CBCS SCHEME

USN | | | | | | | | | | |

18EE71

Seventh Semester B.E. Degree Examination, Jan./Feb. 2023
Power System Analysis – 2

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define the following with example:
   i) Oriented graph
   ii) Basic loop
   iii) Co-tree. **(06 Marks)**

b. What is primitive network? Give the representation of a typical component and arrive at their performance equation in impedance and admittance form. **(06 Marks)**

c. The below Fig.Q.1(c) shows the one-line diagram of a simple four-bus system, Table Q.1(c) gives the line impedances identified by the buses on which these terminates. The shunt admittances at all the buses is assumed to be negligible.
   i) Find Y_{BUS} assuming that the line shown dotted is not connected
   ii) What modification need to be carried out in Y_{BUS} if the line shown dotted is connected?

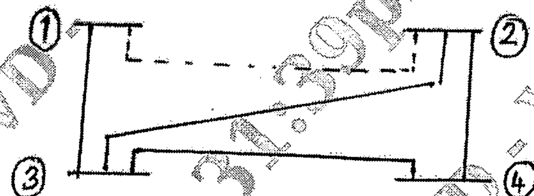


Fig.Q.1(c)

Table Q1(c)

Line (bus to bus)	R (pu)	X (pu)
1 – 2	0.05	0.15
1 – 3	0.10	0.30
2 – 3	0.15	0.45
2 – 4	0.10	0.30
3 – 4	0.05	0.15

(08 Marks)

OR

2 a. With usual notation, deduce the expression for Y_{BUS} using singular transformation method. **(06 Marks)**

b. Determine Y_{BUS} by singular transformation of the system with data as given in Table Q.2(b) **(08 Marks)**

Table Q.2(b)

Element No.	1	2	3	4	5
Bus code (p-q)	0-1	1-2	2-3	3-0	2-0
Self admittance y_{pqpq}	1.4	1.6	2.4	2.0	1.8

c. The bus incidence matrix A for a network of 8 elements and 5 nodes (4-buses) is given in Table Q.2(c). Reconstruct the oriented graph. Hence obtain the one-line-diagram of the system indicating the generator position. **(06 Marks)**

Table Q.2(c)

A =

	1	2	3	4	5	6	7	8
1	1	0	0	0	-1	0	-1	0
2	0	1	0	0	1	-1	0	-1
3	0	0	1	-1	0	1	0	0
4	0	0	0	1	0	0	1	1

Module-2

3 a. Why load flow analysis in power system is necessary? Explain. **(06 Marks)**

b. What is the data required to conduct load flow analysis? Discuss the need of acceleration factor in load flow solution. **(06 Marks)**

c. Obtain the load flow solution using Gauss-Seidal method at the end of one iteration of the power system shown in Fig.Q.3(c). The data is given in Table Q.3(c)-1 and Table Q.3(c)-2.

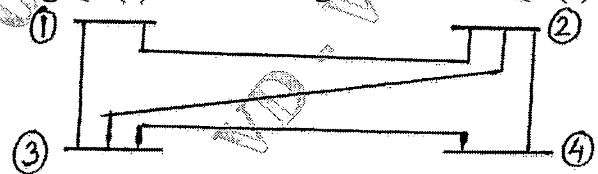


Fig.Q.3(c)

Table Q.3(c) – 1
Line data

S. B.	E. B	R (PU)	X(PU)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table Q.3(c) – 2
Bus data

Bus No	P_i (p.u)	Q_i (p.u)	V_i
1	-	-	$1.04 \angle 0$
2	0.5	-0.2	-
3	-1.0	0.5	-
4	0.3	-0.1	-

(08 Marks)

OR

4 a. Explain how buses are classified for load flow study. **(06 Marks)**

b. Discuss operating constraints in load flow analysis. **(06 Marks)**

c. For the three bus system shown in Fig.Q.4(c), use Gauss-Seidel method and determine the voltages at bus 2 and bus 3 at the end of first iteration. Line impedances marked on the diagram are in p.u. The information relating to bus data is given in Table Q.4(c).

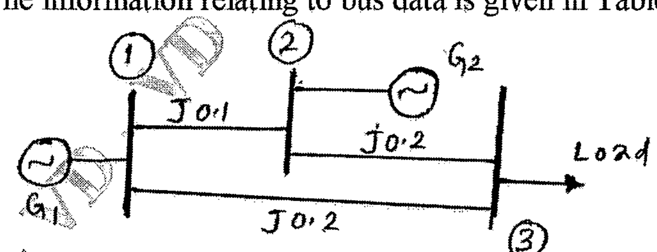


Fig.Q.4(c)

Table Q.4(c)

Bus No.	Type	Generation		Load		Voltage magnitude $\|V\|$	Reactive Power Limit	
		P	Q	P	Q		Q_{min}	Q_{max}
1	Slack	-	-	-	-	1.0	-	-
2	PV	5.32	-	-	-	1.1	0	5.32
3	PQ	-	-	3.64	0.53	-	-	-

(08 Marks)

Module-3

5 a. Discuss the algorithm procedure for load flow analysis using Newton-Raphson's method in polar coordinates. **(06 Marks)**

b. Obtain the voltages at all buses for three bus system shown in Fig.Q.5(b) at the end of first iteration by N-R method. The data is given in Table Q.5(b) – 1 and Table Q.5(b) – 2.

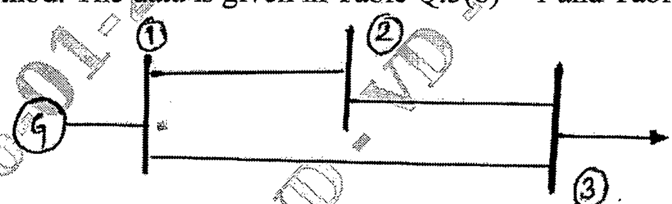


Fig.Q.5(b)

Table Q.5(b) – 1 Line data

SB	EB	R(pu)	X(pu)
1	2	0.0	0.1
1	3	0.0	0.2
2	3	0.0	0.2

Table Q.5(b) – 2 Bus data

Bus No.	P_G	Q_G	P_L	Q_L	V_{sp}
1(slack)	-	-	-	-	1.0
2 (pv)	5.3217	-	-	-	1.1
3 (PQ)	-	-	3.6392	0.5339	-

(08 Marks)

c. Compare load flow methods with standard features. **(06 Marks)**

OR

6 a. Stating all assumptions, deduce FDLF model. Explain the step by step procedure for load flow solution using FDLF method. **(08 Marks)**

b. Draw a flow chart for Fast Decoupled Load Flow (FDLF) method. **(06 Marks)**

c. Derive expression for all elements of Jacobian matrices in polar form. **(06 Marks)**

Module-4

7 a. Explain the followings:

i) Input-output curve

ii) Heat rate curve

iii) Incremental cost curve related to thermal plants. **(06 Marks)**

b. The fuel inputs per hour of plant 1 and 2 are given as $F_1 = 0.2P_1^2 + 40P_1 + 120$ RS/hr ; $F_2 = 0.25P_2^2 + 30P_2 + 150$ RS/hr. Determine the economic operating schedule and the corresponding cost of generation if the maximum and minimum loading units is 100MW and 25MW, the demand is 180MW and transmission line losses are neglected. If the load is equally shared by both units, determine the saving obtained by loading the units as per equal incremental production cost. **(06 Marks)**

c. Discuss the algorithm procedure for priority list method of unit commitment solution. **(08 Marks)**

OR

8 a. With usual notation, derive the generalized transmission loss formula and B-coefficient. **(08 Marks)**

b. A system consists of two plants connected to a transmission line, the load is located at Plant-2. The power transfer of 100MW from station 1 to the load results in a loss of 8MW. Find the required generation at each station and the power received by the load, when the system, is operating with λ = RS 100/MWh. The incremental fuel cost of two plants are $\frac{dc_1}{dp_1} = 0.12P_1 + 65$ RS/MWh and $\frac{dc_2}{dp_2} = 0.25P_2 + 75$ RS/MWh. **(06 Marks)**

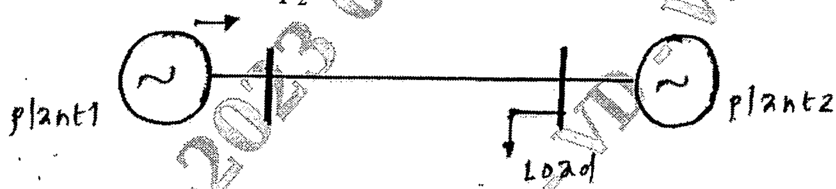


Fig.Q.8(b)

c. Draw the flow chart of dynamic forward DP approach for unit commitment. **(06 Marks)**

Module-5

9 a. Obtain the generalized algorithm expression for bus impedance matrix elements when a link is added to the partial network. Also discuss the special cases. **(10 Marks)**

b. Explain clearly the point-by-point method of solving swing equation. Mention the assumptions made. **(10 Marks)**

OR

10 a. Obtain Z_{bus} by building algorithm for the system shown in Fig.Q.10(a). All values are in p.u. (impedance). Take bus '0' as reference bus. Add the elements in the order of ref. bus to bus1, ref. bus to bus2 and finally bus 1 to bus 2. **(10 Marks)**

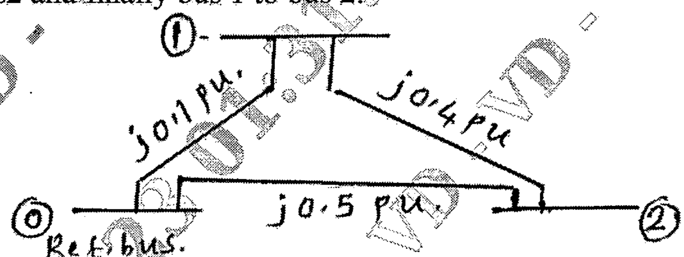


Fig.Q.10(a)

b. Describe the methodology of using Runge-Kutta technique for transient stability studies of a power system. **(10 Marks)**

* * * * *

Solution of VTU Question Paper
Jan | Feb - 2023

Power System Analysis-2 [18EE71]

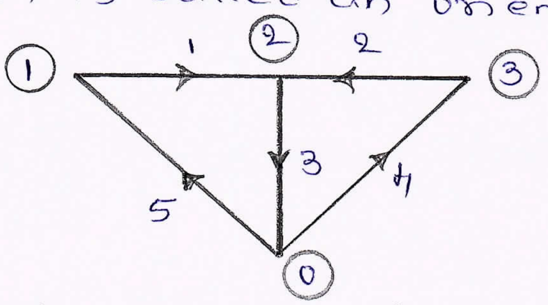
Prepared By :- Varaprasad Gaonkar
Assistant Professor
Dept. of E&E
KLS VDIT Haliyal.

