CBCS SCHEME

USN | | | | | | | | | | | **18ME71**

Seventh Semester B.E. Degree Examination, June/July 2023
Control Engineering

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define control system with block diagram. Explain open loop and closed loop control system. **(08 Marks)**

b. Explain the following controllers :
i) Proportional controllers
ii) Proportional plus integral controller. **(06 Marks)**

c. Obtain the differential equation and determine the transfer function of mechanical networks shown in Fig.Q1(c).

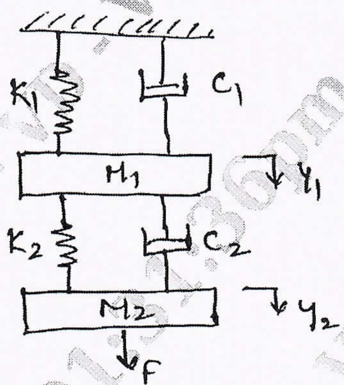

Fig.Q1(c) **(06 Marks)**

OR

2 a. What are the requirements of a Good Control System? Distinguish between open loop and closed loop control system. **(06 Marks)**

b. Explain the following controllers with block diagram :
i) Integral controller
ii) Derivative controller
iii) Proportional plus integral plus differential controllers. **(09 Marks)**

c. Derive an expression for transfer function of hydraulic system. **(05 Marks)**

Module-2

3 a. What are standard test signals? Derive an expression for transient response of first order system subjected to step input I/P. **(05 Marks)**

b. Derive an expression for steady state error and explain error constants. **(05 Marks)**

c. A unity feedback system is characterized by an open loop transfer function : 169-G

$$G(s) = \frac{K}{s(s+10)}$$

Determine the gain K so that, the system will have a damping ratio of 0.5. For this value of K determine the settling time, peak overshoot and time of peak overshoot for unit step input I/P. **(10 Marks)**

OR

4 a. Explain with the help of neat sketch transient response specifications of second order under damped system. **(06 Marks)**

b. An undrerdamed second order system is subjected to a step input of 4 units. If the first peak overshoot of 25% occurs at a time equal to 0.8 seconds. Then determine rise time, settling time, damping co-efficient (factor) and natural frequency. **(08 Marks)**

c. A unity feedback system has $G(s)=\dfrac{40(s+2)}{s(s+1)(s+4)}$. Determine : i) Type of system ii) All error co-efficient iii) Steady state error for ramp input with magnitude 4. **(06 Marks)**

93-B

Module-3

5 a. What is block diagram? Obtain the transfer function C(s)/R(s) for the following Fig.Q5(a) using block diagram reduction rules.

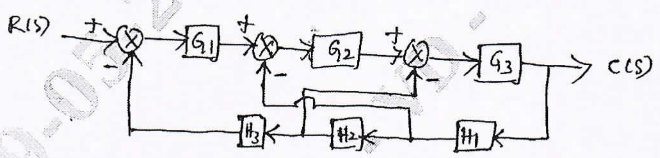

Fig.Q5(a) **(10 Marks)**

b. Define the terms : Node, Branch, Transmittance, Loop gain, Forward path, Source, Non-touching loops and also explain Masson's Gain Formulae. **(10 Marks)**

OR

6 a. Construct the signal flow graph for the following set of system equations and find the transfer function :
$Y_2 = G_1Y_1 + G_3Y_3$; $Y_3 = G_4Y_1 + G_2Y_2 + G_5y_3$ and $Y_4 = G_6Y_2 + G_7Y_3$. **(10 Marks)**

b. Draw the signal flow graph for the system shown in Fig.Q6(b) and determine. The transfer function using Masson's gain formulae.

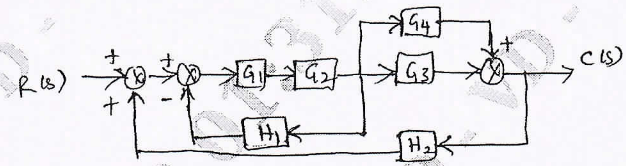

Fig.Q6(b) **(10 Marks)**

Module-4

7 a. State and explain the Routh's stability criterion. The characteristic equation of a system is given by $s^4 + 6s^3 + 23s^2 + 40s + 50 = 0$. Determine the stability using R–H criterion. **(06 Marks)**

b. The characteristics equation of a system is given by $s^4 + 6s^3 + 11s^2 + K = 0$. Determine the range of K for the system to be stable. Use R – H criterion, **(06 Marks)**

c. Sketch the root locus plot of a unity feedback system with an open loop transfer function : $G(s)=\dfrac{K}{s(s+2)(s+4)}$. What is the greatest value of K which can be used before continuous oscillations occurs. Also determine the frequency of continuous oscillations. **(08 Marks)**

OR

8 a. Investigate the stability of the system using Routh Henvitz criterion having the following characteristics in $s^5 + 4s^4 + 12s^3 + 20s^2 + 30s + 100 = 0$. **(08 Marks)**

b. Sketch the root locus plot for the transfer function : $G(s)H(s)=\dfrac{K}{s(s^2+2s+2)}$. For what value of K will the system be unstable? Find the frequency at which the locus crosses the imaginary axis. **(12 Marks)**

P-269
G

Module-5

9 a. Explain Nyquist Stability Criterion. **(04 Marks)**

b. Sketch the polar plot for $GH(s) = \dfrac{12}{s(s+2)(s+4)}$ and ascertain the nature of stability. **(06 Marks)**

c. Sketch the bode plot and determine the gain crossover and phase crossover frequency,

$GH(s) = \dfrac{10}{s(1+0.55)(1+0.1s)}$. **(10 Marks)**

OR

10 a. For a system with an open loop transformation,

$GH(s) = \dfrac{1}{s(1+2s)(1+s)}$ Comment on stability of the system by Nyquist plot. **(08 Marks)**

b. Draw the bode plot for a system having $G(s)H(s) = \dfrac{100}{s(s+1)(s+2)}$. Find, Gain Margin, Phase margin, Gain crossover frequency and phase crossover frequency. **(12 Marks)**

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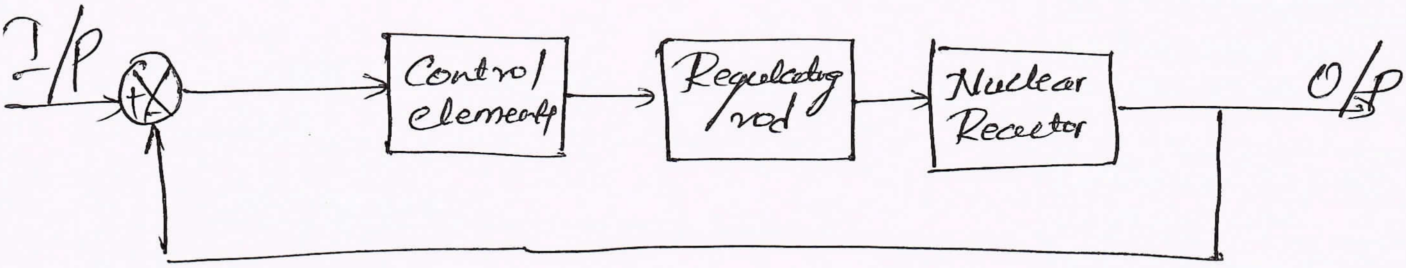
(K S Pujari)

<u>CE</u>

Qno-1(a) Control System is a set of mech, electrical & electronic devices that regulate, other devices or systems by a way of control loops. For ex. the nuclear reactor power level control system as shown in the following diagram.

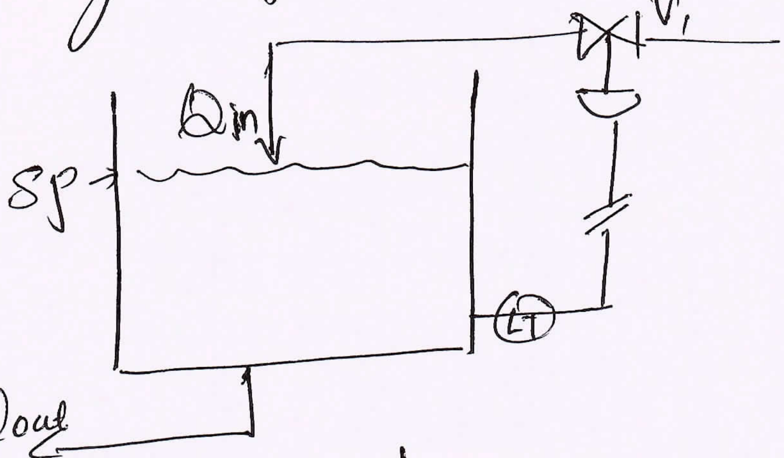


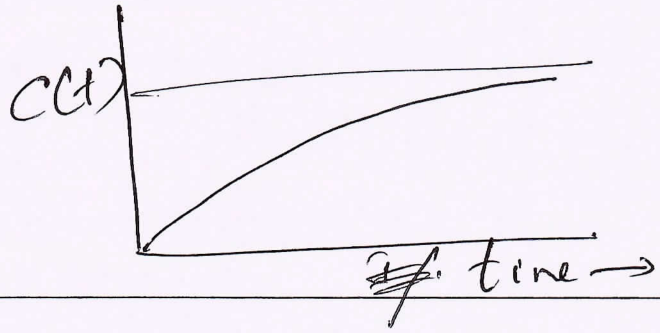
For proper operation the power level of the reactor must be maintened a a desired value or it must vary in prescribed maner. In this system if the power output diff from the reference I/P a positive or negative actuating signal is applied to the control elements which in turn move a regulating rod in the proper direction to achieve the desired O/P power

Q.No.	Solution and Scheme	Marks
Q no-1(b)	Propotional Controller	

It is Control System technology in which o/p is propotional to the difference between ~~ste~~ Setpoint value and Current value of process variable. For ex.

