CBCS SCHEME

USN | | | | | | | | | | | **18EC55**

Fifth Semester B.E. Degree Examination, Jan./Feb. 2023
Electromagnetic Waves

Time: 3 hrs.   Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. The three vertices of a triangle are located at $A(6, -1, 2)$, $B(-2, 3, -4)$ and $C(-3, 1, 5)$. Find (i) $R_{AB} \times R_{AC}$ (ii) Area of triangle **(04 Marks)**

b. Define Electric field intensity. Derive the expression for electric field intensity due to infinite line charge. **(10 Marks)**

c. Given the electric flux density $\overline{D} = 0.3r^2\bar{a}_r \, nC/m^2$ in free space.
- (i) Find E at point $P(r = 2, \theta = 25°, \phi = 90°)$.
- (ii) Find total charge within the sphere $r = 3$.
- (iii) Find total electric flux learing the sphere $r = 4$. **(06 Marks)**

OR

2 a. Four identical 3nC (nano Coulomb) charges are located at $P_1(1, 1, 0)$, $P_2(-1, 1, 0)$, $P_3(-1, -1, 0)$ and $P_4(1, -1, 0)$. Find the electric field intensity $\overline{E}$ at $P(1, 1, 1)$. **(10 Marks)**

b. Infinite uniform line charges of 5 nC/m lie along the (positive and negative) x and y axes in free space. Find $\overline{E}$ at $P_A(0, 0, 4)$. **(04 Marks)**

c. Define Coulomb's law. Make use of this to find the force on Q_1. Given that the point charges $Q_1 = 50\,\mu C$ and $Q_2 = 10\,\mu C$ are located at $(-1, 1, -3)$m and $(3, 1, 0)$m respectively. **(06 Marks)**

Module-2

3 a. Explain Gauss law applicable to the case of infinite line charge and derive the relation used. **(08 Marks)**

b. Evaluate both sides of the divergence theorem for the field $\overline{D} = 2xy\bar{a}_x + x^2\bar{a}_y$ C/m^2 and the rectangular parallelepiped formed by the places $x = 0$ and 1, $y = 0$ and 2 and $z = 0$ and 3. **(08 Marks)**

c. Given the potential field $V = 2x^2y - 5z$ and point $P(-4\ \ 3\ \ 6)$. (i) Find potential V at P. (ii) Field intensity $\overline{E}$, (iii) Volume charge density ρ_V. **(04 Marks)**

OR

4 a. Compute the numerical value for $\text{div}\overline{D}$ at the point specified below:
$\overline{D} = (2xyz - y^2)\bar{a}_x + (x^2z - 2xy)\bar{a}_y + x^2y\bar{a}_z \, C/m^2$ at $P_A(2, 3, -1)$ **(04 Marks)**

b. Show that Electric field is a negative gradient of potential. **(08 Marks)**

c. Let $E = y\bar{a}_x$ V/m at a certain instant of time and calculate the work required to move a 3c charge from $(1, 3, 5)$ to $(2, 0, 3)$ along the straight line segment joining
- (i) $(1, 3, 5)$ to $(2, 3, 5)$ to $(2, 0, 5)$ to $(2, 0, 3)$
- (ii) $(1, 3, 5)$ to $(1, 3, 3)$ to $(1, 0, 3)$ to $(2, 0, 3)$ **(08 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Solve the Laplace's equation for the potential field in the homogenous region between the two concentric conducting spheres with radii 'a' and 'b' such that b>a, if potential $V = 0$ at $r = b$ and $V = V_0$ at $r = a$. Also find the capacitance between two concentric spheres. **(10 Marks)**

b. State and explain Biot-Savart law applicable to magnetic field. **(06 Marks)**

c. Calculate the value of vector current density in a rectangular coordinates at $P_A(2, 3, 4)$ if $\overline{H} = x^2 z\overline{a}_y - y^2 x\overline{a}_z$. **(04 Marks)**

OR

6 a. State and illustrate uniqueness theorem. **(08 Marks)**

b. Define Stoke's theorem. Use this theorem to evaluate both sides of the theorem for the field $\overline{H} = 6xy\overline{a}_x - 3y^2\overline{a}_y$ A/M and the rectangular path around the region, $2 \le x \le 5$, $-1 \le y \le 1$ $z = 0$. Let the positive direction of ds be $\overline{a}_z$. **(12 Marks)**

Module-4

7 a. Obtain the expression for magnetic force between differential current elements. **(06 Marks)**

b. Derive the boundary conditions to apply to $\overline{B}$ and $\overline{H}$ at the interface between two different magnetic materials. **(08 Marks)**

c. The point charge $\theta = 18nC$ has a velocity of 5×10^6 m/s in the direction. $\overline{a}_v = 0.60\overline{a}_x + 0.75\overline{a}_y + 0.30\overline{a}_z$

Calculate the magnitude of the force exerted on the charge by the field,

(i) $\overline{B} = -3\overline{a}_x + 4\overline{a}_y + 6\overline{a}_z$ mT

(ii) $\overline{E} = -3\overline{a}_x + 4\overline{a}_y + 6\overline{a}_z$ kV/m

(iii) B and $\overline{E}$ acting together **(06 Marks)**

OR

8 a. Find the magnetization in a magnetic material, where

(i) $\mu = 1.8\times10^{-5}$ H/m and $H = 120$ A/m

(ii) $\mu_r = 22$, there are 8.3×10^{28} atoms/m³, and each atom has a dipole moment of 4.5×10^{-27} A.m²

(iii) $B = 300\,\mu T$ and $\chi_m = 15$. **(06 Marks)**

b. Let permittivity be 5 μH/m in region A where $x < 0$ and 20 μH/m in region B, where $x > 0$. If there is a surface current density $\overline{K} = 150\overline{a}_y - 200\overline{a}_z$ A/m at $x = 0$, and if $H_A = 300\overline{a}_x - 400\overline{a}_y + 500\overline{a}_z$ A/m. Compute

(i) $|H_{tA}|$ (ii) $|H_{NA}|$ (iii) $|H_{tB}|$ (iv) $|H_{NB}|$ **(08 Marks)**

c. State and explain Faraday's law of electromagnetic induction. **(06 Marks)**

Module-5

9 a. List and explain Maxwell's equations in point and integral form. **(08 Marks)**

b. The time domain expression for the magnetic field of a uniform plane wave travelling in free space is given by,

$H(z,t) = \bar{a}_y 2.5\cos(1.257\times10^9 t - K_0 z)$ mA/m.