Module-01

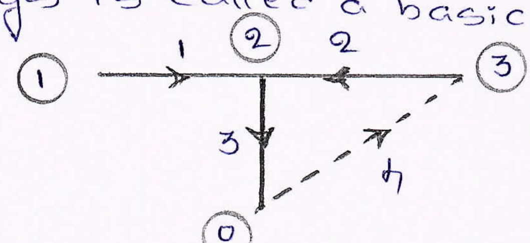
Q1. Define the following with example:
i) Oriented graph. ii) Basic loop iii) Co-tree.

(06 Marks)

The connection of a network topology shown by replacing all physical elements by lines is called a graph. A graph in which a direction is assigned to each branch is called an oriented graph.

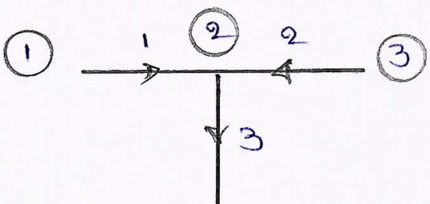


Basic loop- When a link is added to a tree it forms a loop. A loop containing only one link and remaining twigs is called a basic loop.

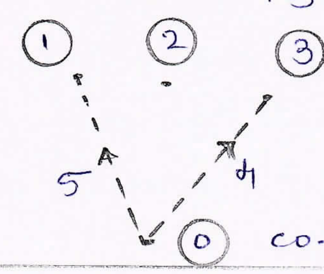


A set {4,2,3} forms a basic loop.

Co-tree- The set of branches of the original graph G not included in the tree is called co-tree.



Tree

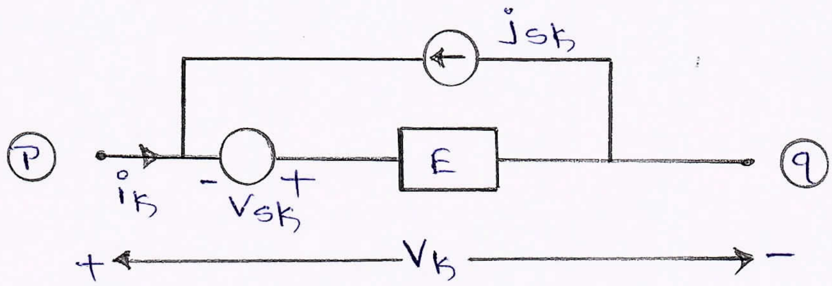


co-tree

01.b. What is primitive network? Give the representation of a typical component and arrive at their performance equation in impedance and admittance form (06 Marks)

A primitive element is a fundamental element which is not connected to any other element. A set of such unconnected elements is defined as a primitive network.

General primitive element is as shown below.



V_k = Voltage across branch k.
i_k = current through branch k.
V_{sk} = independent voltage source in branch k.
j_{sk} = independent current source in branch k.
E = Passive element.

(*) Impedance form of primitive element.

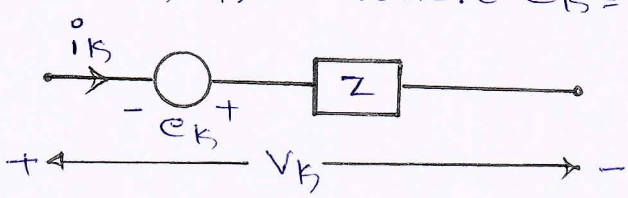
From above fig.

$$V_k + V_{sk} = (i_k + j_{sk}) Z_k$$

$$V_k = -V_{sk} + (i_k + j_{sk}) Z_k$$

$$= (j_{sk} Z_k - V_{sk}) + Z_k i_k$$

$$V_k + e_k = i_k Z_k. \quad \text{where } e_k = V_{sk} - j_{sk} Z_k$$



In general $\bar{V} + \bar{e} = [Z]\bar{i}$

where $\bar{V}$ = vector of voltages across b branches.
$\bar{i}$ = vector of currents through b branches.
$\bar{e}$ = vector of equivalent voltage sources.
$[Z]$ = Primitive impedance matrix.

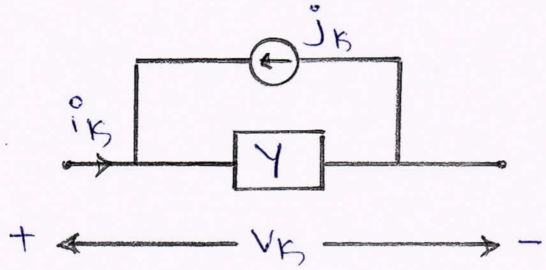
(*) Admittance form of the primitive element.

From general primitive element

$$i_K + j_{sK} = (V_K + V_{sK})\, Y_K$$

$$i_K = (Y_K V_{sK} - j_{sK}) + V_K Y_K$$

$$i_K + j_K = V_K Y_K \quad \text{where } j_K = j_{sK} - Y_K V_{sK}.$$



In general $\bar{i} + \bar{j} = [Y]\,\bar{V}$

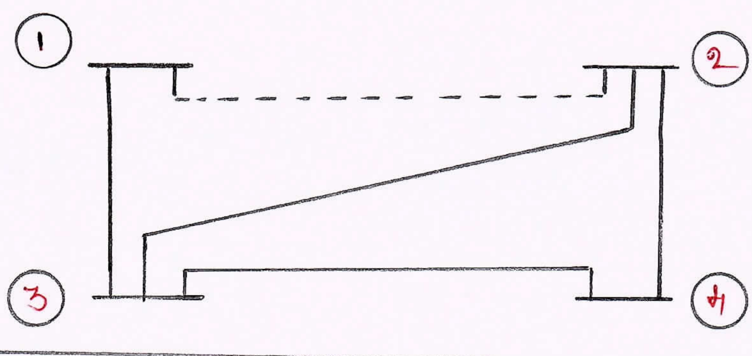
where $\bar{j}$ = vector of injected currents.

$[Y]$ = Primitive admittance matrix.

01.c. The below fig shows the one-line diagram of a simple four-bus system. Table gives the line impedances identified by the buses on which these terminates. The shunt admittances at all the buses is assumed to be negligible.

i) Find Ybus assuming that the line shown dotted is not connected.

ii) What modification need to be carried out in Ybus if the line shown dotted is connected?



Line (bus to bus)	R (pu)	X (pu)
1-2	0.05	0.15
1-3	0.10	0.30
2-3	0.15	0.45
2-4	0.10	0.30
3-4	0.05	0.15

[08 Marks]

Admittance of the lines are.

Line	$Y = {}^1/_Z$
1 - 2	$2 - j6$
1 - 3	$1 - j3$
2 - 3	$0.66 - j2$
2 - 4	$1 - j3$
3 - 4	$2 - j6$

$$
Y_{bus} = \begin{bmatrix} 1-j3 & 0 & -1+j3 & 0 \\ 0 & 1.66-j5 & -0.66+j2 & -1+j3 \\ -1+j3 & -0.66+j2 & 3.66-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}
$$

Ybus modification for addition of line 1-2.

Y_{11}, Y_{12}, Y_{21} and Y_{22} get modified.

$Y_{11}(new) = Y_{11}(old) + y_{12} = 3 - j9.$

$Y_{12}(new) = Y_{12}(old) - y_{12} = -2 + j6.$

$Y_{21}(new) = Y_{21}(old) - y_{21} = -2 + j6.$

$Y_{22}(new) = Y_{22}(old) + y_{21} = 3.66 - j11$

Other elements remain the same.

Q2. a. With usual notation, deduce the expression for Ybus using singular transformation method. (06 Marks)

Performance equation of the primitive network in admittance form is given by

$$\bar{i} + \bar{j} = [Y]\,\bar{V}$$

Pre multiplying by A^T

$$A^T\bar{i} + A^T\bar{j} = A^T[Y]\,\bar{V}$$

but $A^T\bar{i} = 0$ and $A^T\bar{j} = I_{bus}$ and $\bar{V} = A\,E_{bus}$.

Hence $I_{bus} = A^T[Y]\,A\,E_{bus}$

also $I_{bus} = Y_{Bus}\,E_{bus}$.

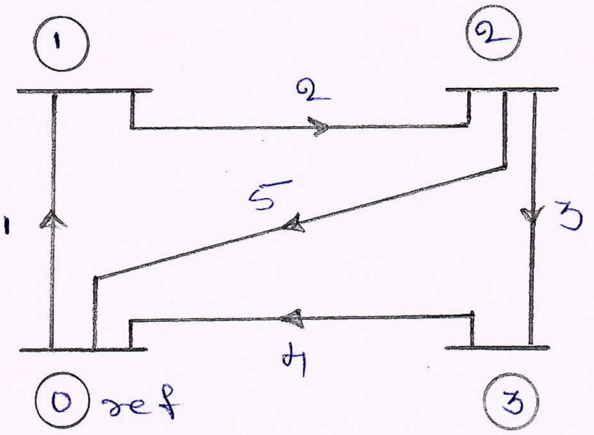
Hence $Y_{Bus} = A^T[Y]\,A$.

Q2.b. Determine Y_{bus} by singular transformation of the system with data as given in table below.

Element No.	1	2	3	4	5
Bus code (p-q)	0-1	1-2	2-3	3-0	2-0
Self admittance Y_{pqpq}	1.4	1.6	2.4	2.0	1.8

(08 Marks)

Oriented graph.



Bus incidense matrix.

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Primmitive admittance matrix

$$Y_{prim} = \begin{bmatrix} 1.4 & 0 & 0 & 0 & 0 \\ 0 & 1.6 & 0 & 0 & 0 \\ 0 & 0 & 2.4 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1.8 \end{bmatrix}$$

$$Y_{bus} = A^T \, Y_{prim} \, A.$$

$$Y_{bus} = \begin{bmatrix} 3 & -1.6 & 0 \\ -1.6 & 5.8 & -2.4 \\ 0 & -2.4 & 4.4 \end{bmatrix}$$

02.c. The bus incidence matrix A for a network of 8 elements and nodes (4-buses) is given in table. Reconstruct the oriented graph. Hence obtain the one-line-diagram of the system indicating the generator position.

