Model Question Paper-I with effect from 2022 (CBCS Scheme)

USN | | | | | | | | | | |

Second Semester B.E Degree Examination
Mathematics-II for CIVIL ENGINEERING STREAM -BMATC201

TIME: 03 Hours **Max. Marks: 100**

Note:
1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **module**.
2. VTU Formula Hand Book is permitted.
3. M: Marks, L: Bloom's level, C: Course outcomes.

		Module -1	**M**	**L**	**C**
Q.01	a	Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx\, dy$	7	L3	CO1
	b	Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ by changing the order of integration	7	L3	CO1
	c	Derive the relation $\beta(m,n) = \frac{\gamma(m)\,\gamma(n)}{\gamma(m+n)}$	6	L2	CO1
		OR			
Q.02	a	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2) dxdy$ by changing into polar coordinates	7	L3	CO1
	b	Using double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	7	L3	CO1
	c	Write a modern mathematical tool program to evaluate the double integral $\int_0^1 \int_0^x (x^2+y^2) dydx$	6	L3	CO5
		Module-2			
Q. 03	a	Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at the point (1, -2, 1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$	7	L2	CO2
	b	Evaluate $Curl(Curl\vec{F})$ and $div(curl\vec{F})$, If $\vec{F} = x^2y\,\hat{\imath} + y^2z\,\hat{\jmath} + z^2x\,\hat{k}$	7	L3	CO2
	c	Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\ \hat{\imath} + (3xz + 2xy)\,\hat{\jmath} + (3xy - 2xz + 2z)\,\hat{k}$ is both solenoidal and irrotational	6	L2	CO2
		OR			
Q.04	a	Find the work done in moving a particle in the force field $\vec{F} = 3x^2\,\hat{\imath} + (2xz - y)\,\hat{\jmath} + z\,\hat{k}$ along the straight line from $(0,0,0)\ to\ (2,1,3)$	7	L2	CO2
	b	Using Green's theorem, Evaluate $\oint[(3x - 8y^2)dx + (4y - 6xy)\,dy]$ over the boundary of the region $x = 0, y = 0, and\ x + y = 1$	7	L3	CO2
	c	Write a modern mathematical tool program to find the gradient of $\varphi = x^2y + 2xz - 4$	6	L3	CO5
		Module-3			
Q. 05	a	Form the partial differential equation from the relation $z = f(x + at) + g(x - at)$	7	L2	CO3

	b	Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 when y is an odd multiple of $\frac{\pi}{2}$.	7	L3	CO3
	c	Solve $x(y^2 - z^2)p + y(z^2 - x^2)q - z(x^2 - y^2) = 0$	6	L3	CO3
		OR			
Q. 06	a	Form the partial differential equation from $f(x + y + z,\ x^2 + y^2 + z^2) = 0$	7	L2	CO3
	b	Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial z}{\partial x} - 4z = 0$, given that when x = 0, z = 1 and $\frac{\partial z}{\partial x} = y$	7	L3	CO3
	c	With usual notations, derive one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$	6	L2	CO3
		Module-4			
Q. 07	a	Find a real root of $x^3 - 9x + 1 = 0\ in\ (2, 3)$ by the Regula-Falsi method in four iterations.	7	L3	CO4
	b	Using Newton's forward interpolation find y at x = 5 from the data $\begin{array}{\|c\|c\|c\|c\|c\|} \hline x & 4 & 6 & 8 & 10 \\ \hline y & 1 & 3 & 8 & 16 \\ \hline \end{array}$	7	L3	CO4
	c	Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta$ by taking 7 ordinates using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule	6	L3	CO4
		OR			
Q. 08	a	Find the real root of the equation $\cos x = xe^x$, which is nearer to $x = 0.5$ by the Newton-Raphson method, correct to four decimal places.	7	L3	CO4
	b	Determine $f(x)$ as a polynomial in x for the data given below by using Newton's divided difference formula $\begin{array}{\|c\|c\|c\|c\|c\|c\|c\|} \hline x & 2 & 4 & 5 & 6 & 8 & 10 \\ \hline f(x) & 10 & 96 & 196 & 350 & 868 & 1746 \\ \hline \end{array}$	7	L3	CO4
	c	Evaluate $\int_{0.2}^{1.4} (\sin x - \log x + e^x)\, dx$ by taking seven ordinates, using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule.	6	L3	CO4
		Module-5			
Q. 09	a	Find an approximate value of y when $x = 0.1$, if $\frac{dy}{dx} = x - y^2$ and y = 1 at $x = 0$ using Taylor's series method.	7	L3	CO4
	b	Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with the initial condition $y = 1$ when $x = 0$. Find approximately y for $x = 0.1$ by Modified Euler's method. Carry out three modifications.	7	L3	CO4

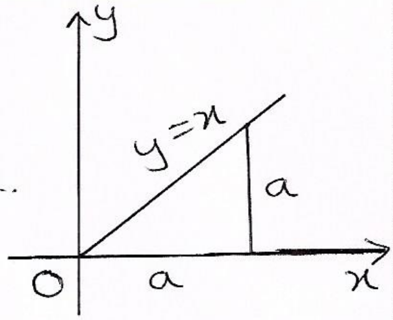
	c	Given $\frac{dy}{dx} = xy + y^2, y(0) = 1, y(0.1) = 1.1169,\ y(0.2) = 1.2773,$ $y(0.3) = 1.5049,$ compute $y(0.4)$ using Milne's Predictor-Corrector method.	6	L3	CO4
		OR			
Q. 10	a	Using modified Euler's formula, compute $y(1.1)$ correct to three decimal places given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and $y = 1$ at $x = 1$.	7	L3	CO4
	b	Using the Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$	7	L3	CO4
	c	Write a modern mathematical tool program to solve $\frac{dy}{dx} = 2x + y,\ y(1) = 2$ by the Runge-Kutta 4th order method.	6	L3	CO5

