Model Question Paper-I with effect from 2022

USN

Fourth Semester B.E Degree Examination

Complex Analysis, Probability & Statistical Methods

All branches Except CS & ME Engg.Allied branches-21MAT41

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

<table>
<tr><th colspan="2">Q.No.</th><th>Question</th><th>M</th><th>L</th><th>CO</th></tr>
<tr><td colspan="3">Module -1</td><td></td><td></td><td></td></tr>
<tr><td>01</td><td>a</td><td>Define analytic function and derive C-R equations in Cartesian form.</td><td>06</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>b</td><td>Show that $f(z) = log z$ is analytic and hence obtain its derivative.</td><td>07</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>c</td><td>Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the curve $y = x^2$.</td><td>07</td><td>L3</td><td>CO1</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>02</td><td>a</td><td>Construct an analytic function, whose imaginary part is $v = e^x(xSiny + yCosy)$ by the Milne-Thomson method</td><td>06</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>b</td><td>State and prove Cauchy's integral formula.</td><td>07</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>c</td><td>Evaluate $\int_C \frac{Sin\pi z^2 + Cos\pi z^2}{(z-1)\ (z-2)}dz$, where $C : |z| = 3$.</td><td>07</td><td>L3</td><td>CO1</td></tr>
<tr><td colspan="5">Module-2</td><td></td></tr>
<tr><td>03</td><td>a</td><td>Obtain the series solution of Bessel's differential equation
$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + n^2)y = 0$$</td><td>06</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>b</td><td>Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.</td><td>07</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>c</td><td>Express $x^3 - 5x^2 + 6x + 1$ in terms of Legendre polynomial</td><td>07</td><td>L2</td><td>CO2</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>4</td><td>a</td><td>Show that $J_{-n}(x) = (-1)^n J_n(x)$</td><td></td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>b</td><td>Show that $P_4(\cos\theta) = \frac{1}{64}(35\cos 4\theta + 2\theta\cos\theta + 9)$.</td><td>07</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>c</td><td>Prove that $x^3 - 2x^2 - x - 3 = \frac{2}{5}P_3(x) + \frac{4}{3}P_2(x) - \frac{2}{5}P_1(x) - \frac{7}{3}P_0(x)$</td><td>07</td><td>L2</td><td>CO2</td></tr>
<tr><td colspan="5">Module-3</td><td></td></tr>
<tr><td>5</td><td>a</td><td>Find Karl Pearson's coefficient of correlation.
<table><tr><td>x:</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>y:</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td></tr></table></td><td>06</td><td>L2</td><td>CO3</td></tr>
</table>

<table>
<tr><td></td><td>b</td><td>Fit a straight line $y = ax + b$ for the data
<table>
<tr><td>x:</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr>
<tr><td>y:</td><td>16</td><td>19</td><td>23</td><td>26</td><td>30</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c</td><td>Find the regression lines of y on x and x on y for the following data
<table>
<tr><td>x:</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y:</td><td>2</td><td>5</td><td>3</td><td>8</td><td>7</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>6</td><td>a</td><td>The participants in a contest are ranked by two judges as follows.
<table>
<tr><td>x:</td><td>1</td><td>6</td><td>5</td><td>10</td><td>3</td><td>2</td><td>4</td><td>9</td><td>7</td><td>8</td></tr>
<tr><td>y:</td><td>6</td><td>4</td><td>9</td><td>8</td><td>1</td><td>2</td><td>3</td><td>10</td><td>5</td><td>7</td></tr>
</table>
Compute the Rank correlation.</td><td>06</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b</td><td>Compute means $\bar{x}$, $\bar{y}$ and the correlation coefficient r from the given regression lines $2x + 3y + 1 = 0$, $x + 6y = 4$.</td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c</td><td>Fit a second degree polynomial $y = ax^2 + bx + c$ for the data.
<table>
<tr><td>x:</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y:</td><td>10</td><td>12</td><td>13</td><td>16</td><td>19</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">Module-4</td></tr>
<tr><td>7</td><td>a</td><td>A random variable X has the following probability function:
<table>
<tr><td>X</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>$P(X)$</td><td>k</td><td>$2k$</td><td>$3k$</td><td>$4k$</td><td>$3k$</td><td>$2k$</td><td>k</td></tr>
</table>
Find k. Also find $P(X \leq 1)$, $P(X > 1)$, $P(-1 < X \leq 2)$,</td><td>06</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>Find the mean and variance of Binomial distribution.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>In a certain factory turning out razor blades there is a small probability of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in a packets of 10. Use Poisson distribution to calculate approximate number of packets containing
i) No defective
ii) Two defective
iii) Three defective in consignment of 10000 packets.</td><td>07</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>8</td><td>a</td><td>A random variable X has density function: $f(x) = \begin{cases} kx^2 & -3 \leq x \leq 3 \\ 0 & Otherwise \end{cases}$,
Find k. Also, find $P(X \leq 2)$, $P(X \geq 2)$ and $P(X > 1)$.</td><td>06</td><td>L2</td><td>CO4</td></tr>
</table>

<table>
<tr><td></td><td>b</td><td>The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that
i) Exactly two pens will be defective
ii) At most two pens will be defective
iii) None will be defective</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>The marks of 1000 students in an examination follow the normal distribution with mean 70 and standard deviation 5. Find the number students whose marks will be
i) Less than 65
i) More than 75
iii) Between 65 and 75.</td><td>07</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">**Module-5**</td></tr>
<tr><td>9</td><td>a</td><td>The joint distribution of two random variables X and Y is as follows.
<table>
<tr><td>Y / X</td><td>-4</td><td>2</td><td>7</td></tr>
<tr><td>1</td><td>$\frac{1}{8}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{8}$</td></tr>
<tr><td>5</td><td>$\frac{1}{4}$</td><td>$\frac{1}{8}$</td><td>$\frac{1}{8}$</td></tr>
</table>
Compute the following.
i) E(X) and E(Y)
ii) E(XY)
iii) $\sigma_X \& \sigma_Y$</td><td>06</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>Define i) Null hypothesis ii) Type-I & Type-II errors iii) Degrees of freedom iv) Level of Significance.</td><td>07</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>Two types of batteries are tested for their length of life and the following results are obtained:
BatteryA: $n_1 = 10 \quad \bar{x}_1 = 500\,Hrs. \quad \sigma_1^2 = 100$
BatteryB: $n_2 = 10 \quad \bar{x}_2 = 506\,Hrs. \quad \sigma_2^2 = 121$
Compute Student's t and test whether there is a significant difference in the two means at 5% significance level.</td><td>07</td><td>L3</td><td>CO5</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>10</td><td>a</td><td>Determine (i) Marginal distributions
(ii) Covariance between the variables X and Y,
If the joint probability distribution is given by:
<table>
<tr><td>Y / X</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>2</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td></tr>
<tr><td>5</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td></tr>
<tr><td>7</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td></tr>
</table></td><td>06</td><td>L2</td><td>CO5</td></tr>
</table>

