Model Question Paper-II with effect from 2022

USN

Fourth Semester B.E Degree Examination

Complex Analysis, Probability & Statistical Methods

All branches Except CS & ME Engg. Allied branches-21MAT41

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Q.No.		**Question**	**M**	**L**	**CO**
		Module -1			
01	a	Define Analytic function and hence derive C-R equations in Polar form.	06	L2	CO1
	b	Show that $w = f(z) = z + e^{z}$ is analytic and hence find its derivative.	07	L3	CO1
	c	Evaluate $\int_{(0,3)}^{(2,4)} (2y+x^2)\,dx+(3y-x)\,dy$ along with the parabola $x = 2t\,, y = t^2 + 3$.	07	L2	CO1
		OR			
02	a	Find analytic function $f(z) = u + iv$ where $u - v = (x - y)(x^2 + 4xy + y^2)$ by the Milne-Thomson method.	06	L3	CO1
	b	State and prove Cauchy's integral formula.	07	L3	CO1
	c	Evaluate $\int_C \frac{e^{2z}}{(z+1)\ (z+2)}dz$, where C is a circle $\lvert z\rvert = 3$.	07	L2	CO1
		Module-2			
03	a	Show that $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$	06	L2	CO2
	b	If α *and* β are two distinct roots of $J_n(x)=0$, then prove that $$\int_0^1 x J_n(\alpha x) J_n(\beta x)\,dx = 0.$$	07	L2	CO2
	c	Show that $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$	07	L2	CO2
		OR			
4	a	Show that $J_{-\frac{1}{2}}(x) = J_{\frac{1}{2}}(x)cotx$	06	L2	CO2
	b	Find the series solution of the Legendre's equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$, leading to Legendre polynomial of order n.	07	L2	CO2
	c	Express $4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials	07	L2	CO2
		Module-3			

<table>
<tr><td>5</td><td>a</td><td>The following table gives the heights of father(x) and sons(y). Calculate the Karl Pearson's coefficient of correlation.
<table>
<tr><td>$x:$</td><td>65</td><td>66</td><td>67</td><td>68</td><td>68</td><td>69</td><td>70</td><td>72</td></tr>
<tr><td>$y:$</td><td>67</td><td>68</td><td>65</td><td>68</td><td>72</td><td>72</td><td>69</td><td>71</td></tr>
</table></td><td>06</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b</td><td>Fit a straight line $y = ax + b$ for the data
<table>
<tr><td>$x:$</td><td>12</td><td>15</td><td>21</td><td>25</td></tr>
<tr><td>$y:$</td><td>50</td><td>70</td><td>100</td><td>120</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c</td><td>Using the method of least square, fit a curve $y = ax^b$ for the following data
<table>
<tr><td>$x:$</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>$y:$</td><td>2.98</td><td>4.26</td><td>5.21</td><td>6.1</td><td>6.8</td><td>7.5</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>6</td><td>a</td><td>The scores for 9 students in Physics (x) and Mathematics (y) are as follows
<table>
<tr><td>$x:$</td><td>35</td><td>23</td><td>47</td><td>17</td><td>10</td><td>43</td><td>9</td><td>6</td><td>28</td></tr>
<tr><td>$y:$</td><td>30</td><td>33</td><td>45</td><td>23</td><td>8</td><td>49</td><td>12</td><td>4</td><td>31</td></tr>
</table>
Compute the Ranks and Rank correlation.</td><td>06</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b</td><td>Compute the means $\bar{x}, \bar{y}$ and the correlation coefficient r from the given regression lines $4x - 5y + 33 = 0, 20x - 9y = 107$</td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c</td><td>Fit a second-degree polynomial $y = ax^2 + bx + c$ for the data.
<table>
<tr><td>$x:$</td><td>20</td><td>60</td><td>100</td><td>140</td><td>180</td><td>220</td></tr>
<tr><td>$y:$</td><td>0.18</td><td>0.37</td><td>0.35</td><td>0.78</td><td>0.56</td><td>0.75</td></tr>
</table></td><td>07</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">Module-4</td></tr>
<tr><td>7</td><td>a</td><td>A random variable X has the following probability function:
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>$P(X)$</td><td>0</td><td>k</td><td>$2k$</td><td>$2k$</td><td>$3k$</td><td>k^2</td><td>$2k^2$</td><td>$7k^2 + k$</td></tr>
</table>
Find k. Also find $P(X < 6), P(X \geq 6)$, $P(3 < X \leq 6)$.</td><td>06</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>Derive the mean and variance of Poisson distribution.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>The number of telephonic lines busy at an instant line is a binomial variate with a probability 0.1. If 10 lines are chosen at random, what is the probability that
(i) No line is busy
(ii) All lines are busy
(iii) At least one line is busy</td><td>07</td><td>L3</td><td>CO4</td></tr>
</table>

