CBCS SCHEME

USN 2 V D 2 1 E C 0 0 8 **21EC33**

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Basic Signal Processing

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Explain vector spaces and its necessary axioms. And also explain four fundamental subspaces with example. **(08 Marks)**

b. Write the vector $V = (1, 3, 9)$ as a linear combination of the vectors $u_1 = (2, 1, 3)$, $u_2 = (1, -1, 1)$ and $u_3 = (3, 1, 5)$ and thereby show that the system is consistent. **(08 Marks)**

c. Let $f : V_1(R) \rightarrow V_2(R)$ be a mapping $f(x) = (3x, 5x)$ show that 'f' is linear transformation. **(04 Marks)**

OR

2 a. Let 'w' be the subspace of R^5 spanned by
$x_1 = (1, 2, -1, 3, 4)$, $x_2 = (2, 4, -2, 6, 8)$, $x_3 = (1, 3, 2, 2, 6)$,
$x_4 = (1, 4, 5, 1, 8)$, $x_5 = (2, 7, 3, 3, 9)$.
Find a subset of vectors which forms a basis of 'w'. **(06 Marks)**

b. Solve $Ax = b$ by least square and find $P = A\hat{x}$ if

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}_{3\times 2} \quad \text{and} \quad b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}_{3\times 1}$$

. Also, write a program to solve linear equation $Ax = b$. **(07 Marks)**

c. Apply Gram – Schmidth process to the vectors $V_1(1, 1, 1)$, $V_2(1, -1, 2)$, $V_3(2, 1, 2)$ to obtain an orthonormal basis for $V_3(R)$ with standard inner product and thereby write a program for Gram – Schmidth process. **(07 Marks)**

Module-2

3 a. If $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ find Eigen values and corresponding Eigen vector for matrix 'A' and diagonalize the matrix. **(10 Marks)**

b. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. Show that matrix 'A' is positive definite matrix using the following approaches :

i) By finding its Eigen value
ii) By finding its pivots. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

4 a. Compute A^TA and AA^T, find Eigen values and Eigen vectors, if $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}_{3\times 2}$, thereby multiply $U \Sigma V^T$ to recover matrix 'A'. Also write a program to find SVD. **(12 Marks)**

b. Diagonalize the matrix A, if $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ by finding its eigen value and eigen vector. **(08 Marks)**

Module-3

5 a. Define signal and system and also explain basic discrete elementary signals with neat sketch and expressions. **(04 Marks)**

b. A discrete time signal x(n) is shown below Fig.Q5(b).

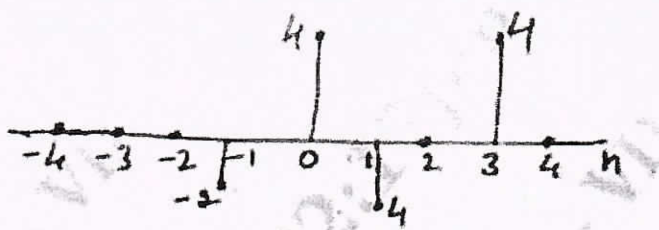

Fig.Q5(b)

Sketch :
i) $2x(n-2)$
ii) $3 - x(n)$
iii) $2x(-n) - 4$. **(08 Marks)**

c. Sketch : $x(n) = \begin{cases} 1; & -1 \le n \le 3 \\ \frac{1}{2}; & n = 4 \\ 0; & \text{otherwise} \end{cases}$ and $y(n) = \begin{cases} \frac{1}{2}n; & |n| \le 4 \\ 0; & \text{othewise} \end{cases}$

Also sketch $x(n+2)\,y(1-2n)$. **(08 Marks)**

OR

6 a. For the following discrete time systems, determine whether the system is linear, time invariance, memoryless, causal and stable :

i) $y(n) = 2x(n) + \dfrac{1}{x(n-2)}$

ii) $y(n) = \ln(3 + |x(n)|)$

iii) $y(n) = \cos x(n)$

iv) $y(n) = r^n x(n)\,;\ r > 1$. **(16 Marks)**

b. Write a program to generate exponential and triangular waveforms. **(04 Marks)**

Module-4

7 a. Compute the discrete time convolution for the sequences $x_1(n)$ and $x_2(n)$ given below $x_1(n) = \alpha^n u(n)$; $x_2(n) = \beta^n u(n)$. (08 Marks)

b. Consider the input signal $x(n)$ and the impulse response $h(n)$ given below ;

$$x(n) = \begin{cases} 1; & 0 \le n \le 4 \\ 0; & \text{othwerise} \end{cases} \quad \text{and} \quad h(n) = \begin{cases} \alpha^n; & 0 \le n \le 6, \alpha > 1 \\ 0; & \text{othwerise} \end{cases}$$

compute the output signal $y(n)$. (12 Marks)

OR

8 a. The following are the impulse responses of discrete time LTI systems. Determine whether each system is memoryless, causal and stable :

i) $h(n) = e^{-n}\cos(n)\cdot u(n)$

ii) $h(n) = (0.99)^n u(n+3)$

iii) $h(n) = n\left(\frac{1}{2}\right)^n u(n)$. (10 Marks)

b. Evaluate the step response of LTI system represented by the impulse response $h(n) = (-1)^n \{u(n+2) - u(n-3)\}$.

Also write a program to compute the step response from the given impulse response. (10 Marks)

Module-5

9 a. Define Z-transform. Explain the properties of ROC. (06 Marks)

b. Let $x(n) = \left(\frac{1}{2}\right)^{|n|}$.

i) Sketch $x(n)$

ii) Find $x(z)$ and sketch pole zero plot and ROC (08 Marks)

c. Find the Z-transform of $x(n) = \left(+\frac{1}{2}\right)^n u(n) * \left(\frac{1}{3}\right)^n u(n)$ (06 Marks)

OR

10 a. Explain the properties of Z-transform with proof :

i) Convolution

ii) Initial value theorem

iii) Final value theorem. (08 Marks)

b. Determine the describe time sequence $x(n)$ of the sequence using partial fraction expression:

$$X(z) = \frac{-1 + 5z^{-1}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}; \text{ROC}: |z| > 1$$ (08 Marks)

c. Write a program to find Z-transform of the sequence. (04 Marks)

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Department of Electronics and Communication Engg

Subject: Basic Signal Processing Jan/Feb 2023

Scheme and Solution.

Subject code: 21 EC33

Sem: 3

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Module 1

1 a Vector Space

It is a non empty set V of objects called vectors on which are two defined two operations called addition and multiplication by scalars.

Axioms:

1. Closed under Addition
2. Commutative under addition
3. Associative under addition
4. Additive Identity : There exist 0 in V, such that $u + 0 = u$, for all u in V.
5. Additive Inverse: for all u in V, there exists $-u$ in V such that $u + (-u) = 0$.
6. closed under scalar multiplication
7. Multiplicative Identity
8. Associative under Scalar multiplication
9. Distributive under scalar multiplication

Fundamental Subspaces:

1. Column space of A, is denoted by C(A). Its dimension is rank r.
2. Null space of A denoted by N(A), Its dimension is $n-r$.
3. Row space of A is column space of A^T. It is $C(A^T)$ and is spanned by rows of A. Its dimension is also r.
4. Left null space of A is null space of A^T. It contains all vectors y such that $A^T y = 0$. and it is written $N(A^T)$. Its dimension is $m-r$.

1 b. $au_1 + bu_2 + cu_3 = v$

$$a\begin{bmatrix}2\\1\\3\end{bmatrix} + b\begin{bmatrix}1\\-1\\1\end{bmatrix} + c\begin{bmatrix}3\\1\\5\end{bmatrix} = \begin{bmatrix}1\\3\\9\end{bmatrix}$$

Equations are,

$$2a + b + 3c = 1$$
$$a - b + c = 3$$
$$3a + b + 5c = 9$$

Solving above equations,

$$a = -12$$
$$b = -5$$
$$c = 10$$

Since components of u_1 are coefficients of a.
Components of u_2 are coefficients of b
Components of u_3 are coefficients of c
The given vector v is a linear combination of u_1, u_2 & u_3
from above equations, it is realized, the system is consistent and it has at least one solutions.

c. Given $f(x) = (3x, 5x)$

i) $f(x_1 + x_2) = \{3(x_1 + x_2), 5(x_1 + x_2)\}$

$= (3x_1 + 3x_2, 5x_1 + 5x_2)$

$= (3x_1, 5x_1) + (3x_2, 5x_2)$

$f(x_1 + x_2) = f(x_1) + f(x_2)$

(ii) $f(ax) = \{3(ax), 5(ax)\}$

$= (3ax, 5ax)$

$= a(3x, 5x)$

$f(ax) = a(f(x)$, f is linear transformation of x.