Compute
(i) The direction of wave propagation.
(ii) Operating frequency
(iii) Phase constant.
(iv) The time domain expression for electric field E(z,t) starting from the Maxwell's equations.
(v) The phasor form of both the electric and magnetic field. **(10 Marks)**

c. For silver the conductivity is $\sigma = 3\times10^6$ S/m. At what frequency will the depth of penetration be 1 mm. **(02 Marks)**

OR

10 a. State and explain Poynting theorem and write the equation both in point and integral form. **(08 Marks)**

b. Simplify the value of K to satisfy the Maxwell's equations for region $\sigma = 0$ and $\rho_v = 0$ if $\bar{D} = 10x\bar{a}_x - 4y\bar{a}_y + kz\bar{a}_z\ \mu C/m^2$ and $B = 2\bar{a}_y$ mT. **(06 Marks)**

c. A plane wave of 16 GHz frequency and E = 10 V/m propagates through the body of salt water having constant $\varepsilon_r = 100$, $\mu_r = 1$ and $\sigma = 100$ s/m. Determine attenuation constant, phase constant, phase velocity and intrinsic impedance and depth and penetration. **(06 Marks)**

*****

Module - 1

1]

(a) (i) $R_{AB} = R_B - R_A = \langle -2,3,-4\rangle - \langle 6,-1,2\rangle = -8a_x + 4a_y - 6a_z$

$R_{AC} = R_C - R_A = \langle -3,1,5\rangle - \langle 6,-1,2\rangle = -9a_x + 2a_y + 3a_z$ - 2M

$R_{AB} \times R_{AC} =$

$$\begin{vmatrix} a_x & a_y & a_z \\ -8 & 4 & -6 \\ -9 & 2 & 3 \end{vmatrix}$$

$R_{AB} \times R_{AC} = 24a_x + 78a_y + 20a_z$

(ii) Area of triangle $= \frac{1}{2}|R_{AB} \times R_{AC}| =$

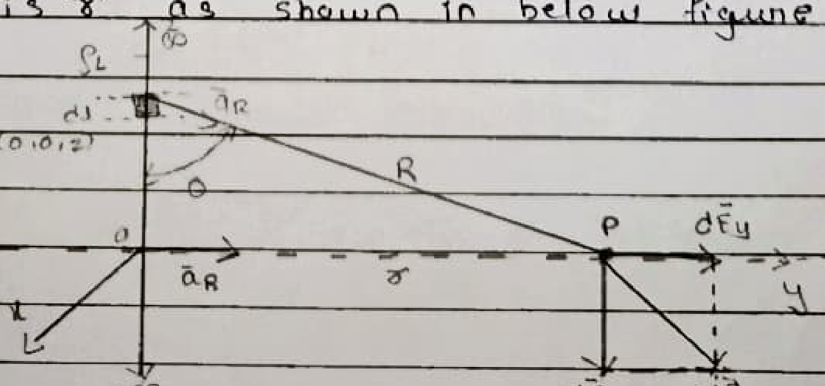
$$= \frac{1}{2}\left[\sqrt{(24)^2 + (78)^2 + (20)^2}\right] = 42 \text{ sq unit}$$ - 2M

1]

(b) Electric field intensity :- The force exerted per unit charge is called electric field intensity.

$$E = \frac{F}{Q} \quad V/m$$

Consider a long straight line carrying uniform line charge having density ρ_L C/m. Let a point P is on y-axis at which electric field intensity is to be determined. The distance of point P from the origin is 'r' as shown in below figure



4M

1 (b)

$$d\theta = \rho_L dl = \rho_L dz \quad -①$$

$$\bar{R} = \bar{r}_P - \bar{r}_{dl} = [r\bar{a}_y - z\bar{a}_z]$$

$$|\bar{R}| = \sqrt{r^2+z^2}$$

$$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{r\bar{a}_y - z\bar{a}_z}{\sqrt{r^2+z^2}} \quad -②$$

$$d\bar{E} = \frac{d\theta}{4\pi\epsilon_0 R^2}\bar{a}_R$$

$$= \frac{\rho_L dz}{4\pi\epsilon_0[\sqrt{r^2+z^2}]^2}\left[\frac{r\bar{a}_y - z\bar{a}_z}{\sqrt{r^2+z^2}}\right] \quad -③$$

For every charge on positive z-axis there is equal charge present on negative z-axis. So z-component will cancel out each other. So; by eliminating z compont

$$d\bar{E} = \frac{\rho_L dz}{4\pi\epsilon_0(\sqrt{r^2+z^2})^2}\frac{r\bar{a}_y}{\sqrt{r^2+z^2}} \quad -④$$

$$E = \int_{-\infty}^{\infty}\frac{\rho_L}{4\pi\epsilon_0(r^2+z^2)^{3/2}} r\,dz\,\bar{a}_y$$

$z = r\tan\theta \quad r = z/\tan\theta \quad dz = r\sec^2\theta\, d\theta$

for $z = -\infty$, $\theta = \tan^{-1}(-\infty) = -\pi/2$; $z = \infty$ $\theta = \tan^{-1}(\infty) = \pi/2$

$$\bar{E} = \frac{\rho_L}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2}\frac{r^2\sec^2\theta\, d\theta}{r^3[1+\tan^2\theta]^{3/2}}\bar{a}_y$$

$$E = \frac{\rho_L}{4\pi\epsilon_0}\int_{-\pi/2}^{\pi/2}\frac{\sec^2\theta\, d\theta}{r\sec^3\theta}\bar{a}_y = \frac{\rho_L}{4\pi\epsilon_0 r}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta\,\bar{a}_y.$$

$$E = \frac{\rho_L}{4\pi\epsilon_0 r}[\sin\theta]_{\pi/2}^{\pi/2}\bar{a}_y = \frac{\rho_L}{4\pi\epsilon_0 r}\left[1-(-1)\right]a_y$$

$$\boxed{E = \frac{\rho_L}{2\pi\epsilon_0 r}\bar{a}_y \quad V/m}$$

6M

Topic : .... Date : ....

KLS VDIT HALIYAL

1]

(c) (i) We know that electric flux density (D) = $\epsilon_0 E$

$E = D/\epsilon$

$D = 0.3\, r^2\, \bar{a}_r$ nC/m² at r = 2 — 2M

$E = 135.2$ V/m

(ii) $D = \dfrac{Q}{4\pi r^2}$ $Q = 4 \times \pi \times r^2 \times D$ — 2M

$D = 4 \times 3.14 \times (3)^2 \times 0.3 \times (3)^2$

$D = 305.208$ nC

(iii) we know that $Q = \phi$ at $r = 4$ — 2M

$D = \dfrac{Q}{4\pi r^2} = Q = 4\pi r^2 \times D$

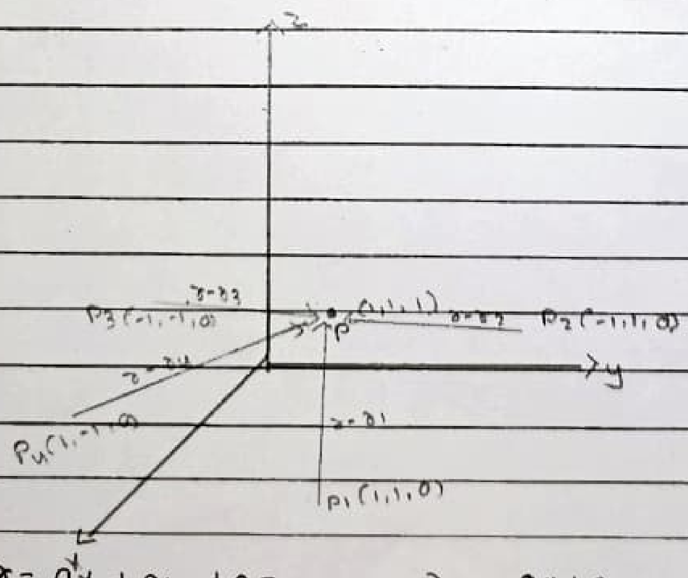
$Q = 4 \times 3.14 \times (4)^2 \times (0.3) \times (4)^2$