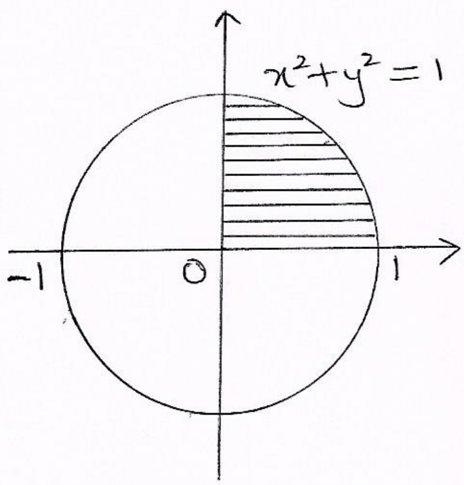
Bloom's Taxonomy Levels	**Lower-order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6

AY: 2022-23

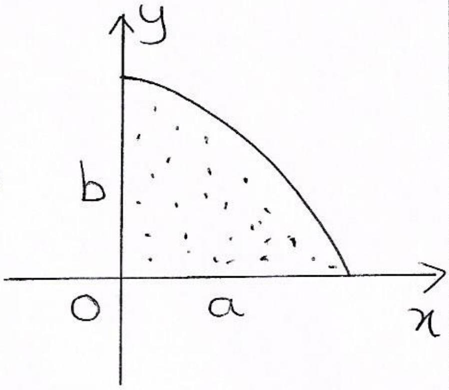
Department: Basic Sciences & Humanities (Mathematics) **Model Question Paper - I**
Branch: Civil **Semester: II**
Subject with Sub. Code: Mathematics-II for Civil Engineering Stream (BMATC201)
Name of Faculty: Dr. Meenal M. Kaliwal

Q.No.	Solution and Scheme	Marks
01 a.	Let, $I = \int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz\, dx\, dy$	
	$= \int_0^1 \left\{ \int_{y^2}^1 \left[x \int_0^{1-x} 1 \, dz \, dx \right\} dy$	1
	$= \int_0^1 \left\{ \int_{y^2}^1 x(1-x) \, dx \right\} dy$	1
	$= \int_0^1 \left\{ \frac{x^2}{2} - \frac{x^3}{3} \right\}_{x=y^2}^1 dy$	1
	$= \int_0^1 \left\{ \frac{1}{2}(1-y^4) - \frac{1}{3}(1-y^6) \right\} dy$	2
	$= \frac{1}{2} - \frac{1}{10} - \frac{1}{3} + \frac{1}{21} = \frac{4}{35}$	1
	$\therefore I = \frac{4}{35}$	1
		7

Q.No.	Solution and Scheme	Marks
1b.	Here y varies from 0 to a, and for each, y, x varies from $x=y$ to $x=a$.  Thus, the lower value of x lies on the line $y=x$ and the upper value on the line $x=a$.	1
	$\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy = \int_{x=0}^{a}\int_{y=0}^{x} \frac{x}{x^2+y^2}\,dy\,dx$	2
	$= \int_0^a \left\{ x. \left[\frac{1}{x} \tan^{-1}\left(\frac{y}{x}\right) \right]_{y=0}^{x} \right\} dx$	2
	$= \int_0^a (\tan^{-1} 1 - \tan^{-1} 0)\,dx$	1
	$= \int_0^a \frac{\pi}{4}\,dx = \frac{\pi a}{4}$	1
		7
1c.	$\Gamma n = 2\int_0^\infty e^{-x^2} x^{2n-1}\,dx$	1
	Similarly, $\Gamma m = 2\int_0^\infty e^{-y^2} y^{2m-1}\,dy$	
	Therefore, $\Gamma m \Gamma n = 4\left\{\int_0^\infty e^{-y^2} y^{2m-1}\,dy\right\}\left\{\int_0^\infty e^{-x^2} x^{2n-1}\,dx\right\}$	1

Q.No.	Solution and Scheme	Marks
	$= 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\, x^{2n-1}\, y^{2m-1}\, dx\, dy$	1
	Transforming the repeated integrals to polar coordinates,	
	$\Gamma m\, \Gamma n = 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2}(r\cos\theta)^{2n-1}(r\sin\theta)^{2m-1}\, r\, d\theta\, dr$	1
	$= 2\int_0^\infty r^{2(m+n)-1} e^{-r^2}\, dr \times 2\int_0^{\pi/2}\sin^{2m-1}\theta\, \cos^{2n-1}\theta\, d\theta$	1
	$= \Gamma(m+n)\, \beta(m,n)$	1
	$\therefore\ \beta(m,n) = \dfrac{\Gamma m\, \Gamma n}{\Gamma(m+n)}$	6
	OR	
02. a.	$I = \int_{y=0}^{1}\int_{x=0}^{\sqrt{1-y^2}} (x^2+y^2)\, dx\, dy$ 	
	$x=0$ & $x=\sqrt{1-y^2}$ $\Rightarrow x^2 = 1-y^2$ $\therefore\ x^2+y^2=1$, is a circle with centre at the origin and radius 1.	1