2 a. $x_1 = (1, 2, -1, 3, 4)$

$x_2 = (2, 4, -2, 6, 8)$

$x_3 = (1, 3, 2, 2, 6)$

$x_4 = (1, 4, 5, 1, 8)$

$x_5 = (2, 7, 3, 3, 9)$

Let $W = \begin{bmatrix} 1 & 2 & -1 & 3 & 4 \\ 2 & 4 & -2 & 6 & 8 \\ 1 & 3 & 2 & 2 & 6 \\ 1 & 4 & 5 & 1 & 8 \\ 2 & 7 & 3 & 3 & 9 \end{bmatrix}$

$R_2 = R_2/2 - R_1 \qquad R_3 \to R_3 - R_1$

$W = \begin{bmatrix} 1 & 2 & -1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & -1 & 2 \\ 1 & 4 & 5 & 1 & 8 \\ 2 & 7 & 3 & 3 & 9 \end{bmatrix}$

$R_4 \to R_4 - R_1 \quad , \quad R_5 \to R_5 - 2R_1$

$W = \begin{bmatrix} 1 & 2 & -1 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 2 & 6 & -2 & 4 \\ 0 & 3 & 5 & -3 & 1 \end{bmatrix}$

$R_4 \to R_4 - 2R_3 \quad , \quad R_2 \leftrightarrow R_3$

$W = \begin{bmatrix} 1 & 0 & -7 & 5 & 0 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 5 & -3 & 1 \end{bmatrix}$

$R_5 - 3R_2 \leftarrow R_5$

$W = \begin{bmatrix} 1 & 0 & -7 & 5 & 0 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & -5 \end{bmatrix}$

$R_3 \longleftrightarrow R_5$

$$W = \begin{bmatrix} 1 & 0 & -7 & 5 & 0 \\ 0 & 1 & 3 & -1 & 2 \\ 0 & 0 & -4 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The above matrix is row reduced, Hence basis of W will be

$\{(1, 0, -7, 5, 0), (0, 1, 3, -1, 2),$
$(0, 0, -4, 0, -5)\}$

2 b.

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A\vec{x} = b$$

$$A^T A \vec{x} = A^T b$$

$$A^T A = \begin{bmatrix} 17 & 1 \\ 1 & 5 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$$17x_1 + 1x_2 = 19 \qquad \begin{bmatrix} 17 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$$x_1 + 5x_2 = 11$$

$$x_1 = 1, \quad x_2 = 2$$

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$P = A\hat{x}$$

$$P = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$p = \begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$$

```
from sympy import symbols , Eq, solve
  x, y = symbols ('x, y')

  # defining equations
    eq1 = Eq ((x + y), 1)
    print ("Equation 1:")
    print (eq1)
    eq2 = Eq ((x - y), 1)
    print ("equation 2:")
    print (eq2)

  # solving the equations
    print ("Values of 2 unknown variable
            are as follows:")
    print (solve ((eq1, eq2), (x, y)))
```

2 c. $V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $V_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ $V_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

$$u_1 = \frac{V_1}{\|V_1\|} = \frac{(1, 1, 1)}{\sqrt{3}}$$

$$u_1 = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$u_2 = \frac{u_2'}{\| u_2' \|}$$

$$u_2' = v_2 - \text{proj}_{u_1}(v_2)$$

$$= v_2 - (u_1 v_2)\, u_1$$

$$= \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} - \left(\tfrac{1}{\sqrt{3}} \cdot 1 - \tfrac{1}{\sqrt{3}} \cdot 1 + \tfrac{1}{\sqrt{3}} \cdot 2 \right) \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2/3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

$$u_2' = \begin{bmatrix} 1/3 \\ -5/3 \\ 4/3 \end{bmatrix}$$

$$\| u_2' \| = \sqrt{14/3} = \sqrt{42}/3$$

$$u_2 = \begin{bmatrix} 1/\sqrt{42} \\ -5/\sqrt{42} \\ 4/\sqrt{42} \end{bmatrix}$$

$$u_3 = \frac{u_3'}{\| u_3 \|}$$

$$u_3' = v_3 - \text{proj}_{u_1}(v_3) - \text{proj}_{u_2}(v_3)$$

$$= v_3 - (u_1 v_3)\, u_1 - (u_2 \cdot v_3) \cdot u_2$$

$$= \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} - \left(2/\sqrt{3} + 1/\sqrt{3} + 2/\sqrt{3} \right) \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} - \left(2/\sqrt{42} - 5/\sqrt{42} + 8/\sqrt{42} \right) \begin{bmatrix} 1/\sqrt{42} \\ -5/\sqrt{42} \\ 4/\sqrt{42} \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} - 5/\sqrt{3} \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} - 5/\sqrt{42} \begin{bmatrix} 1/\sqrt{42} \\ -5/\sqrt{42} \\ 4/\sqrt{42} \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 5/3 \\ 5/3 \\ 5/3 \end{bmatrix} - \begin{bmatrix} 5/42 \\ -25/42 \\ 20/42 \end{bmatrix}$$

$$u_3' = \begin{bmatrix} 3/14 \\ -1/14 \\ -1/7 \end{bmatrix}$$

$$\| u_3' \| = 1/\sqrt{14}$$

$$u_3 = \begin{bmatrix} 3/\sqrt{14} \\ -1/\sqrt{14} \\ -\sqrt{14}/7 \end{bmatrix}$$

$$u_1 = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \quad u_2 = \begin{bmatrix} 1/\sqrt{42} \\ -5/\sqrt{42} \\ 4/\sqrt{42} \end{bmatrix} \quad u_3 = \begin{bmatrix} 3/\sqrt{14} \\ -1/\sqrt{14} \\ -\sqrt{14}/7 \end{bmatrix}$$

```
import numpy as np
def gram-schmidt (A):
 """ Orthogomalize a set of vectors"""
 n = A.shape[1]
 for j in range(n):
     for k in range (j ):
         A [ : , j ]  = np.dot (A [ :, k ], A [ :, j ]) * A [ :, k]
         A [ : , j ] = A [ : , j ] / np. linalg. norm(A [ : , j ])
 return A
 if  name == ' main' :
 A = np. array([[1.0, 1.0, 2.0 ], [1.0, -1.0, 1.0],
                [ 1.0, 2.0, 2.0]])
 print ( gram-schmidt ( A))
```

3 a.

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \lambda^3 - S1\lambda^2 + S2\lambda - S3$$

$$S1 = 4 + 3 + 1 = 8$$

$$S2 = (3-8) + (4-4) + (12+10) = 17$$

$$S3 = 4(3-8) - 2(-5+4) - 2(-20+6)$$

$$= 4(-5) - 2(-1) - 2(-14)$$

$$S3 = 10$$

$$|A - \lambda I| = \lambda^3 - 8\lambda^2 + 17\lambda - 10 = 0$$

By trial & Error method $\lambda = 5$ is a root of a above equation,

$$\begin{array}{c|cccc} 5 & 1 & -8 & 17 & -10 \\ & 0 & 5 & -15 & 10 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

$$\therefore \lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda-2) - (\lambda-2) = 0$$

$$(\lambda-1)(\lambda-2) = 0$$

$$\lambda = 1, \quad \lambda = 2$$

Hence the roots are $\lambda_1 = 5$, $\lambda_2 = 2$, $\lambda_3 = 1$

for $\lambda_1 = 5$

$$A - \lambda_1 I = A - 5I = \begin{bmatrix} -1 & 2 & -2 \\ -5 & -2 & 2 \\ -2 & 4 & -4 \end{bmatrix}$$

$$[A - \lambda_1 I]\,\vec{x} = 0$$

$$\begin{bmatrix} -1 & 2 & -2 \\ -5 & -2 & 2 \\ -2 & 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-x_1 + 2x_2 - 2x_3 = 0$$

$$-5x_1 - 2x_2 + 2x_3 = 0$$

By Cramer's Rule,

$$\frac{x_1}{\begin{vmatrix} +2 & -2 \\ -2 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & -2 \\ -5 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 2 \\ -5 & -2 \end{vmatrix}}$$

$$\frac{x_1}{0} = \frac{-x_2}{-12} = \frac{x_3}{12}$$

$$\frac{x_1}{0} = \frac{+x_2}{1} = \frac{x_3}{1}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

for $\lambda_2 = 2$

$$[A - \lambda_2 I] = A - 2I = \begin{bmatrix} 2 & -2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix}$$

$$[A - 2I]\,\vec{y} = 0$$

$$\begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 0$$

$$2y_1 + 2y_2 - 2y_3 = 0$$
$$-5y_1 + y_2 + 2y_3 = 0$$

$$\frac{y_1}{\begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-y_2}{\begin{vmatrix} 2 & -2 \\ -5 & 2 \end{vmatrix}} = \frac{y_3}{\begin{vmatrix} 2 & 2 \\ -5 & 1 \end{vmatrix}}$$

$$\frac{y_1}{6} = \frac{-y_2}{-6} = \frac{y_3}{12}$$

$$\frac{y_1}{1} = \frac{y_2}{1} = \frac{y_3}{2}$$

$$\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

for $\lambda_3 = 1$

$$A - \lambda_3 I = A - I = \begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix}$$

$$[A - I]\vec{z} = 0$$

$$\begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = 0$$

$$3z_1 + 2z_2 - 2z_3 = 0$$
$$-5z_1 + 2z_2 + 2z_3 = 0$$

$$\frac{z_1}{\begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix}} = \frac{-z_2}{\begin{vmatrix} 3 & -2 \\ -5 & 2 \end{vmatrix}} = \frac{z_3}{\begin{vmatrix} 3 & 2 \\ -5 & 2 \end{vmatrix}}$$

$$\frac{z_1}{8} = \frac{-z_2}{-4} = \frac{z_3}{16}$$

$$\frac{z_1}{2} = \frac{z_2}{1} = \frac{z_3}{4} \qquad \vec{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

