Model Question Paper-I with effect from 2022 (CBCS Scheme)

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Second Semester B.E Degree Examination
Mathematics-II for Electrical & Electronics Engineering-BMATE201

TIME: 03 Hours **Max. Marks: 100**

Note:
1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **module.**
2. VTU Formula Hand Book is permitted.
3. M: Marks, L: Bloom's level, C: Course outcomes.

		Module -1	**M**	**L**	**C**
Q.01	a	Find the directional derivatives of $\emptyset = x^2yz + 4xz^2$ at (1, 2, -1) along $2i - j - 2k$.	7	L2	CO1
	b	Evaluate $Curl(Curl\vec{F})$ and $Div(Curl\vec{F})$, if $\vec{F} = x^2y\,\hat{\imath} + y^2z\,\hat{\jmath} + z^2x\,\hat{k}$.	7	L2	CO1
	c	Show that the vector $\vec{F} = \frac{x\hat{\imath}+y\hat{\jmath}}{x^2+y^2}$ is both solenoidal and irrotational.	6	L3	CO1
		OR			
Q.02	a	Find the total work done by the force $F = 3xy\boldsymbol{I} - y\boldsymbol{J} + 2zx\boldsymbol{K}$ in moving a particle around the circle $x^2 + y^2 = 4$.	7	L3	CO1
	b	Verify Stoke's theorem for the vector field $\boldsymbol{F} = (2x - y)\boldsymbol{I} - yz^2\boldsymbol{J} - y^2z\boldsymbol{K}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy- plane.	7	L2	CO1
	c	Using modern mathematical tools, write a code to find the divergence and curl of the vector $2x^2i - 3yzj + xz^2k$	6	L3	CO5
		Module-2			
Q. 03	a	Prove that in $V_3(R)$, the vectors {(1, 2, 1), (2, 1, 0), (1, -1, 2)} are linearly independent.	7	L2	CO2
	b	If W is the set of all points in R^3 satisfying the equation $lx + my + nz = 0$, then prove that W is a subspace of R^3.	7	L2	CO2
	c	Define an Inner product space. Consider $f(t) = 3t - 5$ and $g(t) = t^2$, the inner product $< f,g > = \int_0^1 f(t)g(t)dt$. Find $< f,g >$.	6	L2	CO2
		OR			
Q.04	a	Express the vector (3, 5, 2) as a linear combination of the vectors (1, 1, 0), (2, 3, 0), (0, 0, 1) of $V_3(R)$.	7	L2	CO2
	b	Find the dimension and basis of the subspace spanned by the vectors (2, 4, 2), (1, -1, 0), (1, 2, 1), and (0, 3, 1) in $V_3(R)$.	7	L2	CO2
	c	Let $T: V \rightarrow W$ be a linear transformation defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Find the range, null space, rank, nullity and hence verify the rank-nullity theorem.	6	L2	CO2

		Module-3			
Q. 05	a	Find the Laplace transform of (i) $te^{-t}sin4t$ (ii) $\frac{1-cosat}{t}$	7	L2	CO3
	b	Find the Laplace transform of the square wave function of period 2a, defined by $f(t) = \begin{cases} k, & 0 < t < a \\ -k, & a < t < 2a \end{cases}$	7	L3	CO3
	c	Express $f(t) = \begin{cases} cost\,, & 0 < t < \pi \\ cos2t, & \pi < t < 2\pi \\ cos3t, & t > 2\pi \end{cases}$ in terms of the Heaviside unit step function and hence find $L\{f(t)\}$.	6	L3	CO3
		OR			
Q. 06	a	Find $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$	7	L2	CO3
	b	Find $L^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\}$ Using the convolution theorem.	7	L2	CO3
	c	Solve by Laplace transform method: $y'' + 4y' + 3y = e^{-t}$, given $y(0) = y'(0) = 1$.	6	L3	CO3
		Module-4			
Q. 07	a	Find the real root of the equation $x\log_{10} x = 1.2$ by the Regula-Falsi method between 2 and 3 (Three iterations).	7	L2	CO4
	b	Using Newton's forward difference formula, find $f(38)$ <table><tr><td>x</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>y</td><td>184</td><td>204</td><td>226</td><td>250</td><td>276</td><td>304</td></tr></table>	7	L3	CO4
	c	The following table gives the values of x and y <table><tr><td>x :</td><td>2.8</td><td>4.1</td><td>4.9</td><td>6.2</td></tr><tr><td>y :</td><td>9.8</td><td>13.4</td><td>15.5</td><td>19.6</td></tr></table>Find y when x = 8 using Lagrange's interpolation formula.	6	L2	CO4
		OR			
Q. 08	a	Using Newton-Raphson Method find the real root of $tanx = x$ near x = 4.5 correct to four decimal places.	7	L3	CO4
	b	Find the interpolating polynomial using Newton's divided difference formula for the following data <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>5</td></tr><tr><td>y</td><td>2</td><td>3</td><td>12</td><td>147</td></tr></table>	7	L2	CO4

	c	Evaluate $\int_{4}^{5.2} \log x \, dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule, taking h = 0.2	6	L3	CO4
		Module-5			
Q. 09	a	Use Taylor series method to find $y(0.2)$ by considering the terms up to 4th degree, given $\frac{dy}{dx} - 2y = 3e^{x}$ & $y(0) = 0$.	7	L3	CO4
	b	Given $\frac{dy}{dx} = 3x + \frac{y}{2}, y(0) = 1$. Compute $y(0.2)$ by taking $h = 0.2$ using Runge-Kutta method of fourth order.	7	L2	CO4
	c	Apply Milne's method to find $y(0.8)$ given $\frac{dy}{dx} + xy^2 = 0$ <table><tr><td>x</td><td>0</td><td>0.2</td><td>0.4</td><td>0.6</td></tr><tr><td>y</td><td>2</td><td>1.9231</td><td>1.7214</td><td>1.4706</td></tr></table>	6	L2	CO4
		OR			
Q. 10	a	Using Modified Euler's method to find y at $x = 0.2$ given $\frac{dy}{dx} = x - y^2$ & $y(0) = 1$ by taking step size h = 0.1	7	L3	CO4
	b	Find $y(2)$ by using Milne's Predictor and Corrector method, given $\frac{dy}{dx} = \frac{x+y}{2}$ and <table><tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td></tr><tr><td>y</td><td>2</td><td>2.636</td><td>3.595</td><td>4.968</td></tr></table>	7	L2	CO4
	c	Using modern mathematical tools, write a code to find $y(0.1)$, given $\frac{dy}{dx} = x - y, y(0) = 1$ by Taylor's Series.	6	L3	CO5