<table>
<tr><td></td><td>b</td><td>Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches at 5% significance level.
($t_{0.05}$ =2.262 for 9 d.f.)</td><td>07</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>In experiments on pea breeding the following frequencies of seeds were obtained:
<table>
<tr><th>Round and Yellow</th><th>Wrinkled and Yellow</th><th>Round and Green</th><th>Wrinkled and Green</th><th>Total</th></tr>
<tr><td>315</td><td>101</td><td>108</td><td>32</td><td>556</td></tr>
</table>
Theory predicts that the frequencies should be in proportions$9:3:3:1$. Examine the correspondence between theory and experiment</td><td>07</td><td>L3</td><td>CO5</td></tr>
</table>

Bloom's Taxonomy Levels	**Lower-order thinking skills**		
	Remembering (knowledge):L_1	Understanding (Comprehension): L_2	Applying (Application):L_3
	Higher-order thinking skills		
	Analyzing (Analysis):L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6

AY: 2022-23

Department: Electronics & Communication Engineering
Subject with Sub. Code: Complex Analysis, Probability & Statistical Methods (21MAT41)
Semester / Branch / Division: IV / ECE / B **Name of Faculty: Dr. Meenal Kaliwal/**Akshata P

Q.No.	Solution and Scheme	Marks
	Module-1	
01a.	Analytic Function: A complex valued function $\omega=f(z)$ is said to be analytic at a point $z=z_0$ if $\frac{d\omega}{dz}=f'(z)=\lim_{\delta z\to 0}\frac{f(z+\delta z)-f(z)}{\delta z}$ exists and is unique at z_0 and in the neighbourhood of z_0.	1
	Cauchy-Reimann equation in Cartesian form Proof: Let $f(z)$ be analytic at a point $z=x+iy$ and hence by the definition, $f'(z)=\lim_{\delta z\to 0}\frac{f(z+\delta z)-f(z)}{\delta z}$ exists & is unique. In the cartesian form $f(z)=u(x,y)+iv(x,y)$ and let δz be the increment in z corresponding to the increments $\delta x, \delta y$ in x, y. $f'(z)=\lim_{\delta z\to 0}\frac{[u(x+\delta x, y+\delta y)-u(x,y)}{\delta z} + i\lim_{\delta z\to 0}\frac{[v(x+\delta x, y+\delta y)-v(x,y)]}{\delta z} \longrightarrow ①$	1
	Since, $\delta z\to 0$, we have the following possibilities Case (i): Let $\delta y=0$ so that $\delta z=\delta x$ and $\delta z\to 0$ imply $\delta x\to 0$. Now ① becomes, $f'(z)=\lim_{\delta x\to 0}\frac{u(x+\delta x,y)-u(x,y)}{\delta x}+i\lim_{\delta x\to 0}\frac{v(x+\delta x,y)-v(x,y)}{\delta x}$	1

Q.No. | **Solution and Scheme** | **Marks**

These limits from the basic definition are the partial derivatives of u and v with respect to 'x'.

Therefore, $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \longrightarrow ②$

Case (ii): Let $\delta x = 0$ so that $\delta z = i\delta y$ and $\delta z \to 0$ imply $i\delta y \to 0$ or $\delta y \to 0$. Now ① becomes

$$f'(z) = \lim_{\delta y \to 0} \frac{u(x,y+\delta y)-u(x,y)}{i\delta y} + i\lim_{\delta y \to 0} \frac{v(x,y+\delta y)-v(x,y)}{i\delta y}$$ 1

But, $1/i = i/i^2 = -i$ and hence we have

$$f'(z) = \lim_{\delta y \to 0} -i.\frac{u(x,y+\delta y)-u(x,y)}{\delta y} + \lim_{\delta y \to 0} \frac{v(x,y+\delta y)-v(x,y)}{\delta y}$$

$$= -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$ 1

Therefore $f'(z) = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} \longrightarrow ③$

Equating the RHS of ② & ③,

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$

Now, by equating the real & imaginary parts we get

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$ 1

or $u_x = v_y$ & $v_x = -u_y$

6

1b. $f(z) = \log z$. Let $z = re^{i\theta}$

$u+iv = \log(re^{i\theta}) = \log r + \log(e^{i\theta}) = \log r + i\theta \log_e e$

$u+iv = \log r + i\theta$ Since $\log_e e = 1$

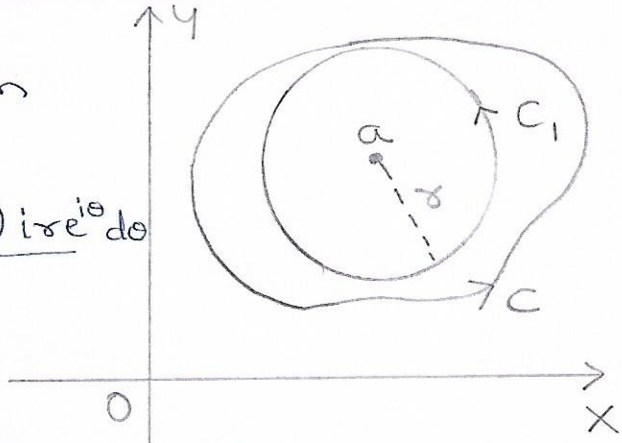
$\therefore u = \log r$ $v = \theta$

$u_r = \frac{1}{r}; u_\theta = 0$ $v_r = 0, v_\theta = 1$ 2

C-R equations in the polar form: $ru_r = v_\theta$ & $rv_r = -u_\theta$ 1

Q.No.	Solution and Scheme	Marks
	are satisfied. Thus, $w = \log z$ is analytic.	1
	Also, $f'(z) = e^{-i\theta}(u_r + iv_r) = e^{-i\theta}\left(\frac{1}{r} + i.0\right) = \frac{1}{re^{i\theta}}$	2
	Since, $z = re^{i\theta}$, $f'(z) = \frac{1}{z}$	1
		7
1 c.	$I = \int_{0}^{1+i} (x^2 - iy)\,dz$ along the curve $y = x^2$	
	$I = \int_{x=0}^{1} (x^2 - ix^2)\,(1+2ix)dx)$ $y = x^2$ $dy = 2x\,dx$ $dz = dx + i\,dy$ $= dx + 2ix\,dx$ $= (1+2ix)\,dx$	2
	$= (1-i)\int_{0}^{1} x^2(1+2ix)\,dx$	
	$= (1-i)\int_{0}^{1} (x^2 + 2ix^3)\,dx$	1
	$= (1-i)\left[\frac{x^3}{3} + 2i\frac{x^4}{4}\right]_{x=0}^{1}$	1
	$= (1-i)\left(\frac{1}{3} + \frac{2i}{4}\right)$	1
	$= (1-i)\left(\frac{1}{3} + \frac{i}{2}\right)$	1
	$= \frac{(1-i)}{6}(2+3i)$	1
		7