<table>
<tr><td colspan="6">OR</td></tr>
<tr><td>8</td><td>a</td><td>The diameter of a electric cable is assumed to be a continuous random variable
with p.d.f $f(x) = \begin{cases} kx(1-x), & 0 \le x < 1 \\ 0, & else\ where \end{cases}$
Find the value of k and also obtain the mean and variance of the variable</td><td>06</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of drivers with
i) No accident in a year
ii) More than three accidents in a year.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>If the life time of a certain types electric bulbs of a particular brand was distributed normally with an average life of 2000 hours and S.D.60 hours. If a firm purchase 2500 bulbs, find the number of bulbs that are likely to last for
(i) More than 2100 hours
(ii) Less than 1950 hours
(iii) Between 1900 and 2100 hours.</td><td>07</td><td>L2</td><td>CO4</td></tr>
<tr><td colspan="6">**Module-5**</td></tr>
<tr><td>9</td><td>a</td><td>The joint distribution of two random variables X and Y is as follows.
<table>
<tr><td>Y
X</td><td>1</td><td>3</td><td>6</td></tr>
<tr><td>1</td><td>$\frac{1}{9}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{18}$</td></tr>
<tr><td>3</td><td>$\frac{1}{6}$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{12}$</td></tr>
<tr><td>6</td><td>$\frac{1}{18}$</td><td>$\frac{1}{12}$</td><td>$\frac{1}{36}$</td></tr>
</table>
Compute the following.
i) Marginal distributions of X and Y
ii) Are X and Y stochastically independent?</td><td>06</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>A set of five similar coins is tossed 320 times and the result is
<table>
<tr><td>No. of heads</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>Frequency</td><td>6</td><td>27</td><td>72</td><td>112</td><td>71</td><td>32</td></tr>
</table>
Test the hypothesis that the data follows a binomial distribution at 5% significance level</td><td>07</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? (*Note* : $t_{0.05}$ for 11 d.f. is 2.201).</td><td>07</td><td>L3</td><td>CO5</td></tr>
</table>

<table>
<tr><td colspan="6">OR</td></tr>
<tr><td>10</td><td>a</td><td>Determine (i) Marginal distributions (ii) Correlation coefficient between the variables X and Y, from the joint probability distribution given by:
<table>
<tr><td>Y / X</td><td>-2</td><td>-1</td><td>4</td><td>5</td></tr>
<tr><td>1</td><td>0.1</td><td>0.2</td><td>0</td><td>0.3</td></tr>
<tr><td>2</td><td>0.2</td><td>0.1</td><td>0.1</td><td>0</td></tr>
</table>
</td><td>06</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>The 9 item of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? ($t_{0.05}$ =2.306 for 8 d.f.)</td><td>07</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>The theory predicts the proportion of beans in the four groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans, the number in the Four groups were 882, 313, 287 and 118. The goodness of fit χ^2 value of above data is approximately equal to?</td><td>07</td><td>L3</td><td>CO5</td></tr>
</table>

Bloom's Taxonomy Levels	**Lower-order thinking skills**		
	Remembering (knowledge):L_1	Understanding (Comprehension): L_2	Applying (Application):L_3
	Higher-order thinking skills		
	Analyzing (Analysis):L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6

①

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Model qp Solution and Scheme for award of marks

AY: 2022-23

Department: Mathematics
Subject with Sub. Complex Analysis, Probability and Statistical Methods(21MAT41)
Semester/Division: IV/EC, CV, EE
Name of Faculty: Dr. Satish P. Hande

Q.No.	Solution and Scheme	Marks
1	a) Analytic function	
	A complex valued function $f(z)$ is said to be analytic at a point $z=z_0$ if it is differentiable at z_0 and also in the neighborhood of z_0	1M
	If (r,θ) be the polar coordinates of a pt. whose cartesian coordinates are (x,y)	1M
	$z=re^{i\theta}$	
	$f(z)=u+iv=f(re^{i\theta}) \longrightarrow ①$	
	diff ① partially w.r.t r,θ	
	$u_r+iv_r=f'(re^{i\theta})\,e^{i\theta}$	
	$u_\theta+iv_\theta=f'(re^{i\theta})\times ire^{i\theta}=ir(u_r+iv_r)$	2M
	Equating the real & imaginary parts	1M
	$\boxed{u_r=\frac{1}{r}v_\theta \text{ and } v_r=\frac{-1}{r}u_\theta}$	$\boxed{6M}$
	b) Given $f(z)=z+e^z$ put $z=x+iy$	1M
	$u+iv=(x+iy)+e^{x+iy}$	
	$=(x+iy)+e^x[\cos y+i\sin y]$	
	$u+iv=(x+e^x\cos y)+i(y+e^x\sin y)$	1M
	$\therefore u=x+e^x\cos y,\quad v=y+e^x\sin y$	1M
	$u_x=1+e^x\cos y\ ;\quad v_x=e^x\sin y$	
	$u_y=-e^x\sin y\ ;\quad v_y=1+e^x\cos y$	2M
	$u_x=v_y$ and $u_y=-v_x$	