A =

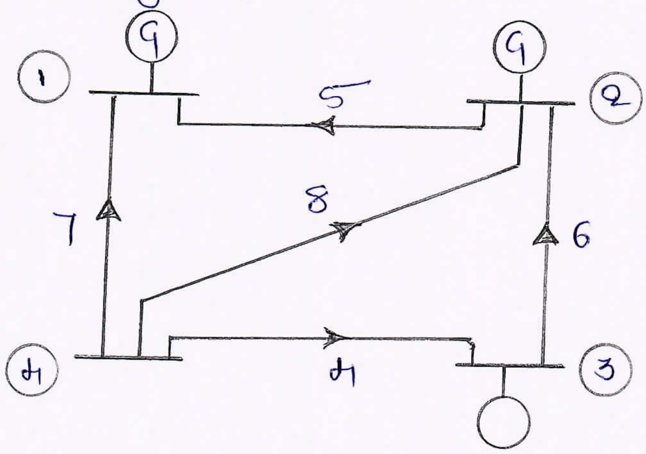
	1	2	3	4	5	6	7	8
1	1	0	0	0	-1	0	-1	0
2	0	1	0	0	1	-1	0	-1
3	0	0	1	-1	0	1	0	0
4	0	0	0	1	0	0	1	1

[06 Marks]

A =

	1	2	3	4	5	6	7	8
1	1	0	0	0	-1	0	-1	0
2	0	1	0	0	1	-1	0	-1
3	0	0	1	-1	0	1	0	0
4	0	0	0	1	0	0	1	1
0	-1	-1	-1	0	0	0	0	0

Oriented graph



Module-02

03.a. Why load flow analysis in power system is necessary? Explain.

[06 Marks]

Load flow gives steady state solutions of the voltages at all the buses for particular load condition.

Load flow studies are important in planning and designing future expansion.

Load flow studies throw ligh on.

- Violation of voltage magnitudes at the buses.
- Overloading of lines.
- Overloading of generators.

* Stability margin reduction.
* Effect of contingencies like line voltages during emergency shutdown of generators etc.

03.b. What is the data required to conduct load flow analysis? Discuss the need of acceleration factor in load flow solution. [06 Marks]

Data for load flow

01. System data.
- number of buses.
- number of PV buses.
- number of load buses
- number of transmission lines.
- number of transformers.
- number of shunt elements.
- Slack bus number
- Voltage magnitude of slack bus.
- tolerance ~~limit~~ limit.
- base MVA
- max. number of iterations.

02. Generator bus data.
for every PV bus, the data required are
- Bus number
- Active power generation
- Specified voltage magnitude
- Reactive power limit.

03. Load data.
for all load bus, the data required are
- bus number.
- active and reactive power demand.

04. Transmission line data.
- Starting and ending bus number.
- resistance, reactance and half line charging admittance.

05. Transformer data.
- Starting and ending bus number.
- resistance and reactance of transformer.
- off-nominal turns ratio.

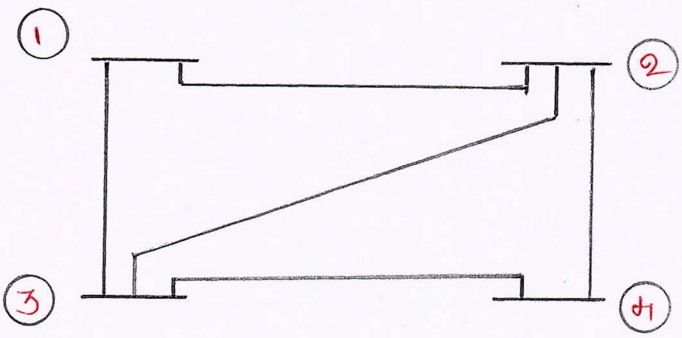
06. Shunt element data.

- Bus number.
- Shunt admittance

* Need of acceleration factor.

Gauss-seidel method uses linear convergence. and it takes more number of iterations to convergence. To reduce number of iteration and to get solution fast we need to use acceleration of convergence. and acceleration factor is generally between 1.6 to 2.

03.c. Obtain the load flow solution using Gauss-Seidal method at the end of one iteration of the power system shown in fig. The data is given in table.

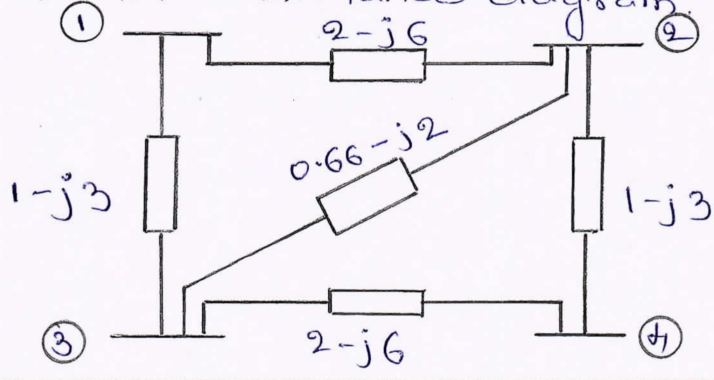


S.B	E.B	R(PU)	X(PU)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Bus no.	P_i(pu)	Q_i(pu)	V_i
1	-	-	$1.04\angle 0$
2	0.5	-0.2	-
3	-1.0	0.5	-
4	0.3	-0.1	-

[08 Marks]

Per unit admittance diagram.



$$Y_{bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.66-j11 & -0.66+j2 & -1+j3 \\ -1+j3 & -0.66+j2 & 3.66-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

$P_2 = 0.5 \qquad Q_2 = -0.2$

$P_3 = -1 \qquad Q_3 = 0.5$

$P_4 = 0.3 \qquad Q_4 = -0.1$

Assume intial voltages as

$V_1^0 = 1.04\angle 0, \quad V_2^0 = 1\angle 0, \quad V_3^0 = 1\angle 0, \quad V_4^0 = 1\angle 0$

we have

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ \neq i}}^{n} Y_{ik}V_k\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - \left[Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4\right]\right]$$

$$V_2^1 = \frac{1}{(3.66-j11)}\left[\frac{0.5+j0.2}{1} - \left[(-2+j6)*1.04 + (-0.66+j2)*1 + (-1+j3)*1\right]\right]$$

$V_2^1 = 1.019 + j0.046 = 1.02\angle 2.6^\circ$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left(\frac{P_3 - jQ_3}{V_3^*} - \left[Y_{31}V_1 + Y_{32}V_2 + Y_{34}V_4\right]\right]$$

$$V_3^1 = \frac{1}{(3.66-j11)}\left(\frac{-1-j0.5}{1} - \left[(-1+j3)*1.04 + (-0.66+j2)*1.02\angle 2.6 + (-2+j6)*1\right]\right]$$

$V_3^1 = 1.02 - j0.087 = 1.03\angle -4.84^\circ$ pu.

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - \left[Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3\right]\right]$$

$$V_4^1 = \frac{1}{(3-j9)}\left[\frac{0.3+j0.1}{1} - \left[0 + (-1+j3)*1.02\angle 2.6 + (-2+j6)*1.03\angle -4.84\right]\right]$$

$V_4^1 = 1.02 - j0.009 = 1.02\angle -0.57$ pu.

at the end of first iteration

$V_1^1 = 1.04 \angle 0^\circ$ PU

$V_2^1 = 1.02 \angle 2.6^\circ$ PU

$V_3^1 = 1.03 \angle -4.84^\circ$ PU

$V_4^1 = 1.023 \angle -0.57$ PU

04.a. Explain how buses are classified for load flow study (06 Marks)

For load flow study buses are classified based on specified parameter at a perticular bus.

01. Load bus (PQ bus)

These are non-generator buses. At this bus real power demand and reactive power demand are specified. At PQ bus $|V_i|$ and δ_i need to be calculated.

02. Voltage controlled bus (PV bus)

At PV bus voltage magnitude and active power generation's are specified. we need to determine reactive power at a bus and voltage angle at PV bus.

03. Slack bus (reference bus)

At a slack bus voltage magnitude and angle δ are specified. we need to determine active and reactive power demand at slack bus.

04.b. Discuss operating constraints in load flow analysis. (06 Marks)

01. Equipments are designed to operate at fixed voltages so operating values of voltage magnitude must lie in certain range.

$$|V_{i\,min}| \leq |V_i| \leq |V_{i\,max}|$$

02. The mechanical power input and the rating of the generator pose a limitation on active power generation.

$$P_{G_{i,min}} \leq P_{G_i} \leq P_{G_{i,max}}$$

03. Limitations of the generator field excitation constrains the reactive power generation.

$$Q_{Gi,min} \leq Q_{Gi} \leq Q_{Gi,max}.$$

04. Stability impose constraints on power angle between two buses

$$|\delta_i - \delta_j| \leq |\delta_i - \delta_j|_{max}.$$

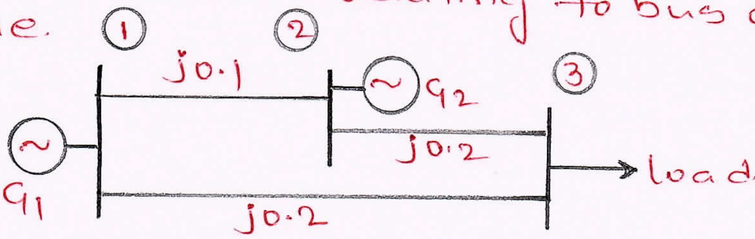
05. Complex power has to be conserved at all buses.

$$\sum_i P_{Gi} = \sum_i P_{Di} + P_L$$

$$\sum_i Q_{Gi} = \sum_i Q_{Di} + Q_L$$

P_L and Q_L are total system real and reactive power loss.