Q.No.	Solution and Scheme	Marks
2 a.	Given, $v = e^x (x\sin y + y\cos y)$	
	$v_x = e^x \sin y + e^x (x\sin y + y\cos y)$	
	$v_x = e^x (\sin y + x\sin y + y\cos y) \longrightarrow ①$	1
	Also, $v_y = e^x (x\cos y - y\sin y + \cos y) \longrightarrow ②$	1
	Consider, $f'(z) = u_x + iv_x$. But, $u_x = V_y$	
	i.e. $f'(z) = V_y + iv_x$	
	$f'(z) = e^x (x\cos y - y\sin y + \cos y) + ie^x (\sin y + x\sin y + y\cos y)$	1
	putting, $x = z$ and $y = 0$ we have	
	$f'(z) = e^z (z+1)$, since $\sin 0 = 0$, $\cos 0 = 1$	1
	$\therefore f(z) = \int (z+1) e^z \, dz + c$	
	Integrating by parts, $f(z) = (z+1)e^z - \int e^z . 1 \, dz + c$	1
	Thus, $\underline{f(z) = ze^z + c}$	1
		6
b.	Statement: If $f(z)$ is analytic inside and on a simple closed curve 'C' and if 'a' is any point within 'C' then $f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} dz$	1
	Proof: Since 'a' is a point within C, we shall enclose it by a circle C_1 with $z = a$ as centre and r as radius such that C_1 lies entirely within C. The function $f(z)\|z-a$ is analytic inside and on the boundary of the annular region between C and C_1.	1
	Now, by a property $\int_C \frac{f(z)}{z-a} dz = \int_{C_1} \frac{f(z)}{z-a} dz$	1
	The equation of C_1 can be written in the form $\|z-a\| = r$. That is, $z - a = re^{i\theta}$ or $z = a + re^{i\theta}$, $0 \le \theta \le 2\pi$	

Q.No.	Solution and Scheme	Marks
	$dz = ire^{i\theta}\,d\theta.$ Using these results in the above property $\int_C \frac{f(z)}{z-a}\,dz = \int_{\theta=0}^{2\pi} \frac{f(a+re^{i\theta})\,ire^{i\theta}\,d\theta}{re^{i\theta}}$  	1
	i.e. $\int_C f(z)\,dz = i\int_{\theta=0}^{2\pi} f(a+re^{i\theta})\,d\theta.$	1
	This is true for any $r>0$ however small. Hence as $r\to 0$ we get $\int_C \frac{f(z)}{z-a}\,dz = i\int_0^{2\pi} f(a)\,d\theta = if(a)[\theta]_0^{2\pi}$	1
	$= 2\pi i f(a)$ Thus, $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz$, Cauchy's integral formula.	1
		7
2c.	Let, $\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$ $1 = A(z-2) + B(z-1)$ put, $z=1 \Rightarrow 1 = A(1-2) + B(1-1)$ $1 = A(-1)$ or $A = -1$ put, $z=2 \Rightarrow 1 = A(z-2) + B(z-1)$ $1 = A(0) + B(2-1) \Rightarrow B = 1$ $I = \int_C \frac{f(z)}{(z-1)(z-2)}\,dz = -\int \frac{f(z)}{z-1}\,dz + \int \frac{f(z)}{z-2}\,dz$	2
	$I = I_1 + I_2$ (say) $\longrightarrow$ ①	
	$I_1 = -[2\pi i f(1)] = -2\pi i(\sin\pi + \cos\pi)$	1
	$= -2\pi i(0-1) = 2\pi i$	1

Q.No.	Solution and Scheme	Marks
	$I_3 = 2\pi i f(2) = 2\pi i\left[\sin 4\pi + \cos 4\pi\right]$	1
	$= 2\pi i\left[0+1\right] = 2\pi i$	1
	From ①, $I = 2\pi i + 2\pi i = \underline{4\pi i}$	1
		7
	Module-2	
3a.	$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-n^2)y = 0 \longrightarrow$ ①	
	Since $x=0$ is a regular singular point of the given differential equation, let its solution be	1
	$y = \sum_{r=0}^{\infty} a_r x^{m+r} = x^m(a_0 + a_1x + a_2x^2 + ---+\infty)$	
	$\Rightarrow \frac{dy}{dx} = \sum_{r=0}^{\infty}(m+r)a_r x^{m+r-1}$ and	
	$\frac{d^2y}{dx^2} = \sum_{r=0}^{\infty}(m+r)(m+r-1)a_r x^{m+r-2}$	1
	Substituting the values of y, dy/dx and d^2y/dx^2 in equation ①, we get	
	$x^2\left(\sum_{r=0}^{\infty}(m+r)(m+r-1)a_r x^{m+r-2}\right) + x\left(\sum_{r=0}^{\infty}(m+r)a_r x^{m+r-1}\right) + (x^2-n^2)\left(\sum_{r=0}^{\infty}a_r x^{m+r}\right) = 0$	1
	$\Rightarrow \sum_{r=0}^{\infty}\left[(m+r)^2-(m+r)+(m+r)-n^2\right]a_r x^{m+r} + \sum_{r=0}^{\infty}a_r x^{m+r+2} = 0$	
	$\Rightarrow \sum_{r=0}^{\infty}\left[(m+r)^2-n^2\right]a_r x^{m+r} + \sum_{r=0}^{\infty}a_r x^{m+r+2} = 0 \longrightarrow$ ②	
	Equating to zero the coefficient of x^m	
	$(m^2-n^2)a_0 = 0 \Rightarrow m^2-n^2$	
	Since, $a_0 \neq 0$	
	$\Rightarrow m = \pm n$	

Equating to zero the coefficient of x^{m+1} (corresponding to $r=1$) in eqn ②, we get $[(m+1)^2-n^2]a_1=0 \Rightarrow a_1=0$, since $[(m+1)^2-n^2] \neq 0$ for $m=\pm n$.