Q//

$S3 = 3(15+1) + (-3+1) + (-1-5)$

$= 48 - 2 - 6$

$S3 = 40.$

$|A - \lambda I| = \lambda^3 - 11\lambda^2 + 38\lambda - 40 = 0$

By trial & Error method,

$\lambda = 2$ is a factor for above quation.

2	1	-11	38	-40
	0	2	-18	40
	1	-9	20	0

$\lambda^2 - 9\lambda + 20 = 0$

$\lambda^2 - 5\lambda - 4\lambda + 20 = 0$

$\lambda(\lambda - 5) - 4(\lambda - 5) = 0$

$(\lambda - 4)(\lambda - 5) = 0$

$\lambda = 4, \quad \lambda = 5$

The roots are $\lambda = 2, 4, 5$

Since the eigen values are greater than zero, the matrix is said to be positive definite.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$R_2 \rightarrow 3R_2 + R_1 \qquad R_3 \rightarrow 3R_3 - R_1$

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 14 & -2 \\ 0 & -2 & 8 \end{bmatrix}$$

$R_2 \rightarrow R_2/2, \quad R_3 \rightarrow R_3/2$

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 7 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 2 & -2 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A = P D P^{-1}$$

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

which is diagonalized matrix

3 b.

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & 1 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & 1 & 3-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \lambda^3 - S1\lambda^2 + S2\lambda - S3 = 0$$

$$S1 = 3 + 5 + 3 = 11$$

$$S2 = (15+1) + (9-1) + (15-1) = 38$$

$R_3 \rightarrow 7R_3 + R_2$

$$A = \begin{bmatrix} 8 & -1 & 1 \\ 0 & 7 & -1 \\ 0 & 0 & 27 \end{bmatrix}$$

The pivots are 8, 7, 27, since pivots are greater then zero matrix is positive definite.

4 a)

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A A^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A A^T = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$A^T A - \lambda I = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$A^T A - \lambda I = \begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix}$$

$$|A^T A - \lambda I| = 0$$

$$(2-\lambda)(3-\lambda) = 0$$

$$\lambda_1 = 3, \quad \lambda_2 = 2$$

for $\lambda_1 = 3$

$$A^T A - \lambda_1 I = A^T A - 3I = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[A^T A - 3I]\,\vec{V_1} = 0$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = 0$$

Let $V_{12} = t$ then $V_{11} = 0$

Thus, $\vec{V_1} = \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

for $\lambda_2 = 2$

$$A^T A - \lambda_2 I = A^T A - 2I = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A^T A - 2I]\,\vec{V_2} = 0$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = 0$$

Let $V_{21} = -1$

then $V_{22} = 0$

$$\vec{V_2} = \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Normalized vectors are

$$\vec{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \vec{V_2} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad V^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix}$$

$$AA^T - \lambda I = \begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{bmatrix}$$

$$\left[AA^T - \lambda I\right] = 0$$

$$\lambda^3 - S1\lambda^2 + S2\lambda - S_3 = 0$$

$$S1 = 2+1+2 = 5$$

$$S2 = (2-1) + (4) + (2-1) = 6$$

$$S3 = 0$$

$$|AA^T - \lambda I| = \lambda^3 - 5\lambda^2 + 6\lambda = 0$$

$$\lambda(\lambda^2 - 5\lambda + 6) = 0$$

$$\lambda(\lambda^2 - 3\lambda - 2\lambda + 6) = 0$$

$$\lambda(\lambda(\lambda-3) - 2(\lambda-3)) = 0$$

$$\lambda(\lambda-3)(\lambda-2) = 0$$

Hence Eigen Values are,

$$\lambda_1 = 3, \quad \lambda_2 = 2, \quad \lambda_3 = 0$$

for $\lambda_1 = 3$

$$AA^T - \lambda_1 I = AA^T - 3I = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$(AA^T - 3I)\vec{u}_1 = 0$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & (-2) & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \\ u_{13} \end{bmatrix} = 0$$

$$-u_{11} + u_{12} + u_{13} \cdot 0 = 0$$

$$u_{11} - 2u_{12} + u_{13} = 0$$

$$\frac{u_{11}}{\begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix}} = \frac{-u_{12}}{\begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix}} = \frac{u_{13}}{\begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix}}$$

$$\frac{u_{11}}{1} = \frac{-u_{12}}{-1} = \frac{u_{13}}{1}$$

$$\frac{u_{11}}{1} = \frac{u_{12}}{1} = \frac{u_{13}}{1}$$

$$\vec{u_1} = \begin{bmatrix} u_{11} \\ u_{12} \\ u_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

for $\lambda_2 = 2$,

$$AA^T - \lambda_2 I = AA^T - 2I = 0$$

$$AA^T - 2I = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[AA^T - 2I]\,\vec{u_2} = 0$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \\ u_{23} \end{bmatrix} = 0$$

$$0 \cdot u_{21} + u_{22} + 0 \cdot u_{23} = 0$$

$$u_{21} - u_{22} + u_{23} = 0$$

$$\frac{u_{21}}{\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix}} = \frac{-u_{22}}{\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}} = \frac{u_{23}}{\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{u_{21}}{-1} = \frac{-u_{22}}{0} = \frac{u_{23}}{+1}$$

$$\frac{u_{21}}{-1} = \frac{u_{22}}{0} = \frac{u_{23}}{+1}$$

$$\vec{u_2} = \begin{bmatrix} u_{21} \\ u_{22} \\ u_{23} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ +1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ +1/\sqrt{2} \end{bmatrix}$$

For $\lambda_3 = 0$,

$$A A^T - \lambda_3 I = A A^T - 0 I = 0$$

$$A A^T - 0 I = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$(A A^T - 0 \cdot I)\vec{u_3} = 0$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{31} \\ u_{32} \\ u_{33} \end{bmatrix} = 0$$

$$2u_{31} + u_{32} + 0 \cdot u_{33} = 0$$

$$u_{31} + u_{32} + u_{33} = 0$$

$$\frac{u_{31}}{\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}} = \frac{-u_{32}}{\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix}} = \frac{u_{33}}{\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}}$$

$$\frac{u_{31}}{1} = \frac{+u_{32}}{-2} = \frac{u_{33}}{1}$$

$$\vec{u_3} = \begin{bmatrix} u_{31} \\ u_{32} \\ u_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

$$U = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & +1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$

$$U \Sigma V^T = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

```
# Singuler Value Decomposition
from numpy import array
from scipy.linalg import svd
# define a matrix
A = array([[3, 1, 1], [-1, 3, 1]])
print ("A=", A)
# SVD
U, S, V_T = svd(A)
# left singular vectors
print ("U=")
print (U)
# singular values
print ("S=")
print (S)
# right singular vectors,
print ('V_T = " ")
print (V_T)
```