Bloom's Taxonomy Levels	**Lower-order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6

AY: 2022-23

Department: Mathematics
Subject with Sub. Code: Mathematics-II for Electrical and Electronics Engineering stream (BMATE201)
Semester/Division: II/A, B, C
Name of Faculty: Dr. Meenal Kaliwal / Mrs. Akshata Patil

Q.No.	Solution and Scheme	Marks

:- MODULE-01 :-

1a. $\phi = x^2yz + 4xz^2$

$$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k.$$

$$\nabla\phi = (2xyz + 4z^2)i + (x^2z)j + (x^2y + 8xz)k. \qquad (2)$$

$$[\nabla\phi]_{(1,-2,-1)} = 8i - j - 10k.$$

The unit vector in the direction of $2i-j-2k$ is,

$$\hat{n} = \frac{2i-j-2k}{\sqrt{4+1+4}} = \frac{2i-j-2k}{3} \qquad (3)$$

$\therefore$ the required directional derivative is,

$$\nabla\phi\cdot\hat{n} = (8i-j-10k)\cdot\frac{(2i-j-2k)}{3}$$

Thus,

$$\nabla\phi\cdot\hat{n} = \frac{(8)(2)+(-1)(-1)+(-10)(-2)}{3} \qquad (2)$$

$$\nabla\phi\cdot\hat{n} = 37/3$$

7M

1b. $\vec{F} = x^2yi + y^2zj + z^2yk.$

$$\text{curl } \vec{F} = \nabla\times\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \partial/\partial y & \partial/\partial z \\ x^2y & y^2z & z^2y \end{vmatrix} \qquad (2)$$

$$= i(z^2 - y^2) - j(0-0) + k(0 - x^2)$$

$$\therefore \text{curl } \vec{F} = (z^2-y^2)i - x^2k.$$

Q.No.	Solution and Scheme	Marks
	Now, $\text{Curl}(\text{curl}\,\vec{F}) = \nabla\times(\nabla\times\vec{F})$ $= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \partial/\partial y & \partial/\partial z \\ (z^2-y^2) & 0 & -x^2 \end{vmatrix} \Rightarrow i(0-0)-j(-2x-2z)+k(0+2y)$ $= j(2x+2z)+2yk.$	(2)
	Thus, $\text{Curl}(\text{curl}\,\vec{F}) = (2x+2z)j+2yk.$	
	<u>Next:</u> $\text{div}(\text{curl}\,\vec{F}) = \nabla\cdot(\nabla\times\vec{F})$ $= \left(\frac{\partial}{\partial x}i+\frac{\partial}{\partial y}j+\frac{\partial}{\partial z}k\right)\cdot\{(z^2-y^2)i-x^2k.$ $= \frac{\partial}{\partial x}(z^2-y^2)+\partial/\partial z(-x^2) = 0+0.$	(3)
	Thus, $\nabla\cdot(\nabla\times\vec{F}) = 0.$	7M
1C	$\text{div}\,\vec{F} = \nabla\cdot\vec{F}$ $= \left(\frac{\partial}{\partial x}i+\frac{\partial}{\partial y}j+\frac{\partial}{\partial z}k\right)\cdot\left(\frac{x}{x^2+y^2}i+\frac{y}{x^2+y^2}j\right)$ $= \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right)+\frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right)$	(1)
	$= \left\{\frac{(x^2+y^2)-2x^2}{(x^2+y^2)^2}\right\}+\left\{\frac{(x^2+y^2)-2y^2}{(x^2+y^2)^2}\right\}$ $= \frac{1}{(x^2+y^2)^2}(y^2-x^2+x^2-y^2)=0$	(2)
	Thus, $\boxed{\text{div}\,\vec{F}=0} \Rightarrow \vec{F}$ is solenoidal.	
	$\text{curl}\,\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} & 0 \end{vmatrix}$	(3)
	$= 0i+0j+k\left\{\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right)-\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)\right\}$ $= k\left[\frac{-2xy}{(x^2+y^2)^2}+\frac{2xy}{(x^2+y^2)^2}\right] = 0$ Thus, $\boxed{\text{curl}\,\vec{F}=0}$ $\vec{F}$ is irrotational.	6M

Q.No.	Solution and Scheme	Marks

2a. Total work done $W = \int_C \vec{F}.d\vec{r}$

$x^2+y^2=4$ can be represented in the parametic form $x = 2\cos\theta$, $y = 2\sin\theta$ and $z=0$, $0 \le \theta \le 2\pi$ (2)

$$W = \int_C \vec{F}.d\vec{r} = \int 3xy\,dx - y\,dy + 2zx\,dz$$

$$W = \int_{\theta=0}^{2\pi} 3(4\cos\theta\sin\theta)(-2\sin\theta)d\theta - \int_{\theta=0}^{2\pi} 4\sin\theta\cos\theta\, d\theta$$

$$W = -24\int_0^{2\pi} \sin^2\theta\cos\theta\, d\theta - 2\int_0^{2\pi} \sin2\theta\, d\theta$$ (3)

$$= -24\left[\frac{\sin^3\theta}{3}\right]_0^{2\pi} - 2\left[\frac{-\cos2\theta}{2}\right]_0^{2\pi} = 0$$ (2)

Thus the total work done is 0. 7M.

2b. $\int_C \vec{F}.d\vec{r} = \iint_S \text{curl}\,\vec{F}.\hat{n}\,ds$

C is the circle ; $x^2+y^2=1$, $z=0$

$\vec{F}.d\vec{r} = (2x-y)dx - yz^2dy - y^2z\,dz = (2x-y)dx \ (\because z=0)$ (1)

Taking,

$x = \cos\theta$, $y = \sin\theta$, where $0 \le \theta \le 2\pi$

LHS.

$$\int_C \vec{F}.d\vec{r} = \int_{\theta=0}^{2\pi} (2\cos\theta - \sin\theta)(-\sin\theta)d\theta$$

$$= \int_0^{2\pi} \left\{-\sin2\theta + \frac{1}{2}(1-\cos2\theta)\right\} d\theta$$ (2)

$$= \left[\frac{\cos2\theta}{2} + \frac{\theta}{2} - \frac{\sin2\theta}{4}\right]_0^{2\pi} = \left(\frac{1}{2}-\frac{1}{2}\right) + (\pi - 0) - 0$$

$= \pi$.