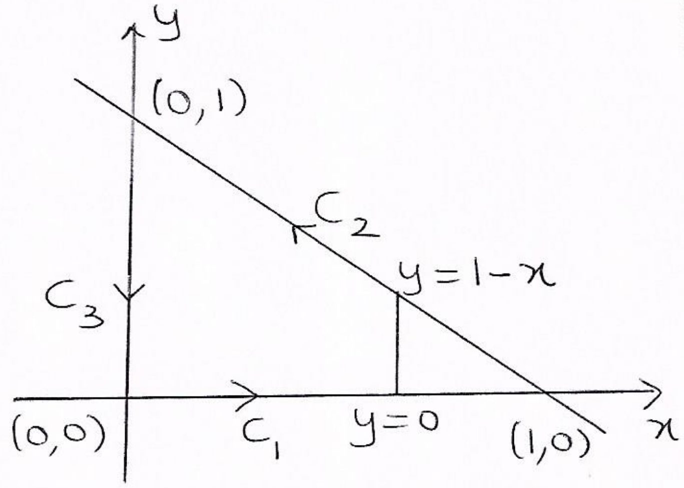
Q.No.	Solution and Scheme	Marks
	Since y varies from 0 to 1, the region of integration is first quadrant of the circle. In polar we have, $x = r\cos\theta$ & $y = r\sin\theta$	1
	$x^2+y^2=r^2$ i.e. $r^2=1^2$ or $r=1$ Also, $x=0$ & $y=0 \Rightarrow r=0$ and hence r varies from 0 to 1.	1
	In the first quadrant θ varies from 0 to $\pi/2$. Also, $dx\,dy = r\,dr\,d\theta$	1
	$I = \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} (r^2\cos^2\theta + r^2\sin^2\theta)\, r\,dr\,d\theta$	1
	$= \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} r^3\,dr\,d\theta = \int_{\theta=0}^{\pi/2} \left(\frac{r^4}{4}\right)_{r=0}^{1} d\theta$	1
	$= \frac{1}{4}\int_{\theta=0}^{\pi/2} 1\,d\theta = \frac{1}{4}\times\frac{\pi}{2}$ $\therefore I = \frac{\pi}{8}$	1
		7

Q.No.	Solution and Scheme	Marks
2 b.	In the given region, x varies from 0 to a and for each x, y varies from 0 to a point on the ellipse i.e. to the point for which $y = b\left(1-x^2/a^2\right)^{1/2}$ 	1
	Hence, the required area is $A = \int_{x=0}^{a} \int_{y=0}^{b(1-x^2/a^2)^{1/2}} dy\, dx$	2
	$= \int_{0}^{a} b\left(1-\frac{x^2}{a^2}\right)^{1/2} dx$	1
	$= \frac{b}{a} \int_{0}^{a} \left(a^2-x^2\right)^{1/2} dx$	1
	$= \frac{b}{a}\left\{\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(x/a\right)\right]_0^a\right\}$	1
	$= \frac{b}{a} \cdot \frac{a^2}{2} \cdot \frac{\pi}{2} = \underline{\underline{\frac{\pi}{4} ab}}$	1
		7
2c.	`from sympy import *`	2
	`x, y, z = symbols('x y z')`	2
	`w1 = integrate(x**2+y**2, (y,0,x), (x,0,1))`	2
	`print(w1)`	6

Q.No.	Solution and Scheme	Marks
	MODULE-2	
03 a.	$\phi = x^2yz + 4xz^2$; $P \equiv (1,-2,1)$	
	$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$	1
	$\nabla\phi = (2xyz + 4z^2)i + (x^2z)j + (x^2y + 8xz)k$	1
	$[\nabla\phi]_{(1,-2,1)} = (-4+4)i + j + (-2+8)k$ $= j + 6k$	2
	The unit vector in the direction of $2i - j - 2k$, $\hat{n} = \frac{2i-j-2k}{\sqrt{4+1+4}} = \frac{2i-j-2k}{3}$	1
	$\therefore$ the required directional derivative is $\nabla\phi.\hat{n} = (j+6k)\cdot\frac{(2i-j-2k)}{3}$	1
	$= \frac{(-1)+(6)(-2)}{3} = \frac{-13}{3}$	1
	Thus, $\boxed{\nabla\phi.\hat{n} = \frac{-13}{3}}$	7
b.	$\vec{F} = x^2y\,\hat{i} + y^2z\,\hat{j} + z^2x\,\hat{k}$	
	curl $\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & y^2z & z^2x \end{vmatrix}$	1
	$= i\left\{\frac{\partial}{\partial y}(z^2x) - \frac{\partial}{\partial z}(y^2z)\right\} - j\left\{\frac{\partial}{\partial x}(z^2x) - \frac{\partial}{\partial z}(x^2y)\right\}$	

Q.No.	Solution and Scheme	Marks
	$+k\left\{\frac{\partial}{\partial x}(y^2z)-\frac{\partial}{\partial y}(x^2y)\right\}$	1
	$= i\{0-y^2\}-j\{z^2-0\}+k\{0-x^2\}$	
	curl $\vec{F} = -y^2\hat{i}-z^2\hat{j}-x^2\hat{k}$	1
	curl (curl $\vec{F}$) $= \nabla\times(\nabla\times\vec{F})$	
	$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y^2 & -z^2 & -x^2 \end{vmatrix}$	1
	$= i\left\{\frac{\partial}{\partial y}(-x^2)-\frac{\partial}{\partial z}(-z^2)\right\}-j\left\{\frac{\partial}{\partial x}(-x^2)-\frac{\partial}{\partial z}(-y^2)\right\}$	
	$+k\left\{\frac{\partial}{\partial x}(-z^2)-\frac{\partial}{\partial y}(-y^2)\right\}$	
	$= i\{0+2z\}-j\{-2x+0\}+k\{0+2y\}$	1
	$= 2z\hat{i}+2x\hat{j}+2y\hat{k}$	
	div (curl $\vec{F}$) $= \nabla\cdot(\nabla\times\vec{F})$	
	$= \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\cdot(2z\hat{i}+2x\hat{j}+2y\hat{k})$	1
	$= \frac{\partial}{\partial x}(2z)+\frac{\partial}{\partial y}(2x)+\frac{\partial}{\partial z}(2y)$	1
	div (curl $\vec{F}$) $= \underline{\underline{0}}$	7
3c.	$\vec{F} = (y^2-z^2+3yz-2x)\hat{i}+(3xz+2xy)\hat{j}$	
	$+(3xy-2xz+2z)\hat{k}$	
	We have, div $\vec{F} = \nabla\cdot\vec{F}$	1