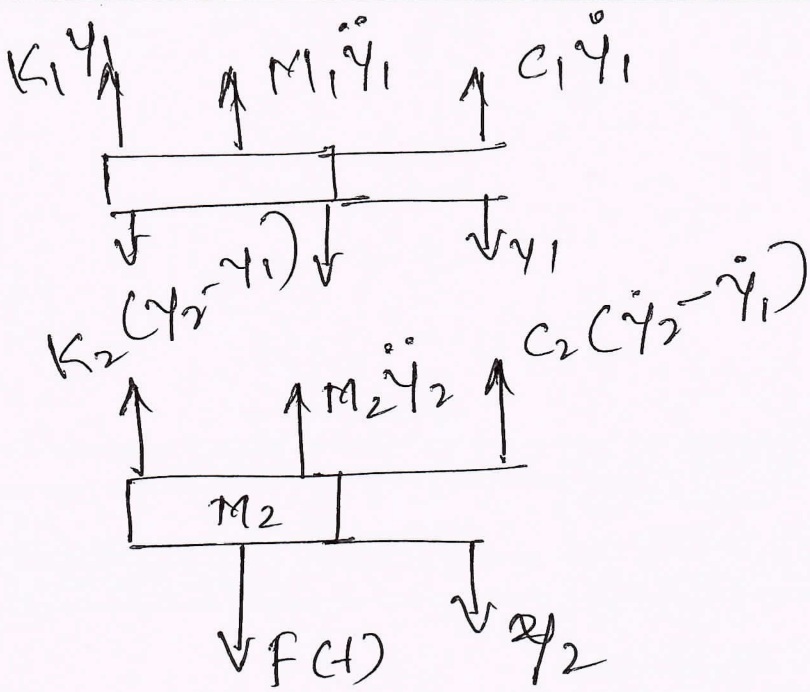
If the outflow Q_0 increases then the level in the tank will fall. The pressure sensed by the level. transmitter, which is representative of the level in the tank, will also fall causing a decrease in the o/p signal from the level transmitter.





Q.No.	Solution and Scheme	Marks
	Propotional plus integral Control System CP In this type of System the Control action ~~of~~ is of both propotional as well as the integral ~~Erro~~ controller, is utilized. The Combination of two different Controllers produces a more efficient Controller which eliminates the disadvantages associated with each one of them. It provides a Controller output propotional to the error signal. Integral action supplies a Controller This type of Controller Commonly used in Control System to Correct for error between the Commanded set point and actual ~~b~~ value based on some feedback. For ex. Cruise ~~on~~ Control on Car where ascending a hill would lower speed if Constant engine power were applied. The PID Controls the over shots.	

Q.No.	Solution and Scheme	Marks
Q no-1 (c)		



<u>at Node y_2</u>

$$F(t) = M_2\ddot{y}_2 + C_2(\dot{y}_2 - \dot{y}_1) + K_2(y_2 - y_1)$$

<u>at Node y_1</u>

$$K_2(y_2 - y_1) + C_2(\dot{y}_2 - \dot{y}_1) = M_1\ddot{y}_1 + C_1\dot{y}_1 + K_1 y_1$$

$$F(t) = M_2\ddot{y}_2 + M_1\ddot{y}_1 + C_1\dot{y}_1 + K_1 y_1$$

Taking Laplace transform on both side

Q.No.	Solution and Scheme	Marks
	$F(s) = M_2 s^2 Y_2(s) + M_1 s^2 Y_1(s) + C_1 s Y_1(s) + K_1 Y_1(s)$	
	~~z~~ Let o/p $Y_1(t)$	
	So $TF = \dfrac{Y_1(s)}{F(s)}$	
	$\dfrac{Y_1(s)}{F(s)} = \dfrac{1}{M_2 s^2 + M_1 s^2 + C_1 s + K_1}$	
Qno-2(a)	Requirement of Ideal Control System (i) Output of the System should be non-oscilatory type (ii) Overall gain of the System must be nearer to the open loop System (iii) while designing closed loop system Consider Stability of the System. ~~Dis~~ Difference betn open & Closed loop Open loop System is simple, least Complex, Cheaper and economical to build Since Stability is not ~~m~~ the major problem to be	

Considered.
Closed loop System is automatically responded to the disturbance variation

Q.No.	Solution and Scheme	Marks

without the presence or indulgence of human operator.

Q no-2(b)(i) <u>Integral Controller</u>

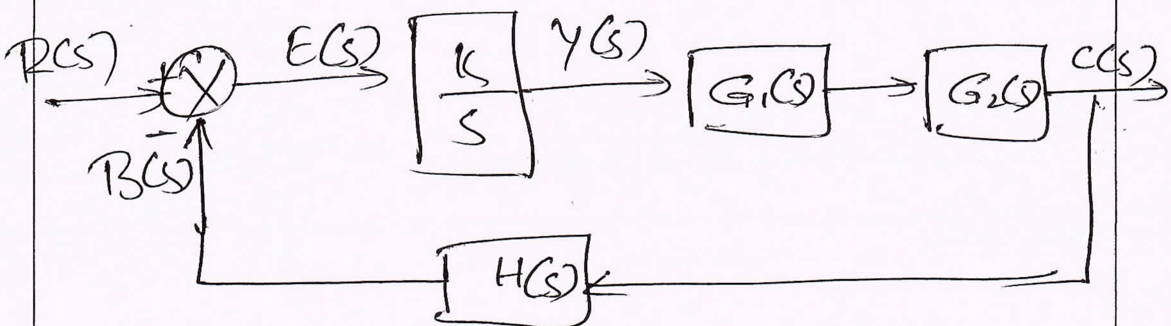
In this the value of the controller output $y(t)$ is altered at rate propotional to the error signal $e(t)$. The output $y(t)$ depends on the integral of the error signal $e(t)$

Mathematically

$$\frac{dy(t)}{dt} = K\, e(t)$$

$$y(t) = K \int_0^t e(t)\, dt$$

$$Y(s) = \frac{K\, E(s)}{s} \qquad \frac{Y(s)}{E(s)} = \frac{K}{s}$$



Q.No.	Solution and Scheme	Marks

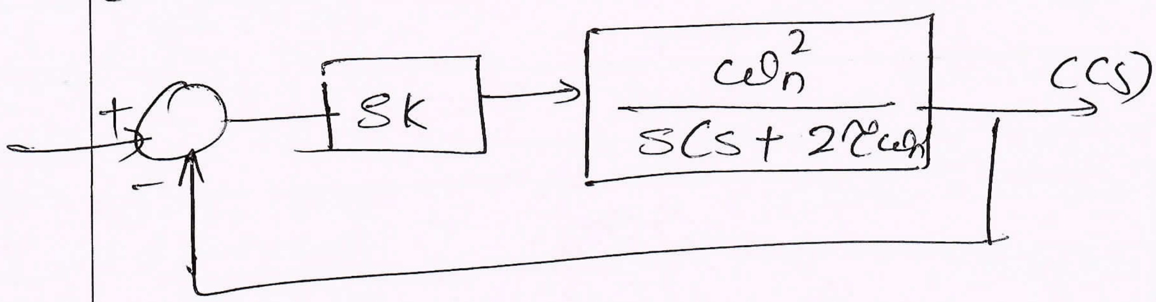
Derivative Controller

A Control System is said to be derivative if output $y(t)$ depends on the rate of Change of error

Mathematically $y(t) = \frac{K_d . e(t)}{dt}$

$$Y(s) = sKE(s)$$

$K \rightarrow$ It is adjustable Constant and called variable of derivative Control action.



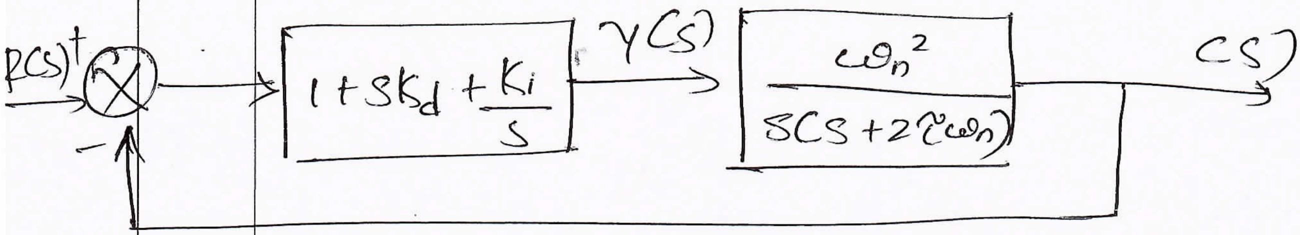
PID Control System

This type of Control action employ propotional, integral & derivative Control action together in Control system so as to derive the advantage of Control action in one.