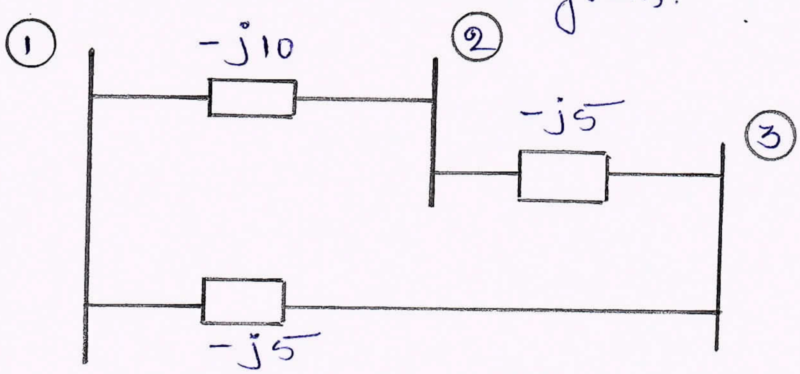
04. c. For the three bus system shown in fig. use Gauss-Seidel method and determine the voltages at bus 2 and bus 3 at the end of first iteration. Line impedances marked on the diagram are in p.u. The information relating to bus data is given in table.



Bus No.	Type	Generation P	Generation Q	Load P	Load Q	Voltage |V|	Reactive Power Limit Q_{min}	Reactive Power Limit Q_{max}
1	Slack	-	-	-	-	1.0	-	-
2	PV	5.32	-	-	-	1.1	0	5.32
3	PQ	-	-	3.64	0.53	-	-	-

[08 Marks]

Per unit admittance diagram.



$$Y_{bus} = \begin{bmatrix} -j15 & j10 & j5 \\ j10 & -j15 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

$P_2 = 5.32$

$P_3 = -3.62$, $Q_3 = -0.53$.

Assume initial voltages as

$V_1^0 = 1\angle 0$, $V_2^0 = 1.1\angle 0$, $V_3^0 = 1\angle 0$.

$$Q_2 = -B_{22}|V_2|^2 + |V_2||V_1|(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) + |V_2||V_3|(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23})$$

$$= 15(1.1)^2 + (1.1\times 1)(0-10) + (1.1\times 1)(0-5)$$

$Q_2 = 1.65$ within the limit

We have

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{K=1 \\ \neq i}}^{n} Y_{iK}V_K\right]$$

$$V_2' = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - [Y_{21}V_1 + Y_{23}V_3]\right]$$

$$= \frac{1}{-j15}\left[\frac{5.32 - j1.65}{1.1} - [(j10\times 1) + (j5\times 1)]\right]$$

$V_2' = 1.1 + j0.32 = 1.14\angle 16.33^\circ$ pu.

$V_2' = 1.1\angle 16.33^\circ$ pu.

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - [Y_{31}V_1 + Y_{32}V_2]\right]$$

$$= \frac{1}{-j10}\left[\frac{-3.62 + j0.53}{1} - [(j5\times 1) + (j5\times 1.1\angle 16.33^\circ)]\right]$$

$V_3' = 0.97 - j0.20 = 0.99\angle -11.65^\circ$ pu.

at the end of first iteration

$V_1' = 1\angle 0^\circ$ pu

$V_2' = 1.1\angle 16.33^\circ$ pu

$V_3' = 0.99\angle -11.65^\circ$ pu.

Module-03.

05.a. Discuss the algorithm procedure for load flow analysis using Newton-Raphson's method in polar coordinates. [06 Marks]

Algorithm for Newton-Raphson's method.

01. Read the data.
02. Formulate Ybus.
03. Assume initial voltages (flat start)
04. At $(r+1)^{th}$ iteration. Calculate P_i^{r+1} at all the PV and PQ buses and Q_i^{r+1} at all the PQ buses, using voltages from previous iteration $V_i^{(r)}$
05. Calculate the power mismatch

$$\Delta P_i^{(r)} = P_{i\,sp} - P_{i\,cal}^{(r+1)} \quad (\text{at PV and PQ buses})$$

$$\Delta Q_i^{(r)} = Q_{i\,sp} - Q_{i\,cal}^{(r+1)} \quad (\text{at PQ buses})$$

06. Calculate the Jacobian matrix usin $V_i^{(r)}$
07. Compute

$$\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

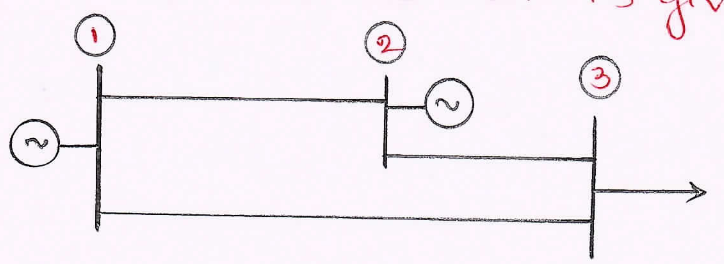
08. Update the variables as follows.

$$\delta_i^{(r+1)} = \delta_i^{(r)} + \Delta\delta_i^{(r)} \quad \text{for all buses.}$$

$$|V_i|^{(r+1)} = |V_i|^{(r)} + \Delta|V_i|^{(r)} \quad \text{for PQ buses.}$$

09. Go to step 04 and iterate till the power mismatches are less than tolerance value.

05.b. Obtain the voltages at all buses for three bus system shown in fig at the end of first iteration by N-R method. The data is given in table



Line data

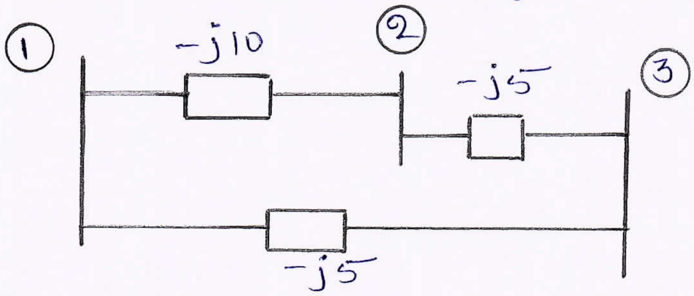
SB	EB	R(pu)	X(pu)
1	2	0.0	0.1
1	3	0.0	0.2
2	3	0.0	0.2

Bus data.

Bus No.	PG	QG	PL	QL	Vsp
1 (slack)	–	–	–	–	1.0
2 (PV)	5.3217	–	–	–	1.1
3 (PQ)	–	–	3.6392	0.5339	–

[08 Marks]

Per unit admittance diagram.



$$Y_{bus} = \begin{bmatrix} -j15 & j10 & j5 \\ j10 & -j15 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

Assume intial voltages as

$V_1 = 1\angle 0^\circ$, $V_2 = 1.1\angle 0^\circ$, $V_3 = 1\angle 0^\circ$

$P_{2sp} = 5.3217$

$P_{3sp} = -3.6392$, $Q_{3sp} = -0.5339$

$$P_{2cal} = P_2 = G_{22}|V_2|^2 + |V_2||V_1|(G_{21}\cos\delta_{21} + B_{21}\sin\delta_{21})$$
$$+ |V_2||V_3|(G_{23}\cos\delta_{23} + B_{23}\sin\delta_{23})$$

$\delta_{21} = \delta_2 - \delta_1 = 0$, $\delta_{23} = \delta_2 - \delta_3 = 0$.

$\therefore P_{2cal} = 0$

$\Delta P_2 = P_{2sp} - P_{2cal} = 5.3217 - 0 = 5.3217.$

$$P_{3cal} = P_3 = G_{33}|V_3|^2 + |V_3||V_1|(G_{31}\cos\delta_{31} + B_{31}\sin\delta_{31})$$
$$+ |V_3||V_2|(G_{32}\cos\delta_{32} + B_{32}\sin\delta_{32})$$

$P_{3cal} = 0$

$\Delta P_3 = P_{3sp} - P_{3cal} = -3.6392 - 0 = -3.6392.$

$$Q_3 cal = -B_{33}|V_3|^2 + |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$\delta_{31} = 0$, $\delta_{32} = 0$

$$Q_3 cal = 10(1)^2 + [1\times1(0-5)] + [(1\times1.1)(0-5)]$$
$$= -0.5$$

$$\Delta Q_3 = Q_{3sp} - Q_3 cal = -0.5339 - (-0.5) = -0.0339.$$

Matrix for solution is given by

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} H_{22} & H_{23} & N_{23} \\ H_{32} & H_{33} & N_{33} \\ M_{32} & M_{33} & L_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \frac{\Delta|V_3|}{|V_3|} \end{bmatrix}$$

$$H_{22} = -Q_2 - B_{22}|V_2|^2$$

$$Q_2 = -B_{22}|V_2|^2 + |V_2||V_1|(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) + |V_2||V_3|(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23})$$

$$= 15(1.1)^2 + (1.1\times1)(0-10) + (1.1\times1)(0-5)$$

$$Q_2 = 1.65$$

$$H_{22} = -1.65 + 15(1.1)^2 = 16.5$$

$$H_{23} = |V_2||V_3|(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23})$$
$$= (1.1\times1)(0-5) = -5.5$$

$$H_{32} = -5.5$$

$$H_{33} = -Q_3 - B_{33}|V_3|^2$$
$$= 0.5 - (-10) = 10.5$$

$$N_{23} = |V_2||V_3|(G_{23}\cos\delta_{23} + B_{23}\sin\delta_{23}) = 0$$

$$N_{33} = P_3 + G_{33}|V_3|^2 = 0$$

$$M_{32} = -N_{32} = 0$$

$$M_{33} = P_3 - G_{33}|V_3|^2 = 0$$

$L_{33} = \theta_3 - B_{33}|V_3|^2 = -0.5 + 10 = 9.5$

So we get

$$\begin{bmatrix} 5.3217 \\ -3.6392 \\ -0.0339 \end{bmatrix} = \begin{bmatrix} 16.5 & -5.5 & 0 \\ -5.5 & 10.5 & 0 \\ 0 & 0 & 9.5 \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \frac{\Delta|V_3|}{|V_3|} \end{bmatrix}$$

solving

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \frac{\Delta|V_3|}{|V_3|} \end{bmatrix} = \begin{bmatrix} 0.25 \\ -0.215 \\ -0.0035 \end{bmatrix}$$

$\therefore \Delta\delta_2 = 0.25^r = 14.32^\circ$

$\Delta\delta_3 = -0.215^r = -12.31^\circ$

$\Delta|V_3| = -0.0035$

$\delta_2^1 = \delta_2^0 + \Delta\delta_2 = 0 + 14.32 = 14.32^\circ$

$\delta_3^1 = \delta_3^0 + \Delta\delta_3 = 0 + (-12.31) = -12.31^\circ$

$|V_3| = V_3^0 + \Delta|V_3| = 1 + (-0.0035) = 0.9965$

At the end of first iteration

$V_1 = 1\angle 0^\circ$ PU

$V_2 = 1.1\angle 14.32^\circ$ PU

$V_3 = 0.9965\angle -12.31^\circ$ PU

05.c. Compare load flow methods with standard features. [06 Marks]

Parameter of comparison	G-S method	N-R method	FDLF method
01. Co-ordinates	works well with rectangular co-ordinates	Polar coordinates are prefered	Polar co-ordinates.
02. Arithmetic operation	Least in number to complete one iteration	Jacobian matrix need to be calculated	Less than N-R method

Parameter of comparison	G-S method	N-R method	FDLF method.
03. Convergence	Linear convergence	Quadratic convergence	Geometric convergence.
04. No. of iterations.	Large and increses with no. of buses	Very less 3 to 5	Generally 2 to 5
05. Accuracy	less accurate	More accurate	Moderate
06. Memory	less memory required.	Large	less compare to N-R method

06. a. Starting all assumptions, deduce FDLF model. Explain the step by step procedure for load flow Solution using FDLF method. [08 Marks]

From decoupled load flow we get

$$\begin{bmatrix} \Delta P \\ \Delta Q_i \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix}$$

Assumptions based on observations of practical power system are.