Q.No.	Solution and Scheme	Marks
	$= \frac{\partial}{\partial x}(y^2 - z^2 + 3yz - 2x) + \frac{\partial}{\partial y}(3xz + 2xy)$	1
	$+ \frac{\partial}{\partial z}(3xy - 2xz + 2z)$	
	$= -2 + 2x - 2x + 2 = 0$	1
	$\therefore \text{div}\,\vec{F} = 0 \Rightarrow \vec{F}$ is solenoidal.	
	$\text{Curl}\,\vec{F} = \nabla \times \vec{F}$	
	$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (y^2 - z^2 + 3yz - 2x) & (3xz + 2xy) & (3xy - 2xz + 2z) \end{vmatrix}$	1
	$= \hat{i}[3x - 3x] - \hat{j}[3y - 2z + 2z - 3y]$	1
	$+ \hat{k}[3z + 2y - 2y - 3z]$	
	$= \vec{0}$	1
	$\Rightarrow \vec{F}$ is irrotational.	6
	OR	
4a.	Given, $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$	
	Let, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then	
	$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$	1
	$\therefore \vec{F} \cdot d\vec{r} = (3x^2)\,dx + (2xz - y)\,dy + z\,dz$	
	$\int_C \vec{F} \cdot d\vec{r} = \int_C 3x^2\,dx + \int_C (2xz - y)\,dy + \int_C z\,dz$	1
	The equations of the straight line from $(0,0,0)$ to $(2,1,3)$ are $x = 2t$, $y = t$ & $z = 3t$	1

Q.No.	Solution and Scheme	Marks
	$\Rightarrow dx = 2dt$, $dy = dt$ and $dz = 3dt$ and t varies from $t=0$ to $t=1$	1
	$\therefore \int_C \vec{F}\cdot d\vec{r} = \int_0^1 [3(2t)^2\, 2dt + \{(4t)(3t) - t\}dt + (3t)3\,dt]$	1
	$= \int_0^1 [36t^2 + 8t]dt$	
	$= 36\left[\frac{t^3}{3}\right]_{t=0}^{1} + 8\left[\frac{t^2}{2}\right]_{t=0}^{1}$	1
	$= 12 + 4 = 16$	
	$\therefore$ Workdone $= 16$	1
		7
4b.	Here, $M = 3x - 8y^2$, $N = 4y - 6xy$	1
	$\therefore \frac{\partial M}{\partial y} = -16y$, $\frac{\partial N}{\partial x} = -6y$	
		
	Equation to C_1 : $y=0$ ($\therefore dy=0$) and x varies from $x=0$ to $x=1$	1
	Equation to C_2 : $y = 1-x$ ($\therefore dy = -dx$) and x varies from $x=1$ to $x=0$.	

Q.No.	Solution and Scheme	Marks
	Equation to C_3 : $x=0$ ($\therefore dx=0$) and y varies from $y=1$ to $y=0$.	1
	Now, $\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right) = -6y+16y = 10y$	1
	$\iint_R \left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right) dx\,dy = \int_0^1 \int_0^{1-x} 10y\,dy\,dx$	1
	$= \int_0^1 (5y^2)_0^{1-x}\,dx = \int_0^1 5(1-x)^2\,dx$	1
	$= \left[-\frac{5}{3}(1-x)^2\right]_0^1 = -\frac{5}{3}$	
	$\therefore \int_C (3x-8y^2)\,dx + (4y-6xy)\,dy = -\frac{5}{3}$	1
		7
4c.	`from sympy.vector import *`	
	`from sympy import symbols`	1
	`N = CoordSys3D('N')`	1
	`x,y,z = symbols('x y z')`	1
	`A = N.x**2*N.y+2*N.x*N.x*N.z-4`	1
	`delop = Del()`	1
	`display(delop(A))`	
	`gradA = gradient(A)`	1
	`print(f"\n Gradient of{A} is \n")`	
	`display(gradA)`	
		6

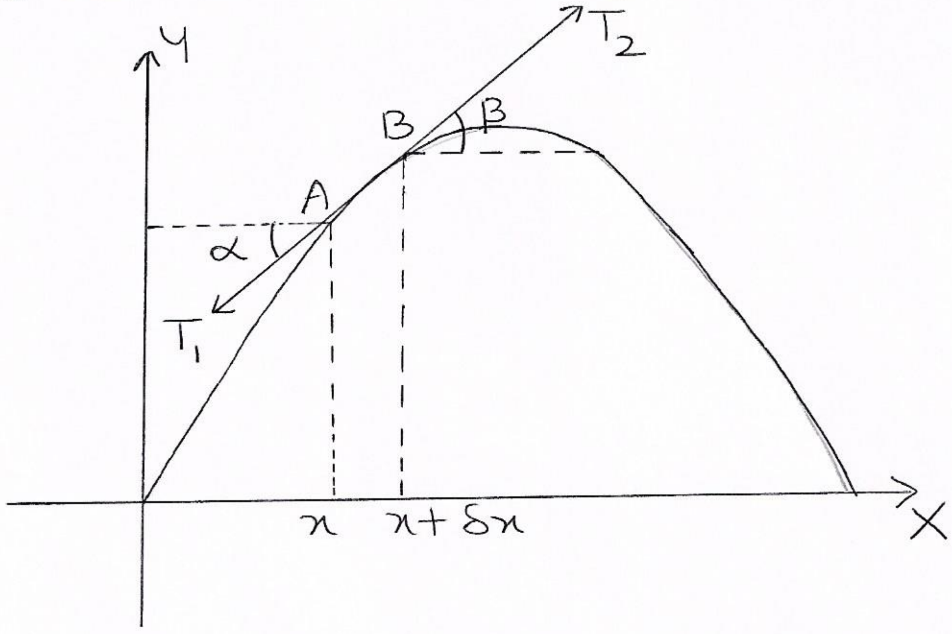
Q.No.	Solution and Scheme	Marks
	Module-3	
05 a	$z = f(x+at) + g(x-at) \longrightarrow (1)$	
	Differentiating eqn (1) partially w.r.t. x and t,	
	$\frac{\partial z}{\partial x} = f'(x+at) + g'(x-at) \longrightarrow (ii)$	1
	and $\frac{\partial z}{\partial t} = af'(x+at) - g'(x-at)a$ (iii)	1
	Also, $\frac{\partial^2 z}{\partial x^2} = f''(x+at) + g''(x-at)$	2
	$\frac{\partial^2 z}{\partial t^2} = a^2 f''(x+at) + a^2 g''(x-at)$	2
	$= a^2 [f''(x+at) + g''(x-at)]$	
	$\Rightarrow \frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$, is the required PDE.	1
		7
b.	Given equation is,	
	$\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y \longrightarrow (1)$	
	Integrating both LHS & RHS of eqn (1) w.r.t. 'x' we get,	
	$\frac{\partial z}{\partial y} = -\cos x \sin y + f(y) \longrightarrow (2)$	1
	Given that $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$	
	substituting in eqn (2) we get	
	$-2\sin y = -\sin y + f(y)$	2
	$f(y) = -\sin y$	