Hence, $\int_C \vec{F}.d\vec{r} = \pi$ ----- (1)

Q.No.	Solution and Scheme	Marks
	Also, $\text{Curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \partial/\partial y & \partial/\partial z \\ 2x-y & -yz^2 & -y^2z \end{vmatrix}$	(2)
	$= i(-2yz+2yz) - j(0) + k(0+1) = k$	
	$\therefore d\vec{s} = \hat{n}\,ds = dydz\,i + dzdx\,j + dxdy\,k$	
	Hence,	
	$RHS = \iint_s \text{curl } \vec{F} \cdot \hat{n}\, ds = \iint dxdy = \pi$ - - - - - (2)	(2)
	$\iint dxdy$ represents the area of the circle $x^2+y^2=1$ which is π.	
	Thus from eq (1) and (2) we conclude that the theorem is verified.	7M
2C	Divergence:	
	from sympy.vector import*	
	from sympy import symbols	
	N=CoordSys3D ('N')	(3)
	x, y, z = symbols ('x y z')	
	A=N.2*X**2*N.-N.3*Y*N*Z*N + N.X*Z**2*N	
	delop = Del( )	
	div A = delop. dot (A)	
	display (div A)	
	Print (f "\n Divergence of {A} is \n")	
	Curl:	
	from sympy. vector import *	
	from sympy import symbols	
	N=CoordSys3D ('N')	(3)
	X, Y, Z = Symbols ('X Y Z')	
	A= N.2*X**2.N.i - N.3*Y.N.Z*N.j+N.X*Z**2.N.k	
	delop = Del ( )	
	curl A = delop. Cross (A)	
	display (curl A)	
	Print (f "\n Curl of {A} is \n")	6M
	display (curl (A))	

Q.No.	Solution and Scheme	Marks
	Module-2	
3a.	Let a, b, c be scalars such that	
	$a(1,2,1) + b(2,1,0) + c(1,-1,2) = (0,0,0)$	1
	$(a+2b+c,\ 2a+b-c,\ a+2c) = (0,0,0)$	1
	$a+2b+c = 0 \longrightarrow (i)$	1
	$2a+b-c = 0 \longrightarrow (ii)$	
	$a+2c = 0 \longrightarrow (iii)$	
	Multiplying eqn (i) by 2 & solving with eqn (ii),	
	$2a+4b+2c = 0$	
	$2a+b-c = 0$	
	$-\quad -\quad +$	
	$3b+3c = 0$	1
	$b+c = 0 \longrightarrow (iv)$	
	Solving equations (i) & (iii),	
	$a+2b+c = 0$	
	$a\quad +2c = 0$	
	$-\quad -$	
	$2b-c = 0 \longrightarrow (v)$	1
	Solving eqns (iv) & (v),	
	$b+c = 0$	
	$2b-c = 0$	
	$3b = 0$	
	$\Rightarrow b = 0$	
	putting $b=0$ in eqn (iv) we get $c=0$	1
	putting $b=0$ & $c=0$ in eqn (i) we get $a=0$.	
	Thus, $a=0, b=0, c=0$ is the only solution of the equations (i), (ii) & (iii).	
	$\therefore a(1,2,1)+b(2,1,0)+c(1,-1,2) = (0,0,0)$	1
	$\Rightarrow a=0,\ b=0,\ c=0$	

Dr. Meenal M. Kaliwal, Asst. Prof.
Department of Mathematics, KLS VDIT, Haliyal

Q.No.	Solution and Scheme	Marks
	Hence, the vectors $(1,2,1)$, $(2,1,0)$ & $(1,-1,2)$ are linearly independent in $V_3(R)$.	7
3b.	Let, $W=\{(x,y,z): lx+my+nz=0\}$	
	Let, $\alpha=(x_1,y_1,z_1)$ and $\beta(x_2,y_2,z_2)$ be any two elements of W such that $lx_1+my_1+nz_1=0$ and $lx_2+my_2+nz_2=0$. for $a,b\in R$	2
	we have	
	$a\alpha+b\beta=a(x_1,y_1,z_1)+b(x_2,y_2,z_2)$	1
	$=(ax_1+bx_2,\ ay_1+by_2,\ az_1+bz_2)$	
	Now, $l(ax_1+bx_2)+m(ay_1+by_2)+n(az_1+bz_2)$	
	$=(lax_1+may_1+naz_1)+(lbx_2+mby_2+nbz_2)$	1
	$=a(lx_1+my_1+nz_1)+b(lx_2+my_2+nz_2)$	1
	$=a.0+b.0=0$	
	$\therefore l(ax_1+bx_2)+m(ay_1+by_2)+n(az_1+bz_2)=0$	1
	$\therefore a\alpha+b\beta\in W$.	
	Thus, $\alpha\in W,\ \beta\in W \Rightarrow a\alpha+b\beta\in W\ \forall\ a,b\in R$	1
	Hence, W is a subspace of R^3.	7
3c.	Inner Product Space: Let V be a vector space over F. An inner product on V is a function $\langle \circ,\circ\rangle : V\times V\longrightarrow F$ which satisfies the following properties:	
	(i) $(\alpha,\alpha)>0$ for all non-zero vectors α in V.	3
	(ii) $(\alpha,\beta)=\overline{(\beta,\alpha)}\ \forall\ \alpha,\beta\in V$	
	(iii) $(a\alpha+b\beta,\gamma)=a(\alpha,\gamma)+b(\beta,\gamma)\ \forall\ \alpha,\beta,\gamma\in V$	

Dr. Meenal M. Kaliwal, Asst. Prof,
Department of Mathematics, KLS VDIT

Q.No.	Solution and Scheme	Marks
	and $a, b \in F$.	
	A vector space V together with an inner product is called an inner product space.	
	Now, $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$	
	$= \int_0^1 (3t-5)(t^2)\, dt$	1
	$= \int_0^1 (3t^3 - 5t^2)\, dt$	
	$= \left[\frac{3t^4}{4} - \frac{5t^3}{3} \right]_{t=0}^{1}$	1
	$= \frac{3}{4} - \frac{5}{3} = \frac{9-20}{12} = -\frac{11}{2}$.	1
		6
	OR	
4a.	Let, $\alpha = (3,5,2)$, $\alpha_1 = (1,1,0)$, $\alpha_2 = (2,3,0)$ and $\alpha_3 = (0,0,1)$.	
	Let, $\alpha = a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3$ where $\alpha_1, \alpha_2, \alpha_3 \in R$.	1
	$(3,5,2) = a_1(1,1,0) + a_2(2,3,0) + a_3(0,0,1)$	1
	$\Rightarrow (3,5,2) = (a_1 + 2a_2, a_1 + 3a_2, a_3)$	1
	$\therefore\ 3 = a_1 + 2a_2 \longrightarrow (1)$	
	$5 = a_1 + 3a_2 \longrightarrow (2)$	
	and $a_3 = 2 \longrightarrow (3)$	1
	Eliminating 'a_1' from (1) & (2),	
	$3 = a_1 + 2a_2$ $5 = a_1 + 3a_2$ $-\quad -\quad -$	
	$-2 = -a_2$ $\quad \boxed{\text{i.e. } a_2 = 2}$	1