Equating to zero the coefficient of x^{m+r+2}, we get the recurrence relation $[(m+r+2)^2-n^2]a_{r+2} + a_r = 0$

$$\Rightarrow a_{r+2} = \frac{-a_r}{(m+r+2)^2-n^2} = -\frac{a_r}{(m-n+r+2)(m+n+r+2)}$$ 1

putting $r=1,3,5$, we get $a_3=a_5=a_7=\text{---}=0$

and putting $r=0,2,4,\text{----}$ we get

$$a_2 = -\frac{a_0}{(m-n+2)(m+n+2)}$$

$$a_4 = \frac{-a_2}{(m-n+4)(m+n+4)}$$

$$= \frac{a_0}{(m-n+4)(m+n+4)(m-n+2)(m+n+2)}$$

and so on,

$$\therefore y = \sum_{r=0}^{\infty} a_r x^{m+r} = x^m(a_0+a_1x+a_2x^2+\text{---}+\infty)$$

$$\Rightarrow y = a_0x^m\left[1-\frac{x^2}{(m+2)^2-n^2}+\frac{x^4}{[(m+2)^2-n^2][(m+4)^2-n^2]} \text{------}\right.$$

We get different solutions, depending upon the values of n.

For, $m=n$ we get

$$y_1 = a_0x^n\left[1-\frac{x^2}{2^2(n+1)}+\frac{x^4}{2^4.2(n+1)(n+2)}-\dots\right]$$ 1

$$= a_0x^n\left[\sum_{r=0}^{\infty}\frac{(-1)^r}{2^{2r}(r!)(n+1)(n+2)\text{----}(n+r)}x^{2r}\right]$$

$$y_1 = a_0 x^n \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma(n+1)}{r!\, 2^{2r}\, \Gamma(n+r+1)} x^{2r}$$

where $\Gamma(n)$ is the gamma function

The second independent solution corresponding to $m=-n$ is

$$y_1 = a_0 x^{-n} \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma(-n+1)}{r!\, 2^{2r}\, \Gamma(-n+r+1)} x^{2r}$$ 1

The complete solution of (1) is $y = c_1 y_1 + c_2 y_2$

Since, a_0 is arbitrary constant, we can choose it as we like. Choosing $a_0 = \frac{1}{2^n \Gamma(n+1)}$

$$y_1 = \frac{x^n}{2^n \Gamma(n+1)} \sum_{r=0}^{\infty} \frac{(-1)^r \Gamma(n+1)}{r!\, 2^{2r}\, \Gamma(n+r+1)} x^{2r}$$

$$= \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\, \Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$$ 1

This is called Bessel function of the first kind or order n and is denoted by $J_n(x)$. $\boxed{6}$

3b. We have, $$J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{(r)!\, \Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$$

putting $n = 1/2$, we get

$$J_{1/2}(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{(r)!\, \Gamma\left(\frac{1}{2}+r+1\right)} \left(\frac{x}{2}\right)^{1/2+2r}$$ 1

$$= \left(\frac{x}{2}\right)^{1/2} \left[\sum_{r=0}^{\infty} \frac{(-1)^r}{(r)!\, \Gamma(1/2+r+1)} \left(\frac{x}{2}\right)^{2r}\right]$$

$$= \left(\frac{x}{2}\right)^{1/2} \left[\frac{1}{\Gamma(3/2)} - \frac{1}{\Gamma(5/2)}\left(\frac{x}{2}\right)^2 + \frac{1}{2!\,\Gamma(7/2)}\left(\frac{x}{2}\right)^4 - \cdots\right]$$ 1

Q.No.	Solution and Scheme	Marks
	$= \left(\frac{x}{2}\right)^{1/2}\left[\frac{1}{\frac{1}{2}\Gamma(1/2)} - \frac{1}{\frac{3}{2}\cdot\frac{1}{2}\Gamma 1/2}\left(\frac{x}{2}\right)^2 + \frac{1 \times}{2!\,\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\Gamma 1/2}\left(\frac{x}{2}\right)^4 - - -\right]$	1
	$= \left(\frac{x}{2}\right)^{1/2}\frac{1}{\Gamma 1/2}\left[\frac{1}{1/2} - \frac{1}{\frac{3}{2}\,\frac{1}{2}}\left(\frac{x}{2}\right)^2 + \frac{1}{2!\,\frac{5}{2}\,\frac{3}{2}\,\frac{1}{2}}\left(\frac{x}{2}\right)^4 - \dots\right]$	1
	$= \frac{\sqrt{x}}{\sqrt{2}} \times \frac{1}{\sqrt{\pi}}\left[\frac{2}{1!} - \frac{2}{3!}x^2 + \frac{2}{5!}x^4 - - - -\right]$	1
	$= \frac{\sqrt{x}}{\sqrt{2}} \cdot \frac{1}{\sqrt{\pi}}\,\frac{2}{x}\left[\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right]$	
	$= \sqrt{\frac{2}{\pi x}}\left[\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - - - -\right]$	1
	$\boxed{J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x}$	1
		7
3c.	Given, $x^3 - 5x^2 + 6x + 1$	1
	We have, $x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$	1
	$x^2 = \frac{2}{3}P_2(x) + \frac{1}{3}P_0(x)$ and $x = P_1(x)$	1
	$\therefore x^3 - 5x^2 + 6x + 1 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$ $-5\left[\frac{2}{3}P_2(x) + \frac{1}{3}P_0(x)\right] + 6P_1(x) + P_0(x)$	2
	$= \frac{2}{5}P_3(x) - \frac{10}{3}P_2(x) + \frac{33}{5}P_1(x) - \frac{2}{3}P_0(x)$	2
		7
	OR	
4a.	By the definition of Bessel function, we have $J_n(x) = \sum_{r=0}^{\infty}(-1)^r\left(\frac{x}{2}\right)^{n+2r}\frac{1}{\Gamma(n+r+1)\,r!}$ $\hookrightarrow (1)$	1

Q.No.	Solution and Scheme	Marks
	$\therefore J_{-n}(x) = \sum_{r=0}^{\infty} (-1)^r \left(x/2\right)^{-n+2r} \frac{1}{r!\,\Gamma{-n+r+1}}$ ↳ (2)	1
	In (2) $\Gamma{-n+r+1} = \Gamma[r-(n-1)]$ is of the form $\Gamma{-k}$ for $r = 0, 1, 2, --- (n-2)$ and $\Gamma 0$ for $r=(n-1)$ Noting that $\Gamma{-k} \to \infty$ or $\frac{1}{\Gamma{-k}} \to 0$, k being a positive integer we can say that $\frac{1}{\Gamma[r-(n-1)]} \to 0$ for $r = 0, 1, 2, --- (n-1)$.	
	Hence, $J_{-n}(x) = \sum_{r=n}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{-n+2r} \frac{1}{\Gamma(-n+r+1)\, r!}$ ↳ (3)	1
	Let, $r-n = s$ or $r = s+n$ so that we have when $r=n$, $s=0$. Now, (3) assumes the form, $J_{-n}(x) = \sum_{s=0}^{\infty} (-1)^{s+n} \left(\frac{x}{2}\right)^{-n+2s+2n} \frac{1}{\Gamma(s+1)\, \Gamma(s+n)!}$ $= \sum_{s=0}^{\infty} (-1)^{s+n} \left(x/2\right)^{n+2s} \frac{1}{\Gamma(s+1)(s+n)!}$	1
	Using the properties of Gamma function we can write $\Gamma{s+1} = s!$ and $(s+n)! = \Gamma(s+n+1)$ $\therefore J_{-n}(x) = \sum_{s=0}^{\infty} (-1)^{s+n} \left(\frac{x}{2}\right)^{n+2s} \frac{1}{s!\,\Gamma(s+n+1)}$	1
	$= (-1)^n \sum_{s=0}^{\infty} (-1)^s \left(x/2\right)^{n+2s} \frac{1}{\Gamma(n+s+1)\, s!}$ Comparing with (1) we observe that the summation in the RHS is $J_n(x)$. Thus, $J_{-n}(x) = (-1)^n J_n(x)$, n being a positive integer	1

