**CBCS SCHEME**

USN | | | | | | | | | | |

BMATE101

# First Semester B.E/B.Tech. Degree Examination, Dec.2023/Jan.2024
## Mathematics – I for EEE Stream

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. VTU Formula Hand Book is permitted.*
*3. M : Marks , L: Bloom's level , C: Course outcomes.*

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| 1 | a. | With usual notations prove that $\tan\phi = r\dfrac{d\theta}{dr}$. | 6 | L2 | CO1 |
| | b. | Find the angle of intersection for the pair of curve $r = a(1 + \sin\theta)$, $r = b(1 - \sin\theta)$. | 7 | L2 | CO1 |
| | c. | Find the radius of curvature of the curve $x^3 + y^3 = 3xy$ at $\left(\dfrac{3}{2}, \dfrac{3}{2}\right)$. | 7 | L2 | CO1 |
| | | OR | | | |
| 2 | a. | Prove that the pair of curves $r = a\sec^2\left(\dfrac{\theta}{2}\right)$, $r = b\,\mathrm{cosec}^2\left(\dfrac{\theta}{2}\right)$ intersect orthogronally. | 8 | L2 | CO1 |
| | b. | Find the Pedal equation of the curve $r^n = a^n \cos n\theta$. | 7 | L2 | CO1 |
| | c. | Using modern mathematical tool write a program/code to plot sine and cosine curves. | 5 | L3 | CO5 |
| | | Module – 2 | | | |
| 3 | a. | Expand $\log(\sec x)$ up to the term containing $x^6$ using Maclaurin's series. | 6 | L2 | CO1 |
| | b. | If $u = \tan^{-1}\left(\dfrac{y}{x}\right)$ where $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$. Find $\dfrac{du}{dt}$. | 7 | L2 | CO1 |
| | c. | If: $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$. Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at$(1,-1,0)$. | 7 | L3 | CO1 |



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| | | | | | |
|---|---|---|---|---|---|
| | | **OR** | | | |
| 4 | a. | Evaluate :<br>i) $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x}{2}\right)^{1/x}$<br>ii) $\lim\limits_{x\to 0}(\sin x)^{\tan x}$. | 7 | L2 | CO1 |
| | b. | If $u=f(y-z, z-x, x-y)$ prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. | 8 | L2 | CO1 |
| | c. | Using modern mathematical tool write a program/ code to evaluate :<br>$\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x$. | 5 | L3 | CO5 |
| | | **Module – 3** | | | |
| 5 | a. | Solve : $\dfrac{dy}{dx}+2\dfrac{y}{x}=\dfrac{y^2\log x}{x}$. | 6 | L2 | CO2 |
| | b. | Find the orthogonal trajectories of the family of Asteroid $x^{2/3}+y^{2/3}=a^{2/3}$. | 7 | L3 | CO2 |
| | c. | Solve : $p^3+2xp^2-y^2p^2-2xy^2p=0$. | 7 | L2 | CO2 |
| | | **OR** | | | |
| 6 | a. | Solve : $y(x+y+1)dx+x(x+3y+2)dy=0$. | 6 | L2 | CO2 |
| | b. | Show that a DE for the current i in an electric circuit containing an inductance L and resistance R in series and acted by an electromotive force $E\sin\omega t$ satisfies the equation : $L\dfrac{di}{dt}+Ri=E\sin\omega t$. Find the value of the current at any time t, if initially there is no current in the circuit. | 7 | L3 | CO2 |
| | c. | Modify the equation into Clairaut's form. Hence find the general and singular solution of $xp^2-py+kp+a=0$. | 7 | L2 | CO2 |
| | | **Module – 4** | | | |
| 7 | a. | Evaluate : $\displaystyle\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$. | 6 | L2 | CO3 |
| | b. | Evaluate by changing the order of integration : $\displaystyle\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}y^2\,dx\,dy$. | 7 | L2 | CO3 |
| | c. | Prove that $\beta(m,n)=\dfrac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$. | 7 | L2 | CO3 |