Dr. Mrunal M. Kaliwal, Asst. Professor
Department of Mathematics, KLS VDIT

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	From (1), $3 = a_1 + 2a_2 \Rightarrow 3 = a_1 + 4 \Rightarrow a_1 = -1$	1
	Hence, $(3,5,2) = -1(1,1,0) + 2(2,3,0) + 2(0,0,1)$	1
		7
4b.	Let, $S = \{(2,4,2), (1,-1,0), (1,2,1), (0,3,1)\}$ and $L(S) = W$.	
	Now, we shall find the maximal linearly independent subsets of S. Let, A be a matrix whose rows are elements of S, then	
	$A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & -1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & 1 \end{bmatrix}$	1
	We shall have to reduce A to an Echelon form by using row transformations.	
	Applying $R_2 \longleftrightarrow R_1$	
	$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \\ 0 & 3 & 1 \end{bmatrix}$	1
	Applying $R_2 \to R_2 - 2R_1;\ R_3 \to R_3 - R_1$	
	$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 6 & 2 \\ 0 & 3 & 1 \\ 0 & 3 & 1 \end{bmatrix}$	2
	Applying $R_3 \to 2R_3 - R_2,\ R_4 \to R_4 - R_3$	
	$A \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 6 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	2

Dr. Mrunal M. Kaliwal, Asst. Professor
Department of Mathematics, KLS VDIT

Mmf.

Q.No.	Solution and Scheme	Marks
	which is in Echelon form, which has 2 non-zero rows representing the coordinate vectors $(1,-1,0)$ and $(0,6,2)$ that form a basis of rows space i.e. $T=\{(1,-1,0),(0,6,2)\}$ is a basis of W.	1
	Thus, $\dim(W)=2$.	7
4c.	Determination of range of T i.e. R_T and rank	
	Since the ordered set $\{(1,0,0),(0,1,0),(0,0,1)\}$ forms a basis of V, then by definition of T, $T(1,0,0)=(1,1,2)$, $T(0,1,0)=(1,-1,0)$ and $T(0,0,1)=(0,0,1)$.	1
	Since, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ generates V, therefore $T(1,0,0)$, $T(0,1,0)$ and $(T(0,0,1)$ will generate $T(V)=R_T$.	
	$\Rightarrow (1,1,2),(1,-1,0),(0,0,1)$ generates R_T i.e. $R_T=\{(1,1,2),(1,-1,0),(0,0,1)\}$	1
	Also, for some scalars $x,y,z\in R$ such that $x(1,1,2)+y(1,-1,0)+z(0,0,1)=(0,0,0)$ $\Rightarrow (x+y,x-y,2x+z)=(0,0,0)$ $\Rightarrow x+y=0,\ x-y=0,\ 2x+z=0$ $\Rightarrow x=0,\ y=0,\ z=0$	1
	$\therefore \{(1,1,2),(1,-1,0),(0,0,1)\}$ are linearly independent and spans R_T, so it forms a basis of R_T. Hence, $\dim(R_T)=3$.	1
	Since T is a linear transformation from V to W, therefore	

Q.No.	Solution and Scheme	Marks
	$\dim(R_T) + \dim(N_T) = \dim(V)$	
	$\Rightarrow \quad 3 + \dim(N_T) = 3$	
	$\Rightarrow \quad \dim(N_T) = 0$	
	Thus, nullity of $T = 0$	
	Since, $\dim(N_T) = 0 \Rightarrow$ Null space of T i.e. N_T	
	is a zero space.	1
	$\Rightarrow \quad N_T = \{0,0,0\}$	
	<u>Verification of Rank-Nullity Theorem</u>	
	If V and W are vector spaces over the field	
	F and T is a linear transformation from V	
	into W and if V is finite dimensional, then	
	$\text{rank}(T) + \text{nullity}(T) = \dim(V)$	
	$\Rightarrow R_T + N_T = \dim(V)$	1
	$3 + 0 = 3$	
	$\Rightarrow \quad 3 = 3$	
	Hence, the rank-nullity theorem is verified.	
		6

Dr. Meenal M. Kaliwal, Asst. Professor
Department of Mathematics, KLS VDIT

Q.No.	Solution and Scheme	Marks

-: MODULE-03 :-

5a. Let,

i] $f(t) = t e^{-t} \sin 4t$

$L[\sin 4t] = \frac{4}{s^2+16}$ $\quad \therefore L[e^{-t} \cdot \sin 4t] = \frac{4}{(s+1)^2+16}$

$= \frac{4}{s^2+1+2s+16}$

Hence,

$L[t e^{-t} \sin 4t] = \frac{-d}{ds}\left\{\frac{4}{s^2+2s+17}\right\}$ (2)

$= \frac{4(2s+2)}{(s^2+2s+17)^2}$

Thus,

$L[t e^{-t} \sin t] = \frac{8(s+1)}{(s^2+2s+17)^2}$

$\frac{1-\cos at}{2}$

ii] $\frac{1-\cos at}{t}$

$\Rightarrow L\left[\frac{1-\cos at}{t}\right]$ (3)

Let,

$f(t) = 1-\cos at$ $\qquad L[f(t)] = F(s)$

$L[1-\cos at] = L(1) - L(\cos at)$

$\therefore L\left[\frac{1-\cos at}{t}\right] = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+a^2}\right) ds$ $\qquad = \frac{1}{s} - \frac{s}{s^2+a^2}$

$= \int_s^\infty \frac{1}{s}\, ds - \int_s^\infty \frac{s}{s^2+a^2}\, ds$

$= \left[\log s - \frac{1}{2}\log(s^2+a^2)\right]_s^\infty$ (2)

$= \frac{1}{2}\left[2\log s - \log(s^2+a^2)\right]_s^\infty$

$= \frac{1}{2}\left[\log s^2 - \log(s^2+a^2)\right]_s^\infty = \frac{1}{2}\left[\log\left(\frac{s^2}{s^2+a^2}\right)\right]_s^\infty$

$= \frac{1}{2}\left[\lim_{s\to\infty} \log\left(\frac{s^2}{s^2+a^2}\right) - \log\left(\frac{s^2}{s^2+a^2}\right)\right]$