$\boxed{6}$

Q.No.	Solution and Scheme	Marks
4b.	Using Rodrigue's formula, $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2-1)^n\right]$	1
	At $n=4$, it gives $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$	
	Taking $x = \cos\theta$, we get $P_4(\cos\theta) = \frac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3)$	
	$= \frac{1}{8}\left[35(\cos^2\theta)^2 - 30\left(\frac{1+\cos 2\theta}{2}\right) + 3\right]$	1
	$= \frac{1}{8}\left[35\left(\frac{1+2\cos 2\theta + \cos^2 2\theta}{4}\right) - 15(1+\cos 2\theta) + 3\right]$	1
	$= \frac{1}{8}\left[35\left(\frac{1+2\cos 2\theta + \left(\frac{1+\cos 4\theta}{2}\right)}{4}\right) - 15\cos 2\theta - 12\right]$	
	$= \frac{1}{8}\left[35\left(\frac{2+4\cos 2\theta + 1 + \cos 4\theta}{8}\right) - 15\cos 2\theta - 12\right]$	1
	$= \frac{1}{8}\left[35\left(\frac{3+4\cos 2\theta + \cos 4\theta}{8}\right) - 15\cos 2\theta - 12\right]$	1
	$P_4(\cos\theta) = \frac{1}{64}\left[105 + 140\cos 2\theta + 35\cos 4\theta - 120\cos 2\theta - 96\right]$	1
	$= \frac{1}{64}\left[35\cos 4\theta + 20\cos 2\theta + 9\right]$	1
		7

Q.No.	Solution and Scheme	Marks
4c.	LHS = $x^3 - 2x^2 - x - 3 = f(x)$	
	We have, $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$	1
	$P_3(x) = \frac{1}{2}(5x^3 - 3x)$	
	Simplifying, $P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$	
	$\Rightarrow \frac{3}{2}x^2 = P_2(x) + \frac{1}{2}P_0(x)$	
	$x^2 = \frac{2}{3}P_2(x) + \frac{1}{3}P_0(x)$	1
	Similarly simplifying $P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$,	
	we get $x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$	1
	Substituting these in $f(x)$, along with $x = P_1(x)$	
	and $1 = P_0(x)$ we have,	
	$f(x) = \left[\frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)\right] - 2\left[\frac{2}{3}P_2(x) + \frac{1}{3}P_0(x)\right]$	1
	$- P_1(x) - 3P_0(x)$	
	$= \frac{2}{5}P_3(x) + P_1(x)\left[\frac{3}{5} - 1\right] - \frac{4}{3}P_2(x)$	2
	$+ P_0(x)\left[-\frac{2}{3} - 3\right]$	
	$f(x) = \frac{2}{5}P_3(x) - \frac{4}{3}P_2(x) - \frac{2}{5}P_1(x) - \frac{11}{3}P_0(x)$	1
		7

Module - 3

5a. $r = \dfrac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}}$ $\quad X = x - \bar{x}$

$Y = y - \bar{y}$

$\bar{x} = \Sigma x / n = 28/7 = 4$

$\bar{y} = \Sigma y / n = 77/7 = 11$

x	y	X	X^2	Y	Y^2	XY
1	9	-3	9	-2	4	6
2	8	-2	4	-3	9	6
3	10	-1	1	-1	1	1
4	12	0	0	1	1	0
5	11	1	1	0	0	0
6	13	2	4	2	4	4
7	14	3	9	3	9	9
			$\Sigma X^2 = 28$		$\Sigma Y^2 = 28$	$\Sigma XY = 26$

2

$r = \dfrac{26}{\sqrt{28}\sqrt{28}} = \dfrac{26}{28} = 0.9287 \simeq 0.93$

2

$\therefore r = 0.93$

The lines of regression are,

$Y = \dfrac{\Sigma XY}{\Sigma X^2}(X) \qquad X = \dfrac{\Sigma XY}{\Sigma Y^2}(Y)$

1

$X = x - \bar{x} \qquad Y = y - \bar{y}$

$(y - \bar{y}) = \dfrac{26}{28}(x - \bar{x}) \Rightarrow (y - 11) = (0.93)(x - 4)$

$y - 11 = (0.93)x - 3.72 \Rightarrow y = 0.93x - 3.72 + 11$

1

$\overline{6}$

Q.No. **Solution and Scheme** **Marks**

$y = 0.93x + 7.28$

$(x-\bar{x}) = \frac{26}{28}(y-\bar{y}) \Rightarrow (x-4) = (0.93)(y-11)$

$x-4 = (0.93)y - 10.23$

$x = 0.93y - 6.23$

Thus, $x = 0.93y - 6.23$ & $y = 0.93x + 7.28$ are lines of regression.

5b. The normal equations for fitting the straight line $y = ax+b$ is,

$$\Sigma y = a\Sigma x + nb$$

$$\Sigma xy = a\Sigma x^2 + b\Sigma x$$

1

x	y	x^2	xy
5	16	25	80
10	19	100	190
15	23	225	345
20	26	400	520
25	30	625	750
75	114	1375	1885

3

$\therefore \Sigma x = 75, \Sigma y = 114, \Sigma xy = 1885, \Sigma x^2 = 1375$

$$114 = 75a + 5b$$

$$1885 = 1375a + 75b$$

2

Solving these equations simultaneously we get $a = 0.7$ & $b = 12.3$

$\therefore$ The required straight line is $y = 0.7x + 12.3$ 1

7

Q.No.	Solution and Scheme	Marks
5c.		

x	y	$z=x-y$	x^2	y^2	z^2
1	2	-1	1	4	1
2	5	-3	4	25	9
3	3	0	9	9	0
4	8	-4	16	64	16
5	7	-2	25	49	4
Σx $=15$	Σy $=25$	Σz $=-10$	Σx^2 $=55$	Σy^2 $=151$	$\Sigma z^2=30$