Q.No.	Solution and Scheme	Marks
	Therefore eqn (2) becomes,	
	$\frac{\partial z}{\partial y} = -\cos x \sin y - \sin y \longrightarrow (3)$	1
	Integrating both sides of eqn (3) w.r.t. 'y' we get	
	$z = \cos x \cos y + \cos y + g(x)$	1
	Using the condition $z=0$ when y is odd multiple of $\pi/2$, it gives	1
	$g(x) = 0$	
	$\therefore$ The complete solution is	1
	$\underline{\underline{z = \cos x \cos y + \cos y}}$	7
5c.	$x(y^2-z^2)p + y(z^2-x^2)q - z(x^2-y^2) = 0$	
	$Pp + Qq = R$	
	$P = x(y^2-z^2)$, $Q = y(z^2-x^2)$ & $R = z(x^2-y^2)$	
	The auxilliary equations are,	1
	$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$	
	$\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)} \longrightarrow (1)$	1
	Using x, y, z as multipliers in (1) we get	1
	$x\,dx + y\,dy + z\,dz = 0$	
	$\Rightarrow \int x\,dx + \int y\,dy + \int z\,dz = 0 + k$	
	$\Rightarrow x^2+y^2+z^2 = k \longrightarrow (2)$	1
	Again using $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multipliers in eqn (1) we get,	

Q.No.	Solution and Scheme	Marks
	$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$	
	$\Rightarrow \int \frac{1}{x}dx + \int \frac{1}{y}dy + \int \frac{1}{z}dz = k_1$	
	$\Rightarrow \log x + \log y + \log z = \log b$	
	$\log(xyz) = \log b$	
	$\Rightarrow xyz = b \longrightarrow (3)$	1
	From equations (2) and (3), the general solution is $\phi(u, v) = 0$.	1
	$\phi(x^2+y^2+z^2, xyz) = 0$	6
	OR	
06 a.	Let, $u = x+y+z$ and $v = x^2+y^2+z^2$	
	$\frac{\partial u}{\partial x} = 1 + \frac{\partial z}{\partial x} = 1+p$ $\quad \frac{\partial u}{\partial y} = 1 + \frac{\partial z}{\partial y} = 1+q$	
	$\frac{\partial v}{\partial x} = 2x + 2z\frac{\partial z}{\partial x} = 2(x+zp)$	2
	$\frac{\partial v}{\partial y} = 2y + 2z\frac{\partial z}{\partial y} = 2(y+zq)$	
	Let, $f(u, v) = 0 \longrightarrow (1)$	
	Differentiating eqn (i) partially w.r.t. 'x'	
	$\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} = 0$	
	$\frac{\partial f}{\partial u}(1+p) + \frac{\partial f}{\partial v}2(x+zp) = 0 \longrightarrow (2)$	1
	Differentiating eqn (2) partially w.r.t. 'y'	
	$\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} = 0$	

Q.No.	Solution and Scheme	Marks
	$\frac{\partial f}{\partial u}(1+q) + \frac{\partial f}{\partial v} 2(y+zq) = 0 \longrightarrow (3)$	1
	Eliminating $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ from eqn (2) and (3), we get	
	$\begin{vmatrix} 1+p & 2(x+zp) \\ 1+q & 2(y+zq) \end{vmatrix} = 0$	2
	$2(y+zq)(1+p) - (x+zp)2(1+q) = 0$	
	$(y+zq)(1+p) - (x+zp)(1+q) = 0$	
	$(y+yp+zq+zpq) - (x+xq+zp+zpq) = 0$	
	$\Rightarrow (y-z)p + (z-x)q = x-y$, is the required PDE.	1
		7
b	The question must be:	
	$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$	
	Let us suppose that z is a function of x only.	
	$D^2 z + 3Dz - 4z = 0, \quad D = \frac{d}{dx}$	
	$(D^2 + 3D - 4)z = 0 \quad D^2 = \frac{d^2}{dx^2}$	
	Auxilliary equation is,	
	$D^2 + 3D - 4 = 0$	
	$D^2 + 4D - D - 4 = 0$	
	$D(D+4) - 1(D+4) = 0$	
	$(D+4)(D-1) = 0$	
	$D = -4, 1$	

Q.No.	Solution and Scheme	Marks
	∴ Solution of ODE is, $z = c_1 e^{-4x} + c_2 e^{x}$	1
	Solution of PDE is obtained by replacing c_1 by $f(y)$ and c_2 by $g(y)$ $z = f(y)e^{-4x} + g(y)e^{x} \longrightarrow (1)$	1
	put $x=0$ & $z=1$ in eqn (1) $1 = f(y) + g(y) \longrightarrow (2)$	1
	Differentiating eqn (1) partially w.r.t 'x', $\frac{\partial z}{\partial x} = -4f(y)e^{-4x} + g(y)e^{x}$ put, $\frac{\partial z}{\partial x} = y$ & $x=0$ in above eqn $y = -4f(y) + g(y) \longrightarrow (3)$	1
	Add & subtract equations (2) & (3), $1+y = -3f(y) + 2g(y)$ $1-y = 5f(y) \Rightarrow f(y) = \frac{1-y}{5}$ ∴ $1+y = -3\left(\frac{1-y}{5}\right) + 2g(y)$ $g(y) = \frac{8+2y}{10} = \frac{4+y}{5}$	2
	∴ Eqn (1) becomes, $z = \left(\frac{1-y}{5}\right)e^{-4x} + \left(\frac{4+y}{5}\right)e^{x}$ $z = \frac{1}{5}\left\{(1-y)e^{-4x} + (4+y)e^{x}\right\}$	1
		7