$= \frac{1}{2}\left[\lim_{s\to\infty} \log\left(\frac{1}{1+a^2/s^2}\right) - \log\left(\frac{s^2}{s^2+a^2}\right)\right]$

$= -\frac{1}{2}\log\left(\frac{s^2}{s^2+a^2}\right) = -\frac{1}{2}\log\left(\frac{s^2}{s^2+a^2}\right)$ 7M

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Q.No.	Solution and Scheme	Marks

5b. We know Laplace transform of periodic function on with period T.

$$L[f(t)] = \frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}} \qquad (1)$$

here $f(t)$ is a periodic function with period $2a$.

$$L[f(t)] = \frac{\int_0^{2a} e^{-st} f(t)\,dt}{1-e^{-s2a}}$$

$$= \frac{1}{1-e^{-2as}}\left[\int_0^a f(t)\cdot e^{-st}\,dt + \int_a^{2a} f(t)\cdot e^{-st}\,dt\right]$$

$$= \frac{1}{1-e^{-2as}}\left[\int_0^a k\cdot e^{-st}\,dt + \int_a^{2a} -k\cdot e^{-st}\,dt\right]$$

$$= \frac{k}{1-e^{-2as}}\left[\int_0^a e^{-st}\,dt - \int_a^{2a} e^{-st}\,dt\right] \qquad (2)$$

$$= \frac{k}{1-e^{-2as}}\left[\left(\frac{e^{-st}}{-s}\right)_0^a - \left(\frac{e^{-st}}{-s}\right)_a^{2a}\right]$$

$$= \frac{k}{1-e^{-2as}}\left[\left(\frac{e^{-sa}}{-s} - \frac{e^{-s\times 0}}{-s}\right) - \left(\frac{e^{-s2a}}{-s} - \frac{e^{-sa}}{-s}\right)\right]$$

$$= \frac{k}{(1-e^{-2as})}\left[\frac{-e^{-sa}}{s} + \frac{1}{s} + \frac{e^{-s2a}}{s} - \frac{e^{-sa}}{s}\right]$$

$$= \frac{k}{s(1-e^{-2as})}\left[-e^{-as} + 1 + e^{-2as} - e^{-as}\right] \qquad (2)$$

$$= \frac{k}{s(1-e^{-2as})}\left[-2e^{-as} + 1 + e^{-2as}\right]$$

$$= \frac{k}{s(1-e^{-2as})}\left[(e^{-as})^2 - 2(1)(e^{-as}) + (1)^2\right]$$

$$= \frac{k}{s(1-e^{-2as})} \times (1-e^{-as})^2$$

Q.No.	Solution and Scheme	Marks
	$= \dfrac{k}{s\left[(1)^2-(e^{-as})^2\right]} \times (1-e^{-as})^2$	
	$= \dfrac{k \times (1-e^{-as})^2}{s(1-e^{-as})(1+e^{-as})} = \dfrac{k(1-e^{-as})}{s(1+e^{-as})}$	(2)
		7M.
5c	$f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$	
	$f(t) = \cos t + (\cos 2t - \cos t)\,u(t-\pi) + (\cos 3t - \cos 2t)$ $u(t-2\pi)$	
	$L[f(t)] = L\{\cos t\} + L\left[(\cos 2t - \cos t)\,u(t-\pi)\right]$ $+ L\left[(\cos 3t - \cos 2t)\,u(t-2\pi)\right]$ ---- (1)	
	Let, $F(t-\pi) = \cos 2t - \cos t$; $G(t-2\pi) = \cos 3t - \cos 2t$	(4)
	$F(t) = \cos 2(t+\pi) - \cos(t+\pi)$ and	
	$G(t) = \cos 3(t+2\pi) - \cos 2(t+2\pi)$	
	$G(t) = \cos 3t - \cos 2t$	
	Therefore, $F(t) = \cos 2t + \cos t$; $G(t) = \cos 3t - \cos 2t$	
	$\overline{F}(s) = \dfrac{s}{s^2+4} + \dfrac{s}{s^2+1}$, $\overline{G}(s) = \dfrac{s}{s^2+9} - \dfrac{s}{s^2+4}$	
	But, $L\left[F(t-\pi)\,u(t-\pi)\right] = e^{-\pi s}\,\overline{F}(s)$ and	(1)
	$L\left[G(t-2\pi)\,u(t-2\pi)\right] = e^{-2\pi s}\,\overline{G}(s)$	
	$L\left[(\cos 2t - \cos t)\,u(t-\pi)\right] = e^{-\pi s}\left(\dfrac{s}{s^2+4} + \dfrac{s}{s^2+1}\right)$	

Q.No.	Solution and Scheme	Marks

And,

$$L\left[(\cos 3t - \cos 2t)\,u(t-2\pi)\right] = e^{-2\pi s}\left[\frac{s}{s^2+9} - \frac{s}{s^2+4}\right]$$

Hence eq^n (1) becomes.

$$L[f(t)] = \frac{s}{s^2+1} + e^{-\pi s}\left(\frac{s}{s^2+4} + \frac{s}{s^2+1}\right) + e^{-2\pi s}\left(\frac{s}{s^2+9} - \frac{s}{s^2+4}\right) \quad (1)$$

Thus,

$$L[f(t)] = \frac{s}{s^2+1} + s e^{-\pi s}\left(\frac{1}{s^2+4} + \frac{1}{s^2+1}\right) - \frac{5 s e^{-2\pi s}}{(s^2+4)(s^2+9)}$$

6M

6a.
$$L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$$

$$\bar{f}(s) = \frac{2s^2-6s+5}{s^3-6s^2+11s-6} = \frac{2s^2-6s+5}{(s-1)(s-2)(s-3)} \quad (2)$$

Using partial fraction

$$\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)} = \frac{A}{s-1} + \frac{B}{(s-2)} + \frac{C}{s-3} \text{ - - - - - (1)}$$

$$2s^2-6s+5 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$$

Put,
$s = 1$; $2-6+5 = A(-1)(-2)$

$1 = 2A \Rightarrow \boxed{A = 1/2}$ (3)

Put,
$s = 2$; $8-12+5 = B(1)(-1)$

$1 = -B \Rightarrow \boxed{B = -1}$

Put,
$s = 3$; $18-18+5 = C(2)(1)$

$5 = 2C \Rightarrow \boxed{C = 5/2}$ (2)

∴ Therefore eq (1)

$$\bar{f}(s) = \frac{1}{2}\left(\frac{1}{s-1}\right) - \frac{1}{(s-2)} + \frac{5}{2}\left(\frac{1}{s-3}\right)$$