2

$$\bar{x}=\frac{\Sigma x}{n}=3\ ;\ \bar{y}=\frac{\Sigma y}{n}=5\ ;\ \bar{z}=\frac{\Sigma z}{n}=-2$$

$$\sigma_x^2=\frac{\Sigma x^2}{n}-(\bar{x})^2=\frac{55}{5}-(3)^2=2$$

$$\sigma_y^2=\frac{\Sigma y^2}{n}-(\bar{y})^2=\frac{151}{5}-(5)^2=5.2$$

3

$$\sigma_z^2=\frac{\Sigma z^2}{n}-(\bar{z})^2=\frac{30}{5}-(-2)^2=2$$

$$r=\frac{\sigma_x^2+\sigma_y^2-\sigma_{x-y}^2}{2\sigma_x\sigma_y}=0.8062$$

$$\boxed{\therefore r=0.81}$$

The equations of the regression lines are,

$$y-\bar{y}=r\frac{\sigma_y}{\sigma_x}(x-\bar{x})\quad ;\ x-\bar{x}=r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

$$y-5=(0.81)\frac{\sqrt{5.2}}{\sqrt{2}}(x-3)\quad ;\ x-3=(0.81)\frac{\sqrt{2}}{\sqrt{5.2}}(y-5)$$

2

$$y=1.306x+1.082\quad ;\ x=0.502y+0.49$$

7

Q.No.	Solution and Scheme	Marks

OR

6a.

A	B	$d^2 = (A-B)^2$
1	6	25
6	4	4
5	9	16
10	8	4
3	1	4
2	2	0
4	3	1
9	10	1
7	5	4
8	7	1
		$\Sigma d^2 = 60$

3

$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)}, \quad n=10$$ 2

1

$$\underline{\underline{\rho}} = 0.636$$ 6

b. Since regression lines passes through $(\bar{x}, \bar{y})$

we have, $2\bar{x} + 3\bar{y} + 1 = 0$

$\bar{x} + 6\bar{y} - 4 = 0$

Solving above equations, $\bar{x} = -2$, $\bar{y} = 1$

To write the regression coefficients identify the regression line of y on x & x on y

$2x + 3y + 1 = 0$ or $3y = -2x - 1$

$\Rightarrow y = -\frac{2}{3}x - \frac{1}{3} \longrightarrow (1)$ 1

& also, $x = -6y + 4 \longrightarrow (2)$

Q.No.	Solution and Scheme	Marks
	The regression coefficients are $-\frac{2}{3}$ & -6 resply.	
	$\therefore r = \sqrt{\left(-\frac{2}{3}\right)\times(-6)} = \sqrt{4} = \pm 2.$	1
	As r lies between -1 & $+1$, we must change the given equations into other possible form.	
	$2x+3y+1=0 \Rightarrow 2x = -3y-1$	1
	$\Rightarrow x = -\frac{3}{2}y - \frac{1}{2}$	
	& $x+6y-4=0$ gives $6y=-x+4$ or	1
	$y = -\frac{1}{6}x + \frac{2}{3}$	
	Now, the regression coefficients will be $-\frac{3}{2}$ and $-\frac{1}{6}$ so that	
	$r = \sqrt{\left(-\frac{3}{2}\right)\times\left(-1/6\right)} = \pm 1/2$	2
	$\therefore r = -0.5$	
	The sign or r must be negative as both the regression coefficients are negative.	1
	Thus, $\bar{x}=-2$, $\underline{\underline{\bar{y}=1}}$, $r=-0.5$	7

Q.No. **Solution and Scheme** **Marks**

6c. Here, $n=5$, The normal equations of a parabola $ax^2+bx+c=y$ is

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$

$$\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

2

x	y	x^2	x^3	x^4	xy	x^2y
1	10	1	1	1	10	10
2	12	4	8	16	24	48
3	13	9	27	81	39	117
4	16	16	64	256	64	256
5	19	25	125	625	95	475
$\Sigma x = 15$	$\Sigma y = 70$	$\Sigma x^2 = 55$	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$	$\Sigma xy = 232$	$\Sigma x^2y = 906$

3

$$55a + 15b + 5c = 70$$

$$225a + 55b + 15c = 232$$

$$979a + 225b + 55c = 906$$

$$a = 0.285, \quad \underline{\underline{b = 0.485}}, \quad c = 9.4$$

2

7

∘: MODULE - 04 :∘

7a. we must have $p(x) \geq 0$ and $\Sigma p(x) = 1$

The first condition is satisfied if $k \geq 0$

Second condition is requires that,

$$k + 2k + 3k + 4k + 8k + 2k + k = 1$$

$$16k = 1 \Rightarrow k = 1/6$$ (2)

x	-3	-2	-1	0	1	2	3
$p(x)$	$1/16$	$2/16$	$3/16$	$4/16$	$3/16$	$2/16$	$1/16$

$$\text{Mean } (\mu) = \Sigma x \cdot p(x) = \frac{1}{16}(-3-4-3+0+3+4+3) = 0$$ (1)

$$\text{Variance } (V) = \Sigma (x-\mu)^2 \cdot p(x)$$

$$V = \frac{1}{16}(9+8+3+0+3+8+9) = \frac{5}{2}$$ (1)

$$S.D = \sqrt{V} = \sqrt{5/2} = 1.58$$

Thus, $k = 1/16$, Mean $= 0$ and $S.D = 1.58$

Also,

$$P(x \leq 1) = P(-3) + P(-2) + P(-1) + P(0) + P(1)$$

$$= \frac{1}{16} + \frac{2}{16} + 3/16 + 4/16 + 3/16$$ (2)