Q.No.	Solution and Scheme	Marks
6c.	$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$ Consider a flexible string tightly stretched between two fixed points at a distance 'l' apart. Let ρ be the mass per unit length of the string. Assume the following (i) The tension T of the string is same throughout, (ii) The effect of gravity can be ignored due to large tension T. (iii) The motion of the string is in small transeverse vibrations.	1
	Let us consider the forces acting on a small element AB of length δx.  Let T_1 and T_2 be the tensions at the points A and B. Since there is no motion in the horizontal direction, the horizontal components T_1 & T_2	1

Q.No.	Solution and Scheme	Marks
	must cancel each other. $\therefore T_1 \cos\alpha = T_2 \cos\beta = T \dashrightarrow (i)$ where α & β are the angles made by T_1 and T_2 with the horizontal, Vertical components of tension are $-T_1 \sin\alpha$ and $T_2 \sin\beta$, where the negative sign is used because T_1 is directed downwards. Hence the resultant force acting vertically upwards is $T_2 \sin\beta - T_1 \sin\alpha$. Applying Newton's second law of motion Force = mass x acceleration	
	$T_2 \sin\beta - T_1 \sin\alpha = (\rho \delta x) \dfrac{\partial^2 u}{\partial t^2}$	1
	$\dfrac{T_2}{T} \sin\beta - \dfrac{T_1}{T} \sin\alpha = \dfrac{\rho}{T} \delta x \dfrac{\partial^2 u}{\partial t^2}$ (Dividing by T) But from (i), $\dfrac{T_1}{T} = \dfrac{1}{\cos\alpha}$; $\dfrac{T_2}{T} = \dfrac{1}{\cos\beta}$ $\therefore \dfrac{\sin\beta}{\cos\beta} - \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\rho}{T} \delta x \dfrac{\partial^2 u}{\partial t^2}$	
	$\tan\beta - \tan\alpha = \dfrac{\rho}{T} \delta x \dfrac{\partial^2 u}{\partial t^2}$	1
	But, $\tan\beta$ and $\tan\alpha$ represent the slopes at $B(x+\delta x)$ and $A(x)$ respectively. $\therefore \tan\beta = \left(\dfrac{\partial u}{\partial x}\right)_{x+\delta x}$ and $\tan\alpha = \left(\dfrac{\partial u}{\partial x}\right)_x$	

Q.No.	Solution and Scheme	Marks
	Now, (ii) becomes $\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_{x} = \frac{\rho}{T}\,\delta x\left(\frac{\partial^2 u}{\partial t^2}\right)$	
	Dividing by δx and taking limit as $\delta x \to 0$, we have $\lim_{\delta x\to 0}\frac{\left(\frac{\partial u}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial u}{\partial x}\right)_{x}}{\delta x} = \frac{\rho}{T}\frac{\partial^2 u}{\partial t^2}$	1
	But, the LHS is derivative of $\frac{\partial u}{\partial x}$ w.r.t. 'x' treating 't' as a constant. $\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{T}\frac{\partial^2 u}{\partial t^2}$ or $\frac{\partial^2 u}{\partial t^2} = \frac{T}{\rho}\frac{\partial^2 u}{\partial x^2}$	
	Denoting T/ρ by c^2 we get $\boxed{\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}}$	1
	This is the wave equation in one-dimension	6
	Module - 4	
7a.	Let, $f(x) = x^3 - 9x + 1$	
	$f(2) = -9 < 0$, $f(3) = +1 > 0$	
	The real root lies in $(2,3)$	
	1st approximation: $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$	2
	$x_1 = 2.9000$	1

Q.No.	Solution and Scheme	Marks
	2nd approximation:	
	$f(2.9) = (2.9)^3 - 9(2.9)+1 = -0.7110$	
	$a = 2.9, \; b = 3$	1
	$\therefore x_2 = 2.9416$	
	3rd approximation:	
	$f(2.9416) = -0.0207.$	1
	$a = 2.9416, \; b = 3$	
	$x_3 = 2.9428$	
	4th approximation: $f(2.9428) = -0.0207$	
	$a = 2.9428, \; b = 3$	1
	$x_4 = 2.9428$	
	$\therefore$ The real roots for the given equation	1
	is $x = 2.9428$	7
b.	$h = 6-4 = 2$	1
	$r = \frac{x-x_0}{h} = \frac{5-4}{2} = 0.5$	
	Forward difference table is	

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$x_0 = 4$	$y_0 = 1$			
		2		
$x_1 = 6$	$y_1 = 3$		3	
		5		0
$x_2 = 8$	$y_2 = 8$		3	
		8		
$x_3 = 10$	$y_3 = 16$			

Marks: 3

Q.No.	Solution and Scheme	Marks
	Newton's forward interpolation formula is,	
	$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + ----$	1
	$= 1 + (0.5)2 + \frac{(0.5)(0.5-1)}{2}(3) + \frac{(0.5)(0.5-1)(0.5-2)}{6} \times 0$	1
	$= 1 + 1 - 0.375$	1
	$y_{(5)} = \underline{\underline{1.6250}}$	7
7c.	Let, $f(\theta) = \sqrt{\sin\theta}$. $a = 0$ & $b = \pi/2$	
	7 ordinates = 6 equal parts $\Rightarrow n = 6$	1
	$h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = \pi/12$	

θ	$\theta_0 = 0$	$\theta_1 = \pi/12$	$\theta_2 = \pi/6$	$\theta_3 = \pi/4$
$f(\theta)$	$y_0 = 0$	$y_1 = 0.5087$	$y_2 = 0.7071$	$y_3 = 0.8409$

$\theta_4 = \pi/3$	$\theta_5 = 5\pi/12$	$\theta_6 = \pi/2$
$y_4 = 0.9306$	$y_5 = 0.9828$	$y_6 = 1$