4 b. $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$$

$$|A - \lambda I] = 0$$

$$\lambda^3 - S1\,\lambda^2 + S2\lambda - S3 = 0$$

$$S1 = -2 + 1 = -1$$

$$S2 = (-12) - (3) + (-2-4) = -21$$

$$S3 = 45$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

5	1	1	-21	-45
	0	5	30	45
	1	6	9	0

$$\lambda^2 - 6\lambda + 9 = 0$$

$$\lambda^2 - 3\lambda - 3\lambda + 9 = 0$$

$$\lambda(\lambda - 3) - 3(\lambda - 3) = 0$$

$$(\lambda - 3)\ (\lambda - 3) = 0$$

$$\lambda_1 = 5, \quad \lambda_2 = 3, \quad \lambda_3 = 3$$

for $\lambda_1 = 5$,

$$A - 5I = \begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}$$

$$[A - 5I]\,\vec{x} = 0$$

$$[A - 5I] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-7x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 - 4x_2 - 6x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 2 & -3 \\ -4 & -6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -7 & -3 \\ 2 & -6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & 2 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-12-12} = \frac{-x_2}{42+6} = \frac{x_3}{28-4}$$

$$\frac{x_1}{-24} = \frac{x_2}{-48} = \frac{x_3}{24}$$

$$\vec{x} = \begin{bmatrix} -24 \\ -48 \\ 24 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

$$[A + 3I]\vec{y} = 0$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 0$$

Reduced row Echelon form of matrix is,

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 0$$

Let $y_2 = t$, $y_3 = s$ then $y_1 = 3s - 2t$

$$\vec{y} = \begin{bmatrix} 3s-2t \\ t \\ s \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} s$$

$$\vec{y} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -1/8 & -1/4 & 3/8 \\ -1/4 & 1/2 & 3/4 \\ 1/8 & 1/4 & 5/8 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$A = P D P^{-1}$$

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} -1/8 & -1/4 & 3/8 \\ -1/4 & 1/2 & 3/4 \\ 1/8 & 1/4 & 5/8 \end{bmatrix}$$

5 a. Signal : A function of one or more independent variables which contain some information is called signal.

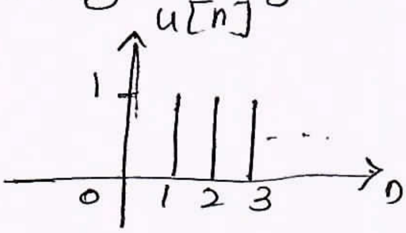
System : A system is a set of elements or functional blocks that are connected together and produces an output in response to an input signal.

Discrete elementary signals are,

i) Unit Step function :

The unit step signal has amplitude of "one" for positive value and "zero" amplitude for negative values.

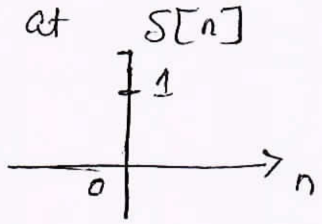
$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



(ii) Unit sample signal $\delta(n)$

Amplitude of unit sample is 1, at $n = 0$ & it has zero values at all other values of n.

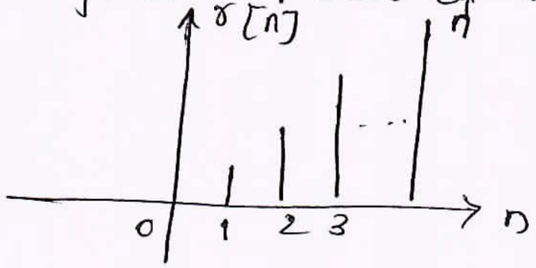
$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & n \neq 0 \end{cases}$$



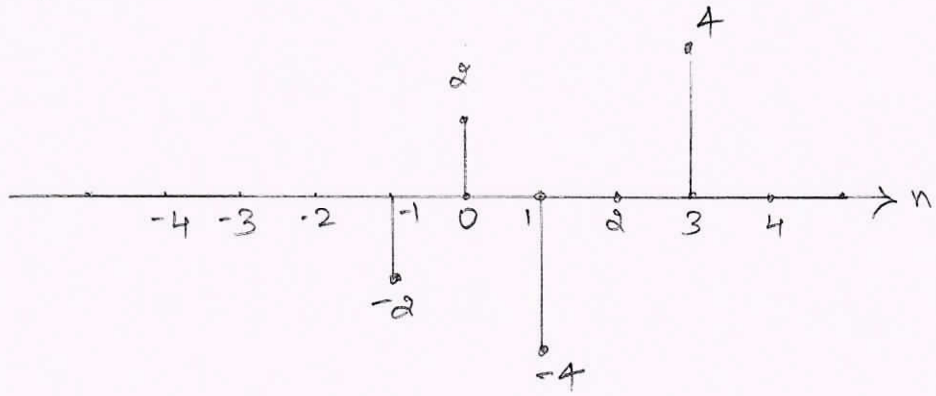
(iii) Unit Ramp function

The amplitude of every sample increases linearly with its number (n) for positive values of n.

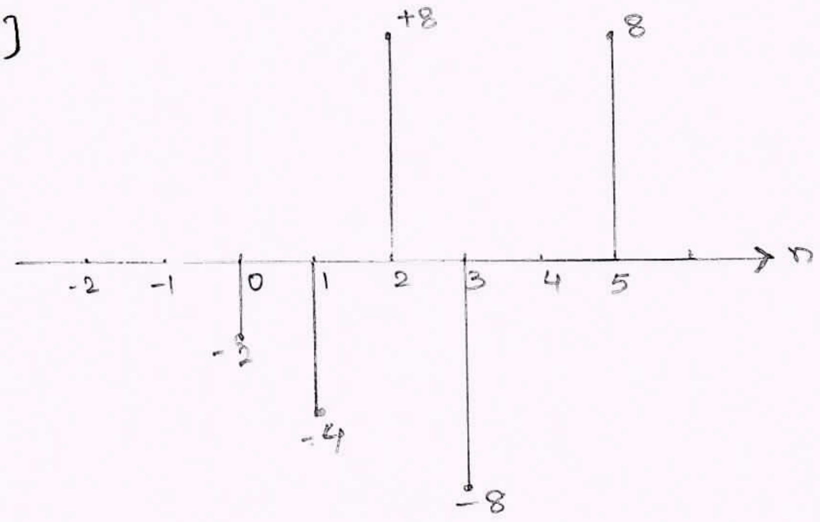
$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & n < 0 \end{cases}$$