$Q = 964.608$ nC

$\phi = 965$ nC

2]

(a)



$r = a_x + a_y + a_z$, $r_1 = a_x + a_y$ $r_2 = -a_x + a_y$

$r_3 = -a_x - a_y$ $r_4 = a_x - a_y$

Teacher's Sign.

KLS, VDIT, Haliyal

2] (a) $|r - r_1| = \langle 1,1,1\rangle - \langle 1,1,0\rangle = \langle 0,0,1\rangle = 1$

$|r - r_2| = \sqrt{5}$, $|r - r_3| = 3$ and $|r - r_4| = \sqrt{5}$ } 4M

$$E = 26.96 \left[\frac{a_z}{1} \cdot \frac{1}{1^2} + \frac{2a_x + a_z}{\sqrt{5}} \cdot \frac{1}{(\sqrt{5})^2} + \frac{2a_x + 2a_y + a_z}{3} \cdot \frac{1}{(3)^2} + \frac{2a_y + a_z}{\sqrt{5}} \cdot \frac{1}{(\sqrt{5})^2} \right]$$

4M

$E = 6.82\bar{a}_x + 6.82\bar{a}_y + 32.8\bar{a}_z$ V/m // 2M

2]

(b) Electric field at uniform line change is given by

2M

$$E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho = \frac{5\times10^{-9}}{2\times3.14\times\epsilon(4)} \cdot \frac{4a_z}{4} + \frac{5\times10^{-9}}{2\times\pi\times\epsilon(4)} \cdot \frac{4\bar{a}_z}{4}$$

$E = 45a_z$ V/m. 2M

2]

(c) The coloumb's law states that the force between two point changes is directly proportional to the product of magnitude of change and inversely proportional to the square of distance between them. 1M

$$F \propto \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$ //

5M

$Q_1 = 50\mu C$ $\quad$ $Q_2 = 10\mu C$

$r_1 = \langle -1, 1, -3\rangle$ $\quad$ $r_2 = \langle 3, 1, 0\rangle$

$r = r_1 - r_2 = -4\bar{a}_x - 3\bar{a}_z$

Teacher's Sign.

2] (c)
$$\bar{a}_r = \frac{\bar{r}}{|\bar{r}|} = \frac{-4a_x - 3a_z}{5}$$

$$F = \frac{Q_1 Q_2}{4 \times \pi \times \epsilon_0 \times r^2} \bar{a}_r .$$

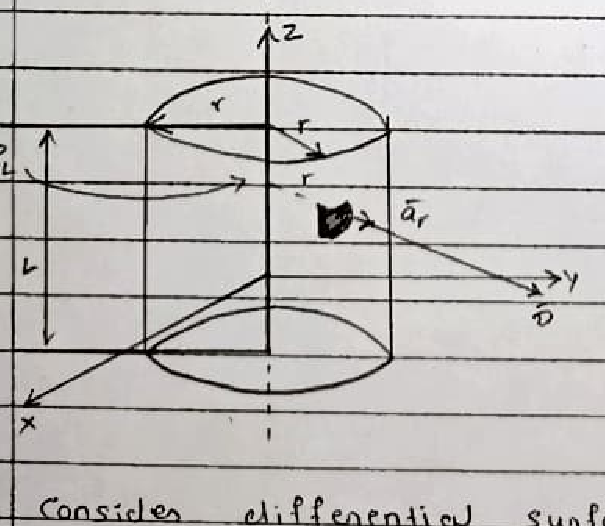
$$F = \frac{50 \times 10^{-9} \times 10 \times 10^{-9} \times (-4a_x - 3a_z)}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (5)^2 \times 5} .$$

$$F = -0.144 \bar{a}_x - 0.108 \bar{a}_z \ N.$$

Module - 2

3]

(a) Consider an infinite line charge of density ρ_L C/m lying along z-axis from $-\infty$ to ∞. Consider the gaussian surface as the right circular cylinder with z-axis as its axis and radius r. The length of cylinder is L



- 2M

Consider differential surface area dS as shown which is at the radial distance r from the line charge.

KLS, VDIT, Haliyal

3] (a) As the line charge is along z-axis there cannot be any component of $\bar{D}$ in z- direction

Now $Q = \oint_S \bar{D} \cdot \bar{ds}$

The integeration is to be evaluated for side, top and bottom surface

$$Q = \oint_{side} \bar{D} \cdot \bar{ds} + \oint_{top} \bar{D} \cdot \bar{ds} + \oint_{bottom} \bar{D} \cdot \bar{ds}$$

$\bar{D} = D_r \bar{a}_r$ as has only radial component

$\bar{ds} = r\, d\phi\, dz\, \bar{a}_r$ normal to $\bar{a}_r$ direction

$\bar{D} \cdot \bar{ds} = D_r\, r\, d\phi\, dz$ 3M

As $\bar{D}$ has only radial component and no z- component along $\bar{a}_z$ and $-\bar{a}_z$ hence integerating over top and bottom surfaces is zero.

$$\oint_{top} \bar{D} \cdot \bar{ds} = \oint_{bottom} \bar{D} \cdot \bar{ds} = 0$$

$$Q = \oint_{side} \bar{D} \cdot \bar{ds} = \oint_{side} D_r\, r\, d\phi\, dz = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} D_r\, r\, d\phi\, dz$$

$$Q = r D_r [z]_0^L [\phi]_0^{2\pi}$$

$$Q = 2\pi r D_r L$$

$$D_r = \frac{Q}{2\pi r L}$$

$$D = D_r \bar{a}_r = \frac{Q}{2\pi r L} \bar{a}_r$$

$$Q/L = \rho_L \quad C/m$$

$$\bar{D} = \frac{\rho_L}{2\pi r} \bar{a}_r \quad C/m^2$$

$$E = \frac{\bar{D}}{\epsilon_0} = \frac{\rho_L}{2\pi \epsilon_0 r} \bar{a}_r \quad V/m$$ 3M

Teacher's Sign.