2

Simpson's $(1/3)^{rd}$ Rule is,

$$\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \frac{h}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right] \quad 1$$

$$= \frac{\pi}{36}\left[(0+1) + 2(0.7071+0.9306) + 4(0.5087+0.8409+0.9828)\right] \quad 1$$

$$= \pi/36\,[1 + 3.2754 + 9.6176]$$

Q.No.	Solution and Scheme	Marks
	$\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \frac{\pi}{36}[13.893]$	1
	$\simeq 1.2124$	6
	OR	
8 a.	Let, $f(x) = \cos x - xe^x$	1
	$f'(x) = -\sin x - xe^x - e^x$	
	Newton Raphson formula is, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	2
	1st approximation: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$	1
	$x_1 \simeq 0.51803$	
	2nd approximation: $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$	1
	$x_2 = 0.51776$	
	3rd approximation: $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$	1
	$x_3 = 0.5177$	
	Thus, the root correct to four decimal places is $x = 0.5177$.	1
		7
b,	Newton's general interpolation formula is, $f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3) + \text{----}$	1

Type text here

Q.No.	Solution and Scheme	Marks
	The divided difference table is	

x	$f(x)$	1st Divided Difference
$x_0=2$	$f(x_0)=10$	$f(x_0,x_1)=\frac{96-10}{4-2}=43$
$x_1=4$	$f(x_1)=96$	$f(x_1,x_2)=\frac{196-96}{5-4}=100$
$x_2=5$	$f(x_2)=196$	$f(x_2,x_3)=\frac{350-196}{6-5}=154$
$x_3=6$	$f(x_3)=350$	$f(x_3,x_4)=\frac{868-350}{8-6}=259$
$x_4=8$	$f(x_4)=868$	$f(x_4,x_5)=\frac{1746-868}{10-8}=439$
$x_5=10$	$f(x_5)=1746$	

2

2nd Divided Differences	3rd Divided Differences
$f(x_0,x_1,x_2)=\frac{100-43}{5-2}=19$	$f(x_0,x_1,x_2,x_3)=\frac{27-19}{6-2}=2$
$f(x_1,x_2,x_3)=\frac{154-100}{6-4}=27$	$f(x_1,x_2,x_3,x_4)=\frac{35-27}{8-4}=2$
$f(x_2,x_3,x_4)=\frac{259-154}{8-5}=35$	$f(x_2,x_3,x_4,x_5)=\frac{45-35}{10-5}=2$
$f(x_3,x_4,x_5)=\frac{439-259}{10-6}=45$	

2

Q.No.	Solution and Scheme	Marks

$$f(x) = 10+(x-2)43+(x-2)(x-4)19 + (x-2)(x-4)(x-5)2$$ 1

$$= 10+(x-2)\{43+(19x-76)+(x^2-9x+20)2\}$$

$$= 10+(x-2)(2x^2+x+7)$$ 1

$$\therefore f(x) = 2x^3-3x^2+5x-4$$ 7

8c. Let, $f(x) = \sin x - \log x + e^x$

$a=0.2$ and $b=1.4$, $n=6$, $h=\dfrac{b-a}{n}$ 1

$\therefore h = 0.2$

x	$x_0 = 0.2$	$x_1 = 0.4$	$x_2 = 0.6$
$f(x)$	$y_0 = 3.0295$	$y_1 = 2.7975$	$y_2 = 2.8976$

2

$x_3 = 0.85$	$x_4 = 1.0$	$x_5 = 1.2$	$x_6 = 1.4$
$y_3 = 3.1660$	$y_4 = 3.5597$	$y_5 = 4.0698$	$y_6 = 4.4042$

Simpson's $(3/8)^{th}$ Rule is,

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+2(y_3+y_6+\cdots+y_{n-3}) + 3(y_1+y_2+\cdots+y_{n-1})\right]$$ 1

$$\int_{0.2}^{1.4} (\sin x - \log x + e^x)\,dx = \frac{3h}{8}\left[(y_0+y_6)+2(y_3) + 3(y_1+y_2+y_4+y_5)\right]$$ 1

$$= \frac{3(0.1)}{8}\left[(3.0295+4.4042)+2(3.166) + 3(2.7975+2.8976+3.5597+4.4042)\right]$$

$$\int_a^b f(x)\,dx = 4.053$$ 1

6

Q.No.	Solution and Scheme	Marks
	Module-5	
9a.	$y' = x - y^2, \quad x_0 = 0, \quad y_0 = 1$	
	$y'(0) = -1$	
	$y'' = 1 - 2yy' \Rightarrow y''(0) = 3$	3
	$y''' = -2yy'' - 2y'y' \Rightarrow y'''(0) = -8$	
	$y'''' = -2[yy''' + y'y'' + 2y'y'']$	
	$= -2[yy''' + 3y'y'']$	
	$y''''(0) = 34$	
	Taylor's series is given by,	
	$y(x) = y_0 + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y''''(x_0) + \cdots$	1
	$y(x) = y_0 + x\,y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y''''(0) + \cdots$	1
	$y(0.1) = 1 + x(-1) + \frac{x^2}{2} \times 3 + \frac{x^3}{6} \times (-8) + \frac{x^4}{24}(34)$	
	$y(0.1) = 1 - x + \frac{3}{2}x^2 - \frac{4}{3}x^3 + \frac{17}{12}x^4$	1
	$= 1 - (0.1) + \frac{3}{2}(0.1)^2 - \frac{4}{3}(0.1)^3 + \frac{17}{12}(0.1)^4$	
	$= 1 - 0.1 + 0.015 - 0.001333 + 0.000141667$	
	$y(0.1) = 0.91381$	1
		7