| | | | Marks | Level | CO |
|---|---|---|---|---|---|
| | | **OR** | | | |
| 8 | a. | Evaluate : $\int_0^a \int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\,dxdy$ by changing into polar form. | 6 | L2 | CO3 |
| | b. | Find the area bounded between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ using double integration. | 7 | L3 | CO3 |
| | c. | Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. | 7 | L2 | CO3 |
| | | **Module – 5** | | | |
| 9 | a. | Find the Rank of the Matrix : $\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$. | 6 | L2 | CO4 |
| | b. | Solve the system of equations by Gauss – Elimination method. $2x+y+z=10$, $3x+2y+3z=18$, $x+4y+9z=16$. | 7 | L3 | CO4 |
| | c. | Using Gauss – Seidel iterative method to solve : $5x+2y+z=12$, $x+4y+2z=15$, $x+2y+5z=20$. Carryout 4 iterations, taking the initial approximation to the solution as (1, 0, 3). | 7 | L3 | CO4 |
| | | **OR** | | | |
| 10 | a. | Find the Rank of the matrix : $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$. | 7 | L2 | CO4 |
| | b. | Solve by Gauss – Jordan method : $2x+y+3z=1$, $4x+4y+7z=1$, $2x+5y+9z=3$. | 7 | L3 | CO4 |
| | c. | Using modern mathematical tool write a program/code to test the consistency of the equations : $x+2y-z=1$, $2x+y+4z=2$, $3x+3y+4z=1$. | 6 | L3 | CO5 |

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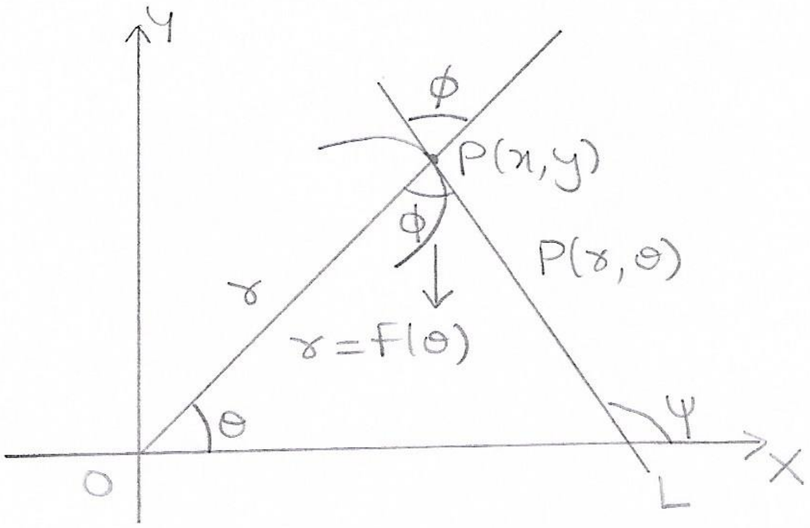
AY: 2023-24

**Department: Electronics & Communication Engineering**
**Subject with Sub. Code: Mathematics-I for Electrical and Electronics Engineering Stream (BMATE101)**
**Semester / Branch / Division: I / ECE & EEE** **Name of Faculty: Dr. Meenal Kaliwal**

| Q.No. | Solution and Scheme | Marks |
|---|---|---|
| 1a. | Let, $P(r,\theta)$ be any point on the curve $r = f(\theta)$. | |
| | $\therefore X\hat{O}P = \theta$ and $OP = r$. | |
| |   | |
| | Let PL be the tangent to the curve at P subtending an angle $\psi$ with the positive direction of the initial line ($x$-axis) and $\phi$ be the angle between the radius vector OP and the tangent PL. That is $OPL = \phi$. | |
| | From the figure, $\psi = \phi + \theta$ | |
| | $\Rightarrow \tan\psi = \tan(\phi+\theta)$ | 1 |
| | or $\tan\psi = \dfrac{\tan\theta + \tan\phi}{1 - \tan\phi\tan\theta} \longrightarrow (1)$ | 1 |
| | Let $(x, y)$ be the cartesian cordinates | |