01. $B_{ij} \gg G_{ij}$

02. $\delta_i - \delta_j$ is very small so $\cos(\delta_i - \delta_j) \simeq 1$, $\sin(\delta_i - \delta_j) \simeq 0$

03. $Q_i << B_{ii} |V_i|^2$

So we can write

$$H_{ik} = L_{ik} = -|V_i||V_k| B_{ik} \quad (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$$

the matrix reduced to

$$[\Delta P] = [|V_i||V_j||B_{ij}'|][\Delta \delta]$$

$$[\Delta Q] = [|V_i||V_j||B_{ij}''|]\left[\frac{\Delta |V|}{|V|}\right]$$

B_{ij}' and B_{ij}'' are -ve of the susceptance of respective elements of the bus admittance matrix. Divide by $|V_i|$ and assume $|V_j| \cong 1.pu$ we get

$$\left[\frac{\Delta P}{|V|}\right] = [B_{ij}'] [\Delta \delta]$$

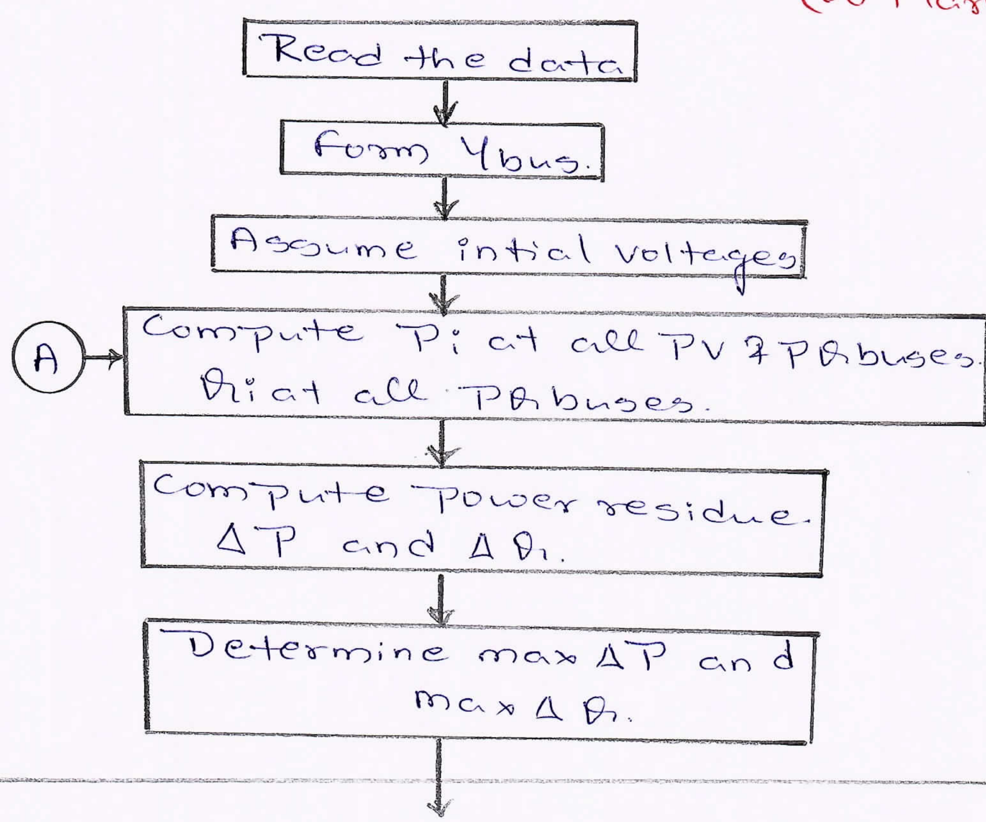
$$\left[\frac{\Delta Q}{|V|}\right] = [B_{ij}''] \left[\frac{\Delta |V|}{|V|}\right]$$

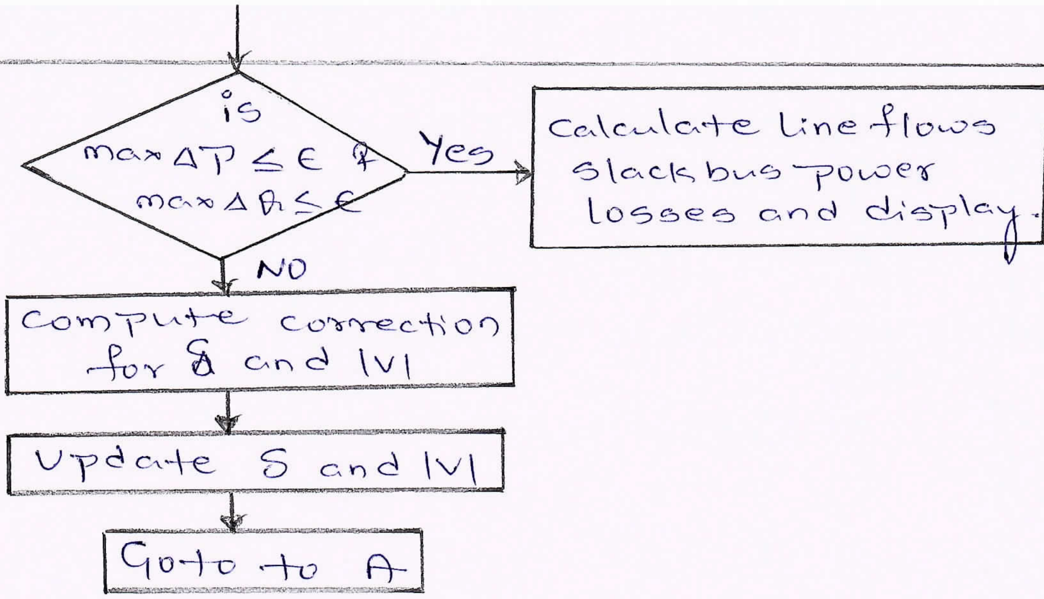
(*) Step by step procedure

01. Read the data.
02. Formulate Ybus.
03. Assume intial voltages (flat start)
04. calculate P_i at all PV and PQ buses and Q_i at all PQ buses.
05. Calculate power residues.
06. $[\Delta \delta] = [B_{ij}']^{-1} \left[\frac{\Delta P}{|V|}\right]$

    $\left[\frac{\Delta |V|}{|V|}\right] = [B_{ij}'']^{-1} \left[\frac{\Delta Q}{|V|}\right]$
07. Update δ_i and V_i
08. Goto step 04 and iterate till power residue is less than tolerance value.

06.b. Draw a flow chart for fast decoupled load flow (FDLF) method.
(06 Marks)





06. C. Derive expression for all elements of Jacobian matrices in polar form. [06 Marks]

We have

$$P_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k| (G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k| (G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

(*) Elements of J_1

Diagonal elements

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k| (-G_{ik}\sin\delta_{ik} + B_{ik}\cos\delta_{ik})$$

$$= -\sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k| (G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$$

off diagonal elements.

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k| (G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

(*) Elements of J_3.

Diagonal elements.

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k| (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

$$= P_i - G_{ii}|V_i|^2$$

Off diagonal elements.

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k| (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

(*) Elements of J_2

Diagonal elements.

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i| G_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_k| (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

$$\frac{\partial P_i}{\partial |V_i|} |V_i| = 2|V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k| (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

$$= P_i + |V_i|^2 G_{ii}$$

Off diagonal elements.

$$\frac{\partial P_i}{\partial |V_k|} = |V_i| (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

$$\frac{\partial P_i}{\partial |V_k|} |V_k| = |V_i||V_k| (G_{ik} \cos\delta_{ik} + B_{ik} \sin\delta_{ik})$$

(*) Elements of J_4

Diagonal elements.

$$\frac{\partial Q_i}{\partial |V_i|} = -2B_{ii} |V_i| + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_k| (G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$$

$$\frac{\partial Q_i}{\partial |V_i|} |V_i| = -2B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ \neq i}}^{n} |V_i||V_k| (G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$$

$$= Q_i - B_{ii}|V_i|^2$$

Off diagonal elements.