Q.No.	Solution and Scheme	Marks
	$\therefore f(z)$ is analytic	
	$\frac{dw}{dz} = u_x + iv_x = 1+e^x\cos y+i(e^x\sin y)$	
	put $x=2,\ y=0$	2m
	$\boxed{\frac{dw}{dz} = 1+e^2}$	$\boxed{7M}$
	c) If $x=0,\ 2t=0 \Rightarrow t=0$ $\quad dx=2dt$	1M
	If $x=2,\ 2t=2 \Rightarrow t=1$	
	t varies from 0 to 1	2m
	$I = \int_{(0,3)}^{(2,4)} (2y+x^2)\,dx + (3x-y)\,dy$	
	$= \int_{t=0}^{1} \{2[t^2+3]+4t^2\}2dt + \{6t-(t^2+3)\}2t\,dt$	1M
	$= \int_{t=0}^{1} (24t^2-2t^3-6t+12)\,dt$	2m
	$= 24\left(\frac{t^3}{3}\right)_0^1 - 2\left[\frac{t^4}{4}\right]_0^1 - 6\left(\frac{t^2}{2}\right)_0^1 + 12(t)_0^1$	1M
	$\boxed{I = 33/2}$	$\boxed{7M}$
2	a) $u-v = (x-y)(x^2+4xy+y^2)$	1M
	$f(z) = u+iv \quad if(z) = iu-v$	
	$(1+i)f(z) = (u-v)+i(u+v)$	1M
	$F(z) = U+iV$	
	$F'(z) = U_x+iV_x$	
	$= U_x - iU_y$	1M
	$U_x = (x-y)(2x+4y)+(x^2+4xy+y^2)$	
	$U_x{}_{(2,0)} = 3z^2$	1M
	$U_y = (x-y)(4x+2y)+(x^2+4xy+y^2)(-1)$	

$U_y(z,0) = 3z^2$

$F'(z) = 3z^2(1-i)$

$F(z) = (1-i)z^3 + C$

$(1+i)f(z) = (1-i)z^3 + C$

$f(z) = \frac{(1-i)}{(1+i)} z^3 + \frac{c}{1+i}$

$\boxed{f(z) = -iz^3 + c_1}$ 2M

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b) Statement: If $f(z)$ is analytic inside and on a simple closed curve 'C' and 'a' is any point ~~whic~~ within C then

$$\int_C \frac{f(z)}{z-a}\,dz = 2\pi i f(a)$$ 1M

Proof: Consider a curve 'C'. Let 'a' be any pt. within 'C' draw a circle C_1 with center at 'a' and radius r such that C_1 lies entirely within 'C'. $\frac{f(z)}{z-a}$ is analytic 1M

By the Consequence of cauchy's theorem 1M

$$\int_C \frac{f(z)}{z-a}\,dz = \int_{C_1} \frac{f(z)}{z-a}\,dz$$

$C_1 : |z-a| = r \Rightarrow z-a = re^{i\theta} \Rightarrow z = a+re^{i\theta}$

$dz = ire^{i\theta}d\theta \quad 0 \le \theta \le 2\pi$ 2M

$$\int_C \frac{f(z)}{z-a}\,dz = \int_{\theta=0}^{2\pi} \frac{f(a+re^{i\theta})}{re^{i\theta}}\, ire^{i\theta}d\theta$$

as $r \to 0$ $f(a+re^{i\theta}) \to f(a)$

$= 2\pi i f(a)$ 2M

$\therefore \boxed{f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}\,dz}$

Q.No.	Solution and Scheme	Marks
2.	c) $f(z)=e^{2z}$	
	$c:\lvert z\rvert=3$ is a circle with center at origin	1M
	and radius '3'	
	$\therefore$ $a=-1$, $a=-2$ lies within C.	1M
	$\int_C \frac{e^{2z}}{(z+1)(z+2)}dz = \int_C \frac{e^{2z}}{(z+1)}dz - \int_C \frac{e^{2z}}{(z+2)}dz$	2M
	$= 2\pi i f(-1) - 2\pi i f(-2)$	2M
	$= 2\pi i[e^{-2}-e^{-4}]$ //	1M
3.	a) By the def^n	
	$J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{\Gamma(n+r+1)\, r!} \longrightarrow$ ①	1M
	put $n=-1/2$ in eq^n ①	1M
	$J_{-1/2}(x) = \left(x/2\right)^{-1/2} \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{2r} \frac{1}{\Gamma(r+1/2)\, r!}$	
	$= \sqrt{\frac{2}{x}}\left[\frac{1}{\sqrt{\pi}\,\Gamma(1/2)} - \left(\frac{x}{2}\right)^2 \frac{1}{\Gamma(3/2)\cdot 1!} + \left(\frac{x}{2}\right)^4 \frac{1}{\Gamma(5/2)\, 2!} - \cdots\right]$	2M
	$= \sqrt{\frac{2}{x}}\left[\frac{1}{\sqrt{\pi}} - \frac{x^2}{4}\frac{2}{\sqrt{\pi}} + \frac{x^4}{16}\frac{4}{3\sqrt{\pi}\cdot 2} - - - -\right]$	
	$= \sqrt{\frac{2}{\pi x}}\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right]$	
	$\boxed{J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x}$	2M
	b) We know that $J_n(\lambda x)$ is a soln. of the eq^{ns}	
	$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (\lambda^2 x^2 - n^2)y = 0 \longrightarrow$ ①	
	If $u=J_n(\alpha x)$ and $v=J_n(\beta x)$ then from ①	
	$x^2u'' + xu' + (\alpha^2 x^2 - n^2)u = 0 \longrightarrow$ ②	
	$x^2v'' + xv' + (\beta^2 x^2 - n^2)v = 0 \longrightarrow$ ③	1M