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| Question Number | Solution | Marks Allocated |
| --- | --- | --- |
| | of P so that we have, $x = r\cos\theta, \; y = r\sin\theta$ | |
| | Also, $\tan\psi = \frac{dy}{dx}$ = slope of the tangent PL. | |
| | $\tan\psi = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta}$ | |
| | $\therefore \tan\psi = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$ where $r' = \frac{dr}{d\theta}$ | 1 |
| | Dividing both Numerator & denominator by $r'\cos\theta$, $\tan\psi = \frac{\frac{r\cos\theta}{r'\cos\theta} + \frac{r'\sin\theta}{r'\cos\theta}}{\frac{-r\sin\theta}{r'\cos\theta} + \frac{r'\cos\theta}{r'\cos\theta}}$ | 1 |
| | $\tan\psi = \frac{\frac{r}{r'} + \tan\theta}{1 - \frac{r}{r'}\tan\theta} \longrightarrow (2)$ | 1 |
| | Comparing equations (1) & (2), $\tan\phi = \frac{r}{r'} = \frac{r}{dr/d\theta}$ | |
| | $\boxed{\tan\phi = r\frac{d\theta}{dr}}$ | 1 |
| | | 6 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| b. | $r = a(1+\sin\theta)$ | |
| | $\log r = \log a + \log(1+\sin\theta)$ | |
| | Differentiating w.r.t. '$\theta$', | |
| | $\frac{1}{r}\frac{dr}{d\theta} = \frac{\cos\theta}{1+\sin\theta}$ | 1 |
| | $\cot\phi_1 = \frac{\cos\theta}{1+\sin\theta}$ | 1 |
| | Consider, $r = b(1-\sin\theta)$ | |
| | $\log r = \log b + \log(1-\sin\theta)$ | |
| | Differentiating wrt '$\theta$', | |
| | $\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{(-\cos\theta)}{1-\sin\theta}$ | 1 |
| | $\therefore \cot\phi_2 = \frac{-\cos\theta}{1-\sin\theta}$ | 1 |
| | $\tan\phi_1 = \frac{1+\sin\theta}{\cos\theta}$ & $\tan\phi_2 = \frac{1-\sin\theta}{-\cos\theta}$ | |
| | $\therefore \tan\phi_1 . \tan\phi_2 = \frac{1-\sin^2\theta}{-\cos^2\theta}$ | 1 |
| | $= \frac{\cos^2\theta}{-\cos^2\theta} = -1$ | 1 |
| | $\therefore$ or $\underline{\underline{\lvert\phi_1-\phi_2\rvert}} = \pi/2$ is the angle of intersection. | 1 |
| | | 7 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| 1 c. | Given curve is, $x^3+y^3=3xy$ | |
| | Differentiating wrt '$x$', | |
| | $3x^2+3y^2\frac{dy}{dx}=3\left(x\frac{dy}{dx}+y\right)$ | |
| | $\therefore x^2+y^2y_1=xy_1+y$ | |
| | $\therefore y_1=\frac{y-x^2}{y^2-x} \longrightarrow (1)$ | 1 |
| | At $(3/2, 3/2)$, $y_1=\frac{3/2-9/4}{9/4-3/2}$ | |
| | $\therefore y_1=\frac{3/2(1-3/2)}{3/2(3/2-1)}=\frac{(1-3/2)}{-(1-3/2)}=-1$ | 1 |
| | $\therefore y_1=-1$ | |
| | Differentiating eqn (1) wrt '$x$', | |
| | $y_2=\frac{(y^2-x)(y_1-2x)-(y-x^2)(2yy_1-1)}{(y^2-x)^2}$ | 1 |
| | At $(3/2, 3/2) \Rightarrow y^2-x=9/4-3/2=\frac{3}{4}$ | |
| | $y_1-2x=(-1)-2\times 3/2=-4$ | |
| | $y-x^2=\frac{3}{2}-\frac{9}{4}=\frac{-3}{4}$ | |
| | $2yy_1-1=\cancel{2}\times\frac{3}{\cancel{2}}\times(-1)-1=-4$ | |
| | & $(y^2-x)^2=\left(\frac{3}{4}\right)^2=\frac{9}{16}$ | |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $\therefore y_2 = \dfrac{\frac{3}{\cancel{4}} \times (-\cancel{4}) - \left(-3/\cancel{4}\right)(-\cancel{4})}{9/16}$ | |
| | $= \dfrac{-3-3}{9/16} = -6 \times \dfrac{16}{9}$ | |
| | $y_2 = -\dfrac{32}{3}$ | 2 |
| | $\rho = \dfrac{(1+y_1^2)^{3/2}}{y_2} = \dfrac{(1+1)^{3/2}}{-32/3}$ | 1 |
| | $= \dfrac{2\sqrt{2} \times 3}{-32} = -\dfrac{3\sqrt{2}}{16} = \dfrac{-3}{8\sqrt{2}}$ | |
| | Thus, $\lvert\rho\rvert = \dfrac{3}{8\sqrt{2}}$ | 1 |