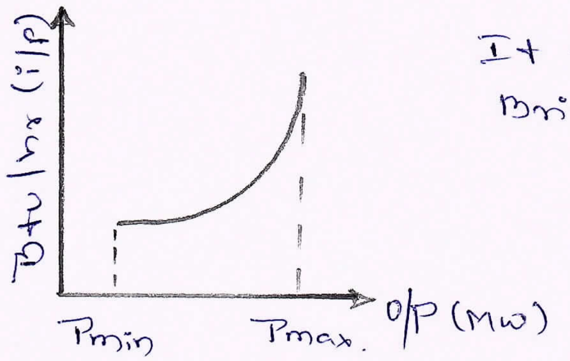
$$\frac{\partial Q_i}{\partial |V_k|} = |V_i| (G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$$

$$\frac{\partial Q_i}{\partial |V_k|} |V_k| = |V_i||V_k| (G_{ik} \sin\delta_{ik} - B_{ik} \cos\delta_{ik})$$

Module-04

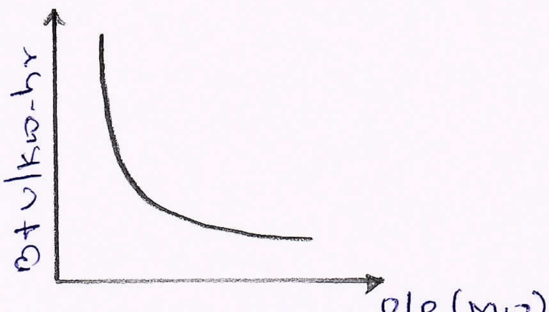
07.a. Explain the followings.
i) Input - output curve
ii) Heat rate curve.
iii) Incremental cost curve related to thermal plants.
[06 Marks]

i) Input - output curve



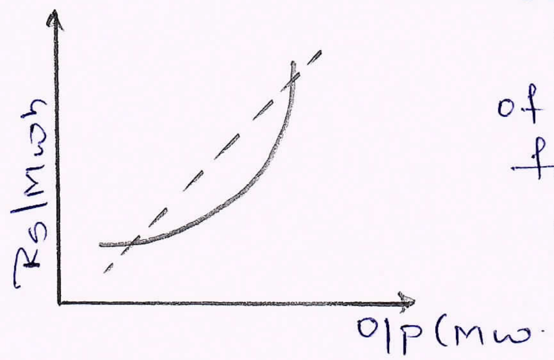
It is a plot of the input in British thermal units/hr v/s Power output in MW.

ii) Heat rate curve.



Heat rate is ratio of fuel input in Btu to energy output in kwh. It is slope of the input-output curve at any point. Reciprocal of heat rate is called fuel efficiency. Heat rate curve is ploted heat rate v/s output in MW.

iii) Incremental cost curve.



Incremental cost is the product of incremental fuel rate and fuel cost Rs/Btu. Unit of the incremental fuel cost is Rs/Mwh.

07. b. The fuel inputs per hour of plant 1 and 2 are given as $F_1 = 0.2P_1^2 + 40P_1 + 120$ Rs/hr. $F_2 = 0.25P_2^2 + 30P_2 + 150$ Rs/hr. Determine the economic operating schedule and the corresponding cost of generation if the maximum and minimum loading units is 100 MW and 25 MW.

the demand is 180MW and transmission line losses are neglected. If the load is equally shared by both units, determine the saving obtained by loading the units as per equal incremental production cost (06 Marks)

$$\frac{dF_1}{dPG_1} = 0.4P_1 + 40$$

$$\frac{dF_2}{dPG_2} = 0.5P_2 + 30.$$

for economic generation

$$\frac{dF_1}{dPG_1} = \frac{dF_2}{dPG_2} \Rightarrow 0.4P_1 + 40 = 0.5P_2 + 30 \longrightarrow (1)$$

also $P_1 + P_2 = 180. \longrightarrow (2)$

$P_2 = 180 - P_1$

$\therefore\ 0.4P_1 + 40 = 0.5(180 - P_1) + 30.$

$P_1 = 88.88$ MW.

$P_2 = 180 - 88.88 = 91.12$ MW.

$F_1 = 0.2(88.88)^2 + 40(88.88) + 120 = 5255.13$ Rs/hr

$F_2 = 0.25(91.12)^2 + 30(91.12) + 150 = 4959.31$ Rs/hr.

Total cost $= 10214.44$ Rs/hr.

With equal load sharing

$P_1 = P_2 = 180/2 = 90$ MW.

$F_1 = 0.2(90)^2 + 40(90) + 120 = 5340$ Rs/hr.

$F_2 = 0.25(90)^2 + 30(90) + 150 = 4875$ Rs/hr.

Total cost $= 10215$ Rs/hr.

Saving $= 10215 - 10214.44$

$= 0.56$ Rs/hr.

07. c. Discuss the algorithm procedure for priority list method of unit commitment solution
(08 Marks)

01. Determine the hourly load forecast for next 24 h.
02. Prioritize the units based on their production cost and prepare a table based on unit combination to meet required load.
03. For the first hour determine the minimum number of units necessary to carry the maximum pridicted load and the spinning reserve.
04. Compare the number of units running in the present hour with the minimum number required for the next hour.
05. If the number required in the next hour is grater than the number of units in the present hour, startup the units according to the priority list.
06. If the minimum number of units required in the next hour is lesser than those running in the present hour, then determine whethere dropping the unit with highest priority number (least efficient) in the present group will leave sufficient generation to supply the load + spinning reserve. If not do not shut down the unit.
07. Else, determine the number of 'H' hours, before which the unit would be needed again.
08. If 'H' is less than the minimum down time of the unit, continue with the present commitment.
09. Else, calculate two costs.
a. Sum of the hourly production cost for the next 'H' hours with unit up.
b. Hourly production costs with unit shut down + the start-up cost of the unit (which is the minimum of cooling or banking cost).
If there is significant saving from shutting down

the unit, shut it down.

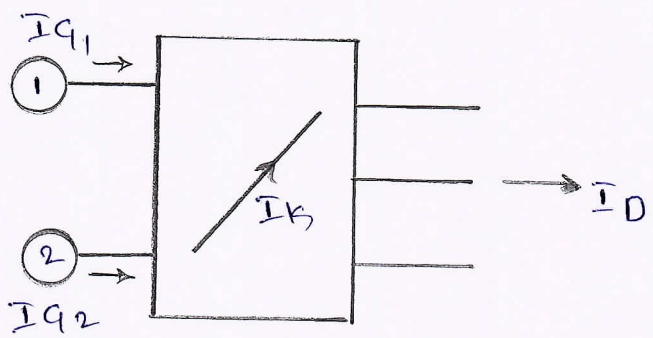
10. Repeat the procedure hour by hour for next 24 hours.

08.a. With usual notation, derive the generalized transmi-ssion loss formula and B-coefficient. (08 Marks)

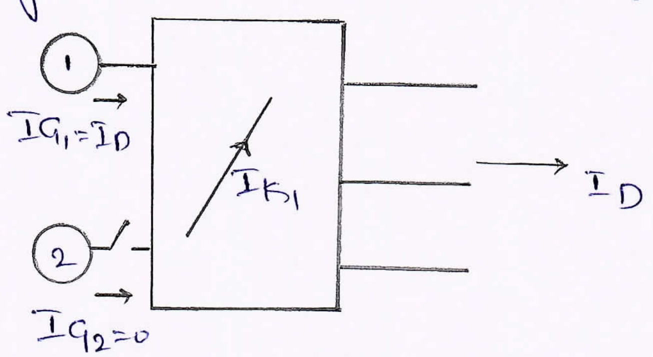
Method of obtaining loss coefficients has been presented by Kron. following assumptions are made.

01. All load currents have same phase angle with respect to a common reference.
02. The ratio X/R is the same for all the network branches.

Consider two generating plants connected to an arbitrary number of loads through a transmission network as shown below.



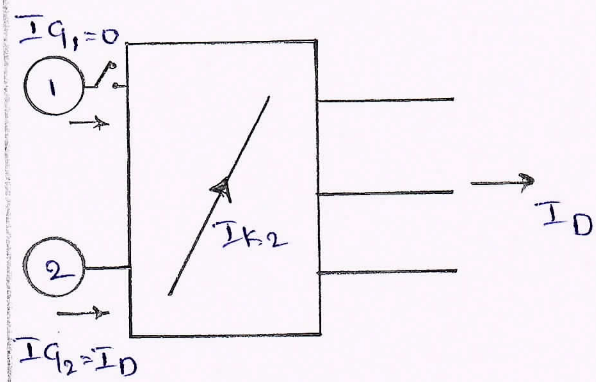
Assume that the total load is supplied by only generator 1, as shown below.



Let current through k^{th} branch in the network be I_{k1}. Let us define current distribution factor

$$N_{k1} = \frac{I_{k1}}{I_{G1}} = \frac{I_{k1}}{I_D}$$

Now assume that I_D is given by only generator 2



So $N_{k2} = \dfrac{I_{k2}}{I_D}$

By principle of superposition.

$I_k = N_{k1} I_{G_1} + N_{k2} I_{G_2}$.

Let $I_{G_1} = |I_{G_1}| \angle \sigma_1$ and $I_{G_2} = |I_{G_2}| \angle \sigma_2$

So $|I_k|^2 = (N_{k1}|I_{G_1}|\cos\sigma_1 + N_{k2}|I_{G_2}|\cos\sigma_2)^2 + (N_{k1}|I_{G_1}|\sin\sigma_1 + N_{k2}|I_{G_2}|\sin\sigma_2)^2$.

$= N_{k1}^2|I_{G_1}|^2(\cos^2\sigma_1 + \sin^2\sigma_1) + N_{k2}^2|I_{G_2}|^2(\cos^2\sigma_2 + \sin^2\sigma_2)$
$+ 2\left[N_{k1}|I_{G_1}|N_{k2}|I_{G_2}|\cos\sigma_1\cos\sigma_2 + N_{k1}|I_{G_1}|N_{k2}|I_{G_2}|\sin\sigma_1\sin\sigma_2\right]$

$= N_{k1}^2|I_{G_1}|^2 + N_{k2}^2|I_{G_2}|^2 + 2N_{k1}|I_{G_1}|N_{k2}|I_{G_2}|\cos(\sigma_1-\sigma_2)$

We know that

$|I_{G_1}| = \dfrac{P_{G_1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G_2}| = \dfrac{P_{G_2}}{\sqrt{3}|V_2|\cos\phi_2}$.

Total transmission loss in the system is given by.

$P_L = \sum_k 3|I_k|^2 R_k$. for all branches of the network.

R_k is branch resistance.

So $P_L = \dfrac{P_{G_1}^2}{|V_1|^2\cos^2\phi_1}\sum_k N_{k1}^2 R_k + \dfrac{P_{G_2}^2}{|V_2|^2\cos^2\phi_2}\sum_k N_{k2}^2 R_k$

$+ \dfrac{2P_{G_1}P_{G_2}\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_k N_{k1}N_{k2}R_k$.