Q.No.	Solution and Scheme	Marks
b.	$y_0 = 1,\ x_0 = 0,\ h = 0.1$	
	$f(x_0, y_0) = 1$	
	Euler's formula ; $y_1^{(0)} = y_0 + hf(x_0, y_0)$	
	$y_1^{(0)} = 1.1$	
	Modified Euler's formula,	1
	$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$	1
	$y_1^{(1)} = 1.0917$	1
	$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$	
	$= 1 + \frac{0.1}{2}\left[1 + \frac{1.0917 - 0.1}{1.0917 + 0.1}\right]$	
	$= 1 + 0.05\left[1.832172527\right]$	2
	$= 1.0916$	
	$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$	2
	$\simeq 1.0916$	
	$\therefore y(0.1) = 1.0916$	7
9c.		

x	y	$y' = xy + y^2$
$x_0 = 0$	$y_0 = 1$	$y_0' = 1$
$x_1 = 0.1$	$y_1 = 1.1169$	$y_1' = 1.3592$
$x_2 = 0.2$	$y_2 = 1.2773$	$y_2' = 1.8869$
$x_3 = 0.3$	$y_3 = 1.5049$	$y_3' = 2.7162$

Marks: 1

Q.No.	Solution and Scheme	Marks
	Milne's Predictor formula is,	
	$y_4^{(p)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$	
	$= 1 + \frac{4(0.1)}{3}[2(1.3592) - 1.8869 + 2(2.7162)]$	
	$y_4^{(p)} = 1.835186667$	1
	$\simeq 1.8352$	
	$y_4' = x_4 y_4 + y_4^2 = (0.4)(1.8352) + (1.8352)^2$	
	$y_4' = 4.10203904 \simeq 4.1020$	
	Milne's corrector formula is,	
	$y_4^{(c)} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$	
	$= 1.2773 + \frac{(0.1)}{3}[1.8869 + 4(2.7162) + 4.1020]$	1
	$y_4^{(c)} = 1.83909$	
	$y_4' = x_4 y_4 + (y_4)^2 = 4.11789$	1
	$y_4^{(c)} = 1.83962$	
	$y_4' = 4.1200$	1
	$y_4^{(c)} = 1.83969$	1
	$\therefore y(0.4) \simeq 1.8396$	
		6

Q.No.	Solution and Scheme	Marks
	OR	
10 a.	$\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x} = \frac{1-xy}{x^2}$; $x_0 = 1$, $y_0 = 1$	
	$x_1 = x_0 + h \Rightarrow 1.1 = 1 + h \Rightarrow h = 0.1$	
	Euler's formula is,	
	$y_1^{(0)} = y_0 + hf(x_0, y_0)$	
	$= 1 + (0.1) f(1, 1)$	
	$= 1 + (0.1)\left(\frac{1-1}{1}\right) = 1 + 0 = 1$	1
	$\therefore y_1^{(0)} = 1$	
	Modified Euler's formula is,	1
	$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$	
	$= 1 + \frac{0.1}{2}\left[f(1, 1) + f(1.1, 1)\right]$	
	$= 1 + (0.05)\left[0 + \frac{1-(1.1)(1)}{(1.1)^2}\right]$	2
	$y_1^{(1)} = 0.9959$	
	$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$	1
	$y_1^{(2)} \simeq 0.99605$	
	$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$	1
	$\simeq 0.99605$	
	Thus, $y(1.1) \simeq 0.99605$	1
		7

Q.No.	Solution and Scheme	Marks
b.	Given, $y' = \frac{y^2-x^2}{y^2+x^2}$, $x_0=0$, $y_0=1$	
	$x_1 = x_0+h \Rightarrow 0.2 = 0+h \Rightarrow h=0.2$	
	$k_1 = hf(x_0,y_0) = (0.2)f(0,1)$	
	$= 0.2\left[\frac{1-0}{1+0}\right] = 0.2$	1
	$k_1 = 0.2$	
	$k_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$	
	$= (0.2)f\left(0+\frac{0.2}{2}, 1+\frac{0.2}{2}\right)$	
	$= (0.2)f(0.1, 1.1) = (0.2)\left[\frac{(1.1)^2-(0.1)^2}{(1.1)^2+(0.1)^2}\right]$	
	$= (0.2)(0.9836065574)$	1
	$k_2 = 0.1967$	
	$k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$	
	$= (0.2)f(0.1, 1.09835)$	
	$= 0.2\left[\frac{(1.09835)^2-(0.1)^2}{(1.09835)^2+(0.1)^2}\right]$	1
	$k_3 = 0.1967$	
	$k_4 = hf(x_0+h, y_0+k_3) = (0.2)f(0.2, 1.1967)$	1
	$k_4 = 0.1891$	
	$y(x_0+h) = y_0+\frac{1}{6}(k_1+2k_2+k_4+2k_3)$	1
	$y(x_1) = y(0.2) = 1+\frac{1}{6}(0.2+0.3934+0.3934+0.1891)$	
	$\underline{\underline{y(0.2) = 1.1959}}$	2
		7

Dr. Meenal M Kaliwal, Assistant Prof., KLS VDIT, Haliyal

Q.No.	Solution and Scheme	Marks

c.	Consider $h=0.2$, $y(1)=2$, $y'=2x+y$
	from sympy import *
	import numpy as np
	def RungeKutta(g, x0, h, y0, xn):
	    x, y = symbols('x,y')
	    f = lambdify([x,y], g)
	    xt = x0+h
	    Y = [y0]
	    while xt <= xn:
	        k1 = h*f(x0, y0)
	        k2 = h*f(x0+h/2, y0+k1/2)
	        k3 = h*f(x0+h/2, y0+k2/2)
	        k4 = h*f(x0+h, y0+k3)
	        y1 = y0+(1/6)*(k1+2*k2+2*k3+k4)
	        Y.append(y1)
	        x0 = xt
	        y0 = y1
	        xt = xt+h
	    return np.round(Y,2)
	RungeKutta('2*x+y', 1, 0.2, 2, 2)
		2
		1
		2
		1
		6

	Faculty: Dr. Meenal M. Kaliwal (Mmk) HOD : Dr. R. S. Munnolli Dean Academics : HOD Basic Sciences & Humanities KLS VDIT. HALIYAL-581329	