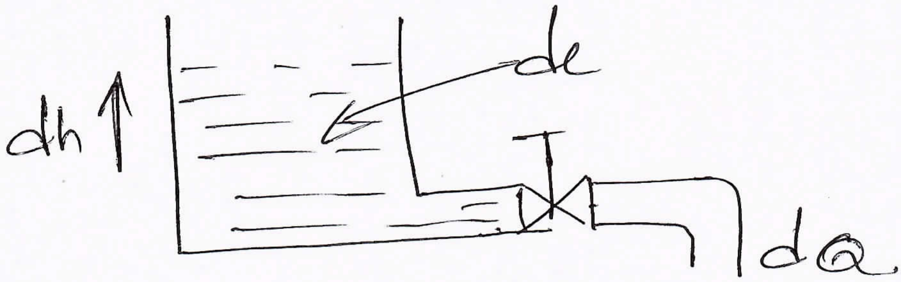
$$y(t) = e(t) + K_d \frac{de(t)}{dt} + K_i \int e(t)dt$$

$$Y(s) = E(s)\left[1 + sK_d + \frac{K_i}{s}\right]$$

Q.No.	Solution and Scheme	Marks



Q no-2 (c) TF for Hydraulic System



$$R = \frac{dh}{dq}$$

$$C = \frac{dv}{dh}$$

$$C\,dh = \left(q_i - \frac{h}{R}\right) dt$$

$$C\,dh = \left(\frac{q_i R - h}{R}\right) dt$$

$$RC\frac{dh}{dt} = q_i R - h$$

$$RC\frac{dh}{dt} + h = q_i R$$

Q.No.	Solution and Scheme	Marks

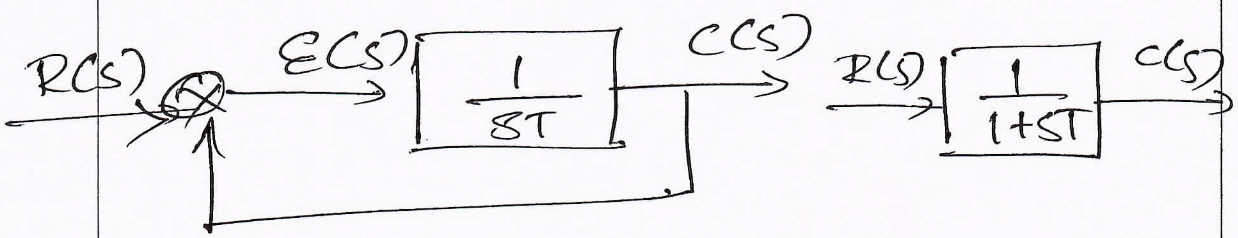
$$R\,C(s)\,h(s) + h(s) = q_i(s)\,R$$

$$h(s)\left[RCs + 1\right] = q_i(s)\,R$$

$$\frac{h(s)}{q_i(s)} = \frac{R}{R(s+1)}$$

Qno-3 (a) A System is generaly designed and analysed for certain anticipated type of inputs generaly Step, Ramp and parabolic type of input signal. These are refered as standart test signal.

First order System block diagram



$$TF = \frac{C(s)}{R(s)} = \frac{1}{1+sT}$$

Unit Step input

$$R(s) = \frac{1}{s}$$

So

$$C(s) = \frac{1}{s(1+sT)}$$

Q.No.	Solution and Scheme	Marks
	$C(s) = \frac{A}{S} + \frac{B}{1+ST}$	
	$A = S\,C(s)\Big\|_{s=0}$	
	$A = S \times \frac{1}{S(1+ST)}\Big\|_{s=0} = 1$	
	$B = (1+ST)\,C(s)\Big\|_{s=-\frac{1}{T}}$	
	$B = (1+ST) \times \frac{1}{S(1+ST)}\Big\|_{s=-\frac{1}{T}}$	
	$B = \frac{1}{S}\Big\|_{s=-\frac{1}{T}} = -T$	
	Therefore $C(s) = \frac{1}{S} + \frac{-T}{(1+ST)}$	
	$C(s) = \frac{1}{S} - \frac{T}{(1+ST)}$	
	Taking inverse laplace	

$$C(t) = 1 - e^{-t/T}$$

Q.No.	Solution and Scheme	Marks
Q no 3(b)		

The error response for unit step I/p is given by expression

$$e(t) = r(t) - c(t)$$

$$\quad " = t - t + T(1 - e^{-t/T})$$

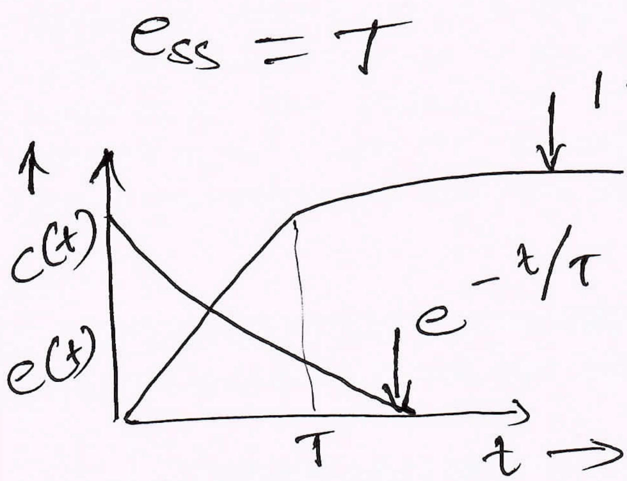
$$\quad " = T(1 - e^{-t/T})$$

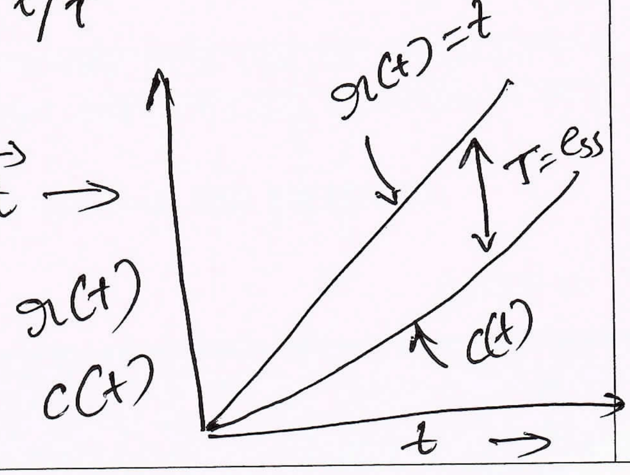
Steady State error $(\cancel{T})$ $\cancel{e(s)}$ e_{ss}

$$e_{ss} = \lim_{t\to\infty} e(t)$$

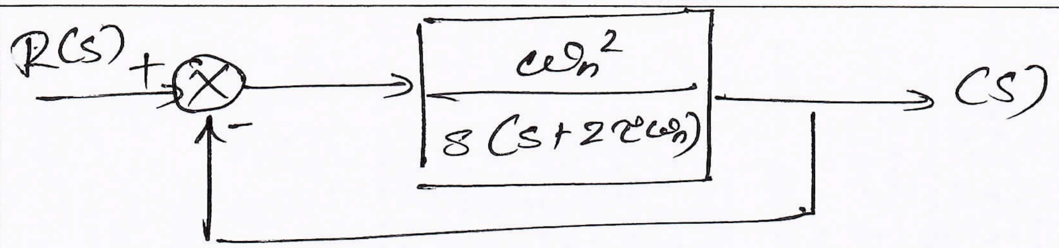
$$e_{ss} = \lim_{t\to\infty} T(1 - e^{-t/T})$$

$$e_{ss} = T$$





Q.No.	Solution and Scheme	Marks
Qno-3 (c)		



The characteristic eqn

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+10)} = 0$$

$$s^2 + 10s + K = 0$$

The characteristic eqn also given by eqn $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$\text{so } 2\zeta\omega_n = 10 \quad \& \quad \omega_n^2 = K$$

given $\zeta = 0.5$ so $\omega_n = \frac{10}{2 \times 0.5}$

$$\omega_n = 10 \text{ rad/sec}$$

$$K = \omega_n^2 = 10^2 = 100$$

Settling time $t_s = \frac{4}{\zeta\omega_n} = 0.8$

$$= \frac{4}{0.5 \times 100} = 0.8 \text{ sec}$$

Peak overshoot $M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$

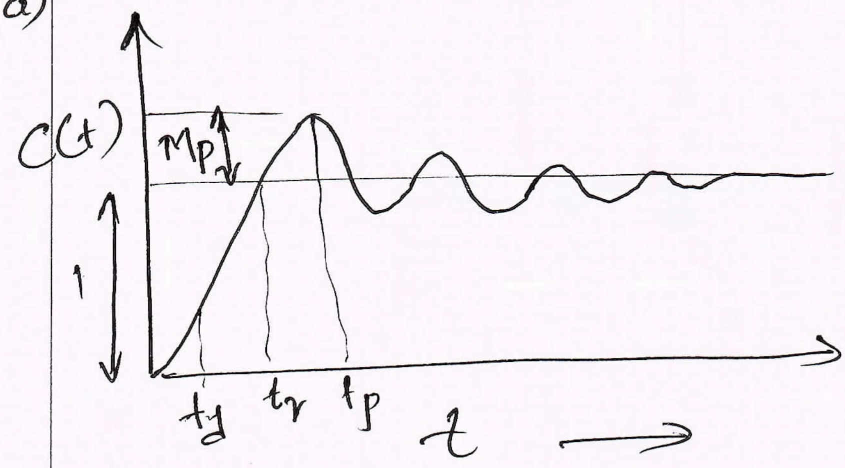
$$M_p = e^{-\frac{\pi \times 0.5}{\sqrt{1-0.5^2}}} = 0.16 = 16\%.$$

Q.No.	Solution and Scheme	Marks

Time for peak overshoot $t_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$

$$t_p = \frac{\pi}{100(1-0.5^2)} = 0.363\ \text{sec}$$

Qno- 4(a)



The transient response of a System to a unit step I/p is shown in the fig. following are the time domain Specification

Delay time t_d = It is the time taken for the response to reach 50% of the final value, for the very first time.

Rise time (t_r) It is the time taken for response to raise 0 to 100%. for very first time

Peak time (t_p) — It is the time taken for the response to reach the peak value the very first time.