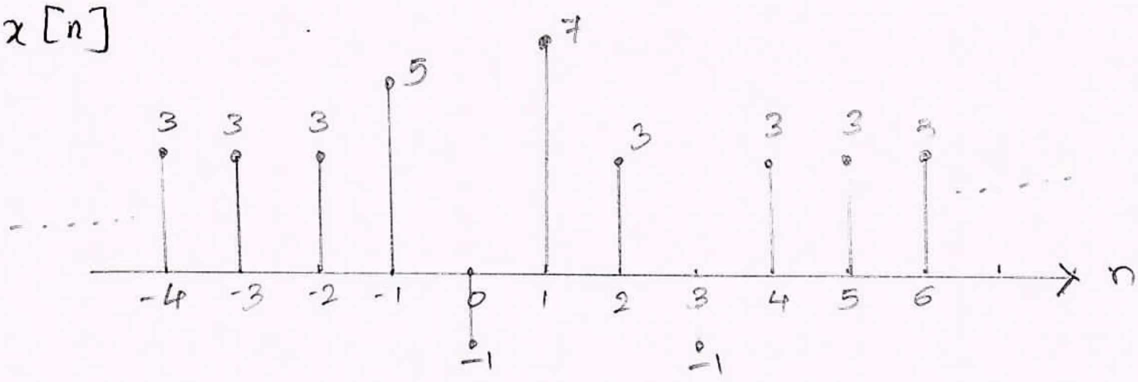
5b $x[n]$



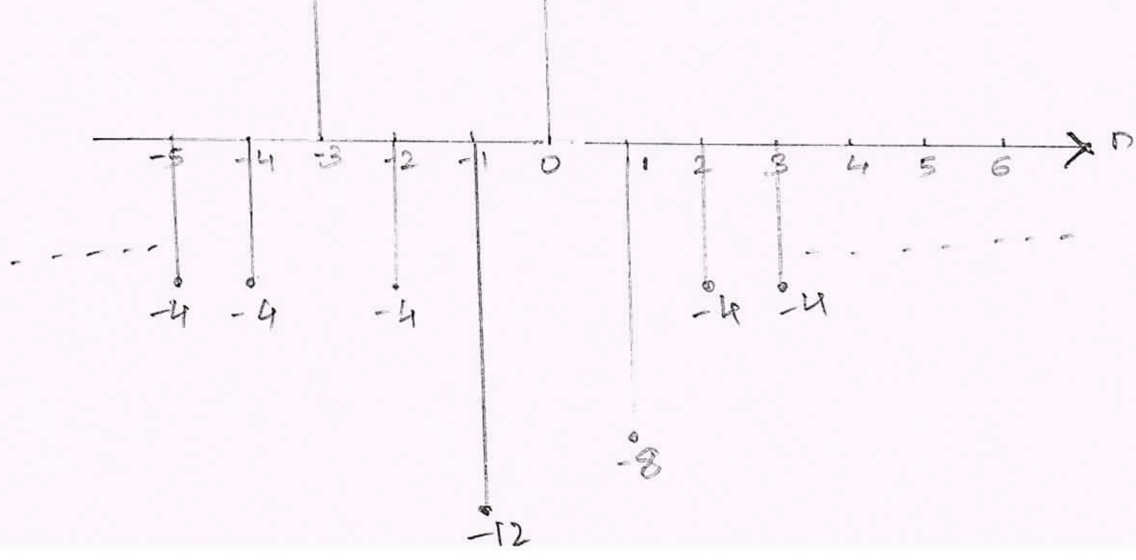
$2x[n-2]$



$3 - x[n]$



$2x[-n] - 4$

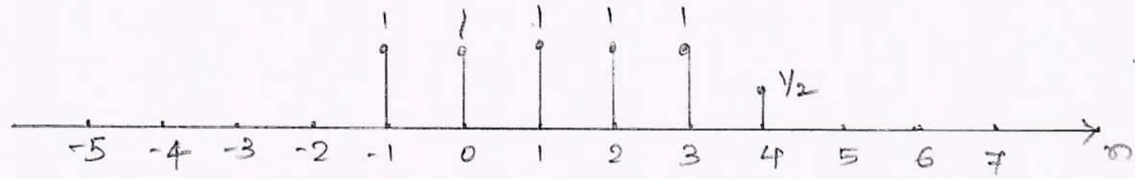


5C. Sketch $x[n] = \begin{cases} 1 & ; \quad -1 \leq n \leq 3 \\ 1/2 & ; \quad n = 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$

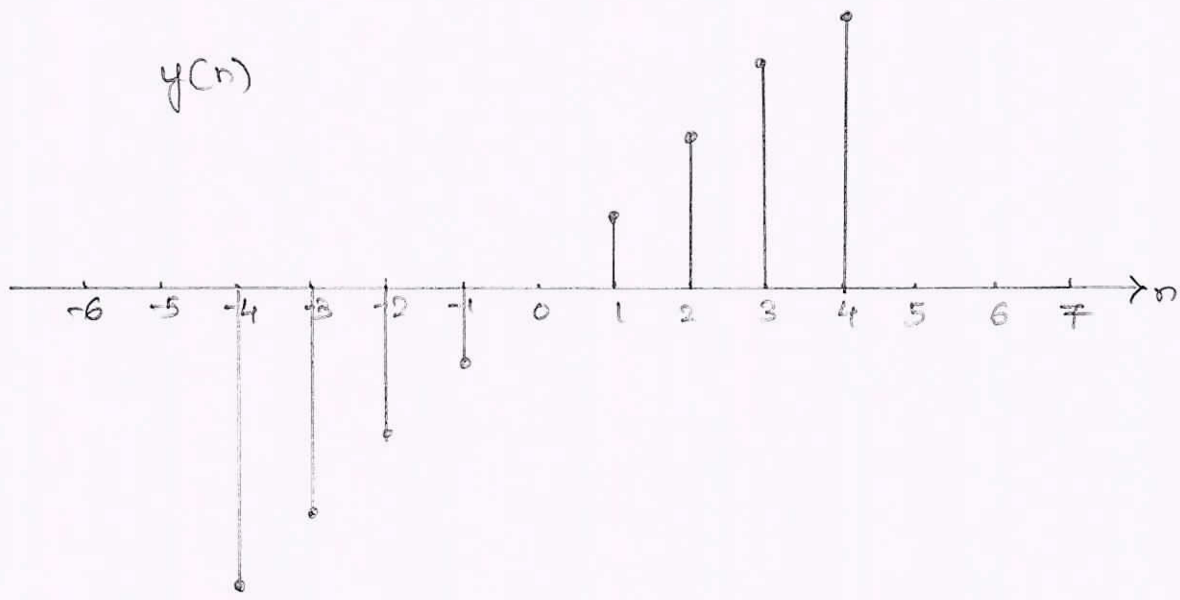
$y[n] = \begin{cases} 1/2\, n & ; \quad |n| \leq 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$

Also sketch $x[n+2]\ y[1-2n]$

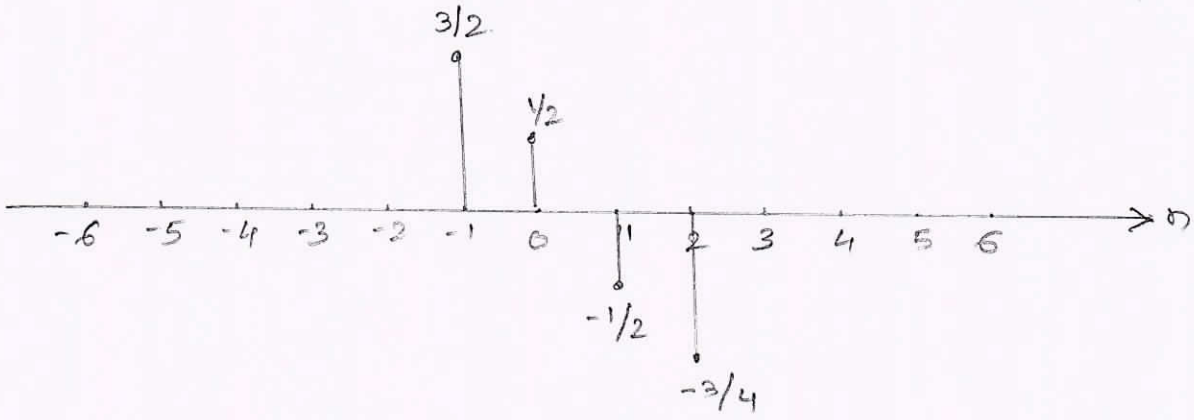
$x(n)$



$y(n)$



$x(n+2)\ \ y(1-2n)$



6 a. (i) $y[n] = 2x[n] + \frac{1}{x[n-2]}$

The $y[n]$ depends upon previous and present input. Hence it is Causal system.

$y[n] = T\left\{2x[n] + \frac{1}{x[n-2]}\right\}$

$y_3[n] \neq T y_3'[n]$

$\therefore$ System is non linear

Since $y[n]$ is a bounded output because $x[n]$ is also bounded therefore system is stable.

Let us apply the delayed input to the system.

$y[n,k] = T\left\{2(n-k) + \frac{1}{n-k-2}\right\}$

$y[n,k] = T\left\{2[n-k] + \frac{1}{n-k-2}\right\}$

$\therefore$ hence $y(n,k) = y(n-k]$

Hence system is time invariant

- system is memory system

ii) $y[n] = \ln[3 + |x[n]|)$

$y_1[n] = \ln[3 + |x_1[n]|)$

$y_2[n] = \ln[3 + |x_2[n]|)$

$y_3[n] = T[a_1 \ln[3 + |x_1[n]|] + a_2 \ln[3 + x_2[n]]$

$y_3'[n] = a_1 y_1[n] + a_2 y_2[n]$

$= a_1 \ln[3 + |x_1[n]|] + a_2 \ln[3 + |x_2[n]|]$

$y_3[n] = \ln[a_1[3 + x_1[n]] + a_2[3 + x_2[n]]$

$y_3'[n] \neq y_3[n]$

$\therefore$ System is non linear.

Since $y[n]$ not depends upon the present input
Hence system is noncausal.

$$y[n, k] = T\{\ln[3 + |x[n-k]|)\}$$

$$y[n, k] = T\{\ln[3 + |x'(n-k)|\}$$

hence $y[n-k] = y[n-k]$

The system is time invariant

Since $x[n]$ is a bounded input, hence $y[n]$ is a bounded output. Hence system is stable

It is memoryless system.

(iii) $y[n] = \cos x[n]$

Since $y[n]$ depends upon the present input, Hence the system is causal.

$$y_1[n] = \cos x_1[n]$$

$$y_2[n] = \cos x_2[n]$$

$$y_3[n] = \cos[x_1[n] + x_2[n]]$$

$$y_3'[n] = \cos x_1[n] + \cos x_2[n]$$

$$y_3[n] \neq y_3'[n]$$

The system is non linear

Since $x[n]$ is bounded input have $y[n]$ is bounded output.

Hence system is stable.

$$y[n, k] = \cos x[n-k]$$

$$y[n-k] = \cos x[n-k]$$

$$y[n-k] = y[n-k]$$

Hence system is time invariant.

It is memoryless system.

iv) $y[n] = \gamma^n x[n] \quad ; \quad \gamma > 1$

$y[n]$ depends upon the present input. Hence system is causal.