3]

(b) Divergence theorem

$$\oint_S \vec{D}\cdot d\vec{s} = \int_{vol} \nabla\cdot\vec{D}\, dv$$

$$\oint \vec{D}\cdot d\vec{s} = \oint_{front} \vec{D}\cdot d\vec{s} + \oint_{back} \vec{D}\cdot d\vec{s} + \oint_{left} \vec{D}\cdot d\vec{s} + \oint_{right} \vec{D}\cdot d\vec{s}$$

$$\int_{front} \vec{D}\cdot d\vec{s} = \int_0^3\int_0^2 (\vec{D})_{x=1}\cdot dy\,dz\,\hat{a_x} = \int_0^3\int_0^2 2y\,dy\,dz$$

$$= \int_0^3 4\,dz = \underline{\underline{12}}$$

$$\int_{back} \vec{D}\cdot d\vec{s} = \int (\vec{D})_{x=0}\cdot dy\,dz(-\hat{a_x}) = \int -(D_x)_{x=0}\,dy\,dz$$

$$= 0$$

$$\int_{left} \vec{D}\cdot d\vec{s} = \int (\vec{D})_{y=0}\cdot dx\,dz(-\hat{a_y}) = \int -(D_y)_{y=0}\,dx\,dz$$

$$= -\int_0^3\int_0^1 x^2\,dx\,dz = -\int_0^3 \left[\frac{x^3}{3}\right]_0^1 dz = -3\times\frac{1}{3} = \underline{\underline{-1}}$$

4M

$$\int_{right} \vec{D}\cdot d\vec{s} = \int (\vec{D})_{y=2}\cdot dx\,dz(\hat{a_y}) = \int (D_y)_{y=2}\,dx\,dz$$

$$= \int_0^3\int_0^1 x^2\,dx\,dz = \int_0^3 \left[\frac{x^3}{3}\right]_0^1 dz = 3\times\frac{1}{3} = \underline{1}$$

$$\oint \vec{D}\cdot d\vec{s} = 12+0-1+1 = 12 \text{ // LHS}$$

$$\nabla\cdot\vec{D} = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z$$

$$\frac{\partial}{\partial x}D_x = \frac{\partial}{\partial x}(2xy) = 2y$$

$$\frac{\partial}{\partial y}(D_y) = \frac{\partial}{\partial y}(x^2) = 0 \qquad \frac{\partial}{\partial z}(D_z) = 0$$

3](b) $\int_0^3\int_0^2\int_0^1 \nabla \cdot \vec{D}\, dv$

$\int_0^3\int_0^2\int_0^1 2y\, dx\, dy\, dz$

$\int_0^3\int_0^2 2y\, dy\, dz$

$\int_0^3 \left[\frac{2y^2}{2}\right]_0^2 dz$

$\int_0^3 4\, dz = 12$ 4M

$\iiint \nabla \cdot \vec{D}\, dv = 12 = RHS$

LHS = RHS Hence divegerence theorem is verified on the both side of integral

3]
(c) $V = 2x^2y - 5z$, @ $P(-4,3,6)$

$V = 2[-4]^2 \times 3 - 5\times 6 = 66\ V$

$\bar{E} = -\nabla V = -\left[\frac{\partial v}{\partial x}\hat{a_x} + \frac{\partial u}{\partial y}\hat{a_y} + \frac{\partial u}{\partial x}\hat{a_z}\right]$ 2M

$= -[4xy\,\hat{a_x} + 2x^2\hat{a_y} + -5\hat{a_z}]$ -(1)

@ P, $\vec{E} = 48\hat{a_x} - 32\hat{a_y} + 5\hat{a_z}$ V/m

D @ P $= \vec{E}$ @ P $\times \epsilon_0 = 0.425\hat{a_x} - 0.2833\hat{a_y} + 0.0442\hat{a_z}\ nC/m^2$

$\rho_v = \nabla\cdot\bar{D} = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z$ 2M

$= \epsilon_0[-4y + 0 + 0] = -4\epsilon_0 y\ C/m^3 = -35.4y\ pC/m^3$

Teacher's Sign.

4]

(a) $D = (2xyz - y^2)\,\hat{a_x} + (x^2 z - 2xy)\,\hat{a_y} + x^2 y\,\hat{a_z}$ C/m²

@ point $A = (2, 3, -1)$

2M

$$\nabla \cdot D = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$\nabla \cdot D = \frac{\partial}{\partial x}(2xyz - y^2) + \frac{\partial}{\partial y}(x^2 z - 2xy) + \frac{\partial}{\partial z}(x^2 y)$$

$$\nabla \cdot D = 2yz - 2x$$

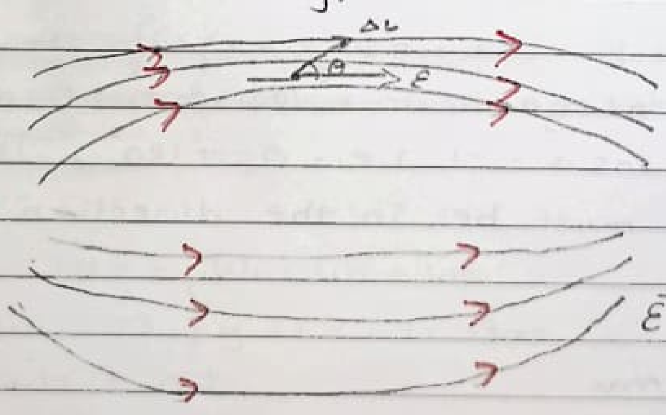
2M

$$\nabla \cdot D = 2 \times 3(-1) - 2(2)$$

$$\nabla \cdot D = -6 - 4 = -10$$

4]

(b) Consider a $\vec{E}$ due to particular charge distribution in space. The electric field $\vec{E}$ and potential V is changing from point to point in space. Consider a vector incremental length $\Delta \bar{L}$ making an angle θ with respect to the direction of $\bar{E}$, as shown in below fig.



4M

To, find incremental potential we use,

$$\Delta V = -\bar{E} \cdot \Delta \bar{L} \quad - (1)$$

$$\Delta \bar{L} = \Delta L\, a_L \quad - (2)$$

Teacher's Sign.

KLS; VDIT, Haliyal

4] (b) where $\bar{a}_L$ = Unit vector in the direction of ΔL

$$\vec{E} \cdot \vec{\Delta L} = (E_L \bar{a}_L) \cdot (\Delta L \bar{a}_L)$$

$$\vec{E} \cdot \vec{\Delta L} = E_L \Delta L$$

$$\Delta V = - E_L \Delta L$$

where;

E_L = Component of $\vec{E}$ in the direction of $\bar{a}_L$

In other words dot product is expressed in terms of $\cos\theta$ as

$$\Delta V = - E \Delta L \cos\theta$$

$$\frac{\Delta V}{\Delta L} = - E \cos\theta$$

4M

To find ΔV at a point, take limit $\Delta L \to 0$

$$\therefore \lim_{\Delta L \to 0} \frac{\Delta V}{\Delta L} = - E \cos\theta$$

But $\lim_{\Delta L \to} \frac{\Delta V}{\Delta L} = \frac{dV}{dL}$ = Potential gradient

The potential gradient dV/dL can be maximum only when $\cos\theta = -1$ i.e., $\theta = +180^\circ$. This indicates that ΔL must be in the direction opposite to $\vec{E}$

$$\therefore \left.\frac{dV}{dL}\right|_{max} = E$$

$$E = - \left.\frac{dV}{dL}\right|_{max} \hat{a}_n$$

MAYUR

Teacher's Sign.