Taking Inverse Laplace Transform on both sides.

$$L^{-1}[\bar{f}(s)] = \tfrac{1}{2} L^{-1}\left(\tfrac{1}{s-1}\right) - L^{-1}\left(\tfrac{1}{s-2}\right) + \tfrac{5}{2} L^{-1}\left(\tfrac{1}{s-3}\right)$$

7M

$$L^{-1}[\bar{f}(s)] = \tfrac{1}{2} e^{t} - e^{2t} + \tfrac{5}{2} e^{3t}$$

Q.No.	Solution and Scheme	Marks

6b. $L^{-1}\left[\frac{1}{s^3(s^2+1)}\right]$

$= L^{-1}\left(\frac{1}{s^3}\right) + L^{-1}\left(\frac{1}{s^2+1}\right)$

Let,

$F(s) = 1/s^3 \quad ; \quad G(s) = 1/s^2+1$

Now,

$F(S) = L^{-1}[F(s)] = L^{-1}[1/s^3] = \frac{t^{3-1}}{(3-1)!} = t^2/2.$ (2)

$G(s) = L^{-1}[G(s)] = L^{-1}\left[\frac{1}{s^2+1}\right] = \sin t.$

Now by convolution theorem

$L^{-1}[F(s) * G(s)] = \int_0^t F(\,).g(t-u)\,du$

$= \int_0^t \left(\frac{u^2}{2}\right).[\sin(t-u)]\,du$

$= 1/2 \int u^2 . \sin(t-u)\,du$. By int by parts. (3)

$= 1/2\left\{(u^2)\left[-\frac{\cos(t-u)}{-1}\right] - (2u)\left[\frac{\sin(t-u)}{-1}\right] + 2\left[\frac{-\cos(t-u)}{(-1)(-1)}\right]\right\}_0^t$

$= 1/2\left[u^2\cos(t-u) + 2u\sin(t-u) - 2\cos(t-u)\right]_0^t$

$= 1/2\left\{\left[t^2\cos(0) + 2t\sin(0) - 2\cos(0)\right] - \left[(0) + (0) - 2\cos t\right]\right\}$ (2)

$= 1/2\left[(t^2 + 0 - 2) + 2\cos t\right]$

$= 1/2\,(t^2 + 2\cos t - 2)$ //

7M.

Apatil

Mrs. Akshata Patil, Dept. of Mathematics
KLS Vishwanathrao Deshpande Institute of Technology

Q.No.	Solution and Scheme	Marks
6c	Taking Laplace transform on both sides of the given eq $L[y''(t)] + 4L[y'(t)] + 3L[y(t)] = L(e^{-t})$ $\{s^2 L[y(t)] - sy(0) - y'(0)\} + 4\{sL[y(t)] - y(0)\}$ $+ 3L[y(t)] = 1/(s+1)$ Using the given initial conditions $(s^2+4s+3)L[y(t)] - s - 1 - 4 = 1/(s+1)$ $(s^2+4s+3)L[y(t)] = (s+5) + 1/(s+1)$ $(s+1)(s+3)L[y(t)] = \frac{s^2+6s+6}{s+1}$ $L[y(t)] = \frac{s^2+6s+6}{(s+1)^2(s+3)} \Rightarrow L^{-1}\left[\frac{s^2+6s+6}{(s+1)^2(s+3)}\right]$	(2)
	Let, $\frac{s^2+6s+6}{(s+1)^2(s+3)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+3)}$ Multiplying by $(s+1)^2(s+3)$ we get, $s^2+6s+6 = A(s+1)(s+3) + B(s+3) + C(s+1)^2$ ---(1)	(2)
	Put, $s=-1$; $1 = B(2)$ $\therefore B = 1/2$ $s=-3$; $-3 = C(4)$ $\therefore C = -3/4$ Equating the coefficient of s^2 on both sides of (1) $1 = A + C$ $\therefore A = 7/4$	(1)
	Hence, $L^{-1}\left[\frac{s^2+6s+6}{(s+1)^2(s+3)}\right] = \frac{7}{4}L^{-1}\left[\frac{1}{s+1}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{(s+1)^2}\right] - \frac{3}{4}L^{-1}\left[\frac{1}{s+3}\right]$ Thus, $y(t) = \frac{7}{4}e^{-t} + \frac{1}{2}e^{-t}.t - \frac{3}{4}e^{-3t}$.	(1)
		6M.

Mrs. Akshata Patil, Dept. of Mathematics
KLS Vishwanathrao Deshpande Institute of Technology

Q.No.	Solution and Scheme	Marks
	MODULE 04	
7a	Let, $f(x) = x\log_{10}x - 1.2$	
	a real root of $f(x)=0$ lies in $(2,3)$	
	I st Approximation:	
	Let, $a=2$; $b=3$	(2)
	$f(a) = -0.6$; $f(b) = 0.23$	
	$\therefore x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(2)(0.23)-(3)(-0.6)}{0.23-(-0.6)}$	
	$\boxed{x_1 = 2.723}$	
	Now, $f(2.723) = -0.0154$	
	II nd Approximation:	(2)
	Let $a=3$; $f(a) = 0.23$	
	$b = 2.723$; $f(b) = -0.0154$	
	$\therefore x_2 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{3(-0.0154)-2.723(0.23)}{-0.0154-0.23}$	
	$\boxed{x_2 = 2.7404}$	
	Now, $f(2.7404) = -0.00021$	
	III rd Approximation:	(1)
	Let, $a = 2.723$; $f(a) = -0.0154$	
	$b = 2.7404$; $f(b) = -0.00021$	
	$\therefore x_3 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{2.723(-0.00021)-2.7404(-0.0154)}{-0.00021-(-0.0154)}$	
	$\boxed{x_3 = 2.7406}$	

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Q.No.	Solution and Scheme	Marks
	Now, $f(2.7406) = -0.00004$ IVth Approximation:- Let $a = 2.7404$; $f(a) = -0.00021$ $b = 2.7406$; $f(b) = -0.00004$	(1)
	$x_4 = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = \dfrac{2.7404(-0.00004) - 2.7406(-0.00021)}{-0.00004 - (-0.00021)}$ $\boxed{x_4 = 2.74065}$ Thus the required real root of the equation is 2.7407	(1) 7M
7b	find $f(38)$	

x	40	50	60	70	80	90
y	184	204	226	250	276	304

we have, Newton's forward difference formula. (2)

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \cdots$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$x_0=40$	$y_0=184$			
		20		
$x_1=50$	$y_1=204$		2	
		22		0
$x_2=60$	$y_2=226$		2	
		24		0
$x_3=70$	$y_3=250$		2	
		26		0
$x_4=80$	$y_4=276$		2	
		28		
$x_5=90$	$y_5=304$			