Q.No. **Solution and Scheme** **Marks**

Multiplying ② by $\frac{v}{x}$ and ③ by $\frac{u}{x}$ and substracting

$$\frac{d}{dx}\{x(vu'-uv')\} = (\beta^2-\alpha^2)\, x\, u v$$

Integrating both the sides w.r.t x betwⁿ 0 to 1. 1M

$$(vu'-uv')_{x=1} - 0 = (\beta^2-\alpha^2)\int_0^1 x u v\, dx \longrightarrow ④$$

Since $u = J_n(\alpha x)$, $v = J_n(\beta x)$

$u' = \alpha J_n'(\alpha x)$, $v' = \beta J_n'(\beta x)$ 2M

Equation ④ becomes

$$\left[J_n(\beta x)\,\alpha \cdot J_n'(\alpha x) - J_n(\alpha x)\,\beta J_n'(\beta x)\right]_{x=1} = (\beta^2-\alpha^2)\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx$$

Hence

$$\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = \frac{1}{\beta^2-\alpha^2}\left[\alpha J_n(\beta) J_n'(\alpha) - \beta J_n(\alpha) J_n'(\beta)\right] \longrightarrow ⑤$$ 1M

Since α & β are distinct roots of $J_n(x)=0$

$J_n(\alpha)=0$, & $J_n(\beta)=0$ 1M

Eqⁿ ⑤ becomes zero provided $\beta^2-\alpha^2 \neq 0$

$$\therefore \boxed{\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = 0}$$ 1M

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c) By Rodrigues formula.

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n \longrightarrow ①$$ 1M

$$P_0(x) = \frac{1}{2^0 0!}\frac{d^0}{dx^0}(x^2-1)^0 = 1$$ 1M

$$P_1(x) = \frac{1}{2\times 1!}\frac{d}{dx}(x^2-1) = \frac{1}{2}[2x] = x.$$ 1M

$$P_2(x) = \frac{1}{2^2 2!}\frac{d^2}{dx^2}[(x^2-1)^2] = \frac{1}{8}[12x^2-4] = \frac{1}{2}(3x^2-1)$$ 1M

$$P_3(x) = \frac{1}{2^3 3!}\frac{d^3}{dx^3}[(x^2-1)^2] = \frac{1}{2}[5x^3-3x]$$ 1M

$$P_4(x) = \frac{1}{8}[35x^4-30x^2+3]$$ 2M

Q.No.	Solution and Scheme	Marks
4	a) WKT.	
	$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\, \sin x \longrightarrow ①$	1M
	and $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x \longrightarrow ②$	1M
	② ÷ ① gives	
	$\frac{J_{-1/2}(x)}{J_{1/2}(x)} = \frac{\sqrt{\frac{2}{\pi x}}\cos x}{\sqrt{\frac{2}{\pi x}}\sin x} = \frac{\cos x}{\sin x} = \cot x.$	2m
	$\therefore \boxed{J_{-1/2}(x) = J_{1/2}(x)\cot x}$	2M
	b) $P_0(x) = 1-x^2 \neq 0$ at $x=0$	
	Assume that the Series solution of equation as	
	$y = \sum_{r=0}^{\infty} a_r x^r \longrightarrow ①$	1M
	$\therefore (1-x^2)\sum_0^{\infty} a_r r(r-1)x^{r-2} - 2x\sum_0^{\infty} a_r r x^{r-1} + n(n+1)\sum_0^{\infty} a_r x^r = 0$	
	Equating the coeff of x^r to Zero $(r \geq 0)$	
	$a_{r+2} = \frac{-[n(n+1)-r^2-r]}{(r+1)(r+2)}a_r$ putting $r=0,1,2,3\ldots$	1M
	$a_2 = \frac{-n(n+1)}{2}a_0$; $a_3 = \frac{-(n^2+n-2)}{6}a_1 = \frac{-(n+1)(n+2)}{6}a_1$	1M
	$a_4 = \frac{n(n+1)(n-2)(n+3)a_0}{24}$	1M
	$a_5 = \frac{(n-1)(n+2)(n-3)(n+4)}{120}a_1$ & so on	1M
	$\therefore y = a_0\left[1 - \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n-2)(n-3)}{4!}x^4 - \cdots\right]$	
	$+a_1\left[x - \frac{(n-1)(n+2)}{3!}x^3 + \frac{(n-1)(n+2)(n-3)(n+4)}{5!}x^5 + \cdots\right]$	
	ie $\boxed{y = a_0 u(x) + a_1 v(x)}$	2m
	is the series solution of the Legendres diff. Eqn.	