4](b) cont: The max value of rate of change of potential with distance (dV/dL) is called gradient of V.

The mathematical operation on V by which $\bar{E}$ is ~~obta~~ obtained is called gradient and denoted as.

Gradient of V = grad V = ∇V

$$\nabla V = \text{grad } V = -\bar{E} \ \text{V/m}$$

$$\boxed{\bar{E} = -\nabla V = -(\text{grad } V)}$$

4]
(c) (i) $W = -Q \oint E \cdot dL$

$W = W_1 + W_2 + W_3$

$W_1 = -3 \int (4a_x + 0a_y + 0a_z)\, dx\, a_x = -9J$

$W_2 = -3 \int_{2}^{8} (4a_x + 0a_y + 0a_z)\, dy\,(a_y) = 0J$

$W_3 = -3 \int_{2}^{5} (4a_x + 0a_y + 0a_z)\, dz\, a_z = 0J$

$W = -9J$

4M

(ii) $W = W_1 + W_2 + W_3$

$W_1 = -3 \int y\, dx = -3x = 0J$

$W_2 = 0J$

$W_3 = 0J$

$W = 0 + 0 + 0 = 0J$

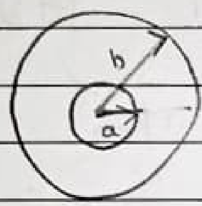
4M

KLS, VDIT, Haliyal

Module - 3

5]
(a) The concentric sphere conductors are shown in the below figure

At $r=b$, $V=0$, hence the outer sphere is shown at zero potential.



According to laplace's equation

$\nabla^2 V = 0$ $\quad \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = 0$ i.e.,

5M

$\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] = 0$ as V is function of r only

Integrate, $r^2\frac{\partial V}{\partial r} = \int 0 + C_1 = C_1 \Rightarrow \frac{\partial V}{\partial r} = \frac{C_1}{r^2} = C_1 r^{-2}$ — (1)

Integrate $V = \int C_1 r^{-2}\,dr + C_2 = \frac{C_1}{-1} r^{-1} + C_2$ — (2)

$V = \frac{-C_1}{r} + C_2$

Use the boundry conditions

$V = 0$ @ $r=b$ and $V = V_0$ at $r = a$

$0 = \frac{-C_1}{b} + C_2$ and $V_0 = -\frac{C_1}{a} + C_2$

Substracting the two equations

$$-V_0 = \frac{-C_1}{b} - \left[\frac{-C_1}{a}\right] \quad \text{i.e.,} \quad -V_0 = C_1\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\therefore C_1 = \frac{-V_0}{\left[\frac{1}{a} - \frac{1}{b}\right]} = \frac{V_0}{\left[\frac{1}{b} - \frac{1}{a}\right]} \quad \text{and} \quad C_2 = \frac{C_1}{b} = \frac{V_0}{b\left[\frac{1}{b} - \frac{1}{a}\right]}$$

$$V = -\frac{V_0}{r\left[\frac{1}{b} - \frac{1}{a}\right]} + \frac{V_0}{b\left[\frac{1}{b} - \frac{1}{a}\right]} \ V$$

This is the potential field in the region between the two spheres

3M

$$\bar{E} = -\nabla V = -\frac{\partial v}{\partial r}\bar{a}_r = \frac{-\partial}{\partial r}\left[\frac{-V_0}{r\left[\frac{1}{b} - \frac{1}{a}\right]}\right]\bar{a}_r$$

$$\vec{E} = \frac{-V_0}{\left[\frac{1}{b} - \frac{1}{a}\right] r^2}\hat{a}_r \ V/m$$

$$\vec{D} = \varepsilon\vec{E} = \frac{-\varepsilon V_0}{\left[\frac{1}{b} - \frac{1}{a}\right] r^2}\hat{a}_r = \frac{\varepsilon V_0}{\left[\frac{1}{a} - \frac{1}{b}\right] r^2} \ C/m^2$$

KLS, VDIT, Haliyal

As per the boundry conditions between conductor and dielectric, the $\vec{D}$ is always normal to the surface hence $\bar{D}_N$.

$$\rho_s \; |\bar{D}_N| = |\bar{D}| = \frac{\epsilon V_0}{\left[\frac{1}{a} - \frac{1}{b}\right] r^2} \quad c/m^2$$

2M

Q = Total charge on surface of sphere of radius r

$$Q = \frac{\epsilon V_0}{\left[\frac{1}{a} - \frac{1}{b}\right] r^2} \times \text{surface area of sphere of radius } r$$

$$Q = \frac{\epsilon V_0}{\left(\frac{1}{a} - \frac{1}{b}\right) r^2} \times 4\pi r^2$$

$$Q = \frac{4\pi\epsilon V_0}{[1/a - 1/b)} \; C$$

$C = Q/V$ where V = Potential between two spheres $= V_0$

$$C = \frac{Q}{V_0} = \frac{\frac{4\pi\epsilon V_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}}{V_0}$$

$$\boxed{C = \frac{4\pi\epsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]} \; F}$$

5]

(b) The magnetic field intensity dH produced at a point P due to differential element Idl is directly proportional to current I and differential length dl and sine of the angle between the element and the line joining point P to the element, and inversely proportional to the square of the distance R between point P and the element.

$$d\bar{H} \propto \frac{I\,dL \sin\theta}{R^2}$$

$$d\bar{H} = \frac{K I\,dl \sin\theta}{R^2}$$

3M

K - constant of proportionality.

$$K = \frac{1}{4\pi}$$

$$d\bar{H} = \frac{I\,dL \sin\theta}{4\pi R^2}$$

dL - magnitude of vector length $d\bar{L}$ and

$\bar{a}_R$ - Unit vector in the direction from differential current element to point P

From rule of cross product,

3M

$$d\bar{L} \times \bar{a}_R = dL\,|\bar{a}_R| \sin\theta = dL \sin\theta$$

$$d\bar{H} = \frac{I\,d\bar{L} \times \bar{a}_R}{4\pi R^2} \quad A/m$$

$$\boxed{d\bar{H} = \frac{I\,d\bar{L} \times \bar{R}}{4\pi R^3} \quad A/m}$$

Topic : KLS, VDIT, Haliyal

Date :

Page No. 6

5]

(c) Given ;

$$\vec{H} = x^2 z \bar{a_y} - y^2 x \bar{a_z}$$

$$J = \nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & x^2 z & -y^2 x \end{vmatrix}$$ 2M

$$\bar{J} = -16 \bar{a_x} + 9 \bar{a_y} + 16 \bar{a_z} \quad A/m^2$$ 2M

6]

(a) Assume the Laplace's equation has two solutions say V_1 and V_2, both are function of the coordinate of the system used. These solutions must be satisfying Laplace equation.