Q.No.	Solution and Scheme	Marks
	Peak overshoot(M_p): It is defined as the ratio of the maximum peak value to the final value Settling time (T_s) It is defined as the time taken by the response to reach and stay within the specified error.	
Q.no-4(b)	Underdamped 2nd order system step input = 4 units $M_p = 0.25 \quad t_p = 0.8\,sec$ $\cancel{t_s = t_d = ?} \quad t_r = ? \quad t_s = ?$ $\zeta = ? \quad \omega_n = ?$ $M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.25$ $t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.8\,sec.$ $\omega_n\sqrt{1-\zeta^2} = \frac{\pi}{0.8} = 3.92 \quad \therefore \sqrt{1-\zeta^2} = \frac{3.92}{\omega_n}$ $\ln 0.25 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$	

$$-1.38 = \frac{-\zeta\pi}{(3.92/\omega_n)}$$

Q.No.	Solution and Scheme	Marks
	$1.38 \times 3.98 = \zeta \pi \omega_n$	
	$5.49 = \zeta \pi \omega_n$	
	$t_s = \dfrac{4}{\zeta \omega_n}$ | $\zeta \omega_n = \dfrac{5.49}{\pi}$; $\zeta \omega_n = 1.74$	
	$t_s = \dfrac{4}{1.74}$	
	$t_s = 2.29 \text{ sec} = \dfrac{4}{\zeta \omega_n}$ | $\omega_n = \dfrac{4}{\zeta \times 2.29}$; $\omega_n = \dfrac{1.74}{\zeta}$	
	~~$t_p =$~~ $\sqrt{1-\zeta^2} = \dfrac{3.92}{\omega_n}$	
	$\sqrt{1-\zeta^2} = \dfrac{3.92 \times \zeta}{1.74}$	
	Squaring on both	
	$1-\zeta^2 = \dfrac{15.36 \times \zeta^2}{3.02}$	
	$1-\zeta^2 = 5\zeta^2$	
	$1 = 6\zeta^2$ $\quad \zeta^2 = \dfrac{1}{6}$	

$\zeta^2 = 0.16$

$\boxed{\zeta = 0.4}$

Q.No.	Solution and Scheme	Marks
	$\zeta\omega_n = 1.74$	
	$$\boxed{\omega_n = \frac{1.74}{0.4} = 4.35 \text{ rad/sec}}$$	
Qn-4 (c)	$$G(s) = \frac{40(s+2)}{s(s+1)(s+4)}$$	
	given that Unity feedback $H(s)=1$	
	The open loop System has a pole at origin Hence System is a type-1 System. In System with number-1, the velocity (ramp) I/p will give a constant Steady State error	
	The Steady State error with unit velocity I/p $e_{ss} = \frac{1}{K_v}$	
	~~In System with~~	
	Velocity error Constant K_v	
	$$K_v = \lim_{s \to 0} s\,G(s)\,H(s)$$	
	$$= \lim_{s \to 0} s\,G(s)$$	

Q.No.	Solution and Scheme	Marks

$$K_v = \lim_{s \to 0} s \, \frac{40(s+2)}{s(s+1)(s+4)}$$

Subst $s = 0$

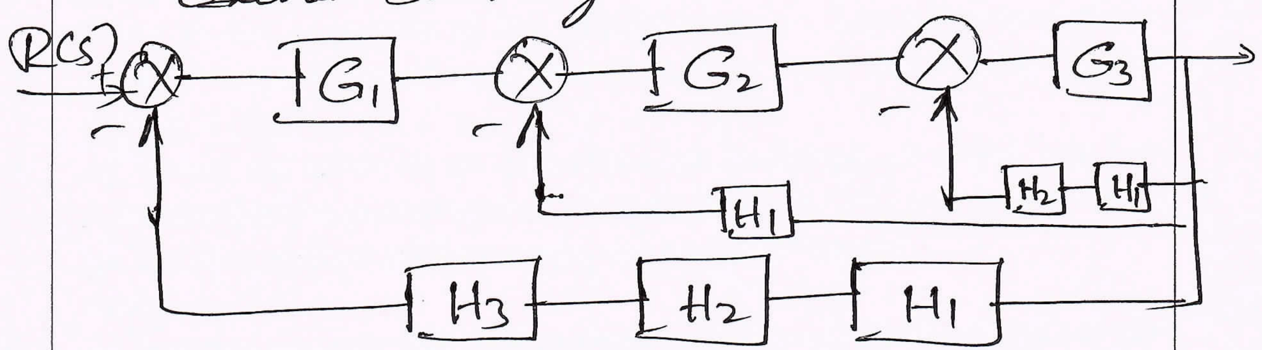
$$K_v = \frac{40 \times 2}{1 \times 4} = \frac{80}{4} = 20$$

Steady State error $e_{ss} = \frac{1}{K_v} = \frac{1}{20}$

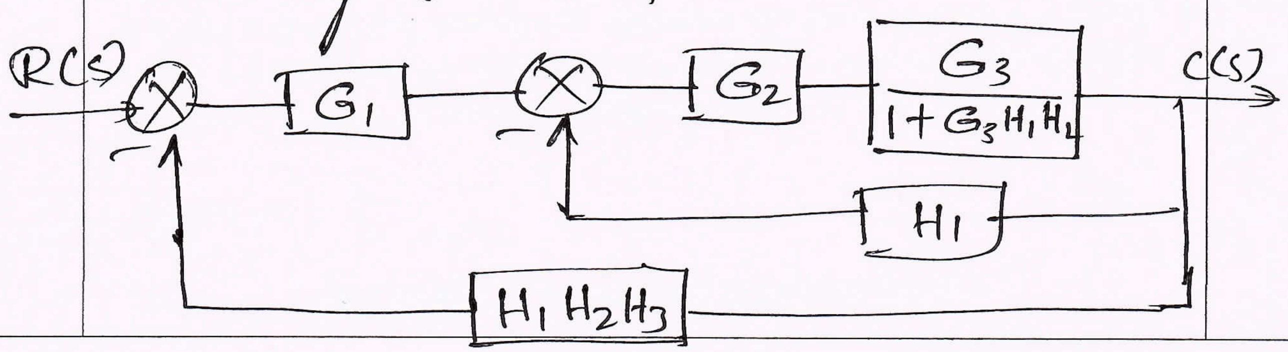
$$e_{ss} = 0.05$$

Qn-5 (a) Block diagram is pictorial representation of the given system it mainly containing elements like rectangular blocks, Summation point, Take off points & Arrows.