We have

$$P_L = P_{G_1}^2 B_{11} + P_{G_2}^2 B_{22} + 2 P_{G_1} P_{G_2} B_{12}$$

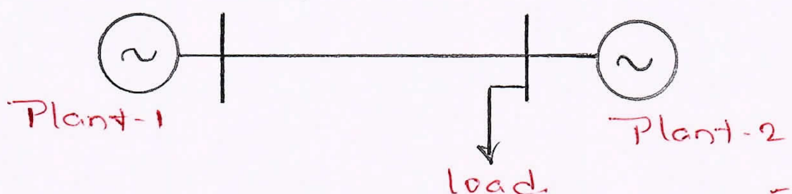
So

$$B_{11} = \frac{1}{|V_1|^2 \cos^2\phi_1} \sum_K N_{K_1}^2 R_K.$$

$$B_{22} = \frac{1}{|V_2|^2 \cos^2\phi_2} \sum_K N_{K_2}^2 R_K.$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K_1} N_{K_2} R_K.$$

08. b. A system consists of two plants connected to a transmission line, the load is located at plant-2. The power transfer of 100 MW from station 1 to the load results in a loss of 8 MW. Find the required generation at each station and the power received by the load, when the system is operating with λ = Rs 100/MWh. The incremental fuel cost of two plants are $\frac{dC_1}{dP_1} = 0.12P_1 + 65$ Rs/MWh and $\frac{dC_2}{dP_2} = 0.25P_2 + 75$ Rs/MWh.



[06 Marks]

Since the load is connected at bus 2, no loss is incurred when plant 2 supplies the load.

$\therefore B_{12} = 0$, $B_{22} = 0$

We have $P_L = B_{11}P_1^2 + 2B_{12}P_1P_2 + B_{22}P_2^2$

$$P_L = B_{11}P_1^2$$

$$\frac{\partial P_L}{\partial P_1} = 2B_{11}P_1$$

$$\frac{\partial P_L}{\partial P_2} = 0.$$

from the given data

$P_L = 8\,MW$ if $P_1 = 100\,MW$.

$$8 = B_{11}(100)^2$$

$$B_{11} = 0.0008\,MW^{-1}$$

co-ordination equation with loss is

$$\frac{dF_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda$$

for plant 1 $\quad \frac{dC_1}{dP_1} + \lambda \frac{\partial P_L}{\partial P_1} = \lambda.$

$$(0.12P_1 + 65) + 100(2 \times 0.0008)P_1 = 100.$$

$$P_1 = 125\,MW.$$

for plant 2 $\quad \frac{dC_2}{dP_2} + \lambda \frac{\partial P_L}{\partial P_2} = \lambda.$

$$\frac{dF_2}{dP_2} = \lambda \Rightarrow 0.25\,P_2 + 75 = 100$$

$$P_2 = 100\,MW.$$

total loss $= B_{11} PG_1^2 = 0.0008 \times 125^2$

$$P_L = 12.5\,MW.$$

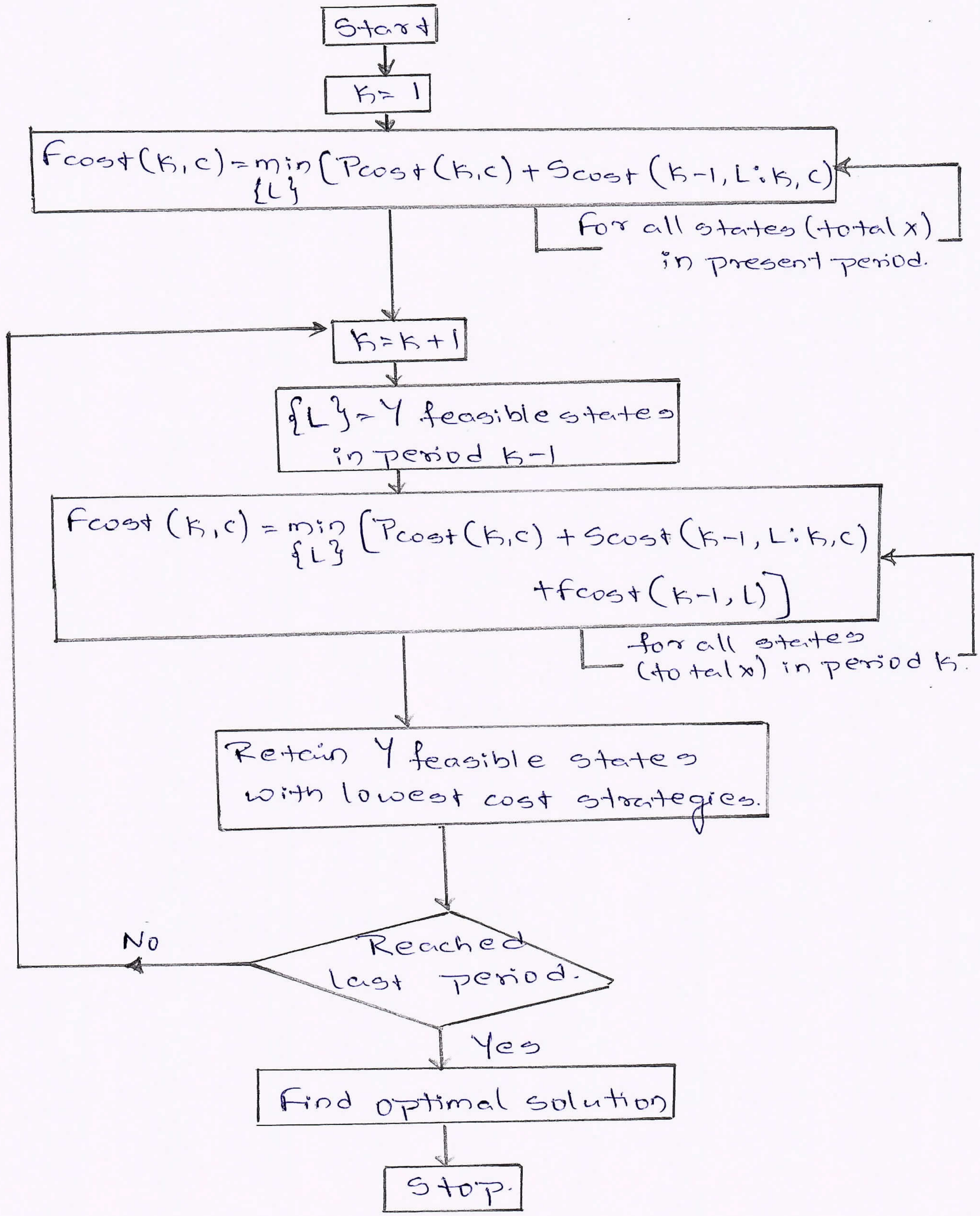
total generation $= P_1 + P_2 = 125 + 100 = 225\,MW.$

total load $P_D = P_1 + P_2 - P_L$

$$= 125 + 100 - 12.5$$

$$P_D = 212.5\,MW$$

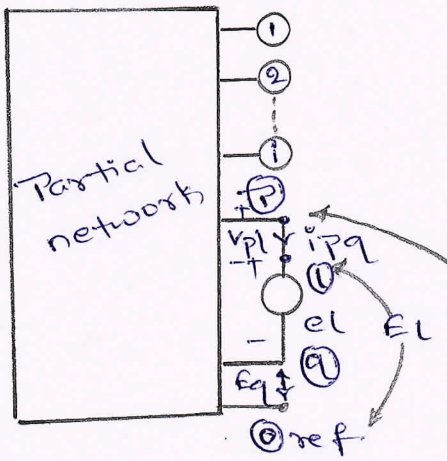
08.c. Draw the flow chart of dynamic forward DP approach for unit commitment. (06 Marks)


Start
K = 1
Fcost(K,c) = min {L} [Pcost(K,c) + Scost(K-1, L:K, c)]
For all states (total x) in present period.
K = K+1
{L} = Y feasible states in period K-1
Fcost (K,c) = min {L} [Pcost(K,c) + Scost(K-1, L:K,c) + Fcost(K-1, L)]
for all states (total x) in period K.
Retain Y feasible states with lowest cost strategies.
Reached last period.
No
Yes
Find optimal solution
Stop.

Module-05

09.a. Obtain the generalized algorithm expression for bus impedance matrix elements when a link is added to the partial network. Also discuss the special cases. (10 Marks)

The link is an element added between two existing buses. The element is modeled as an element p-l in series with a source e_l as shown below.



a fictitious node 'l' is introduced. The source e_l is between buses l and q. e_l is so chosen that the current $i_{pq}=0$, so that the element p-l can be treated as a branch. The performance equation given by.

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ e_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1l} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2l} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pl} \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{ml} \\ Z_{l1} & Z_{l2} & \cdots & Z_{lp} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix}$$

e_l is voltage between node l with reference to q

So $e_l = E_l - E_q = E_p - V_{pl} - E_q \longrightarrow (1)$

Z_{li} can be determined by injecting a current at the i^{th} bus and calculating the voltage at l^{th} node with respect to bus q with all other current injections set to zero.

$E_k = Z_{ki} I_i \qquad k = 1, 2 \ldots m$

$e_l = Z_{li} I_i$

if $I_i = 1.0$ pu

$e_l = Z_{li}$

Since $i_{pq}=0$, $i_{pl}=0$

$$\therefore i_{pl} = y_{plpl} V_{pl} + \bar{y}_{plrs} \bar{V}_{rs} = 0$$

$$\therefore V_{pl} = -\frac{\bar{y}_{plrs} \bar{V}_{rs}}{y_{plpl}}$$

$\bar{y}_{plrs} = \bar{y}_{pqrs}$ and $y_{plpl} = y_{pqpq}$.

$$\therefore V_{pl} = -\frac{\bar{y}_{pqrs} \bar{V}_{rs}}{y_{pqpq}}$$

$$E_p - E_q - e_l = -\frac{\bar{y}_{pqrs}(\bar{E}_r - \bar{E}_s)}{y_{pqpq}}.$$

$$e_l = E_p - E_q + \frac{\bar{y}_{pqrs}(\bar{E}_r - \bar{E}_s)}{y_{pqpq}}$$

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pqrs}(\bar{Z}_{ri} - \bar{Z}_{si})}{y_{pqpq}}. \quad \begin{array}{l} i = 1, 2 \ldots m \\ i \neq l \end{array}$$

To calculate Z_{ll}, inject a current I_l of 1pu at the l^{th} bus with bus q as reference and calculate the voltage of bus l with respect to bus q. Since all other current injections are zero we can write

$$E_k = Z_{kl} I_l \qquad k = 1 \ldots m$$

$$E_l = Z_{ll} I_l$$

let $I_l = 1.0$ pu. $\quad i_{pl} = -I_l = -1.0$ pu.

$$\therefore i_{pl} = y_{plpl} V_{pl} + \bar{y}_{plrs} \bar{V}_{rs} = -1$$

$$\text{so } V_{pl} = -\left(\frac{1 + \bar{y}_{plrs} \bar{V}_{rs}}{y_{plpl}}\right)$$

$$e_l = E_p - E_q + \frac{1 + \bar{y}_{plrs} \bar{V}_{rs}}{y_{plpl}}.$$

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pqrs}(\bar{Z}_{rl} - \bar{Z}_{sl})}{y_{pqpq}}.$$

Case 01.