Q.No.	Solution and Scheme	Marks
4	c) WKT.	
	$P_0(x)=1,\ P_1(x)=x,\ P_2(x)=\frac{1}{2}(3x^2-1)\cancel{\phi},\ P_3(x)=\frac{1}{2}(5x^3-3x)$	1m
	$P_4(x)=\frac{1}{8}[35x^4-30x^2+3]$	1m
	$\therefore x^2=\frac{1}{3}P_0(x)+\frac{2}{3}P_2(x)$	1m
	$x^3=\frac{1}{5}[2P_3(x)+3P_1(x)]$	1m
	$x^4=\frac{8}{35}P_4(x)+\frac{20}{25}P_2(x)+\frac{7}{35}P_0(x)$	
	$\therefore 4x^3+6x^2+7x+2=$	
	$=\frac{8}{5}P_3(x)+\frac{12}{5}P_1(x)+\cancel{8}2P_0(x)+6P_2(x)+7P_1(x)+2P_0(x)$	
	$=\frac{8}{5}P_3(x)+6P_2(x)+\frac{47}{5}P_1(x)+4P_0(x)$	3m
5	a)	
	$n=8$	
	$\bar{x}=\frac{\Sigma x}{n}=\frac{545}{8}=68.12\approx 68$	
	$\bar{y}=\frac{\Sigma y}{n}=\frac{552}{8}=69.$	2M
	$X=x-\bar{x}=x-68,\quad Y=y-\bar{y}=y-69$	

x	y	X	Y	X^2	Y^2	XY
65	67	-3	-2	9	4	6
66	68	-2	-1	4	1	2
67	65	-1	-4	1	16	~~4~~ 4
68	68	0	-1	0	1	0
68	72	0	3	0	9	0
69	72	1	3	1	9	3
70	69	2	0	4	0	2
72	71	4	2	16	4	8

2M

$\Sigma XY = 25, \ \Sigma X^2 = 35, \ \Sigma Y^2 = 44$

$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y2}} = \frac{25}{\sqrt{35} \times \sqrt{44}} = \frac{25}{39.24} =$

$\boxed{r = 0.63}$ 2M

b) The normal Eqn are

$\Sigma Y = a\Sigma x + nb$

$\Sigma xy = a\Sigma x^2 + b\Sigma x$ where $n=4$. 1M

x	y	x^2	xy
12	50	144	600
15	70	225	1050
21	100	441	2100
25	120	625	3000
Σ 73	340	1435	6750

2M

Substitutions in normal Eqn

$340 = 73a + 4b$

$6750 = 1435a + 73b$ Solving $\boxed{a = 5.30, \ b = -11.80}$ 2M

$\therefore \boxed{y = (5.30)x + (-11.80)}$ is the straight line of best fit. 2M

c) $y = ax^b \longrightarrow$ ① Taking logarithm on both sides

$\log y = \log a + b \log x$ 1M

$Y = A + bX$ where $A = \log A, X = \log x, Y = \log y$.

The normal Eqn are

$\Sigma Y = nA + b\Sigma X$ 1M

$\Sigma XY = A\Sigma X + b\Sigma X^2$

No.	Solution and Scheme	Marks

x	y	$X=\log x$	$Y=\log y$	XY	X^2
1	2.98	0	1.0919	0	0
2	4.26	0.6931	1.4492	1.004	0.4803
3	5.21	1.0986	1.6505	1.8132	1.2069
4	6.1	1.3862	1.8082	2.5065	1.9215
5	6.8	1.6094	1.9169	3.0850	2.5901
6	7.5	1.7917	2.0149	3.61	3.2101
	Σ	6.579	9.9316	12.0187	9.4089

2M

From normal equation

$$9.9316 = 6A + 6.579\,B$$

$$12.0187 = 6.579\,A + 9.4089\,B \quad \text{solving}$$

$$A = 1.0914, \quad b = 0.5142$$

2M

$$a = e^A = e^{1.0914} = 2.9784$$

∴ The curve of best fit is

$$\boxed{y = 2.9784\,x^{0.5142}}$$

1M

6 a)

x	Rank(x)	y	Rank(y)	$d=x-y$	d^2
35	3	30	5	-2	4
23	5	33	3	2	4
47	1	45	2	-1	1
17	6	23	6	0	0
10	7	8	8	-1	1
43	2	49	1	1	1
9	8	12	7	1	1
6	9	4	9	0	0
28	4	31	4	0	0
					$\Sigma d^2 = 12$