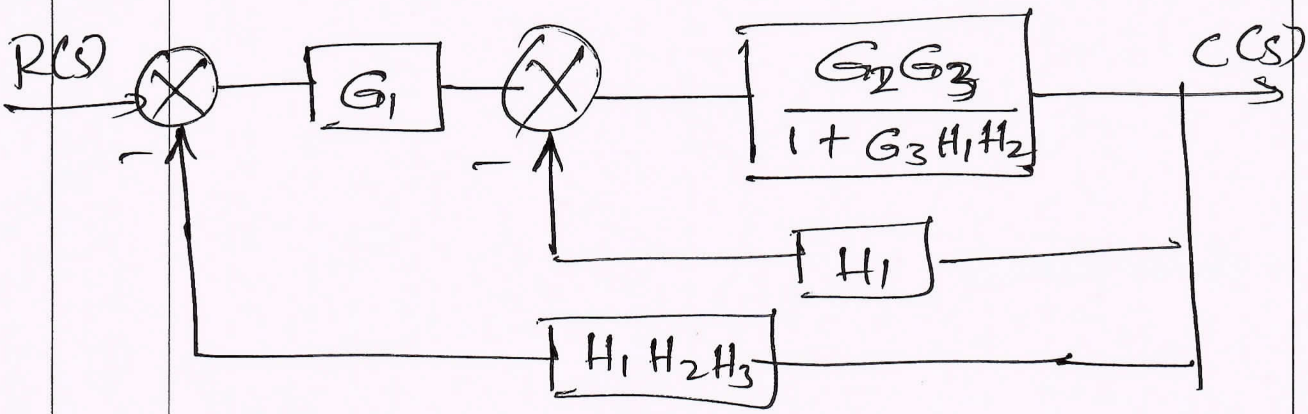
~~Exchan~~ shifting Summation point

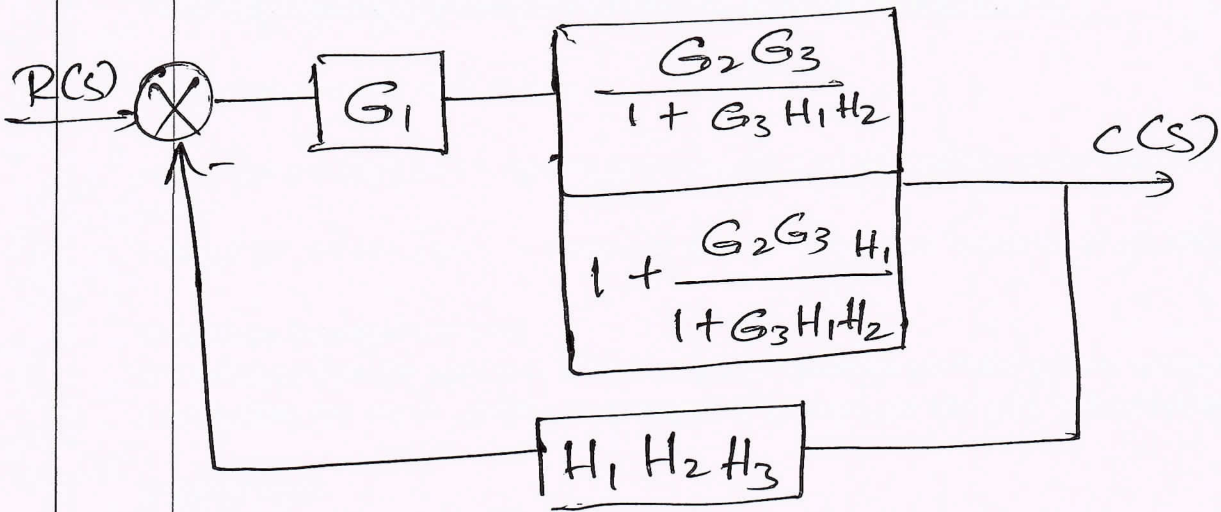


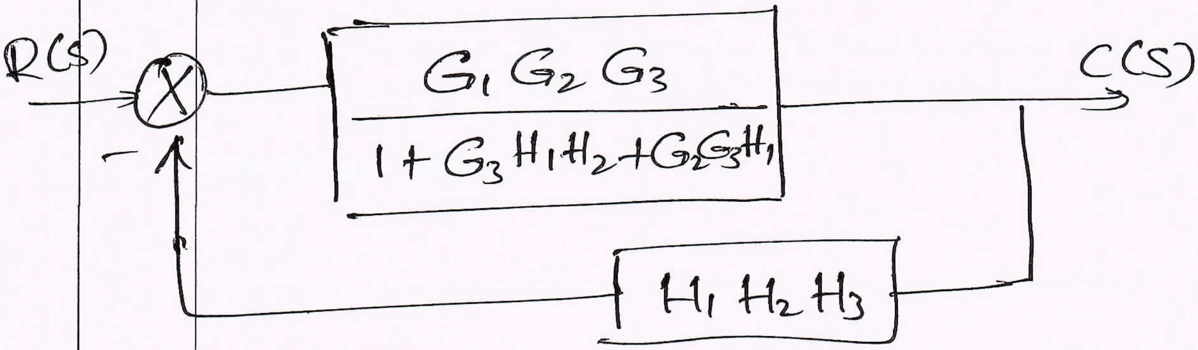
Removing feedback

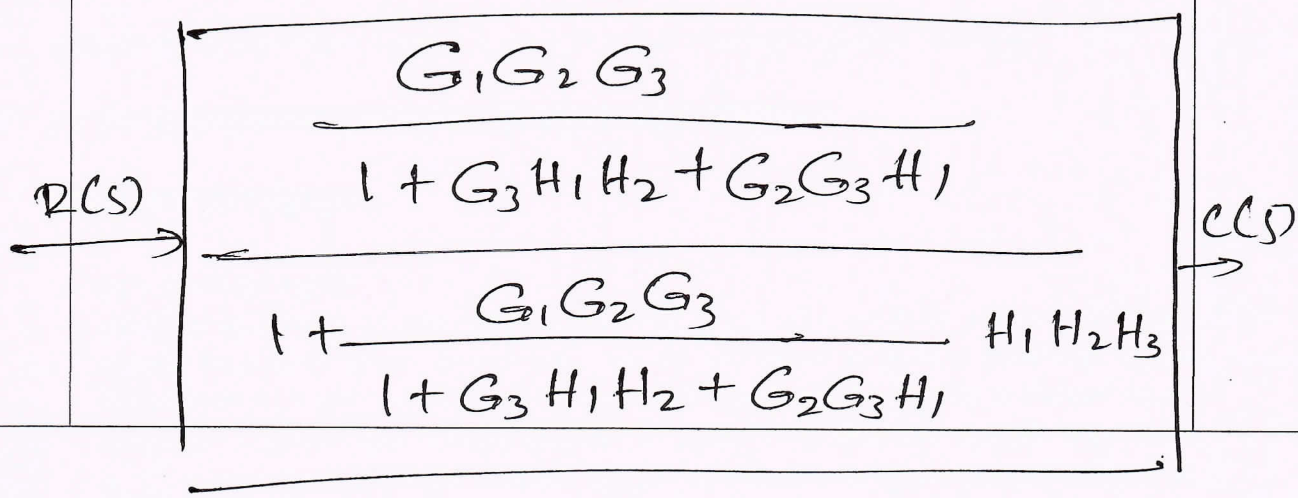


Q.No.	Solution and Scheme	Marks







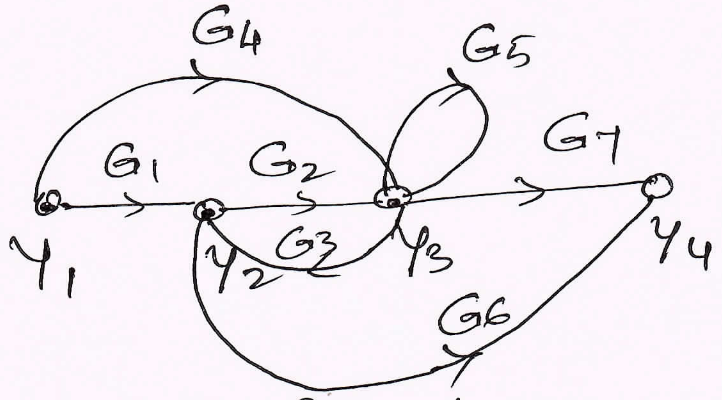


Q.No.	Solution and Scheme	Marks
	$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_3 H_1 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 H_1 H_2 H_3}$$	
Qn-5 (b)	Node: It is a point representing variable or Signal. Branch: A branch is directed line segment joining two nodes. Transmittence: The gain required by the signal when it travels from one node to another. is called as transmittence. Loop gain: It is product of the branch transmittances of the loop. Forward path: It is a path from an I/p node to an output node that does not cross any node more than once. Source: It is practice of tracking and managing changes to Code. It gives input & output of the System.	

Q.No.	Solution and Scheme	Marks
	<u>Non touching loops</u>: If the loops does not have a common node then they are said to be non touching loops. <u>Maisory gain formula</u>: This formula determines the transfer function of the system from the signal flow graph. It is given by Overall gain (TF) $T = \frac{1}{\Delta} \sum_{k} P_k \Delta_k$ P_k = forward path gain K = Number of forward path gain $\Delta = 1 -$ (sum of individual loop gain) + (sum of gain of two non touching loop) — — — — — — $\Delta_k = \Delta$ for that part of graph which is not touching k^{th} forward path.	

Q.No.	Solution and Scheme	Marks

Q no- 6 (a)



Number of forward path = K = 4

$$T.F = \sum_{K=1}^{4} \frac{T_K \Delta_K}{\Delta}$$

Forward path

$T_1 = G_1 G_2 G_7$ $\quad T_2 = G_4 G_7$

$T_3 = G_1 G_6$ $\quad T_4 = G_4 G_3 G_6$

Individual loop & its gain

$L_1 = G_2 G_3$ $\quad L_2 = G_5$

$\Delta = 1 - [L_1 + L_2] = 1 - G_2G_3 - G_5$

$\Delta_1 = 1$ $\quad \Delta_2 = 1$ $\quad \Delta_3 = 1 - G_5$

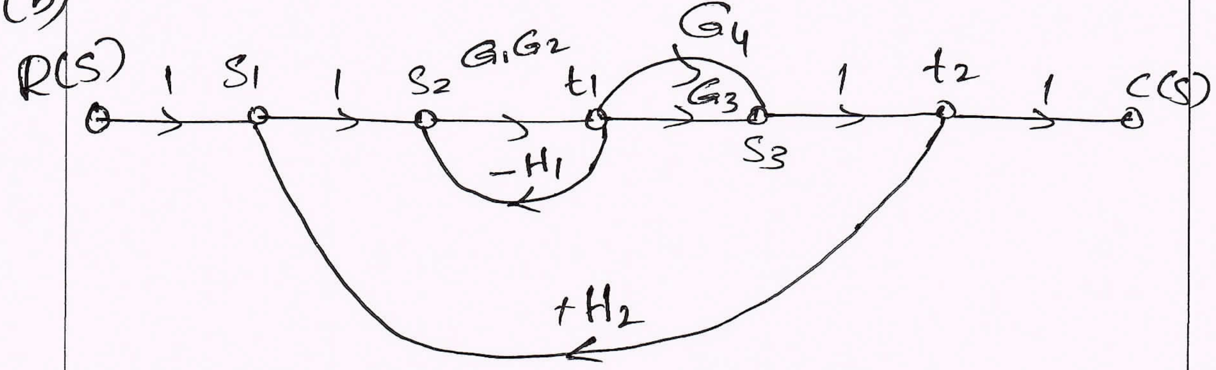
$\Delta_4 = 1$

$$\frac{Y_4}{Y_1} = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta}$$

Q.No.	Solution and Scheme	Marks

$$\frac{y_4}{y_1} = \left[\frac{G_1G_2G_7 \times 1 + G_4G_7 \times 1 + G_1G_6(1-G_5) + G_4G_3G_6 \times 1}{1 - G_2G_3 - G_5} \right]$$

Q no 6(b)



$$\frac{C(s)}{R(s)} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$$

$$T_1 = G_1G_2G_3 \qquad T_2 = G_1G_2G_4$$

$$\frac{C(s)}{R(s)} = \frac{(G_1G_2G_3 \times 1) + (G_1G_2G_4 \times 1)}{(1 + G_1G_2H_1 - G_1G_2G_3H_2 - G_1G_2G_4H_2)}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_2G_4}{1 - G_1G_2G_3H_2 - G_1G_2G_4H_2 + G_1G_2H_1}$$