$$= 13/16$$

$$P(x > 1) = P(2) + P(3)$$

$$= 2/16 + 1/16 = 3/16.$$

$$P(-1 < x \leq 2) = P(0) + P(1) + P(2)$$

$$= \frac{4}{16} + \frac{3}{16} + \frac{2}{16} = \frac{9}{16}.$$

6M

Q.No.	Solution and Scheme	Marks
7b.	Mean and Variance of Binomial Distribution	
	Mean $(\mu) = \sum_{x=0}^{n} x\,p(x)$	(1)
	$\mu = \sum_{x=0}^{n} x \cdot {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$	
	$\mu = \sum_{x=0}^{n} x \cdot \frac{n!}{x!\,(n-x)!} p^{x} \cdot q^{n-x}$	
	$\mu = \sum_{x=0}^{n} \frac{n \cdot (n-1)!}{(x-1)!\,(n-x)!} \cdot p \cdot p^{x-1} \cdot q^{n-x}$	(2)
	$\mu = np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!} \cdot p^{x-1} \cdot q^{(n-1)-(x-1)}$	
	$\mu = np \sum_{x=1}^{n} {}^{(n-1)}C_{(x-1)} \cdot p^{x-1} \cdot q^{(n-1)-(x-1)}$	
	$\mu = np\,(q+p)^{n-1} = np \Longrightarrow \boxed{\mu = np}$	(1)
	Variance : $(V) = \sum_{x=0}^{n} x^{2} p(x) - \mu^{2}$ ------ (i)	
	Now,	
	$\sum_{x=0}^{n} x^{2} p(x) = \sum_{x=0}^{n} [x(x-1)+x]\,p(x)$	
	$= \sum_{x=0}^{n} x(x-1)\,p(x) + \sum_{x=0}^{n} x\,p(x)$	
	$= \sum_{x=0}^{n} x(x-1)\,{}^{n}C_{x} \cdot p^{x} \cdot q^{n-x} + np$	
	$= \sum_{x=0}^{n} x(x-1) \cdot \frac{n!}{x!\,(n-x)!} \cdot p^{x} \cdot q^{n-x} + np$	(3)
	$= \sum_{x=0}^{n} \frac{n \cdot (n-1)(n-2)!}{(x-2)!\,(n-x)!} \cdot p^{2} \cdot p^{x-2} \cdot q^{n-x} + np$	
	$= n(n-1)p^{2} \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,[(n-2)-(x-2)]!} \cdot p^{x-2} \cdot q^{(n-2)-(x-2)} + np$	
	$= n(n-1)p^{2} \sum_{x=2}^{n} {}^{(n-2)}C_{(x-2)} \cdot p^{x-2} \cdot q^{(n-2)-(x-2)} + np$	
	$= n(n-1)\,p^{2}\,(q+p)^{n-2} + np$	7M.
	$\sum x^{2} p(x) = n(n-1)p^{2} + np$	

Q.No.	Solution and Scheme	Marks
	Using this result (1) along with $\mu = np$ we have, $V = \{n(n-1)p^2 + np\} - (np)^2$ $= n^2p^2 - np^2 + np - n^2p^2 = np(1-p) = npq$ $\therefore \boxed{\text{Variance } (V) = npq}$	7M.
7c]	Solution:- p is the probability of a defective blade $= 0.002$ In a packet of 10, The mean number of defective blade is	(1)
	$m = np = 10 \times 0.002 = 0.02$ Poisson distribution:	(1)
	$p(x) = \frac{m^x \cdot e^{-m}}{x!} = \frac{e^{-0.02}(0.02)^x}{x!}$ Let, $f(x) = 10{,}000\, p(x)$; where $e^{-0.02} \simeq 0.9802$	(1)
	$\therefore f(x) = \frac{9802\,(0.02)^x}{x!}$ i] No defective $= f(0) = \frac{9802\,(0.02)^0}{0!} = 9802$.	(1)
	ii] Two defective $= f(2) = \frac{9802\,(0.02)^2}{2!} \simeq 1.9604$	(1)
	$\therefore f(2) \simeq 2$.	(1)
	iii) Three defective $= f(3) = \frac{9802\,(0.02)^3}{3!}$ $f(3) = 0.013069$. Thus, $f(0) = 9802$, $f(2) = 2$, $f(3) = 0.013$ //	(1) 7M.

Q.No.	Solution and Scheme	Marks
8a	Let,	
	$p(x) \geq 0$ if $k \geq 0$ and also we must have $\int_{-\infty}^{\infty} p(x)dx = 1$	
	That is,	
	$\int_{-3}^{3} kx^2 dx = 1$ or $\left[\frac{kx^3}{3}\right]_{-3}^{3} = \frac{k}{3}\left[3^3-(-3)^3\right] \Rightarrow \boxed{k = 1/18}$	(1)
	i] $p(x \leq 2)$	
	$= \int_{-3}^{2} \frac{1}{18}x^2 dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2} = \frac{1}{54}\left[(2)^3-(-3)^3\right] = \frac{1}{54}\left[8+27\right]$	(2)
	$p(x\leq 2) = \frac{35}{54}$	
	ii] $p(x \geq 2)$	
	$= \int_{2}^{3} \frac{1}{18}x^2 dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{2}^{3} = \frac{1}{18\times 3}\left[(3)^3-(2)^3\right] = \frac{1}{54}\left[27-8\right]$	(2)
	$p(x\geq 2) = \frac{19}{54}$.	
	iii] $p(x>1)$	(1)
	$= \int_{1}^{3} \frac{1}{18}x^2 dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{1}^{3} = \frac{1}{54}\left[27-1\right] = \frac{26}{54}$.	
	$p(x>1) = 26/54$.	6M.
8b	probability of a defective pen is $p = \frac{1}{10} = 0.1$	
	probability of a non defective pen is $q = 1-p$	(1)
	$q = 1-0.1 = 0.9$	
	we have,	(1)
	$p(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$ where $n=12$	
	i] Prob. [Exactly two pen will be defective]	
	$p[x=2] = 12C_2 (0.1)^2 (0.9)^{10}$	(1)
	$= \frac{12!}{(12-2)!\,2!} \times 0.01 \times (0.9)^{10} = 0.2301$	

Q.No.	Solution and Scheme	Marks
	ii) Prob [atmost 2 pen will be defective]	
	$=P[x \geq 2] = P(x=0)+P(x=1)+P(x=2)$	
	$=12C_0(0.1)^0(0.9)^{12-0}+12C_1(0.1)^1(0.9)^{12-1}+12C_2(0.1)^2(0.9)^{10}$	(2)
	$=1(1)(0.9)^{12}+12(0.1)(0.9)^{11}+\frac{12!}{(12-2)!2!}\times 0.01\times(0.9)^{10}$	
	$=0.2824+0.3765+0.2301$	
	$=0.8890$	
		(2)
	iii) Prob. [no defective pen will be $P(x=0)$]	
	$P(x=0)=12C_0(0.1)^0(0.9)^{12}=0.2824$	
		7M.
8 C]	Let x represent the marks of students.	
	By the data,	(1)
	$\mu=70, \sigma=5$. Hence S.N.V $=Z=\frac{x-\mu}{\sigma}=\frac{x-70}{5}$	
	i] If $x=65$, $z=-1$ and we have to find $P(Z<-1)$	
	$P(Z<-1)=P(Z>1)$	
	$=P(Z\geq 0)-P(0\leq Z\leq 1)$	
	$=0.5-\phi(1)=0.5-0.3413$	(2)
	$=0.1587$	
	∴ number of students scoring less than 65 marks.	
	$=1000\times 0.1587=158.7\simeq 159$	
	ii) If $x=75$, $Z=1$ and we have to find $P(Z>1)$	
	$P(Z>1)=P(Z\geq 0)-P(0\leq Z\leq 1)$	(2)
	$=0.5-\phi(1)=0.5-0.3413=0.1587$	
	∴ number of students scoring More than 75 marks	
	$=1000\times 0.1587=158.7\simeq 159$	
	iii) we have to find $P(-1<Z<1)$	(2)
	$P(-1<Z<1)=2P(0<Z<1)\Rightarrow 2\phi(1)=2(0.3413)$	
	$=0.6826$	
	∴ number of students scoring marks bet^n 65 & 75	
	$=1000\times 0.6826=682.6\simeq 683$.	7M

MODULE - 05.