4M

$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times 12}{9(81-1)} = 1 - \frac{72}{720}$$

1M

$$\boxed{\rho = 0.90}$$

1M

x	y	X = log x	Y = log y	XY	X^2
1	2.98	0	1.0919	0	0
2	4.26	0.6931	1.4492	1.004	0.4803
3	5.21	1.0986	1.6505	1.8132	1.2069
4	6.1	1.3862	1.8082	2.5065	1.9215
5	6.8	1.6094	1.9169	3.0850	2.5901
6	7.5	1.7917	2.0149	3.61	3.2101
	Σ	6.579	9.9316	12.0187	9.4089

2M

From normal equation

$9.9316 = 6A + 6.579\,B$

$12.0187 = 6.579\,A + 9.4089\,B$ solving 2M

$A = 1.0914, \quad b = 0.5142$

$a = e^{A} = e^{1.0914} = 2.9784.$

∴ The curve of best fit is

$$\boxed{y = 2.9784\,x^{0.5142}}$$ 1M

6 a)

x	Rank(x)	y	Rank(y)	d = x − y	d^2
35	3	30	5	−2	4
23	5	33	3	2	4
47	1	45	2	−1	1
17	6	23	6	0	0
10	7	8	8	−1	1
43	2	49	1	1	1
9	8	12	7	1	1
6	9	4	9	0	0
28	4	31	4	0	0
					$\Sigma d^2 = 12$

4M

$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times12}{9\,(81-1)} = 1 - \frac{72}{720}$$ 1M

$$\boxed{\rho = 0.90}$$ 1M

b) We know that regression lines passes through $(\bar{x}, \bar{y})$

$\therefore 4\bar{x} - 5\bar{y} = -33$ Solving
$20\bar{x} - 9\bar{y} = 107$

3M

$\boxed{\bar{x} = 13 \quad \bar{y} = 17}$

1M

Consider $4x - 5y + 33 = 0 \Rightarrow y = (4/5)x + 33/5$

1M

Also $20x - 9y = 107 \Rightarrow x = (9/20)y + 107/20$

$\therefore r = \sqrt{(4/5) \times (9/20)} = 3/5$

2M

$\boxed{r = 0.6}$

---

c) The normal equations are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$$
$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$
$$\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$

1M

Preparing the table ~~for~~ ~~Σ~~

	x	y	x^2	x^3	x^4	xy	x^2y
	20	0.18	400	8000	160000	3.6	72
	60	0.37	3600	216000	12960000	22.2	1332
	100	0.35	10000	1000000	100000000	35	3500
	140	0.78	19600	2744000	384160000	109.2	15288
	180	0.56	32400	5832000	1049760000	100.8	18144
	220	0.75	48400	10648000	2342560000	165	36300
Σ	720	~~10~~2.64	114400	20448000	3889600000	3172	44480

3M

From normal eqⁿ

$$2.64 = 114400\cancel{22}a + 720b + 6C$$
$$3172 = \cancel{8}20448000a + 114400b + 720C$$
$$44480 = 3889600000a + 20448000b + 114400C$$

2M

Solving

$a = -0.00723$; $b = 1.8374$, $C = -82.1796$

$\therefore$ The second degree parabola of best fit is

$$y = (-0.00723)x^2 + (1.8374)x - 82.1796$$

1M

Q.No.	Solution and Scheme	Marks
7 a)	The first condition satisfies if $k \geq 0$	
	$\Sigma P(x) = 1 \Rightarrow 10k^2 + 9k - 1 = 0$	1M
	$(10k-1)(k+1) = 0 \Rightarrow k = 1/10$ and $k = -1$ but $k \geq 0$	1M
	$\Rightarrow k = 1/10 = 0.1$	
	x 0 1 2 3 4 5 6 7	2M
	$P(x)$ 0 $1/10$ $1/5$ $1/5$ $3/10$ $1/100$ $1/50$ $17/100$	
	$P(x<6) = P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6) = 0.81$	1M
	$P(x \geq 6) = P(6)+P(7) = 0.19$	1M
	$P(3 < x \leq 6) = P(4)+P(5)+P(6) = 0.33$	1M
b)	$P(x) = \frac{\mu^x e^{-\mu}}{x!}$	
	$\mu = \sum_{x=0}^{\infty} x p(x) = \sum_{x=0}^{\infty} x \frac{\mu^x e^{-\mu}}{x!}$	1M
	$= \sum_{x=1}^{\infty} \frac{\mu^{x-1} e^{-\mu} \cdot \mu}{(x-1)!} = \mu e^{-\mu}\left[\sum_{x=1}^{\infty} \frac{\mu^{x-1}}{(x-1)!}\right]$	1M
	$= \mu e^{-\mu}\left[1 + \mu + \frac{\mu^2}{2!} + \frac{\mu^3}{3!} + \cdots\right]$	1M.
	Mean $= \mu e^{-\mu} \cdot e^{\mu} = \mu$	1M
	Variance $= \sum_{x=0}^{\infty} x^2 p(x) - \mu^2 \longrightarrow ①$	
	consider $\sum_{x=0}^{\infty} x^2 p(x) = \sum_{x=0}^{\infty} [x(x-1)+x]\, p(x)$	
	$= \sum_{x=0}^{\infty} x(x-1) p(x) + \sum_{x=0}^{\infty} x p(x)$	1M
	$= \sum_{x=2}^{\infty} \frac{\mu^{x-2} \cdot \mu^2 e^{-\mu}}{(x-2)!} + \mu$	1M
	$= \mu^2 e^{-\mu}\left[1 + \mu + \frac{\mu^2}{2} + \cdots\right] + \mu$	
	$= \mu^2 + \mu$ from ①	1M
	Variance $= \mu^2 + \mu - \mu^2 = \mu$	
c)	Let x denotes the number of telephone line busy	1M
	Given $p = 0.1,\ q = 0.9,\ n = 10$	2M
	$p(x) = nC_x p^x q^{n-x} = {}^{10}C_x (0.1)^x (0.9)^{10-x}$	