Q.No.	Solution and Scheme	Marks
Q no-7 (a)	$s^4 + 6s^3 + 23s^2 + 40s + 50 = 0$	

$$\begin{array}{c|ccc} s^4 & 1 & 23 & 50 \\ s^3 & 6 & 40 & 0 \\ s^2 & 16 & 50 & \\ s^1 & 21 & 0 & \\ s^0 & 50 & & \end{array}$$

As there is no sign change System is Stable

(b) $s^4 + 22s^3 + 10s^2 + s + K = 0$

$$\begin{array}{c|ccc} s^4 & 1 & 10 & K \\ s^3 & 22 & 1 & 0 \\ s^2 & 9 & K & 0 \\ s^1 & \left(\frac{9 - 22K}{9}\right) & 0 & \\ s^0 & K & & \end{array}$$

$$9 - 22K = 0$$

$$K = 0.45$$

$$A(s) = 9s^2 + K = 0$$

$$9s^2 + 0.45 = 0 \qquad s^2 = -0.04$$

$$s = \pm j\,0.21$$

Q.No.	Solution and Scheme	Marks

Q no-7(c)

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+4)}$$

$$1 + G(s)\,H(s) = 0$$

$$s^3 + 5s^2 + 4s + K = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 5 & K \\ s^1 & \frac{20-K}{5} & 0 \\ s^0 & K & \end{array}$$

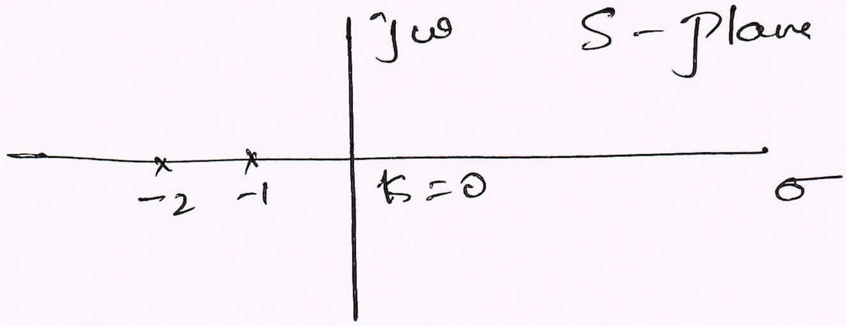
$$K_{max} = 20$$

$$A(s) = 5s^2 + K = 0$$

let $K = K_{max}$

$$5s^2 + 20 = 0$$

$$s^2 = -4 \qquad s = \pm j2$$



at $K = 0$ $\quad s = 0, -1, -2$

$K = \infty$ $\quad s = \infty, \infty, \infty$

Q.No.	Solution and Scheme	Marks

Number of Asymptote $= P - Z = 3 - 0 = 3$

Centroid of Asymptote

$$\sigma_A = \frac{\Sigma \text{finite pole} - \Sigma \text{finite zero}}{P - Z}$$

$$= \frac{(0 - 1 - 2) - 0}{3 - 0} = -1$$

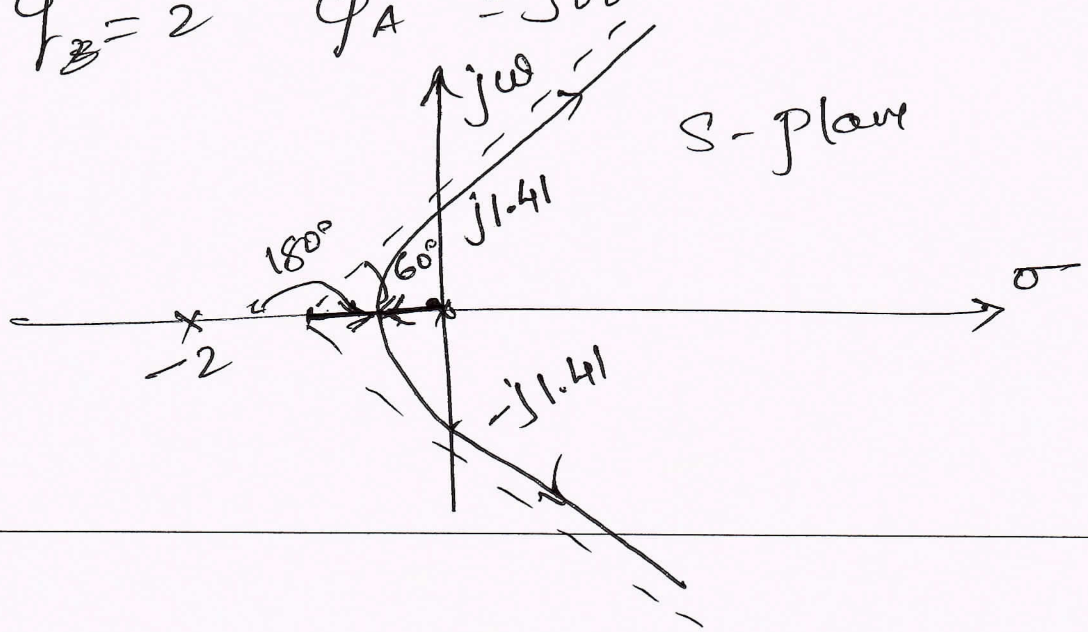
Angle made by Asymptote

$$\phi_A = \frac{(2q+1)\,180^\circ}{P - Z} \qquad q = 0, 1, 2$$

$q = 0 \qquad \phi_A = 60^\circ$

$q = 1 \qquad \phi_A = 180^\circ$

$q = 2 \qquad \phi_A = 300^\circ$



Q.No.	Solution and Scheme	Marks

Break away point

$$\frac{dK}{ds} = 0$$

$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

$$K = -s^3 - 3s^2 - 2s$$

$$\frac{dK}{ds} = 0$$

$$3s^2 + 6s + 2 = 0$$

$$s = -0.42 \ \ \& \ \ -1.58$$

Breakaway point -0.42

K at Breakaway point

$$K\Big|_{s=-0.42} = -s^3 - 3s^2 - 2s = 0.38$$

$$s^3 + 3s^2 + 2s + K = 0$$

$$j^3\omega^3 + 3j^2\omega^2 + 2j\omega + K = 0$$

$$(K - 3\omega^2) + j(2\omega - \omega^3) = 0$$

$$K - 3\omega^2 = 0 \ \ \& \ \ 2\omega - \omega^3 = 0$$

$$\omega \neq 0 \qquad \omega = \pm\sqrt{2}$$

$$K = 3\omega^2 = 6$$

for Stable $\quad 0 < K < 6$

Root locus on right side of s-plane so it is instable

Q.No.	Solution and Scheme	Marks
Q no-8 (a)		

$$s^5+s^4+2s^3+2s^2+3s+5=0$$

$$\begin{array}{c|ccc}
s^5 & 1 & 2 & 3 \\
s^4 & 1 & 2 & 5 \\
s^3 & 0 & -2 & \\
s^2 & & & \\
s^1 & & & \\
s^0 & & &
\end{array}$$

$$s=\frac{1}{z}$$

$$\left(\frac{1}{z}\right)^5+\left(\frac{1}{z}\right)^4+2\left(\frac{1}{z}\right)^3+2\left(\frac{1}{z}\right)^2+3\left(\frac{1}{z}\right)+5=0$$

$$5z^5+3z^4+2z^3+2z^2+z+1=0$$

$$\begin{array}{c|ccc}
z^5 & 5 & 2 & 1 \\
z^4 & 3 & 2 & 1 \\
z^3 & -4/3 & -2/3 & \\
z^2 & 1/2 & 1 & \\
z^1 & 2 & & \\
z^0 & 1 & &
\end{array}$$

Two Sign changes, 2 roots with + real Parts so system ~~is~~ Unstable

Q.No.	Solution and Scheme	Marks
Qn- 8(b)	$G(s)\,H(s) = \dfrac{K}{s(s^2+2s+2)}$ $P=3, Z=0$ Number of asymptotes $= P-Z = 3-0 = 3$ The Centroid of the asymptotes $\sigma_A = \dfrac{\Sigma \text{finite poles} - \Sigma \text{finite zero}}{P-Z}$ $\sigma_A = \dfrac{(0-1+j1-1-j1)-0}{3-0} = -0.66$ Angle made by asymptotes $\phi_A = \dfrac{(2q+1)180^\circ}{P-Z} \quad q=0,1,2$ $\phi_A = 60^\circ, 180^\circ, 300^\circ$ $1+G(s)\,H(s)=0$ $1+\dfrac{K}{s(s^2+2s+2)}=0$ $K = -s^3-2s^2-2s$	