| | | 7 |
| | OR | |
| 2a. | $r = a \sec^2(\theta/2)$ | |
| | $\log r = \log a + 2 \log \sec(\theta/2)$ | |
| | Differentiating wrt '$\theta$', | |
| | $\dfrac{1}{r}\dfrac{dr}{d\theta} = 0 + 2 \times \dfrac{1}{\sec(\theta/2)} \times \sec(\theta/2)\tan(\theta/2) \times 1/2$ | 1 |
| | $\dfrac{1}{r}\dfrac{dr}{d\theta} = \tan(\theta/2)$ | 1 |
| | $\cot\phi_1 = \cot(\pi/2 - \theta/2)$ | 1 |
| | $\Rightarrow \phi_1 = \pi/2 - \theta/2$ | 1 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | Consider, $r = b\,\mathrm{cosec}^2(\theta/2)$ | |
| | $\log r = \log b + 2\log \mathrm{cosec}(\theta/2)$ | |
| | Differentiating wrt '$\theta$', | |
| | $\frac{1}{r}\frac{dr}{d\theta} = 0 + 2\times\frac{(-\mathrm{cosec}\,\theta/2\,\cot\theta/2)}{\mathrm{cosec}(\theta/2)}\cdot\frac{1}{2}$ | 1 |
| | $\cot\phi_2 = -\cot(\theta/2) = \cot(-\theta/2)$ | 1 |
| | $\Rightarrow \phi_2 = -\theta/2$ | 1 |
| | $\lvert\phi_1-\phi_2\rvert = \lvert\pi/2-\theta/2+\theta/2\rvert = \pi/2$ | 1 |
| | $\therefore$ The curves intersect each other orthogonally. | 8 |
| 2b. | $r^n = a^n\cos n\theta$ | |
| | $n\log r = n\log a + \log\cos n\theta$ | |
| | Differentiating wrt '$\theta$', | |
| | $\frac{n}{r}\frac{dr}{d\theta} = -\frac{n\sin n\theta}{\cos n\theta}$ | 1 |
| | $\frac{1}{r}\frac{dr}{d\theta} = -\tan n\theta$ | 1 |
| | $\therefore \cot\phi = \cot(\pi/2+n\theta)$ | |
| | $\Rightarrow \phi = \pi/2+n\theta$ | 1 |
| | Consider, $p = r\sin\phi = r\sin(\pi/2+n\theta)$ | 1 |
| | Now, $r^n = a^n\cos n\theta \longrightarrow (1)$ | 1 |
| | $p = r\cos n\theta \longrightarrow (2)$ | 1 |
| | $\therefore \cos n\theta = p/r$ | |
| | $\therefore r^n = a^n(p/r)$ | 1 |
| | Thus, $r^{n+1} = pa^n$, is the required pedal equation. | 7 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| 2c. | `import numpy as np` | 1 |
| | `import matplotlib.pyplot as plt` | |
| | `x = np.arange(-10, 0, 0.001)` | |
| | `y1 = np.sin(x)` | 1 |
| | `y2 = np.cos(x)` | |
| | `plt.plot(x, y1, x, y2)` | 1 |
| | `plt.title("sine curve and cosine curve")` | |
| | `plt.xlabel("Values of x")` | |
| | `plt.ylabel("Values of sin(x) and cos(x)")` | 1 |
| | `plt.grid()` | 1 |
| | `plt.show()` | |
| | | 5 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| 3a | Maclaurin's series is given by, | |
| | $y(x) = y(0) + x\,y_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0) + \frac{x^4}{4!} y_4(0) + \text{----}$ | |
| | $y = \log(\sec x) \quad \therefore y(0) = \log 1 = 0$ | 1 |
| | $y_1 = \frac{\sec x \tan x}{\sec x}$ i.e. $y_1 = \tan x$ | |
| | $\therefore y_1(0) = 0$ | 1 |
| | $y_2 = \sec^2 x \quad \therefore y_2(0) = 1$ | |
| | Now, $y_2 = 1 + \tan^2 x = 1 + y_1^2$ | |
| | Differentiating this w.r.t '$x$' successively, | 1 |
| | $y_3 = 2y_1y_2 \quad \therefore y_3(0) = 0$ | 1 |
| | $y_4 = 2(y_1y_3 + y_2^2) \quad \therefore y_4(0) = 2$ | |
| | $y_5 = 2(y_1y_4 + y_2y_3 + 2y_2y_3)$ | |
| | $= 2y_1y_4 + 6y_2y_3$ | |
| | $\therefore y_5(0) = 0$ | |
| | $y_6 = 2y_1y_5 + 2y_2y_4 + 6y_2y_4 + 6y_3^2$ | 1 |
| | $\therefore y_6(0) = 16$ | |
| | Substituting these values in $y(x)$, we get | |
| | $\log(\sec x) = \frac{x^2}{2} \cdot 1 + \frac{x^4}{24} \cdot 2 + \frac{x^6}{720} \cdot 16$ | |
| | Thus, $\log(\sec x) = \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45}$ | 1 |
| | | 6 |