$\nabla^2 V_1 = 0$ and $\nabla^2 V_2 = 0$ - (1)

Both the solutions must satisfy the boundary conditions as well. At boundary the potentials at different points are same due to equipotential surface then, 4M

$V_1 = V_2$ - (2)

Let the difference between the two solutions is V_d.

$V_d = V_2 - V_1$ - (3)

Using Laplace's equation for the difference V_d.

$\nabla^2 V_d = \nabla^2 (V_2 - V_1) = 0$ - (4)

$\therefore \nabla^2 V_2 - \nabla^2 V_1 = 0$ - (5)

on the boundary $V_d = 0$ from the equation (2) and (3)

MAYUR

Teacher's Sign

6] (a) Now the divergence theorem states that.

$$\int_{vol} \nabla\cdot\bar{A}\, dv = \oint_S \bar{A}\cdot d\bar{s} \qquad -(6)$$

Let $\bar{A} = V_d \nabla V_d$ and from vector algebra

$\nabla\cdot(\alpha\bar{B}) = \alpha(\nabla\cdot\bar{B}) + \bar{B}\cdot(\nabla\alpha)$

Now use this for $\nabla\cdot(V_d \nabla V_d)$ with $\alpha = V_d$ and $\nabla V_d = B$

$\therefore \nabla\cdot(V_d\nabla V_d) = V_d(\nabla\cdot\nabla V_d) + \nabla V_d\cdot(\nabla V_d)$

$\therefore \nabla\cdot(V_d\nabla V_d) = V_d\nabla^2 V_d + \nabla V_d\cdot\nabla V_d \qquad -(7)$

Using eq(4)

$\nabla\cdot(V_d\nabla V_d) = \nabla V_d\cdot\nabla V_d \qquad -(8)$

$\nabla\cdot V_d\nabla V_d = \nabla\cdot\bar{A} = \nabla V_d\cdot\nabla V_d$

$$\int_{vol} \nabla V_d\cdot\nabla V_d\, dv = \oint_S V_d\nabla V_d\cdot ds \qquad -(9)$$

4M

But $V_d = 0$ on boundry.

$$\therefore \int_{vol} \nabla V_d\cdot\nabla V_d\, dv = 0 \qquad -(10)$$

$$\therefore \int_{vol} |\nabla V_d|^2\, dv = 0 \quad \text{as } \nabla V_d \text{ is vector} \qquad -(11)$$

$|\nabla V_d|^2 = 0$ i.e., $\nabla V_d = 0 \qquad -(12)$

$V_d = V_2 - V_1$

$V_2 - V_1 = 0$

$\boxed{V_1 = V_2}$

6] (b) The line integral of a vector $\bar{A}$ around a closed path L is equal to the integral of curl of A over the open surface S enclosed by the closed path L

$$\boxed{\oint_L \bar{H}\cdot d\bar{L} = \int_S (\nabla\times\bar{H})\cdot d\bar{s}}$$

→ 2M

MAYUR

Teacher's Sign.

6] (b) According to Stoke's theorem

$$\oint_L \bar{H}\cdot d\bar{L} = \int_S (\nabla\times\bar{H})\cdot d\bar{S}$$

$$\oint \bar{H}\cdot d\bar{L} = \int_{ab} + \int_{bc} + \int_{cd} + \int_{da} \bar{H}\cdot d\bar{L}$$

$$\int_{ab} \bar{H}\cdot d\bar{L} = \int_{x=2}^{5} (6xy\,\bar{a}_x - 3y^2 \bar{a}_y)\cdot dx\,\bar{a}_x$$

$$= \int_{x=2}^{5} 6xy\,dx = 6y\left[\frac{x^2}{2}\right]_2^5 = \frac{6y}{2}[25-4] = 63y$$

5M

Now $y=-1$ for path ab, $\int_{ab} \bar{H}\cdot d\bar{L} = 63(-1) = -63$

Similarly $\int_{bc} \bar{H}\cdot d\bar{L} = \int_{y=-1}^{1} -3y^2\,dy = -\frac{3y^3}{3} = -[y^3]_{-1}^{1}$

$$= -[1-(-1)] = -2$$

$$\int_{cd} \bar{H}\cdot d\bar{L} = \int_{x=5}^{2} 6xy\,dx = 6\left[\frac{x^2}{2}\right]_5^2 (y) = \frac{6y}{2}[4-25] = -63y$$

But $y=1$ for path cd hence $\int_{cd} \bar{H}\cdot d\bar{L} = -63$

$$\int_{da} \bar{H}\cdot d\bar{L} = \int_{y=1}^{-1} -3y^2\,dy = -[y^3]_1^{-1} = -[(-1)^3-(1)^3] = -[-1-1]$$
$$= +2$$

$$\therefore \oint \bar{H}\cdot d\bar{L} = -63-2-63+2 = -126\text{ A}$$

Teacher's Sign.

6](b) Now evaluate right hand side

$$\nabla \times \bar{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 6xy & -3y^2 & 0 \end{vmatrix}$$

5M

$= \bar{a}_x [0-0] + \bar{a}_y [0-0] + \bar{a}_z [0-6x] = -6x\bar{a}_z$

$\therefore \int_S (\nabla \times \bar{H}) \cdot d\bar{s} = \int_S (-6x\bar{a}_z) \cdot (dx\,dy\,\bar{a}_z)$

$d\bar{s} = dx\,dy\,\bar{a}_z$ normal to direction $\bar{a}_z$

$\therefore \int_S (\nabla \times \bar{H}) \cdot d\bar{s} = \int_{y=-1}^{1} \int_{x=2}^{5} -6x\,dx\,dy = -6\left[\frac{x^2}{2}\right]_2^5 [y]_{-1}^{1}$

$= -\frac{6}{2}[25-4][1-(-1)] = -3 \times 21 \times 2 = -126\,A$

Thus both the side are same, hence Stoke's theorem is verified.

Module - 4

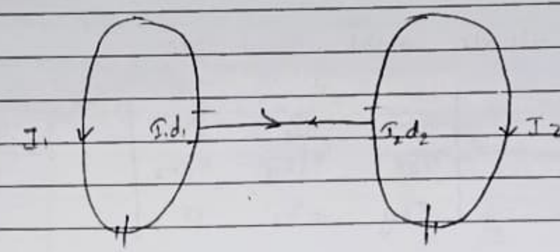
7]
(a) Let us now consider two current elements $I_1 d\bar{L}_1$ and $I_2 d\bar{L}_2$ as shown in below figure. Note that the directions of I_1 and I_2 are same.