Here $\gamma > 1$, Hence $n \to \infty$, $\gamma^n \to \infty$

Therefore $y[n]$ will be unbounded even if $x[n]$ is bounded. Hence this is unstable.

$$y[n] = T\{\gamma^n x[n]\}$$

$$y_1[n] = T\{x_1[n]\} = \gamma^n x_1[n]$$

$$y_2[n] = T\{x_2[n]\} = \gamma^n x_2[n]$$

$$y_3[n] = T\{a_1 x_1[n] + a_2 x_2[n]\}$$

$$= a_1 \gamma^n x_1[n] + a_2 \gamma^n x_2[n]$$

$$y_3'[n] = a_1 y_1[n] + a_2 y_2[n]$$

$$y_3'[n] = a_1 \gamma^n x_1[n] + a_2 \gamma^n x_2[n]$$

$$\therefore y_3[n] = y_3'[n]$$

$\therefore$ The system is linear

$$y[n, k] = T\{x[n-k]\}$$

$$= \gamma^n x[n-k]$$

$$y[n-k] = \gamma^{n-k} x[n-k]$$

$$y[n, k] \neq y[n-k]$$

Hence the system is time variant

It is memoryless system.

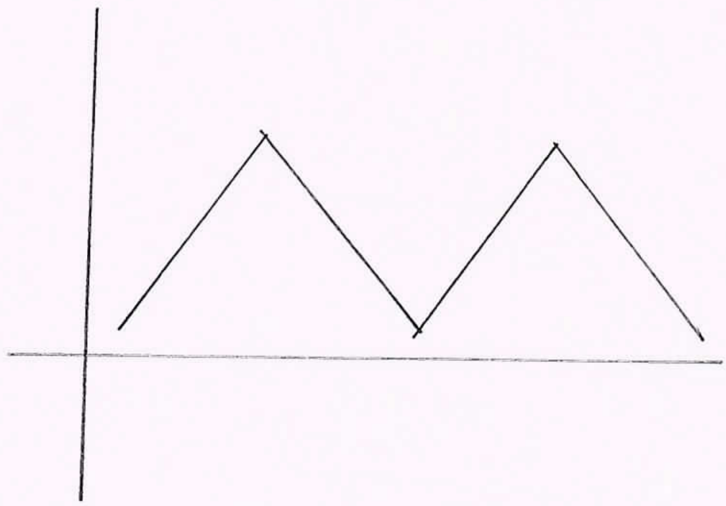
6 b.

```
import numpy as np
import matplotlib.pyplot as plt
def exponential (a,n):
    expo =[n]
    for sample in n:
        expo.append (np.exp (a * sample))
    return (expo)

a=2
UL=1
LL=-1
n=np.arrange (LL, UL, 0.1)
x= exponential (a,n)
plt.stem (n, x)
plt.xlabel ('n')
plt.xticks (np.arrange (LL, UL,0.2))
# plt.yticks [[0, UL,1]]
plt.ylabel ('x[n]')
plt.title (' Exponential signal e**(an)')
plt.savefig ("exponential.png")
```

6 b

```
# Trianguler Waveform Generation
import matplotlib.pyplot as plt
import numpy as np
% matplotlib inline
x = np.arange (0, 101)
def tri(x):
    res = x % 50
    if res < 25:
        return res
    else:
        return 50-res
plt.plot (x, list (map (tri, x)))
```



Module 4

- a
Sol^n

$$y[n] = \sum_{k=-\infty}^{\infty} x[n]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} a^n u[k]\, b^{n-k} u[n-k]$$

$$u[k] = \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$$

$$y[n] = \sum_{k=0}^{\infty} a^k b^{n-k} u[n-k]$$

$$u[n-k] = \begin{cases} 1 & \text{for } n \geq k \\ 0 & \text{for } n < k \text{ or } k > n \end{cases}$$

$$y[n] = \sum_{k=0}^{n} a^k b^{n-k} \quad \text{for } n \geq k \geq 0$$

$$= \sum_{k=0}^{n} a^k \cdot b^k \cdot b^{-k}$$

$$= b^n \sum_{k=0}^{n} (a \cdot b^{-1})^k$$

$$y[n] = b^n \frac{(ab^{-1})^{n+1} - 1}{(ab^{-1}) - 1}$$

$$\because \sum_{k=0}^{N} = \frac{a^{N+1} - 1}{a-1}$$

$$y[n] = b^n \frac{(a/b)^{n+1} - 1}{a/b - 1}$$

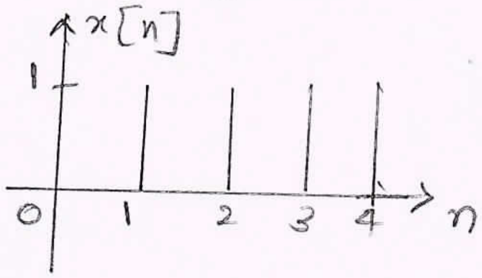
$$= b^n \cdot b \frac{(a/b)^{n+1} - 1}{a-b}$$

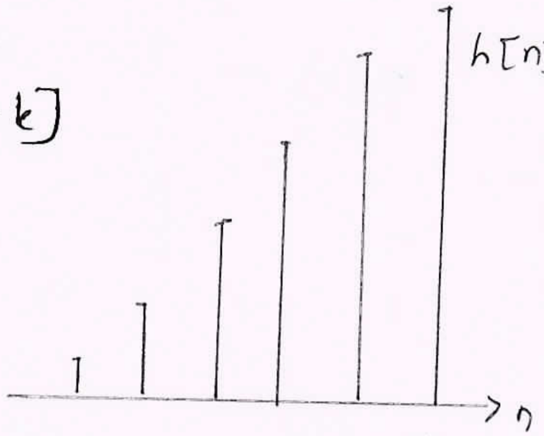
$$= \frac{b^{n+1}\left(\frac{a^{n+1}}{b^{n+1}} - 1\right)}{a-b}$$

$$y[n] = \frac{a^{n+1} - b^{n+1}}{a-b} \quad \text{for } n \geq 0 \text{ and } a \neq b.$$

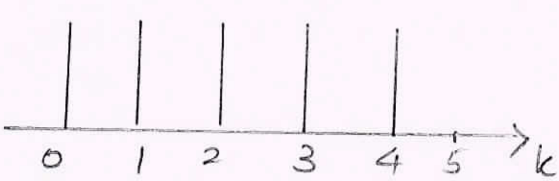
7 b $y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

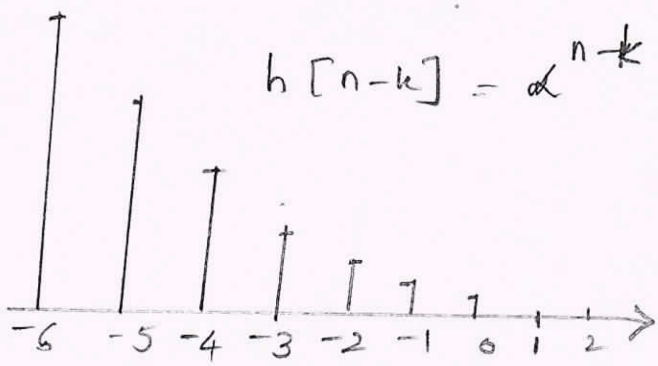




$h[n] = \alpha^n$



$h[n-k] = \alpha^{n-k}$



Case 1>

$n < 0$

$y[n] = 0$;

Case 2>

$n \geq 0$, $n \leq 4$ $\quad y[n] = \dfrac{1-\alpha^{n+1}}{1-\alpha}$

case 3>

$n > 4 \quad n-6 \leq 0 \quad (4 < n \leq 6)$

$$y[n] = \frac{\alpha^{n-4} - \alpha^{n+1}}{1-\alpha}$$

Case 4)

$n > 6 \quad n-6 < 4 \quad (6 < n \leq 10)$

$$y[n] = \frac{\alpha^{n-4} - \alpha^{7}}{1-\alpha}$$

Case 5> $n-6 > 4 \quad (n > 10) \quad y[n] = 0$

$$y[n] = \begin{cases} 0 & ; \quad n<0 \\ 1 - \dfrac{\alpha^{n+1}}{1-\alpha} & ; \quad 0 \leq n \leq 4 \\ \alpha^{n-4} - \dfrac{\alpha^{n+1}}{1-\alpha} & ; \quad 4 < n \leq 6 \end{cases}$$

$$y[n] = \begin{cases} \alpha^{n-4} - \dfrac{\alpha^{7}}{1-\alpha} & ; \quad 6 < n \leq 10 \\ 0 & \quad n > 10 \end{cases}$$

a)

(a) (*) $h(n) = e^{-n} \cos(n) \cdot u(n)$

Multiplication of $u[n]$ ensures that $h[n] = 0$ for $n < 0$

Hence given system is causal.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} e^{-n} \cos(n) \cdot u(n)$$

$$= \sum_{n=0}^{\infty} |\cos[n]| \cdot e^{-n}$$

$$-1 \le \cos[n] \le 1$$

$$|\cos[n]| < 1$$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} |\cos[n]| \cdot e^{-n}$$

for every n, value of $\cos[n]$ is finite, but summation is going for $n \to \infty$, which causes

$$\sum_{n=-\infty}^{\infty} |h[n]| \to \infty$$

So, given system is not stable

6 (ii) $h[n] = (0.99)^n u(n+3)$

- System is dynamic, It requires memory since $h[n]$ is not interms of $\delta(n)$
- The system is non causal since $h(n) \neq 0$ for $n<0$ because of presence of $u(n+3)$
- Consider $\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=-\infty}^{\infty} (0.99)^k u(k+3)$

$= \sum_{k=-3}^{\infty} (0.99)^k$ since $u(k+3) = \begin{cases} 1 & \text{for } k \\ 0 & \text{for } k < \end{cases}$

$= (0.99)^{-3} + (0.99)^{-2} + (0.99)^{-1} + \sum_{k=0}^{\infty} (0.99)$

$= 3.061 + \frac{1}{1-0.99}$ By geometric series

$= 103.061$ which is finite.