Q.No.	Solution and Scheme	Marks
	i) Prob. that no. line is busy $= P(0) = 0.3487$	1M
	ii) Prob. that all lines are busy $= P(10) = (0.1)^{10}$	1M
	iii) Prob. that atleast one line is busy $= 1 - P(0) = 0.6513$	2M
8 a)	Given $f(x) = \begin{cases} kx(1-x) & 0 \le x < 1 \\ 0 & \text{else where} \end{cases}$	
	By the defn of p.d.f $f(x) \geq 0 \Rightarrow k \geq 0$	
	Also, $\int_{-\infty}^{\infty} f(x)\,dx = 1$	
	ie $\int_0^1 kx(1-x)\,dx = 1$	1M
	$k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$	
	$k\left[\frac{1}{2} - \frac{1}{3}\right] = 1 \Rightarrow k(1/6) = 1 \Rightarrow \boxed{k = 6}$	1M
	$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = 6\int_0^1 x^2(1-x)\,dx = 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$	2M
	$V = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2 = 6\int_0^1 x^3(1-x)\,dx = 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 - \frac{1}{4}$ $(-1/4)$	2M
	$V = \frac{6}{20} - \frac{1}{4} = 1/20$	
b)	Given $\mu = 3 \Rightarrow p(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{3^x e^{-3}}{x!}$	
	Let $f(x) = 1000\,p(x)$ $= 1000 \times \frac{3^x . e^{-3}}{x!} = \frac{50 \times 3^x}{x!}$	2M
	i) No. of drivers with no accidents $= f(0) = 50$	
	ii) Prob. of more than 3 accidents $= 1 - P(x \le 3)$	2M
	$= 1 - [P(0) + P(1) + P(2) + P(3)] = 0.35$	
	Number of drivers out of 1000 with more than 3 accidents in a year. $= 1000 \times 0.35 = 350$.	3M
c)	By the data $\mu = 2000, \sigma = 60$	
	S.N.V $=$ $z = \frac{x - \mu}{\sigma} = \frac{x - 2000}{60}$	1M

i) To find $P(x>2100)$

If $x=2100$, $z=100/60=1.67$

$P(z>1.67)=0.5-\phi(1.67)=0.5-0.4525$

$P(z>1.67)=0.0475$

∴ The number of bulbs likely to last for more than 2100 hrs $= 2500\times0.0475 = 119$ 2m

ii) $P(x<1950)$

If $x=1950$, $z=-5/6=-0.183$

$P(z<-0.83) = P(z>0.83) = 0.5-\phi(0.83)=0.2033$

∴ The number of bulbs likely to last more than 1950 hrs $= 2500\times0.2033 = 508$ 2m

iii) To find $P(1900<x<2100) = P(-1.67<z<1.67)$

$=2P(0<z<1.67)$

$=2\phi(1.67) = 2\times0.4525=0.905$

∴ number of bulbs that are likely to last between 1900 to 2100 hrs is $=2500\times0.905 = 2263$ 2m

9a) Marginal distribution of X is

X	1	3	6
f(x)	1/3	1/2	1/6

2m

Marginal distribution of Y is

Y	1	3	6
$g(y_j)$	1/3	1/2	1/16

2m

$P(X=1, Y=1)=\frac{1}{9}$ $\quad P(X=1)\times P(Y=1) = \frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

∴ $P(X=1, Y=1) = P(X=1)\times P(Y=1)$ 2m

∴ X and Y are statistically independent.