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|---|---|---|

3b. The total derivative rule is,

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt}$$

$$\frac{\partial u}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \left(\frac{-y}{x^2}\right) = \frac{x^2}{x^2+y^2} \times \left(\frac{-y}{x^2}\right)$$

$$\therefore \frac{\partial u}{\partial x} = \frac{-y}{x^2+y^2} \qquad 2$$

Similarly, $\frac{\partial u}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{1}{x}$

$$= \frac{x^2}{x^2+y^2} \times \frac{1}{x} = \frac{x}{x^2+y^2}$$

$$\therefore \frac{du}{dt} = \frac{-y}{x^2+y^2} \times (e^t+e^{-t}) + \frac{x}{x^2+y^2} \times (e^t-e^{-t}) \qquad 2$$

$$= \frac{-(e^t+e^{-t})(e^t+e^{-t})}{(e^t-e^{-t})^2+(e^t+e^{-t})^2} + \frac{(e^t-e^{-t})(e^t-e^{-t})}{(e^t-e^{-t})^2+(e^t+e^{-t})^2}$$

$$= \frac{-(e^t+e^{-t})^2+(e^t-e^{-t})^2}{(e^t+e^{-t})^2+(e^t-e^{-t})^2} \qquad 2$$

$$\frac{du}{dt} = \frac{-4}{2e^{2t}+2e^{-2t}} = \frac{-2}{e^{2t}+e^{-2t}}$$

$$\therefore \frac{du}{dt} = \frac{-2}{e^{2t}+e^{-2t}} \qquad 1$$

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| 3c. | $u = x + 3y^2 - z^3,\ v = 4x^2yz$ <br> $w = 2z^2 - xy$ <br> $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$ | 1 |
| | $= \begin{vmatrix} 1 & 6y & -3z^2 \\ 8xyz & 4x^2z & 4x^2y \\ -y & -x & 4z \end{vmatrix}$ | 3 |
| | $\therefore \dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0) = \begin{vmatrix} 1 & -6 & 0 \\ 0 & 0 & -4 \\ 1 & -1 & 0 \end{vmatrix}$ | 2 |
| | Expanding by the second row we get, <br> $-0 + 0 + 4(-1+6) = \underline{\underline{20}}$ | 1 |
| | | 7 |
| | OR | |
| 4a. (i) | Let, $k = \lim_{x \to 0} \left(\dfrac{a^x + b^x}{2}\right)^{1/x} \dashrightarrow 1^\infty$ | |
| | $\Rightarrow \log_e k = \lim_{x \to 0} \dfrac{\log\{(a^x+b^x)/2\}}{x} \dashrightarrow \left(\dfrac{0}{0}\right)$ | |
| | $\log_e k = \lim_{x \to 0} \dfrac{\frac{2}{a^x+b^x} \cdot \frac{1}{2}(a^x \log a + b^x \log b)}{1}$ | 1 |