$$\frac{dK}{ds}=0$$

Q.No.	Solution and Scheme	Marks

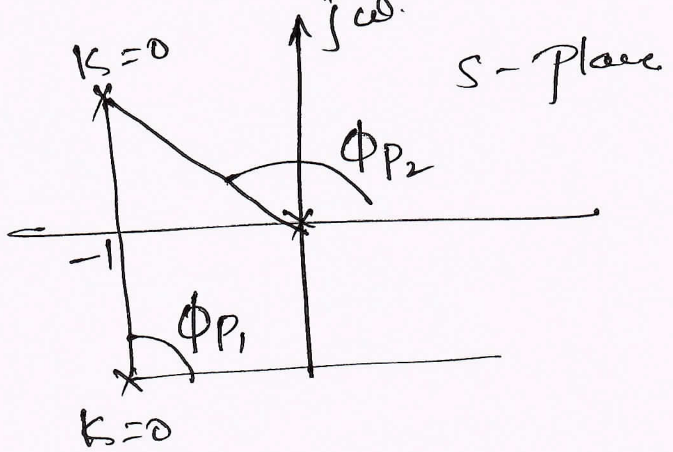
$$3s^2 + 4s + 2 = 0$$

$$s = -0.66 \pm j0.47$$

K at $s = -0.66 \pm j.0.47$

$$K = -s^3 - 3s^2 - 2s = 0.52 \pm j0.41$$

Angle of departure



$$\phi_{p_1} = 90^\circ$$

$$\phi_{p_2} = 180^\circ - \tan^{-1}\left(\frac{1}{1}\right) = 135^\circ$$

$$\phi_d = 180^\circ - \left[\Sigma \phi_{pole} - \Sigma \phi_{zero}\right]$$

$$\phi_d = 180^\circ - \left[(\phi_{p_1} + \phi_{p_2}) - 0\right] = -45^\circ$$

at $s = -1 - j1$ $\phi_d' = 45^\circ$

Q.No.	Solution and Scheme	Marks
	x^n with root locus	

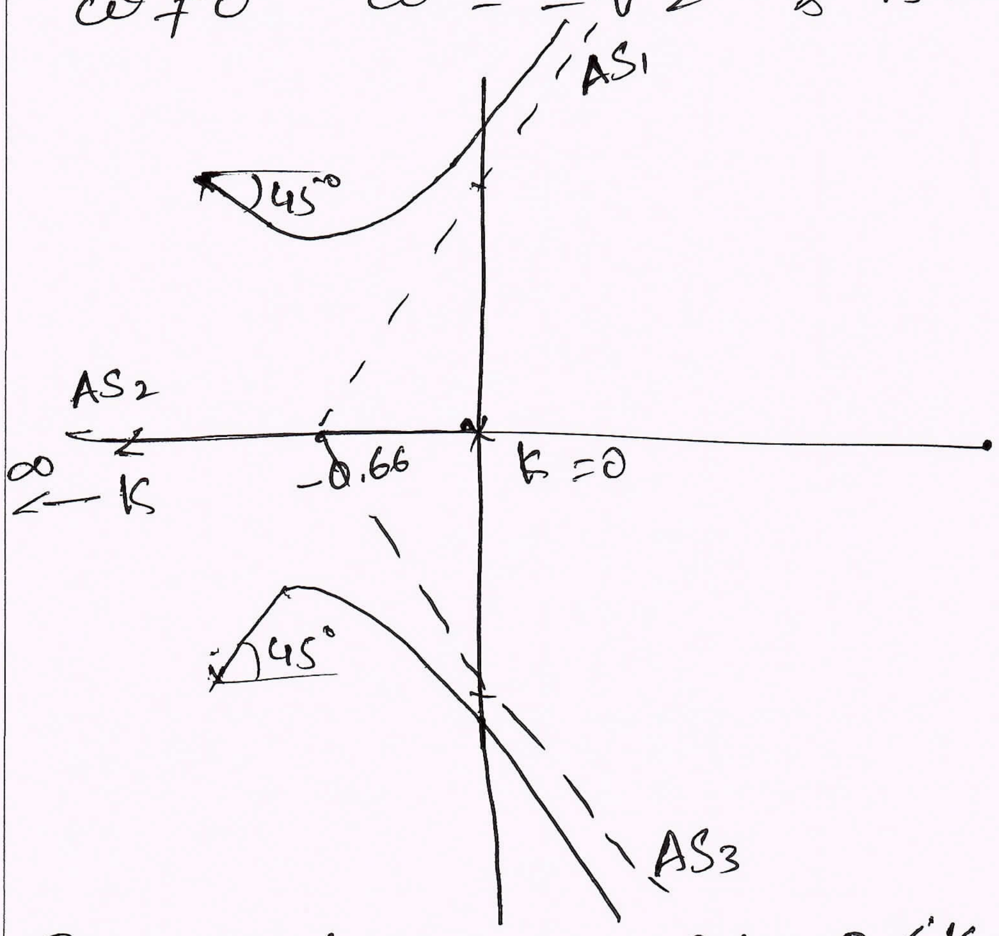
$$j^3\omega^3 + 2j^2\omega^2 + 2j\omega + K = 0$$

$$-j\omega^3 - 2\omega^2 + 2j\omega + K = 0$$

$$(K - 2\omega^2) + j(2\omega - \omega^3) = 0$$

$$K - 2\omega^2 = 0 \quad \& \quad \omega(2-\omega^2) = 0$$

$$\omega \neq 0 \quad \omega = \pm\sqrt{2} \quad \& \quad K = 4$$



For closed loop stability $0 < K < 4$

Unstable $0 > K > 4$

Q.No.	Solution and Scheme	Marks
Qn-9(a)	If $G(s)H(s)$ Contour in $G(s)H(s)$ plane Corresponding to Nyquist Contour in S-plane encircles the point $-1+j0$ in the anticlockwise direction as many times as the number of right half S-plane poles of $G(s)H(s)$ Then the Closed loop System is Stable	
Qno-9(b)	$G(s)H(s) = \dfrac{10}{s(s+1)(s+2)}$ $s = j\omega$ $G(j\omega)H(j\omega) = \dfrac{10}{j\omega(1+j\omega)(2+j\omega)}$ $\lvert G(j\omega)H(j\omega)\rvert = \dfrac{10}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$ $\phi = \dfrac{\tan^{-1}\frac{0}{10}}{\left(\tan^{-1}\frac{\omega}{0}\right)\left(\tan^{-1}\frac{\omega}{1}\right)\left(\tan^{-1}\frac{\omega}{2}\right)}$ $\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$ $\omega \to 0 \qquad \infty \angle -90^\circ$	

$\omega \to \infty \qquad 0 \angle -270^\circ$

Q.No.	Solution and Scheme	Marks

X^n negative real axis

$$G(j\omega)H(j\omega) = \frac{10(-j\omega)(1-j\omega)(2-j\omega)}{j\omega(-j\omega)(1+j\omega)(1-j\omega)(2+j\omega)(2-j\omega)}$$

$$'' = \frac{-10j\omega(2-3j\omega-\omega^2)}{\omega^2(1+\omega^2)(4+\omega^2)} = \frac{-30\omega^2}{D} - \frac{10j\omega(2-\omega^2)}{D}$$

$$D = \omega^2(1+\omega^2)(4+\omega^2)$$

$$10\omega(2-\omega^2) = 0$$

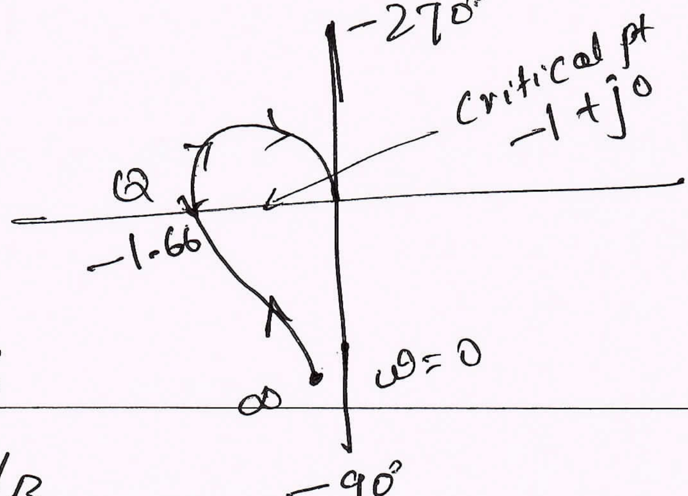
$$\omega = 0 \quad 2-\omega^2 = 0 \quad \omega^2 = 2$$

$$\omega = 0, \sqrt{2}$$

$$\omega_{pc} = \sqrt{2} = \omega$$

$$G(j\omega)H(j\omega) = \frac{-30\times 2}{2\times(1+2)\times(4+2)} + j0 = -1.66 + j0$$

$$GM = \frac{1}{|OQ|}$$



$$GM = \frac{1}{1.66} = 0.6$$

$$GM = 20\log 0.6$$

$$GM = -4.43\,dB$$

GM −ve Critical pt enclosed So system is Unstable

Q.No.	Solution and Scheme	Marks
Q no-9(c)	Bode diagram for TF	

$$G(s) = \frac{10}{s(1+0.4s)(1+0.1s)}$$

$$G(j\omega) = \frac{10}{j\omega(1+0.4j\omega)(1+j0.1\omega)}$$

Magnitude plot

Corner frequency

$$\omega_{c_1} = \frac{1}{0.4} = 2.5 \text{ rad/sec}$$

$$\omega_{c_2} = \frac{1}{0.1} = 10 \text{ rad/sec}$$

Term	Corner freque	Slope db/dec	Change in slope
$\frac{10}{j\omega}$	-	-20	
$\frac{1}{1+j0.4\omega}$	$\omega_{c_1} = \frac{1}{0.4} = 2.5$	-20	$-20-20=-40$
$\frac{1}{1+j0.1\omega}$	$\omega_{c_2} = \frac{1}{0.1} = 10$	-20	$-40-20=-60$