9a. The marginal distribution of X and Y is as follows.

x_i	1	5
$f(x_i)$	$1/2$	$1/2$

y_j	-4	2	7
$g(y_j)$	$3/8$	$3/8$	$1/4$

(1)

a] $E(X) = \Sigma x_i f(x_i)$

$= (1)(1/2) + 5(1/2)$

$= 3$ (1)

$E(Y) = \Sigma y_j g(y_j)$

$= (-4)(3/8) + (2)(3/8) + (7)(1/4)$

$= 1.$

b] $E(XY) = \Sigma x_i y_j J_{ij}$ (2)

$= (1)(-4)(1/8) + (1)(2)(1/4) + (1)(7)(1/8) + 5(-4)(1/4)$

$+ 5(2)(1/8) + 5(7)(1/8)$

$= -\frac{1}{2} + \frac{1}{2} + \frac{7}{8} - 5 + \frac{5}{4} + \frac{35}{8} = 3/2$

Thus, $E(XY) = 3/2$

c] $\sigma_x^2 = E(X^2) - \mu_x^2$ and $\sigma_y^2 = E(Y^2) - \mu_y^2$

Now, (2)

$E(X^2) = \Sigma x_i^2 f(x_i)$

$E(X^2) = (1)(1/2) + 25(1/2) = 13$

$E(Y^2) = \Sigma y_j^2 g(y_j)$

$E(Y^2) = 16(3/8) + 4(3/8) + 49(1/4) = 79/4$

Hence,

$\sigma_x^2 = 13 - (3)^2 = 4$; $\sigma_y^2 = (79/4) - (1)^2 = 75/4$ 6M.

Thus,

$\sigma_x = 2$ and $\sigma_y = \sqrt{75/4} = 4.33.$

Q.No.	Solution and Scheme	Marks
9b.	i) Null Hypothesis: The hypothesis formulated for the purpose of its rejection under the assumption that it is true is called the Null Hypothesis and it is denoted by Ho.	(2)
	ii] Type I & Type II Error:- If a hypothesis is rejected while it should have been accepted is known as Type I Error. If a hypothesis is accepted while it should have been rejected is known as Type II Error.	(2)
	iii] Degree of freedom: The number of degrees of freedom usually denoted by V. is the number of values in a set which may be assigned arbitrarily. It can be interpreted as the number of independent values generated by a sample of small size for estimating a population parameter.	(2)
	iv) Level of significance:- The probability level below which we reject the hypothesis is known as the level of significance.	(1)
		7M.
9c]	Battery A : $n_1 = 10$, $\bar{x}_1 = 500$ hrs , $\sigma_1^2 = 100$ Battery B : $n_2 = 10$, $\bar{x}_2 = 560$ hrs , $\sigma_2^2 = 121$ Studet t distribution is.	

$$S^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2-2}$$ (2)

$$S^2 = \frac{(10\times100)+(10\times121)}{18} = 122.78$$

$\therefore S = 11.0805$

we have,

$$t = \frac{\bar{x}_2 - \bar{x}_1}{S\sqrt{1/n_1 + 1/n_2}} = \frac{60}{11.0805\sqrt{0.1+0.1}}$$ (3)

$t = 12.1081 \simeq 12.11$

This value of t is greater than the table value of t for 18 d.f at all levels of significance. (2)

The null hypothesis that there is no significant diff in the two means is rejected at all significance levels.

7M

10a Solution:-

i] Marginal Distribution

x_i	2	5	7
$f(x_i)$	1/2	1/4	1/4

y_j	3	4	5
$g(y_j)$	1/3	1/3	1/3

(2)

ii) $Cov(X,Y) = E(XY) - \mu_x\mu_y$

Consider, (1)

$\mu_x = E(X) = \sum_i x_i f(x_i)$

$= (2)(1/2) + (5)(1/4) + (7)(1/4)$

$= 4.$

Q.No.	Solution and Scheme	Marks
	$\mu_y = E(Y) = \sum_j y_j\, g(y_j)$	
	$= (3)(1/3) + (4)(1/3) + (5)(1/3) = 4$	(2)
	$E(XY) = \sum x_i y_j J_{ij}$	
	Jij value already there in the question	
	$= (2)(3)(1/6) + (2)(4)(1/6) + (2)(5)(1/6) + (5)(3)(1/12)$	(2)
	$+ (5)(4)(1/12) + (5)(5)(1/12) + (7)(3)(1/12) + (7)(4)(1/12)$	
	$+ (7)(5)(1/12)$	
	$= 16.$	(1)
	Hence, $cov(X,Y) = E(XY) - \mu_x \mu_y$	
	$= 16 - (4)(4)$	
	Thus, $cov(X,Y) = 0$	6M
10b.	we have,	
	$\mu = 66$, $n = 10$	(1)
	$\bar{x} = \frac{\sum x}{n} = \frac{678}{10} = 67.8$	
	$S^2 = \frac{1}{n-1} \sum (x-\bar{x})^2$	
	$s^2 = \frac{1}{9}\left[(63-67.8)^2 + \ldots + (71-67.8)^2\right]$	(2)
	$s^2 = 9.067 \Longrightarrow \boxed{s = 3.011}$	
	we have,	(3)
	$t = \frac{\bar{x}-\mu}{s} \cdot \sqrt{n} = \frac{(67.8-66)}{3.011}\sqrt{10}$	
	$t = 1.89 < 2.262$	
	Thus the hypothesis is accepted at 5% level of significance	(1) 7M

10C.

Round & Yellow	Wrinkled & Yellow	Round & Green	Wrinkled & Green	Total
315	101	108	32	556.

ratio : 9:3:3:1

$\Rightarrow \frac{9}{16} \times 556 = 313$

$\Rightarrow \frac{3}{16} \times 556 = 104$ (2)

$\Rightarrow \frac{3}{16} \times 556 = 104$

$\Rightarrow \frac{1}{16} \times 556 = 35$

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i}$$ (3)

$$\chi^2 = \left[\frac{(315-313)^2}{313} + \frac{(101-104)^2}{104} + \frac{(108-104)^2}{104} + \frac{(32-35)^2}{35}\right]$$

$$\chi^2 = \frac{4}{313} + \frac{9}{104} + \frac{16}{104} + \frac{9}{35}$$

$\chi^2 = 0.51 < \chi^2_{0.05} = 7.815$ for 3 d.f (2)

Hence there is a very high degree of agreement between theory & experiment.

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7M

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