Date : ....

Topic : .... Page No. ...29.......

KLS, VDIT, Haliyal

7] (a)



3M

From the equation of force the force extended on differential current element is given by

$$d(d\bar{F}_1) = I_1 d\bar{L}_1 \times d\bar{B}_2 \quad -(1)$$

According to Biot - Savart's law, the magnetic field produced by current element $I_2 d\bar{L}_2$ is given by for free space

$$d\bar{B}_2 = \mu_0 d\bar{H}_2 = \mu_0 \left[\frac{I_2 d\bar{L}_2 \times a_{R21}}{4\pi R_{21}^2}\right] \quad -(2)$$

Substituting value of $d\bar{B}_2$ in eq (1) we can write

$$d(d\bar{F}_1) = \mu_0 \frac{I_1 d\bar{L}_1 \times (I_2 d\bar{L}_2 \times \bar{a}_{R21})}{4\pi R_{21}^2} \quad -(3)$$

3M

$$\bar{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\bar{L}_1 \times (d\bar{L}_2 \times \bar{a}_{R21})}{R_{21}^2} \quad -(4)$$

Exactly following same steps, we can ~~calcu~~ calculate the force $\bar{F}_2$ extended on the current element 2.

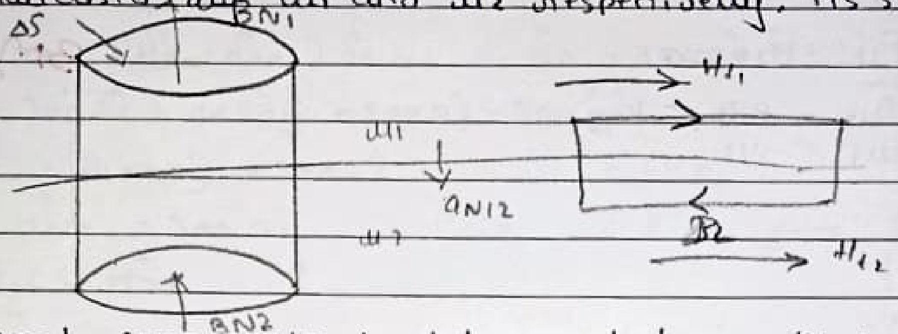
$$\bar{F}_2 = \frac{\mu_0 I_2 I_1}{4\pi} \oint_{L_2} \oint_{L_1} \frac{d\bar{L}_2 \times (d\bar{L}_1 \times \bar{a}_{R12})}{R_{12}^2} \quad -(5)$$

MAYUR

. Teacher's Sign.

KLS; VDIT; Haliyal

7] (b) Let us consider boundry between two isotropic media 1 and 2 characterised by μ_1 and μ_2 respectivelly. As shown in below fig.



2M

Normal components is determined by applying gauss law to the pill box and allowing $\Delta h \to 0$

$B_{1n} \Delta S - B_{2n} \Delta S = 0$

$\oint B \cdot d\vec{s} = B_{1n}\Delta S - B_{2n}\Delta S = 0$

$B_{1n} = B_{2n}$

$\mu_1 \vec{H}_{1n} = \mu_2 \vec{H}_{2n} \Rightarrow H_{2n} = \mu_1 / \mu_0 \; \vec{H}_{1n}$

To determine tangential component let us apply Ampere's circuit law about small closed path abcda

$\oint \vec{H} \cdot dl = I$

$(H_{1t} - H_{2t}) \Delta L = K \Delta L$

$H_{1t} - H_{2t} = K$

$(\vec{H}_1 - \vec{H}_2) \times \bar{a}_{n12} = \bar{k}$

An equivalent formule in terms of the vector tangential components

$\bar{H}_{1t} - \bar{H}_{2t} = \hat{a}_{n12} \times \bar{k}$

for tangential $\vec{B}$ we have

$\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = k$

3M

Tangential component of magnetaisation can be obtained as

$H_{1t} - H_{2t} = k$

$M_{2t} = \frac{X_{m2}}{X_{m1}} M_{1t} - X_{m2} k$

7](b) If boundry is free of current or the media are not conductors.

$$\bar{H}_{1t} - \bar{H}_{2t} = 0$$

$$\frac{\bar{B}_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$$

3M

7]

(c) (i) $\vec{F}_m = Q(\vec{v} \times \vec{B})$

$$\vec{v} \times \vec{B} = 5\times10^3 \begin{vmatrix} a_x & a_y & a_z \\ 0.6 & 0.7 & 0.3 \\ -3 & 4 & 6 \end{vmatrix}$$ — 3M

$\vec{v}\times\vec{B} = (3.3\hat{a}_x - 4.5\hat{a}_y + 4.05\hat{a}_z)\,5\times10^3$

$\vec{F}_m = Q\vec{v}\times\vec{B} = 8\times10^{-9}\times5\times10^3(3.3\hat{a}_x - 4.5\hat{a}_y + 4.65\hat{a}_z)$

$|\vec{F}_m| = 660\mu N$

(ii) $\vec{F}_e = Q\vec{E}$

$\vec{F}_e = 8\mu\langle -3, 4, 6\rangle = 140.6\ \mu N$

(iii) $\vec{F} = Q\vec{E} + Q\vec{v}\times\vec{B}$ — 3M

$\vec{F} = 18\mu\langle -3, 4, 6\rangle + 18\mu\times5\langle 3.3, -4.5, 4.65\rangle$

$\vec{F} = 18\mu\langle 13.5, -18.5, 29.25\rangle$

$\vec{F} = 668.7\mu N$

8]

1(a) (i) $M = \chi_m H$ $\quad \chi_m = \mu/\mu_0 - 1 \quad \mu = \mu_0\mu_r \Rightarrow \mu_r = \mu/\mu_0$

$M = \left[\frac{\mu}{\mu_0} - 1\right]H \quad M = 1599\ A/m$ 2M

(ii) $M = \lim_{\Delta v \to 0} \Sigma \vec{m}_k / \Delta v$

$M = 8.3\times10^{28}\times4.5\times10^{-27} = 3734\ A/m$ 2M

(iii) $B = 300\mu T$, $\chi_m = 15$ $\quad \mu_r = 1+\chi_m = 16$

$M = \chi_m \vec{H}$

$M = \chi_m \frac{\vec{B}}{\mu_0\mu_r} \Rightarrow 16\times\frac{300\mu}{4\pi\times10^{-7}\times16} = 224\ A/m$ 2M

KLS ; VDIT ; Haliyal

8]

(b) $\vec{H}_{ta} = \vec{H}_A - \vec{H}_{NA}$

$\vec{H}_{ta} = \vec{H}_A - (\vec{H}_A \cdot \hat{a}_{n12})\hat{a}_{n12} = \vec{H}_A - 300\hat{a}_x$