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|---|---|---|
| | $= \frac{1}{2}(\log a + \log b) = \frac{1}{2}\log(ab)$ | 1 |
| | $= \log\sqrt{ab}$ | |
| | $\log_e k = \log\sqrt{ab}$ | 1 |
| | Thus, $\underline{\underline{k = \sqrt{ab}}}$ | |
| | (ii) Let, $k = \lim\limits_{x\to\pi/2} (\sin x)^{\tan x} \dashrightarrow 1^\infty$ | |
| | $\Rightarrow \log_e k = \lim\limits_{x\to\pi/2} \tan x \log(\sin x)$ $(\infty\times 0)$ | 1 |
| | $= \lim\limits_{x\to\pi/2} \dfrac{\log(\sin x)}{\cot x} \dashrightarrow \left(\dfrac{0}{0}\right)$ | |
| | Applying L'Hospital's rule, | |
| | $\log_e k = \lim\limits_{x\to\pi/2} \dfrac{\cos x/\sin x}{-\operatorname{cosec}^2 x}$ | 1 |
| | $= \lim\limits_{x\to\pi/2} -\sin x \times \cos x = 0$ | 1 |
| | $\log_e k = 0$ | 1 |
| | Thus, $\underline{\underline{k = e^0}} = 1$ | 7 |
| 4b. | Let, $u = f(p,q,r)$ where $p = y - z$, $q = z - x$ & $r = x - y$ | 1 |
| | $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial p}\cdot\dfrac{\partial p}{\partial x} + \dfrac{\partial u}{\partial q}\cdot\dfrac{\partial q}{\partial x} + \dfrac{\partial u}{\partial r}\cdot\dfrac{\partial r}{\partial x}$ | 1 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot 0 + \frac{\partial u}{\partial q}(-1) + \frac{\partial u}{\partial r} \times 1$ | 1 |
| | $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} - \frac{\partial u}{\partial q} \longrightarrow (1)$ | 1 |
| | Similarly, by symmetry we have | |
| | $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p} - \frac{\partial u}{\partial r} \longrightarrow (2)$ | 2 |
| | $\frac{\partial u}{\partial z} = \frac{\partial u}{\partial q} - \frac{\partial u}{\partial p} \longrightarrow (3)$ | 1 |
| | Adding, (1), (2) & (3), | |
| | $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ | 1 |
| | | 8 |
| 4c. | Program to evaluate $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ | |
| | from sympy import * | 1 |
| | from math import inf | 1 |
| | x = Symbol ('x') | 1 |
| | l = Limit ((1+1/x)** x, x, inf).doit() | 1 |
| | display (l) | 1 |
| | | 5 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | Module 3 | |
| 5a. | $\frac{dy}{dx} + 2\frac{y}{x} = \frac{y^2 \log x}{x}$ | |
| | $\frac{1}{y^2}\frac{dy}{dx} + \frac{2}{xy} = \frac{\log x}{x}$, dividing by $y^2$ $\longrightarrow$ (1) | 1 |
| | put, $\frac{1}{y} = t$, $\therefore -\frac{1}{y^2}\frac{dy}{dx} = dt/dx$ | |
| | (1) becomes, $-\frac{dt}{dx} + \frac{2t}{x} = \frac{\log x}{x}$ | |
| | $P = -2/x$ and $Q = -\log x/x$ | |
| | $\frac{dt}{dx} - \frac{2t}{x} = -\frac{\log x}{x}$ | 1 |
| | $I.F. = e^{\int P dx} = e^{-\int 2/x\, dx} = e^{-2\log x} = 1/x^2$ | 1 |
| | The solution is, $te^{\int P dx} = \int Q e^{\int P dx} dx + c$ | |
| | $\frac{t}{x^2} = -\int \log x \cdot \frac{1}{x^3} dx + c$ | |
| | $t/x^2 = -\left\{\log x \cdot \frac{x^{-2}}{-2} - \int \frac{x^{-2}}{-2} \cdot \frac{1}{x} dx\right\} + c$ | 1 |
| | $t/x^2 = -\left\{-\frac{\log x}{2x^2} + \frac{1}{2} \times \frac{x^{-2}}{-2}\right\} + c$ | |