Since impulse response is absolutely summable the system is stable.

$\mathcal{Q}$=

8 (a) (c)

$$h[n] = n\left(\frac{1}{2}\right)^n u[n]$$

(i) To check stability

The discrete time system is stable if

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

Let us check this condition for given system, ie

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} k\left[\frac{1}{2}\right]^k u[k]$$

Since $u[k] = 1$ for $k \geq 0$, above equation becomes,

$$\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=0}^{\infty} k\left(\frac{1}{2}\right)^k$$

$$\sum_{k=0}^{\infty} k \cdot a^k = \frac{a}{(1-a)^2}, \text{ if } |a|<1$$

Then, above summation becomes,

$$\sum_{k=-\infty}^{\infty} |h(k]| = \frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} = 2$$

Then $\sum_{k=-\infty}^{\infty} |h(k)| = 2 < \infty$

Hence the system is stable.

(ii) To check for Causality

A LTI system is causal if

$h[n] = 0$ for $n<0$

given impulse response is

$$h[n] = n\left[\frac{1}{2}\right]^n u[n]$$

$u[n] = 1$ for $n \geq 0$

$u[n] = 0$ for $n < 0$

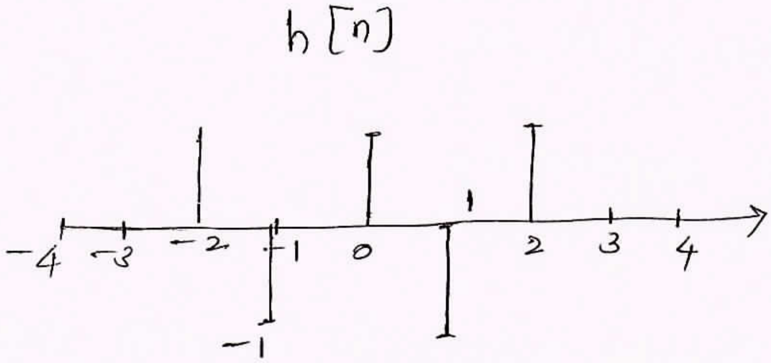
Equation will be

$$h[n] = \begin{cases} n(1/2)^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

Thus the system is causal since $h(n) = 0$, $n < 0$

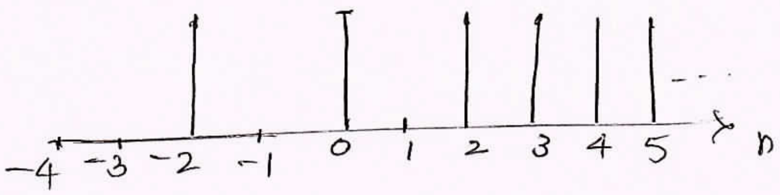
8 b.

$$h[n] = (-1)^n \{u(n+2) - u(n-3)\}$$



$$s[n] = \begin{cases} 0 & ; \quad n \leq -3 \\ 1 & ; \quad n = -2,\ 0,\ 0,\ n \geq 2 \\ 0 & ; \quad -1,\ 1 \end{cases}$$

plot of s (n)



```
# Step Response from given impulse response.
import numpy as np.
import matplotlib.pyplot as plt

# Given impulse response
impulse_response = [0.1, 0.2, 0.3, 0.4]

# convert the impulse response to a step response
step_response = np.cumsum (impulse_response)

# Plot the step response
plt.plot (step_response)
plt.xlabel ('Time')
plt.ylabel ('Response')
plt.title ('Step Response from Impulse Response')
plt.grid()
plt.show()
```

9 a Z transform is a mathematical tool, which is used to convert difference equation in time domain, into algebraic equations in Z domain.

Z transform of $x(n)$ is denoted by $X(z)$ which is defined as,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad \text{——} \; (A)$$

Types of Z transform

i) Bilateral Z transform: The Z transform defined in Eqn (A) has both sided summation, hence it is called bilateral Z transform.

(ii) Unilateral Z transform: It is defined as,

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

Here summation is from 0 to ∞. That is one sided.

Properties of ROC

→ ROC cannot contain any poles.

→ If $x[n]$ is finite causal sequence then ROC is entire Z plane except at $z=0$

→ If $x[n]$ is finite non causal sequence, then ROC is entire Z plane except at $z=\infty$

→ If $x[n]$ is finite double sided sequence, then ROC is entire z plane except at $z=0$ and ∞.

→ If $x[n]$ is causal infinite length sequence then ROC is of the form, $|z| > r_{max}$, where r_{max} equals largest magnitude of any of the poles of $X(z)$

→ If $x[n]$ is non causal infinite length sequence then ROC is of the form $|z| < r_{min}$,

→ If $x[n]$ is two sided sequence of infinite, duration, then ROC is of the form $r_1 < |z| < r_2$

→ ROC of an LTI stable system contains the unit circle, in the z plane.

→ ROC must be connected region (circle)

9 b. $x[n] = (1/2)^n$

(i) When $n = 0$, $x[n] = 1$

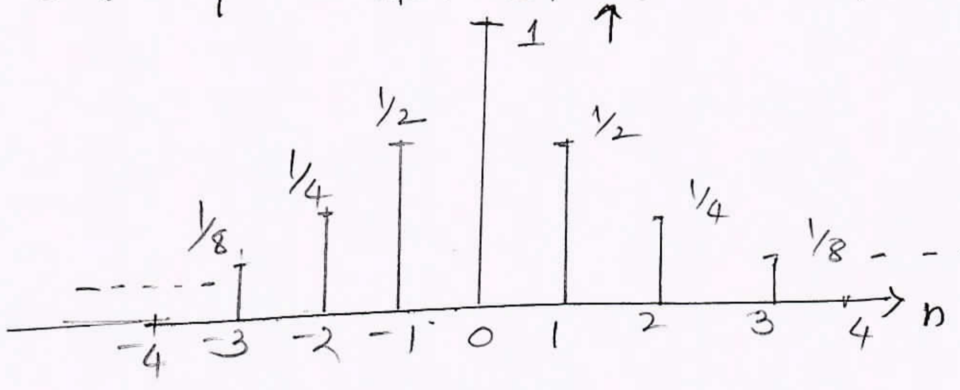
$n = 1$ $x[n] = 1/2$

$n = 2$ $x[n] = 1/4$

$n = -1$ $x[n] = 1/2$

$n = -2$ $x[n] = 1/4$

$x[n] = \{ \ldots 1/4, 1/2, \underset{\uparrow}{1}, 1/2, 1/4 \ldots \}$



(ii) $x(n) = \frac{1}{2}^{(n)}$

$x(n) = (1/2)^n u(n) + (1/2)^{-n} u(-n-1)$

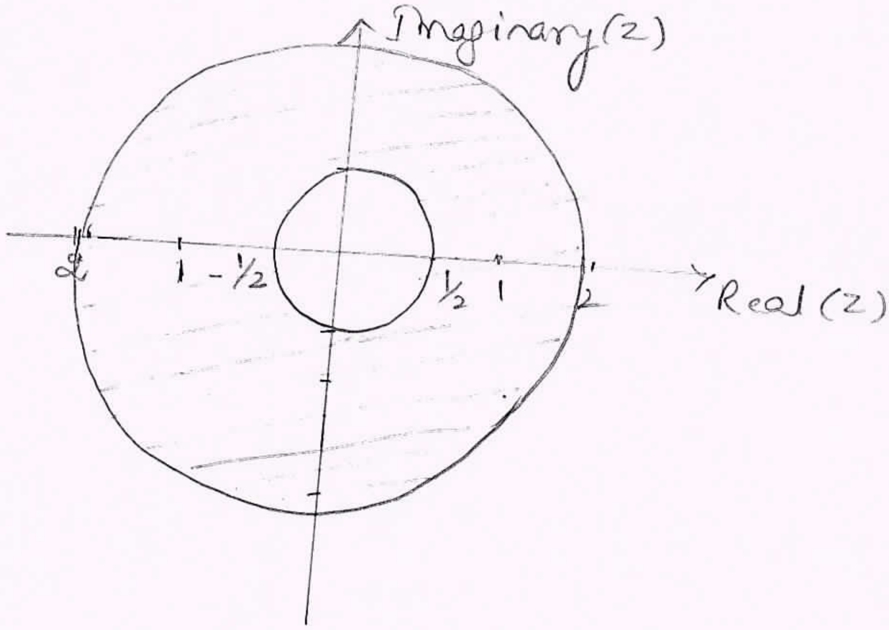
$X(z) = \frac{z}{z - 1/2} - \frac{z}{z-2}$ ROC $= 1/2 < |z| < 2$