If there is no mutual coupling between the added element and elements of the partial network.

$\bar{y}_{pqrs} = 0$ and $Z_{pqpq} = 1/y_{pqpq}$.

So $Z_{li} = Z_{pi} - Z_{qi}$ $i = 1 \ldots m$.

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pqpq}$

Case 02.

If there is no mutual coupling and P is the reference node $Z_{pi} = Z_{pl} = 0$.

$\therefore Z_{li} = -Z_{qi}$ $i = 1, 2 \ldots m$

$Z_{ll} = -Z_{ql} + Z_{pqpq}$.

09.b. Explain clearly the point-by-point method of solving swing equation. Mention the assumptions made.

(10 Marks)

Consider the swing equation

$$M \frac{d^2\delta}{dt^2} = P_m - P_{max} \sin\delta = P_a.$$

or $$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}.$$

In pu $M = \frac{H}{180f}\ s^2/°\text{elec}$ or $\frac{H}{\pi f}\ s^2/\text{rad}$

H = stored kinetic energy in mega joules.

The solution of $\delta(t)$ is obtained in discrete intervals of time, with a uniform interval of Δt. Discretization is done as follows.

01. The acceleration calculated at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered. This assumption also implies that the accelerating power P_a computed at the beginning of an interval is.

assumed constant from the middle of the preceding interval to the middle of the interval being considered.

02. The speed deviation $\omega = \frac{d\delta}{dt}$ is assumed constant over the entire interval computed at the middle of the interval. ω is the speed above synchronous speed ω_s.

Consider the nth interval beginning at $t=(n-1)\Delta t$. The angular position is δ_{n-1}. The acceleration $\alpha(n-1)$ as calculated at this interval is assumed constant from $t=(n-3/2)\Delta t$ to $(n-1/2)\Delta t$. The change in speed during this time is given by $\Delta\omega(n-1/2) = \Delta t\, \alpha(n-1)$
$= \Delta t \frac{P_a(n-1)}{M}$.

where $P_a(n-1) = P_m - P_{max} \sin(\delta(n-1))$

the speed at the end of this interval is

$$\omega_{n-1/2} = \omega_{n-3/2} + \Delta\omega_{n-1/2}$$

From our assumption, the speed is constant throughout the nth interval. The change in angular position during the nth interval is.

$$\Delta\delta_n = \Delta t\left(\omega_{n-1/2}\right) = \Delta t\left[\omega_{n-3/2} + \Delta\omega_{n-1/2}\right]$$

$$= \Delta t\, \omega_{n-3/2} + \Delta t^2 \frac{P_a(n-1)}{M} \longrightarrow (1)$$

lll'y we can write

$$\Delta\delta_{n-1} = \Delta t\, \omega_{(n-3/2)} \longrightarrow (2)$$

Subtracting (2) from (1)

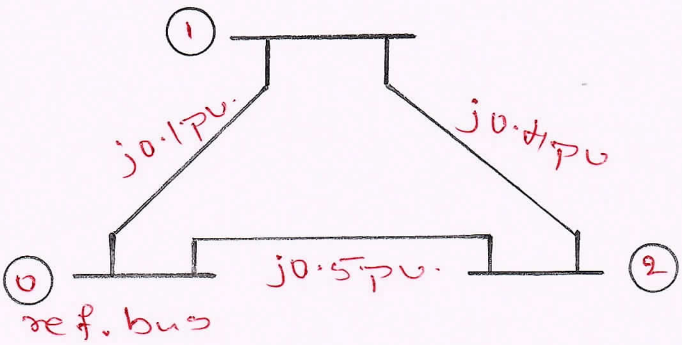
$$\text{So } \Delta\delta_n = \Delta\delta_{n-1} + \Delta t^2 \frac{P_a(n-1)}{M}.$$

The angular position at the end of the nth interval is given by.

$$\delta_n = \delta_{n-1} + \Delta\delta_n.$$

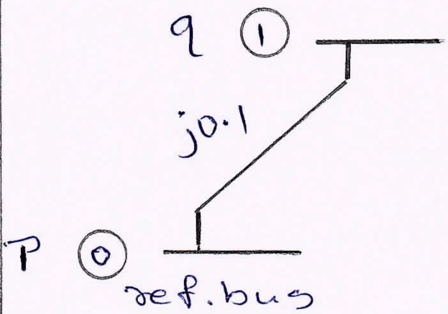
The process is repeated to obtain $P_a(n)$, $\Delta\delta_{n+1}$ and δ_{n+1}. Greater accuracy can be obtained by considering smaller time intervals.

10. a. Obtain Zbus by building algorithm for the system shown in fig. All values are in P.U. (impedance). Take bus '0' as reference bus. Add the elements in the order of ref. bus to bus 1, ref. bus to bus 2 and finally bus 1 to bus 2.



[10 Marks]

Adding elements between ref. bus to bus 1



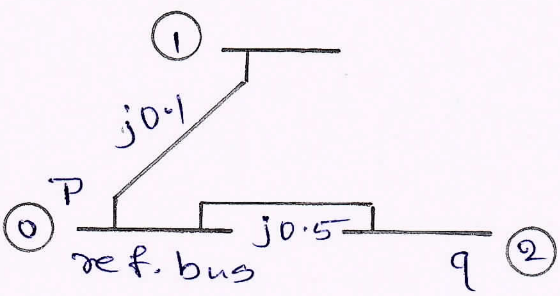
$Z_{bus} = [Z_{11}]$

$Z_{qq} = Z_{pqpq}$

$Z_{11} = Z_{0101} = j0.1$

$Z_{bus} = [j0.1]$

Adding elements between ref. bus to bus 2.



$$Z_{bus} = \begin{bmatrix} j0.1 & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$Z_{qi} = 0$

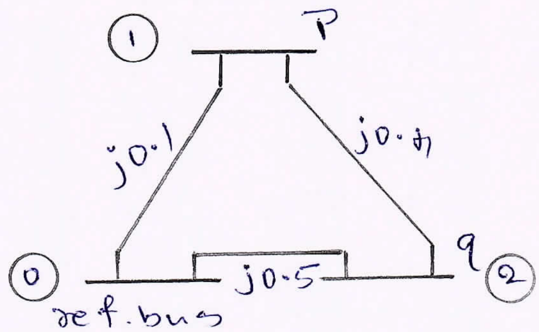
$Z_{21} = 0 = Z_{12}$

$Z_{qq} = Z_{pqpq}$

$Z_{22} = Z_{0202} = j0.5$

$$Z_{bus} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.5 \end{bmatrix}$$

Adding element between bus 1 to bus 2.



$$Z_{bus} = \begin{bmatrix} j0.1 & 0 & Z_{1l} \\ 0 & j0.5 & Z_{2l} \\ Z_{l1} & Z_{l2} & Z_{ll} \end{bmatrix}$$

$Z_{li} = Z_{pi} - Z_{qi}$

$Z_{l1} = Z_{11} - Z_{21} = j0.1 - 0 = j0.1 = Z_{1l}$

$Z_{l2} = Z_{12} - Z_{22} = 0 - j0.5 = -j0.5 = Z_{2l}$

$Z_{ll} = Z_{pl} - Z_{ql} + Z_{pqpq}$

$= Z_{1l} - Z_{2l} + Z_{1212}$

$= j0.1 - (-j0.5) + j0.7$

$Z_{ll} = j1$

$$Z_{bus}(new) = Z_{bus}(old) - \frac{Z_{il} Z_{li}}{Z_{ll}}$$

$$= \begin{bmatrix} j0.1 & 0 \\ 0 & j0.5 \end{bmatrix} - \frac{1}{j1} \begin{bmatrix} j0.1 \\ -j0.5 \end{bmatrix} \begin{bmatrix} j0.1 & -j0.5 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.09 & j0.05 \\ j0.05 & j0.25 \end{bmatrix}$$

10.b. Describe the methodology of using Runge-Kutta technique for transient stability studies of a power system.

[10 Marks]

In Runge-Kutta method, the changes in dependent variables are calculated from a given set of formulae derived using an approximation to replace a truncated Taylor's series expansion. The formulae for Runge-Kutta fourth order approximation for solution of two simultaneous differential equations are given below.

Given $\frac{dx}{dt} = f_x(x, y, t)$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values x_0, y_0, t_0 and step size h, the updated values are

$$x_1 = x_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

where $k_1 = f_x(x_0, y_0, t_0)h$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + h/2\right)h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + h/2\right)h$$

$$k_4 = f_x(x_0 + k_3, y_0 + l_3, t_0 + h)h$$

$$l_1 = f_y(x_0, y_0, t_0)h$$

$$l_2 = f_y\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right)h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right)h$$

$$l_4 = f_y(x_0 + k_3, y_0 + l_3, t_0 + h)h$$

we have

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_m - P_{max}\sin\delta}{M}$$

Starting from intial value δ_0, ω_0, t_0, and step size of Δt the formulae are as follows.

$$k_1 = \omega_0 \Delta t$$

$$l_1 = \left[\frac{P_m - P_{max}\sin\delta_0}{M}\right]\Delta t$$

$$k_2 = \omega_0 + \frac{l_1}{2}\Delta t$$

$$l_2 = \left[\frac{P_m - P_{max}\sin\left(\delta_0 + \frac{k_1}{2}\right)}{M}\right]\Delta t$$

$$k_3 = \omega_0 + \frac{l_2}{2}\Delta t$$

$$l_3 = \left[\frac{P_m - P_{max}\sin\left(\delta_0 + \frac{k_2}{2}\right)}{M}\right]\Delta t$$

$$k_4 = \omega_0 + l_3\Delta t$$

$$l_4 = \left[\frac{P_m - P_{max}\sin\left(\delta_0 + k_3\right)}{M}\right]\Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$\omega_1 = \omega_0 + \frac{1}{6}\left(l_1 + 2l_2 + 2l_3 + l_4\right)$$