$\vec{H}_{ta} = 300\hat{a}_x - 400\hat{a}_y + 500\hat{a}_z - 300\hat{a}_x$

$\vec{H}_{ta} = -400\hat{a}_y + 500\hat{a}_z \qquad \vec{H}_{ta} = 640\ A/m$

$\vec{H}_{NA} = 300\hat{a}_x \qquad H_{NA} = 300\ A/m$ ← 4M,

$\bar{H}_{1t} - \bar{H}_{2t} = \hat{a}_{n12} \times \vec{K}$

$\bar{H}_{At} - \bar{H}_{Bt} = \hat{a}_{n12} \times \vec{K}$

$\vec{H}_{Bt} = -\hat{a}_{n12} \times \vec{K} + \vec{H}_{At}$

$\vec{H}_{Bt} = -150\hat{a}_z - 200\hat{a}_y + (-400\hat{a}_y + 500\hat{a}_z) = -600\hat{a}_y + 350\hat{a}_z$

$\vec{H}_{Bt} = 695\ A/m$

$\frac{H_{NA}}{H_{NB}} = \frac{\mu_2}{\mu_1}$

$\frac{300\,a_x}{H_{NB}} = \frac{20}{5}$ ← 4M

$H_{NB} = 75\hat{a}_x\ A/m$

8]

(c) The EMF induced in a closed path is proportional to the rate of change of magnetic flux enclosed by the closed path.

$e = -N\frac{d\phi}{dt}$ volts — 3M,

N-> no. of turns in the circuit e-> Induced EMF

$Emf = \oint \vec{E} \cdot d\vec{L} = -\int_S \frac{\partial B}{\partial t} \cdot d\bar{s}$

$\Rightarrow \oint_S (\nabla \times \bar{E}) \cdot d\bar{s} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$

KLS's VDIT, Haliyal

8]

(c) Assuming that both the surface integral taken over identical surfaces.

$(\nabla \times \bar{E}) \cdot d\bar{s} = \frac{-\partial \vec{B}}{\partial t} \cdot d\bar{s}$

Hence finally

$\nabla \times \vec{E} = -\partial \vec{B} / \partial t$

3M

Module - 5

9]

(a) Point form

$\nabla \cdot \vec{D} = \rho$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\partial \vec{B} / \partial t$

$\nabla \times \vec{H} = \partial \vec{D} / \partial t$

(2+2) 4M

Integral form

$\oint \vec{D} \cdot d\vec{s} = Q$

$\oint \vec{B} \cdot d\vec{s} = 0$

$\oint \vec{E} \cdot d\vec{l} = -\int_S \partial \vec{B} / \partial t \cdot d\vec{s}$

$\oint \vec{H} \cdot d\vec{l} = \int_S \partial \vec{D} / \partial t \cdot d\vec{s}$

(2+2) 4M

Explanation carrier 2M

9]

(c) $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ — 2M

$\delta = \frac{1}{\sqrt{\pi f (\mu_r \mu_0) \sigma}}$ — 2M

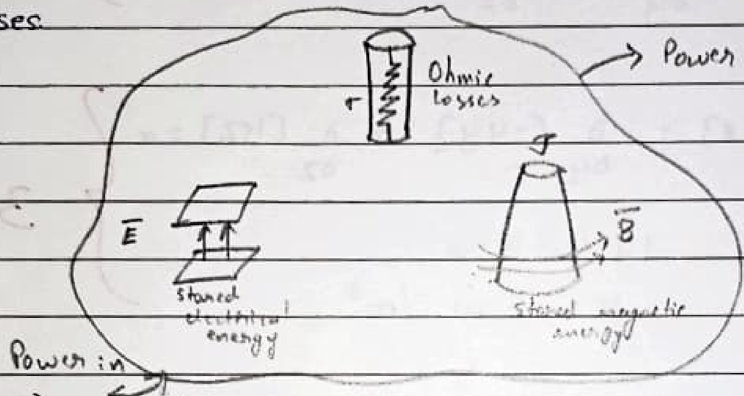
$1 \times 10^{-3} = \frac{1}{\sqrt{\pi \times f \times 1 \times 4 \times \pi \times 10^{-7} \times 3 \times 10^{6}}}$ — 2M

$f = 84.4 \text{ kHz}$

KLS, VDIT, Haliyal

1Q]

(a) Theorem states that the net power flowing out of the given volume v is equal to the time rate of decrease in the energy stored within v minus the ohmic losses.



4M

As $\vec{E}$ and $\vec{H}$ are both are vectors, to get power density we may carry out either dot product or cross product. As power flows in certain direction, it is a vector quantity so we use the cross product as the output of dot product is scalar.

∴ The power density is given by

$\vec{P} = \vec{E} \times \vec{H}$

where $\vec{P}$ is called poynting vector.

Poynting theorem in point form

$$-\nabla \cdot \vec{P} = \sigma E^2 + \frac{1}{2}\frac{\partial}{\partial t}[\mu H^2 - \epsilon E^2]$$

Poynting theorem in integral from

$$-\oint_S \vec{P} \cdot d\vec{s} = \int_v \sigma E^2 dv + \frac{\partial}{\partial t}\int_v \frac{1}{2}[\mu H^2 + \epsilon E^2] dv$$

4M

10]
(b) $\nabla \cdot \vec{D} = \rho_v = 0$

$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0$ } 3M

$\frac{\partial}{\partial x}[10x] + \frac{\partial}{\partial y}[-4y] + \frac{\partial}{\partial z}[kz] = 0$

$10 - 4 + k = 0$

$k = -6 \ \mu C/m^3$ } 3M

9]
(b) The wave propogates along +z direction -

$f = 200$ MHz —2M

phase constant = 4.19 rad/sec —2M

$E(z,t) = a_x \, 0.94 \cos(1.257 \times 10^9 t - 4.19z)$ V/m —2M

$E_s(z) = a_x \, 0.94 \, e^{-j4.19z}$ V/m —2M

$H_s(z) = a_y \, 2.5 \, e^{-j4.19z}$ mA/m —2M

[10] (c) $\frac{\sigma}{\omega \epsilon} > 1$ hence it is a good conductor

$\alpha = \beta = \sqrt{\frac{\omega \mu_0 \mu_r \sigma}{2}} = 2513$

$\eta = \sqrt{\frac{\omega \mu_0 \mu_r}{\sigma}} \angle 45 = 35.5 \angle 45$ } 2M

$v = \frac{\omega}{\beta} = 4 \times 10^7$ m/s

$\delta = \frac{1}{\sqrt{\pi f \mu_0 \mu_r \sigma}} = 2513$ } 4M

Scheme and solution prepared by. Prof. Nikhil K

(Nikhil K)
Staff

HoD
Head of the Department
Dept. of Electronic & Communication Engg.
KLS V.D.I.T., HALIYAL (U.K.)

Dean(A).
Dean, Academics
KLS VDIT, HALIYAL

Teacher's Sign.