(3)

$$r = \frac{x - x_0}{h} = \frac{38 - 40}{10} = -0.2 \quad (2)$$

$$y(38) = 184 + (-0.2)20 + \frac{(-0.2)(-0.2-1)}{2!} \times 2$$

$$y(38) = 184 - 4 + 0.24 = 180.24 \quad \therefore \boxed{y(38) = 180.24}$$

7M

Q.No.	Solution and Scheme	Marks
7c.	$x=8$	

x	2.8	4.1	4.9	6.2
y	9.8	13.4	15.5	19.6

Let,

$x_0=2.8$, $x_1=4.1$, $x_2=4.9$, $x_3=6.2$ } $x=8$ (1)

$y_0=9.8$, $y_1=13.4$, $y_2=15.5$, $y_3=19.6$] $y=2$.

we have Lagrange's Interpolation formula

$$y=f(x)=\frac{(x-x_1)(x-x_2)(x-x_3)y_0}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}+\frac{(x-x_0)(x-x_2)(x-x_3)y_1}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$ (2)

$$+\frac{(x-x_0)(x-x_1)(x-x_3)y_2}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}+\frac{(x-x_0)(x-x_1)(x-x_2)y_3}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$f(8)=\frac{(8-4.1)(8-4.9)(8-6.2)}{(2.8-4.1)(2.8-4.9)(2.8-6.2)}\times 9.8+\frac{(8-2.8)(8-4.9)(8-6.2)}{(4.1-2.8)(4.1-4.9)(4.1-6.2)}\times 13.4$$ (2)

$$+\frac{(8-2.8)(8-4.1)(8-6.2)}{(4.9-2.8)(4.9-4.1)(4.9-6.2)}\times 15.5+\frac{(8-2.8)(8-4.1)(8-4.9)}{(6.2-2.8)(6.2-4.1)(6.2-4.9)}\times 19.6$$

$=-22.976+178.0285-259.071+132.752$ (1)

$=28.7336$

$\boxed{f(8)=28.7336}$

6M

8a. Let,

$f(x)=\tan x-x$

$x_0=4.5$ (1)

$f'(x)=\sec^2 x-1 \implies f'(x)=\tan^2 x.$

By Newton Raphson method.

I[st] Approximation:

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=4.5-\frac{f(4.5)}{f'(4.5)}$$ (2)

APatil

Q.No.	Solution and Scheme	Marks
	$\boxed{x_1 = 4.4936}$	
	II nd Approximation:	
	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4.4936 - \frac{f(4.4936)}{f'(4.4936)}$	(2)
	$\boxed{x_2 = 4.4934}$	
	III rd Approximation:	
	$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 4.4934 - \frac{f(4.4934)}{f'(4.4934)}$	(2)
	$\boxed{x_3 = 4.4934.}$	
	Thus the required approximation result is 4.4934 //	7M
8b	(divided difference table below)	

x	y	$\Delta f(x_0)$	$\Delta^2 f(x_0)$	$\Delta^3 f(x_0)$
0	2	$\frac{8-2}{1-0} = 1$		
			$\frac{9-1}{2-0} = 4$	
1	3	$\frac{12-3}{2-1} = 9$		
				$\frac{9-4}{5-0} = \frac{5}{5} = 1.$
2	12	$\frac{147-12}{5-2} = 45$	$\frac{45-9}{5-1} = 9.$	
5	147			

(3)

we have Newton's divided difference formula. (1)

$$y = f(x_0) = y_0 + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0)$$

$$y = 2 + (x-0)(1) + (x-0)(x-1)4 + (x-0)(x-1)(x-2)(1)$$ (3)

$$f(x) = 2 + x + x(x-1)4 + x(x-1)(x-2)$$
$$= 2 + x + (x^2 - x)4 + x(x^2 - 2x - x + 2)$$
$$= 2 + x + 4x^2 - 4x + x^3 - 2x^2 - x^2 + 2x$$
$$= x^3 + x^2 - x + 2 //$$

7M.

Mrs. Akshata Patil, Dept. of Mathematics
KLS Vishwanathrao Deshpande Institute of Technology

Q.No.	Solution and Scheme	Marks
8C.	Let, $\int_4^{5.2} \log x \, dx$	
	width, $h = \frac{b-a}{n} = \frac{5.2-4}{6} = \frac{1.2}{6} = 0.2$	(1)
	we have Simpson's $3/8$th rule.	
	$y = f(x) = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$	(2)
	we have prepare the table below.	
	(table below)	(2)
	$y = \log x = \frac{3}{8}(0.2)\left[(1.3863+1.6487) + 3(1.4351+1.4816+1.5686+1.6094) + 2(1.5261)\right]$	(2)
	$y = \log x = 1.8278$.	
	Thus, $\boxed{\log x = 1.8278}$	
		6M

x	4	4.2	4.4	4.6	4.8	5.0	5.2
$\log x$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Mrs. Akshata Patil, Dept. of Mathematics
KLS Vishwanathrao Deshpande Institute of Technology

Q.No.	Solution and Scheme	Marks
	Module-5	
9a.	Given, $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$; $x_0 = 0$ & $y_0 = 0$	
	The Taylor's series is,	
	$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \cdots$	1
		1
	$y' = 2y + 3e^x$; $y'(0) = 3$	1
	$y'' = 2y' + 3e^x$; $y''(0) = 6+3 = 9$	1
	$y''' = 2y'' + 3e^x$; $y'''(0) = 18+3 = 21$	1
	$y^{iv} = 2y''' + 3e^x$; $y^{iv}(0) = 42+3 = 45$	1
	$y(x) = 0 + 3x + \frac{9}{2}x^2 + \frac{7}{2}x^3 + \frac{15}{8}x^4 + \cdots$	1
	$\boxed{\therefore y(0.2) \simeq 0.8110}$	7
b	Given, $f(x,y) = 3x + y/2$, $x_0 = 0$, $y_0 = 1$, $h = 0.2$	1
	$k_1 = hf(x_0, y_0) = 0.1$	
	$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.2)f(0.1, 1.05)$	1
	$= 0.2\left[(3\times 0.1) + (1.05/2)\right] = 0.165$	
	$k_2 = 0.165$	
	$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.2)f(0.1, 1.0825)$	
	$= 0.2\left[(3\times 0.1) + (1.0825/2)\right]$	1
	$k_3 = 0.16825$	