Q.No.	Solution and Scheme	Marks
9 b)	Let x denotes the number of heads and f be the corresponding frequency	
	Mean $= \mu = \frac{\Sigma f x}{\Sigma f} = \frac{0+27+144+336+284+160}{320}$	1M
	$\boxed{\mu = 2.97}$ But $\mu = np$.	#
	$5p = 2.97 \Rightarrow \boxed{p = 0.59} \Rightarrow \boxed{q = 0.41}$	1M
	$\therefore P(x) = nC_x p^x q^{n-x} = 5C_x (0.59)^x (0.41)^{5-x}$	
	$F(x) = 320 \times P(x) = 320 \times 5C_x (0.59)^x (0.41)^{5-x}$.	
	$F(0) = 3.70 \approx 4$, ~~$F(0)$~~ $\cdot F(1) = 27$, $F(2) = 77$	2M
	$F(3) = 110$, $F(4) = 79$, $F(5) = 23$	
	$\begin{array}{ccccccc} O_i & 6 & 27 & 72 & 112 & 71 & 32 \\ E_p & 4 & 27 & 77 & 110 & 79 & 23 \end{array}$	
	$\chi^2 = \Sigma \frac{(O_i - E_p)^2}{E_i} = \frac{4}{4} + 0 + \frac{25}{77} + \frac{4}{110} + \frac{64}{79} + \frac{81}{23}$	2M
	$= 1 + 0.3246 + 0.03636 + 0.8101 + 3.2577$	
	$\boxed{\chi^2 = 5.4227 > 11.070}$	1M
	Thus the hypothesis can be rejected.	
c)	$\bar{x} = \frac{\Sigma x}{n} = \frac{31}{12} = 2.5833 \approx 2.58$	2M
	$s^2 = \frac{1}{n-1} \Sigma (x - \bar{x})^2 = \frac{1}{11} [(5-2.58)^2 + (2-2.58)^2 + \cdots]$	
	$s^2 = 9.538$; $\boxed{S = 3.088}$	2M
	We have $t = \frac{\bar{x} - \mu}{s} \sqrt{n} = \frac{2.5833 - 0}{3.088} \sqrt{12}$.	2m
	$\boxed{t = 2.8979 > 2.201}$	
	Hence the hypothesis is rejected at 5% level of significance.	1m

Q.No. | **Solution and Scheme** | **Marks**

10 a) i) Marginal distributions

x_i	1	2
$f(x_i)$	0.6	0.4

y_j	-2	-1	4	5
$g(y_j)$	0.3	0.3	0.1	0.3

1M

$\mu_x = E(X) = \Sigma x_i f(x_i) = 1.4$

$\mu_y = E(Y) = \Sigma y_j g(y_j) = 1$

$E(XY) = \Sigma x_i y_j p_{ij} = 0.9$ 2M

$\sigma_x^2 = E(X^2) - \mu_x^2 = 2.2 - (1.4)^2 = 0.24 \Rightarrow \sigma_x = 0.49$

$\sigma_y^2 = E(Y^2) - \mu_y^2 = 10.6 - (1)^2 = 9.6 \Rightarrow \sigma_y = 3.1$ 2M

$Cov(X,Y) = E(XY) - \mu_x \mu_y$

$= 0.9 - (1.4)(1) = -0.5$ 1M

$$Cor(X,Y) = \rho(X,Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y} = \frac{-0.5}{(0.49)(3.1)}$$

$\rho(X,Y) = -0.3$ ~~2M~~ 1M

b) $\bar{x} = \frac{\Sigma x}{n} = \frac{442}{9} = 49.11 \approx 49$ 1M

$s^2 = \frac{1}{n-1} \Sigma (x-\bar{x})^2 = \frac{1}{8}[16+4+1+9+1+4+0+16+4] = \frac{55}{8}$ 2M

$s^2 = 6.875 \Rightarrow \boxed{s = 2.62}$

$t = \frac{\bar{x} - \mu}{s} \sqrt{n} = \frac{49 - 47.5}{2.62} \sqrt{9} = \frac{1.5}{2.62} \times 3$ 2M

$t = \frac{4.5}{2.62} = 1.71$

$t = 1.71 < 2.306 = t_{0.05}$ at 8 d.f 1M

∴ The hypothesis is accepted at 5% level of significance. 1M

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Q.No.	Solution and Scheme	Marks
c)	The expected frequencies are in the ration	
	$9:3:3:1$ $\quad 9+3+3+1=16$	1M
	$\therefore \frac{9}{16}\times 1600, \frac{3}{16}\times 1600, \frac{3}{16}\times 1600, \frac{1}{16}\times 1600$	1M
	ie $900, 300, 300, 100$	2M
	O_i: 882 313 287 118	
	E_i: 900 300 300 100	
	$\chi^2 = \frac{\Sigma(O_i-E_i)^2}{E_i} = \left[\frac{324}{900}+\frac{169}{300}+\frac{169}{300}+\frac{324}{100}\right]$	
	$=0.36+0.56+0.56+0.36$	2m
	$\chi^2 = \underline{\underline{1.84}} < 7.815 = t_{0.05}$ at 3 d.f.	
	$\therefore$ The test of goodness of fit is accepted	1m
	END.	

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