$$= \frac{z(z-2) - z(z-\frac{1}{2})}{(z-\frac{1}{2})(z-2)}$$

$$= \frac{z^2 - 2z - z^2 + \frac{1}{2}z}{z^2 - 2z - \frac{1}{2}z + 1}$$

$$= \frac{-3z/2}{z^2 - 5/2 z + 1}$$

$$X(z) = \frac{-3z}{2(z-\frac{1}{2})(z-2)}$$ ROC $= \frac{1}{2} < |z| < 2$



c. $x(n) = (\frac{1}{2})^n u(n) * (\frac{1}{3})^n u(n)$

$$X(z) = \frac{1}{1-(\frac{1}{2})z^{-1}} \cdot \frac{1}{1-(\frac{1}{3})z^{-1}}$$

$$X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$$ ROC : $|z| > \frac{1}{2}$

$|z| > \frac{1}{3}$

10 a) Convolution:

(i) If $Z\{x[n]\}$; $R_x^- < |z| < R_x^+$

and $Z\{y[n]\}$; $R_y^- < |z| < R_y^+$

Then $Z\{x[n]\,y[n]\} = \frac{1}{2\pi j} \oint_{C_1} X(v)\, Y(z/v)\, v^{-1} \cdot dv$

where $\oint_{C_1}$ is complex contour integral and C_1 is closed contour in the intersection of the ROC's of $X(v)$ and $Y(z/v)$

ROC for resulting transform is,

$$R_x^- R_y^- < |z| < R_x^+ R_y^+$$

$$Z\{x[n]\,y[n]\} = \sum_{n=-\infty}^{\infty} x[n]\,y[n]\,z^{-n}$$

$$x[n] = \frac{1}{2\pi j} \oint_{C_1} X[z]\, \frac{z^n}{z} \cdot dz$$

$$Z\{x[n]\,y[n]\} = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi j} \oint_{C_1} X(v)\, \frac{v^n}{v} dv \cdot y[n]\, z^{-n}$$

$$= \frac{1}{2\pi j} \oint_{C_1} \frac{X(v)}{v} \cdot dv \sum_{n=-\infty}^{\infty} y[n]\, v^{n} z^{-n}$$

$$= \frac{1}{2\pi j} \oint_{C_1} \frac{X(v)}{v} \cdot dv \sum_{n=-\infty}^{\infty} y[n] \left(\frac{z}{v}\right)^{-n}$$

$$Z\{x[n]\,y[n]\} = \frac{1}{2\pi j} \oint_{C_1} X(v)\, Y\left(\frac{z}{v}\right) v^{-1} \cdot dv$$

If C_1 is a unit circle, above equation can be written in a different form, so that it looks more like a convolution.

Q//

Let $v = e^{j\theta}$ & $z = e^{j\Omega}$

$dv = je^{j\theta}.d\theta$

$Z = \{x[n]y[n]\} = \frac{1}{2\pi j}\oint_{c_1} x(e^{j\theta})\, y(e^{j(\Omega-\theta)})\, je^{j\theta} e^{-j\theta}.d\theta$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x(e^{j\theta})\, y(e^{j(\Omega-\theta)}).d\theta.$$

denoting $x(e^{j\theta})$ as $x(\theta)$ and $y(e^{j(\Omega-\theta)})$ as $y(\Omega-\theta)$, we get,

$$Z = \{x[n]\, y[n]\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(\theta)\, y(\Omega-\theta).d\theta$$

(ii) Initial Value Theorem:-

If $x[n]$ is a causal sequence with z transform $X(z)$, then $x[0] = \lim_{z\to\infty} X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n].z^{-n}$$

Since $x[n]$ is a causal sequence, the lower limit in the above summation is taken as zero.

Hence $X(z) = \sum_{n=0}^{\infty} x[n].z^{-n}$

Taking limit $z \to \infty^+$ on both sides

$$\lim_{z\to\infty} X(z) = x[0] + 0$$

$$x[0] = \lim_{z\to\infty} X(z)$$

iii) <u>Final Value</u> Theorem:

If $z\{x(n)\} = X(z)$ and the poles of $X(z)$ are all inside the unit circle, then final value of $x[n]$ as $n \to \infty$ given by

$$\lim_{n\to\infty} x[n] = \lim_{z\to 1}\left[(z-1)X(z)\right]$$

from the sequence,

$x(n+1) - x(n)$ and take its z transform,

$$Z\{x(n+1) - x(n)\} = \sum_{n=-\infty}^{\infty} x(n+1) - x(n)]\, z^{-n}$$

The sequence $x[n]$ is causal, hence

$$Z\{x(n+1) - x(n)\} = \sum_{n=0}^{\infty} \{x(n+1) - x(n)\} z^{-n}$$

$$= \lim_{n\to\infty}\left[\sum_{k=0}^{\infty}\left[x[k+1] - x[k]\, z^{-k}\right]\right]$$

$$= \lim_{n\to\infty}\left[\sum_{k=0}^{\infty} x[k+1]\, z^{-k} - \sum_{k=0}^{n} x[k]\, z^{-k}\right]$$

$$= \lim_{n\to\infty}\left[-x[0]\right] + x[1](1-z^{-1}) + (x[2](z^{-1}-z^{-2}))$$
$$+ \cdots + x(n+1)\, z^{-n}$$

Taking limit $z \to 1$ on both sides we get

$$\lim_{z\to 1} Z\{\{x(n+1)\} - x(n)\}$$
$$= Z\{x(n+1)\} - Z\{x(n)\}$$
$$= zX(z) - z(x(0)) - X(z)$$
$$= (z-1)X(z) - zX(0)$$

Taking $\lim z \to 1$

$$\lim_{z\to 1} Z\{x(n+1) - x(n)\} = \lim_{z\to 1}\left[(z+1)X(z) - X(0)\right]$$

Equating above Equations,

$$-x[0] + \lim_{n\to\infty} x[n] = -x[0] + \lim_{z\to 1}(z-1)X(z)$$

Hence, $\lim_{n\to\infty} x[n] = \lim_{z\to 1}(z-1)X(z)$

10 b. Given $X(z) = \dfrac{-1+5z^{-1}}{1-3/2\,z^{-1}+1/2\,z^{-2}}$

$$X(z) = \frac{z(5-z)}{z^2 - 3/2\,z + 1/2}$$

$$\frac{X(z)}{z} = \frac{(5-z)}{(z-1/2)(z-1)}$$

$$\frac{5-z}{(z-1/2)(z-1)} = \frac{A}{(z-1/2)} + \frac{B}{(z-1)}$$

$$(5-z) = A(z-1) + B(z-1/2)$$

for $z = 1/2$

$\frac{9}{2} = A(-1/2)$

$A = -9$

for $z = 1$

$4 = B(1/2)$

$B = 8$

$$\frac{X(z)}{z} = \frac{-9}{z-1/2} + \frac{8}{z-1}$$

$$X(z) = \frac{-9z}{(z-1/2)} + \frac{8z}{(z-1)}$$

$$X(z) = \frac{-9}{1-1/2\,z^{-1}} + \frac{8}{1-z^{-1}}$$

for ROC: $|z|>1$ $\quad x(n) = -9(1/2)^n u(n) + 8(1)^n u(n)$

$$= \left[-9(1/2)^n + 8\right] u(n)$$

10 C. # Z Transform

```
k = [2, 4, 3, 7, 5]     # Discrete Sequence

ztop = 0
f = 0
for t in range(5):

    # print(k[t])
        ztop = ztop + k[t] * 2 ** -t
print(ztop)
```