 2(2.92)] = 2.016$ (2)

$z^{(2)} = \frac{1}{5}[20 - 1.096 - 2(2.016)] = 2.9744$.

Third iteration:

$x^{(3)} = \frac{1}{5}[12 - 2(2.016) - 2.9744] = 0.99872$

$y^{(3)} = \frac{1}{4}[15 - 0.99872 - 2(2.9744)] = 2.01312$ (2)

$z^{(3)} = \frac{1}{5}[20 - 0.99872 - 2(2.01312)] = 2.995$

Fourth iteration:

$x^{(4)} = \frac{1}{5}[12 - 2(2.01312) - 2.995] = 0.995752$

$y^{(4)} = \frac{1}{4}[15 - 0.995752 - 2(2.995)] = 2.003562$

$z^{(4)} = \frac{1}{5}[20 - 0.995752 - 2(2.003562)] = 2.9994268$ (2)

Thus the solution after four iteration correct to four decimal places is

$\boxed{x = 0.9958, \quad y = 2.0036, \quad z = 2.9996.}$

7M

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