| | $t/x^2 = 1/2x^2 \left\{\log x + 1/2\right\} + c$ | 1 |
| | Thus, $\frac{1}{x^2 y} = \frac{1}{2x^2}(\log x + 1/2) + c$ | 1 |
| | | 6 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| 5b. | Consider $x^{2/3} + y^{2/3} = a^{2/3}$ | |
| | Differentiating wrt '$x$', | |
| | $\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \frac{dy}{dx} = 0$ | 1 |
| | $x^{-1/3} + y^{-1/3} \frac{dy}{dx} = 0$, is the differential equation of the given family. | |
| | Replacing $\frac{dy}{dx}$ by $-dx/dy$ | |
| | $x^{-1/3} + y^{-1/3}\left(-dx/dy\right) = 0$ | 1 |
| | i.e. $x^{-1/3}\, dy = y^{-1/3}\, dx$ | 1 |
| | $y^{1/3}\, dy = x^{1/3}\, dx$, by separating the variables | 1 |
| | $\Rightarrow \int y^{1/3}\, dy - \int x^{1/3}\, dx = c$ | 1 |
| | $\frac{y^{4/3}}{4/3} - \frac{x^{4/3}}{4/3} = c$ | 1 |
| | or $x^{4/3} - y^{4/3} = -\frac{4c}{3} = k$ | |
| | Thus, $x^{4/3} - y^{4/3} = k$, is the required orthogonal trajectory. | 1 |
| | | 7 |
| 5c. | $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$ | |
| | $p(p^2 + 2xp - y^2p - 2xy^2) = 0$ | |
| | $p\{p(p+2x) - y^2(p+2x)\} = 0$ | |
| | $p\{(p+2x)(p-y^2)\} = 0$ | 2 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $p=0, \quad p-y^2=0, \quad p+2x=0$ | |
| | $\frac{dy}{dx}=0 \Rightarrow dy=0$ | |
| | Integrating $\Rightarrow y=c \longrightarrow (i)$ | 1 |
| | $\frac{dy}{dx}-y^2=0 \Rightarrow \frac{dy}{y^2}-dx=0$ | |
| | $\int \frac{dy}{y^2}-\int 1.dx=c$ | |
| | $\Rightarrow -\frac{1}{y}-x=c \Rightarrow x+\frac{1}{y}=c \longrightarrow (ii)$ | 1 |
| | Consider, $p+2x=0$ | |
| | $\frac{dy}{dx}+2x=0$ | 1 |
| | $dy+2x\,dx=0$ | |
| | $\int dy+2\int x\,dx=c$ | 1 |
| | $\Rightarrow y+x^2=c \longrightarrow (iii)$ | |
| | Hence, The general solution is, | |
| | $(y-c)(y+x^2-c)\left(x+\frac{1}{y}-c\right)=0$ | 1 |
| | | 7 |
| | OR | |
| 6a. | $y(x+y+1)dx+x(x+3y+2)dy=0.$ | |
| | $M=xy+y^2+y \quad ; \quad N=x^2+3xy+2x$ | |
| | $\frac{\partial M}{\partial y}=2y+x+1 \quad ; \quad \frac{\partial N}{\partial x}=2x+3y+2$ | |
| | $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=x+2y+1-(2x+3y+2)$ | |
| | $=-x-y-1 \dashrightarrow$ close to M | 1 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $\frac{1}{M}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{1}{xy+y^2+y}(-x-y-1)$ | |
| | $=\frac{1}{y(x+y+1)}\times(-1)(x+y+1)$ | |
| | $=-1/y=g(y)$ | 2 |
| | $\therefore$ Integrating factor $=e^{-\int g(y)\,dy}$ | |
| | $=e^{-\int -1/y\,dy}$ | |
| | $=e^{\log y}=y$ | 1 |
| | Multiplying by 'y' to the given eqn, | |
| | $y(xy+y^2+y)dx+y(x^2+3xy+2x)dy=0$ | 1 |
| | Now, $M=xy^2+y^3+y^2$ | |
| | $\frac{\partial M}{\partial y}=2xy+3y^2+2y$ | |
| | & $N=x^2y+3xy^2+2xy$ | |
| | $\frac{\partial N}{\partial x}=2xy+3y^2+2y$ | |
| | Hence, $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$. | |
| | The solution is, $\int M\,dx+\int N\,dy=c$ | |
| | $\int(xy^2+y^3+y^2)dx+\int 0\,dy=c$ | |
| | $\frac{x^2}{2}y^2+xy^3+xy^2=c$, is the required solution | 1 |
| | | 6 |