Dr. Mrunal M. Kaliwal, Asst. Professor
Department of Mathematics, KLS VDIT

Mmf

Q.No.	Solution and Scheme	Marks
	$k_4 = hf(x_0+h, y_0+k_3)$	
	$= (0.2) f(0.2, 1.16825)$	
	$= 0.2\left[(3\times 0.2) + (1.16825/2)\right]$	2
	$k_4 = 0.136825$	1
	$\therefore y(x_0+h) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$	1
	$\underline{\underline{y(0.2) = 1.15055}}$	7
9c.	Given, $\frac{dy}{dx} + xy^2 = 0$	
	We shall prepare the following table	

x	y	$y' = -xy^2$
$x_0 = 0$	$y_0 = 2$	$y_0' = 0$
$x_1 = 0.2$	$y_1 = 1.9231$	$y_1' = -0.7397$
$x_2 = 0.4$	$y_2 = 1.7214$	$y_2' = -1.1853$
$x_3 = 0.6$	$y_3 = 1.4706$	$y_3' = -1.2976$
$x_4 = 0.8$	$y_4 = ?$	

Marks: 2

Peridictor Corrector Formula,

$$y_4^{(P)} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 2 + \frac{4\times(0.2)}{3}\left[(2\times -0.7397) + 1.1853 - (2\times 1.2976)\right]$$

$y_4^{(P)} = 1.22952$ — 1

Now, $y_4' = -x_4(y_4^{(P)})^2 = -1.20938$

Dr. Meenal M. Kaliwal, Asst. Professor
Dept. of Mathematics, KLS VDIT

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Q.No.	Solution and Scheme	Marks
	Next, we apply the Corrector Formula,	
	$y_4^{(c)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$	
	$= 1.7214 + \left(\frac{0.2}{3}\right)[-1.1853 + 4(-1.2976) - 1.20938]$	
	$\therefore y_4^{(c)} \simeq 1.2157$	1
	Now, $y_4' = -x_4[y_4^{(P)}]^2 = -(0.8)(1.2157)^2$	1
	$y_4' = -1.18234$	
	$y_4^{(c)} = 1.7214 + \frac{(0.2)}{3}[-1.1853 - (4 \times 1.2976) - 1.2157]$	
	$y_4^{(c)} \simeq 1.2153$	1
	Thus, $\underline{\underline{y_4 = y(0.8) = 1.2153}}$	6
	OR	
10a.	Given, $f(x,y) = x - y^2$; $x_0 = 0$, $y_0 = 1$, $h = 0.1$	
	Stage 1: To find $y(0.1)$	
	By Euler's formula : $y_1^{(0)} = y_0 + hf(x_0, y_0)$	
	$y_1^{(0)} = 1 + (0.1) f(0, 1) = 0.9$	1
	By Modified Euler's formula,	
	$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$	
	$= 1 + \frac{0.1}{2}[f(0,1) + f(0.1, 0.9)]$	
	$y_1^{(1)} = 0.9145$	1
	$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$	

Dr. Munal M. Kaliwal, Asst. Professor
Dept. of Mathematics, KLS VDIT

Q.No.	Solution and Scheme	Marks
	$y_1^{(2)} = 0.9132$	1
	$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$	
	$y_1^{(3)} = 0.9133$	1
	$\therefore y(0.1) = 0.9133$	
	Stage II : $x_0 = 0.1,\ y_0 = 0.9133,\ x_1 = 0.2$	
	By Euler's formula, $y_1^{(0)} = y_0 + hf(x_0, y_0)$	
	$y_1^{(0)} = 0.9133 + (0.2) f(0.1, 0.9133)$	1
	$y_1^{(0)} = 0.7665$	
	By Modified Euler's formula,	
	$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$	
	$= 0.9133 + \frac{(0.2)}{2}\left[f(0.1, 0.9133) + f(0.2, 0.7665)\right]$	1
	$y_1^{(1)} = 0.8012$	
	$y_1^{(2)} = 0.7957$	1
	$y_1^{(3)} = 0.7966$	
	$\therefore y(0.2) = 0.7966$	7

Dr. Mrunal M. Kaliwal, Asst. Professor

Dept. of Mathematics, KLS VDIT

Q.No.	Solution and Scheme	Marks
10b.	Prepare the following table	

x	y	$y' = x+y/2$
$x_0 = 0$	$y_0 = 2$	$y'_0 = 1$
$x_1 = 0.5$	$y_1 = 2.636$	$y'_1 = 1.568$
$x_2 = 1$	$y_2 = 3.595$	$y'_2 = 2.2975$
$x_3 = 1.5$	$y_3 = 4.968$	$y'_3 = 3.234$

2

Milne's Predictor formula,

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3)$$

$$= 2 + \left(\frac{4\times0.5}{3}\right)[2(1.568) - 2.2975 + 2(3.234)]$$

$$= 6.871$$ 1

$$y'_4 = \frac{x_4 + y_4^{(P)}}{2} = \frac{2+6.871}{2} = 4.4355$$ 1

$$y_4^{(C)} = y_2 + \frac{h}{3}(y'_2 + 4y'_3 + y'_4)$$

$$= 3.595 + \frac{0.5}{3}[2.2975 + 4(3.234) + 4.4355]$$

$$= 6.8732$$ 1

$$y'_4 = \frac{x_4 + y_4^{(C)}}{2} = \frac{2+6.8732}{2} = 4.4366$$ 1

Again using Milne's corrector formula,

$$y_4^{(C)} = 6.8734$$ 1

$$\therefore y(2) = 6.8734$$

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Dr. Meenal M. Kaliwal, Asst. Professor

Dept. of Mathematics, KLS VDIT

Q.No.	Solution and Scheme	Marks
10c.	from sympy import *	
	x, y = symbols ('x, y')	1
	x0, x1 = 0, .1	
	yd0 = 1	
	yd1 = x - y	
	yd2 = 1 - yd1	
	yd3 = yd2	
	yd1 = yd1.subs({x:0, y:1})	2
	yd2 = yd2.subs({x:0, y:1})	
	yd3 = yd3.subs({x:0, y:1})	
	ts = yd0 + (x-x0)*yd1 + (x-x0)**2*yd2 /2 + (x-x0)**3*yd3/6	1
	tsx1 = ts.subs(x, x1)	1
	print ('Taylors series is', ts)	
	print ('y(', x1, ') =', round (tsx1, 4))	1
		6
	Faculty: Dr. Meenal M. Kaliwal (Mky) Prof. Akshata Patil HOD, BSH Department: Dr. R.S. Munnolli HOD Basic Sciences & Humanities KLS VDIT. HALIYAL-581329 Dean, Academics KLS VDIT, HALIYAL	