Choosing $\omega_l = 0.1$ rad/sec

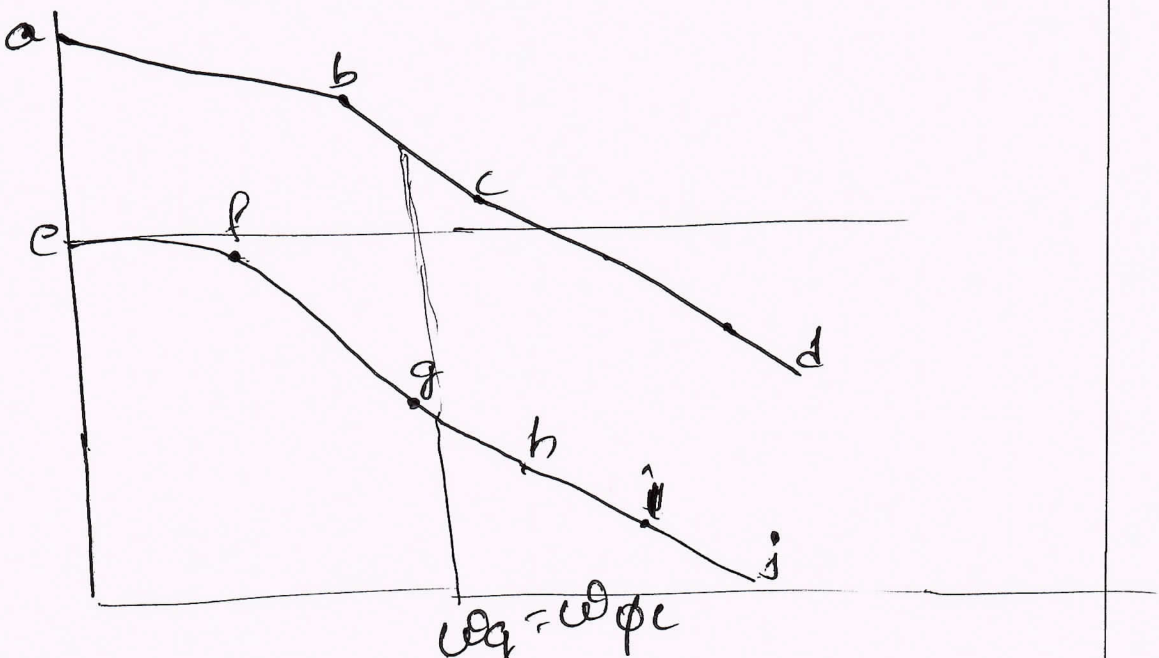
$\omega_h = 50$ " "

Q.No.	Solution and Scheme	Marks

Phase plot $\phi = -90^\circ - \tan^{-1} 0.4\omega - \tan^{-1} 0.1\omega$

ω rad/sec	ϕ degree
0.1	-92
1	-118
2.5	-150
4	-170
10	-210
20	-236

Nature of bode plot



Gain cross over frequency = 5 rad/sec

Phase " " " = 5 "

Q.No.	Solution and Scheme	Marks
Qno-10(a)	Nyquest plot for open loop T.F	
	$G(s)H(s) = \dfrac{1}{s(1+s)(1+2s)}$	
	$M = \dfrac{1}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$	
	$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$	
	$\omega = 0 \quad M = \infty \quad \phi = -90^\circ$	
	$\omega = \infty \quad M = 0 \quad \phi = -270^\circ$	
	$G(j\omega)H(j\omega) = \dfrac{1}{j\omega(1+2j\omega)(1+j\omega)}$	
	$'' = \dfrac{-3\omega}{\omega[(1-2\omega^2)^2+9\omega^2]} - j\dfrac{(1-2\omega^2)}{\omega[(1-2\omega^2)^2+9\omega^2]}$	
	equating imaginary part to zero	
	$(1-2\omega)^2 = 0 \quad \omega = \frac{1}{\sqrt{2}} = 0.7$	
	$M\Big\vert_{\omega=0.7} = \dfrac{1}{0.7\sqrt{1+\frac{1}{2}}\sqrt{1+2}} = 0.66$	
	So OX $= 0.66$ Critical pt $-1+j0$ not encircled	
	$N=0 \quad P=0 \quad N = P - Z$	

$0 = 0 - Z \quad Z = 0$

Q.No.	Solution and Scheme	Marks

$Z = 0$ implies Closed loop Stability and $P = 0$ implies open loop Stability

$$GM = 20 \log \frac{1}{a} = 20 \log \frac{1}{0.66} = 3.61\ db$$

Frequency at which Magnitude will become unity given by

$$\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2} = 1$$

$$\omega^2(1+\omega^2)(1+4\omega^2) = 1$$

$$x(1+x)(1+4x) = 1$$

By trial $x = 0.33$ $\omega = \sqrt{0.33}$

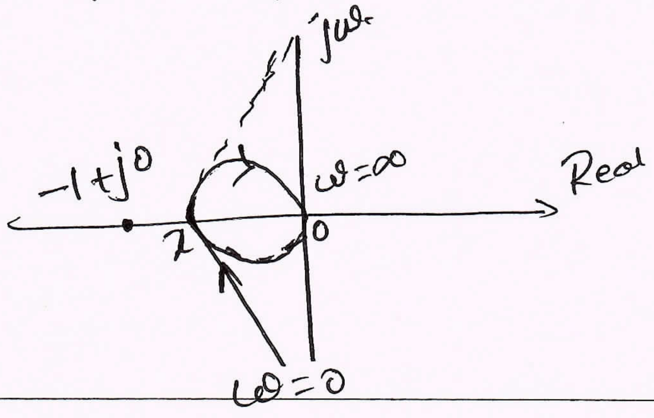
$\omega = 0.57$ rad/sec

PM =

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega + 180^\circ$$

$$\phi = -90^\circ - \tan^{-1}0.57 - \tan^{-1}2(0.57) + 180^\circ$$

$$\phi = 11^\circ$$



Q.No.	Solution and Scheme	Marks
Qno-10(b)	Bode plot	

$$G(s)H(s) = \frac{100}{s(s+1)(s+2)}$$

Std form

$$G(s)H(s) = \frac{50}{s(s+1)(1+0.5s)}$$

Magnitude plot M

Factor	Corner Frequen rad/sec	M db	Slope db/sec
50	–	34	0
$\frac{1}{s}$	–	$\omega = 1$	20
$\frac{1}{1+s}$	1	$\omega = 1$	-20
$\frac{1}{1+0.5s}$	2	$\omega = 2$	-20

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}0.5\omega$$

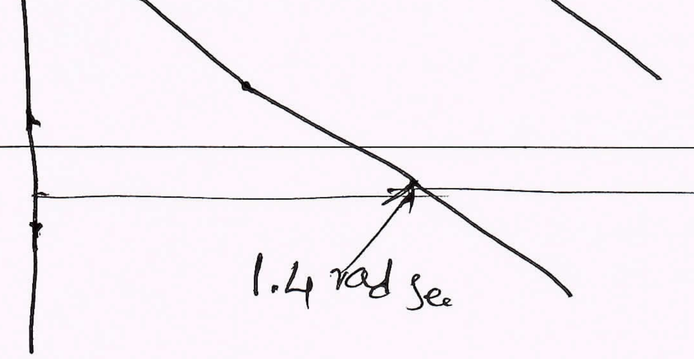
Q.No.	Solution and Scheme	Marks

Phase plot ϕ

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1} 0.5\omega$$

Sl No	ω rad/sec	ϕ
1	0	-90°
2	0.1	-98°
3	0.2	-107°
4	0.5	-130°
5	1	-161°
6	1.3	-175°
7	1.4	-179°
8	1.5	-183°
9	2	-198°
10	4.45	-233°

Nature of Bode plot



$\omega_{gc} = 4.45$ rad/sec

$\omega_{pc} = 1.40$ "

$GM = 27db$

$PM = 53^\circ$

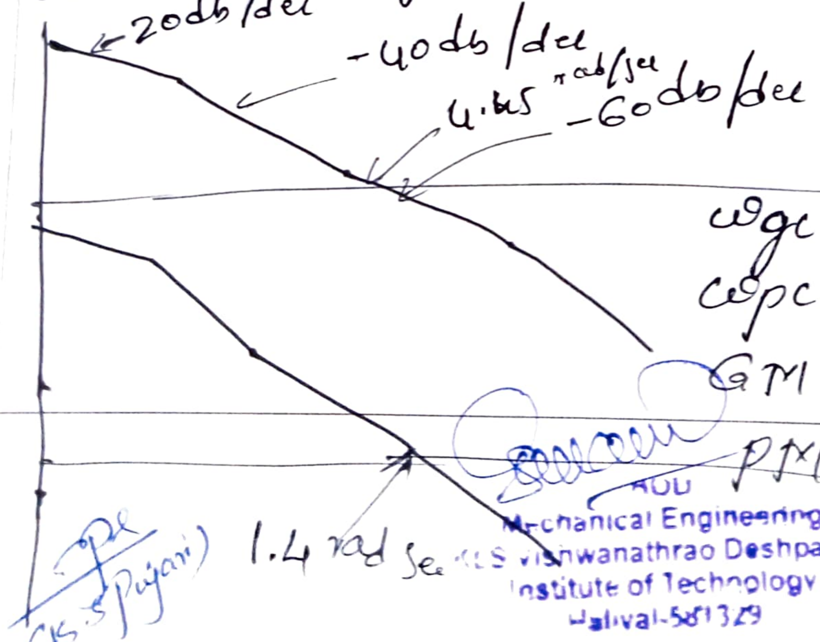
Q.No.	Solution and Scheme	Marks

Phase plot ϕ

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}0.5\omega$$

Sl No	ω rad/sec	ϕ
1	0	-90°
2	0.1	-98°
3	0.2	-107°
4	0.5	-130°
5	1	-161°
6	1.3	-175°
7	1.4	-179°
8	1.5	-183°
9	2	-198°
10	4.45	-233°

Nature of Bode plot



$\omega_{gc} = 4.45$ rad/sec

$\omega_{pc} = 1.40$,,

$GM = 27db$

$PM = 53^\circ$

(K.S Pujari)