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DEPT : ECE

| Question Number | Solution | Marks Allocated |
|---|---|---|
| 6b. | Given, $\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L} \sin \omega t$ | |
| | Applying the solution for the linear differential equation, $\frac{dy}{dx} + Py = Q$, | |
| | $ie^{Rt/L} = \int \frac{E}{L} \sin \omega t \, e^{Rt/L} \, dt + c$ | 1 |
| | Using the standard result, | |
| | $\int e^{at} \sin bt \, dt = \frac{e^{at}}{\sqrt{a^2+b^2}} \sin[bt - \tan^{-1}(b/a)]$ | 1 |
| | $ie^{Rt/L} = \frac{E}{L} \times \frac{e^{Rt/L}}{\sqrt{(R/L)^2+\omega^2}} \times \sin[\omega t - \tan^{-1}(\omega L/R)] + c$ | 1 |
| | Denoting, $\tan^{-1}(\omega L/R) = \phi$ | |
| | or $\tan\phi = \omega L/R$ | |
| | $i = \frac{E}{L} \times \frac{L}{\sqrt{R^2+\omega^2 L^2}} \sin(\omega t - \phi) + ce^{-Rt/L}$ → (1) | 1 |
| | Using the initial conditions, $i=0$ when $t=0$ in (1) we have | |
| | $0 = \frac{E \sin(-\phi)}{\sqrt{R+\omega^2 L^2}} + c$ | |
| | or $c = \frac{E \sin\phi}{\sqrt{R^2+\omega^2 L^2}}$, $\sin(-\phi) = -\sin\phi$ | 1 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | Substituting this value of 'c' in eqn ① we get the value of current at any time t. | |
| | $i = \dfrac{E\sin(\omega t-\phi)}{\sqrt{R^2+\omega^2L^2}} + \dfrac{E\sin\phi}{\sqrt{R^2+\omega^2L^2}}e^{-Rt/L}$ | 1 |
| | Thus, $i = \dfrac{E}{\sqrt{R^2+\omega^2L^2}}\left\{\sin(\omega t-\phi)+e^{-Rt/L}\right\}$ | 1 |
| | where, $\phi = \tan^{-1}(\omega L/R)$ | 7 |
| 6c. | Given, $xp^2-py+kp+a=0$ | |
| | $xp^2+kp+a=py$ | |
| | $y=\dfrac{p(xp+k)+a}{p}$ or | 1 |
| | $y=xp+k+\dfrac{a}{p}$ | |
| | $y=px+(k+a/p) \longrightarrow ①$ | 1 |
| | Here, (1) is in the Clairaut's form $y=px+f(p)$, whose general solution | 1 |
| | is, $y=cx+f(c)$. | 1 |
| | Thus, $y=cx+(k+a/c)$ | |
| | Differentiating partially w.r.t. 'c' we have, | |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $0 = x - \frac{a}{c^2}$ or $\frac{a}{c^2} = x$ or $c^2 = \frac{a}{x}$ | |
| | $\Rightarrow c = \sqrt{a/x}$ | 1 |
| | Hence, the general solution becomes, | |
| | $y = (\sqrt{a/x})\,x + k + a(\sqrt{x/a})$ | 1 |
| | or $y - k = 2\sqrt{ax}$ | |
| | $\Rightarrow (y-k)^2 = 4ax$, is the singular solution. | 1 |
| | | 7 |
| | MODULE - 4 | |
| 7a. | Let, $I = \int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$ | |
| | $= \int_{z=-1}^{1}\int_{x=0}^{z}\left[xy+\frac{y^2}{2}+zy\right]_{y=x-z}^{x+z} dx\,dz$ | 1 |
| | $= \int_{z=-1}^{1}\int_{x=0}^{z}\left\{x(\overline{x+z}-\overline{x-z})+\frac{1}{2}\left[(x+z)^2-(x-z)^2\right] + z(\overline{x+z}-\overline{x-z})\right\}dx\,dz$ | 1 |
| | $I = \int_{z=-1}^{1}\int_{x=0}^{z}(2xz+2xz+2z^2)\,dx\,dz$ | |
| | $= \int_{z=-1}^{1}\int_{x=0}^{z}(4xz+2z^2)\,dx\,dz$ | 1 |
| | $I = \int_{z=-1}^{1}\left[z(2x^2)+2z^2(x)\right]_{x=0}^{z} dz$ | 1 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $I = \int_{z=-1}^{1} (2z^3 + 2z^3)\, dz$ | |
| | $I = \int_{z=-1}^{1} 4z^3\, dz = \left[ z^4 \right]_{z=-1}^{1} = 0$ | 1 |
| | Thus, $\underline{\underline{I = 0}}$ | 1 |
| | | 6 |
| 7b. | $I = \int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} y^2\, dy\, dx$ | |
| | on changing the order of integration | |
| | $I = \int_{y=0}^{1} \int_{x=0}^{\sqrt{1-y^2}} y^2\, dx\, dy$ | 2 |
| | $I = \int_{y=0}^{1} y^2 \left[ x \right]_{x=0}^{\sqrt{1-x^2}} dy$ | 1 |
| | $= \int_{y=0}^{1} y^2 \sqrt{1-y^2}\, dy$ | 1 |
| | put, $y = \sin\theta \quad \therefore dy = \cos\theta\, d\theta$ & $\theta$ varies from 0 to $\pi/2$. | |
| | $I = \int_{\theta=0}^{\pi/2} \sin^2\theta \cos^2\theta\, d\theta$ | 1 |
| | $= \frac{1 \times 1}{4 \times 2} \cdot \frac{\pi}{2} = \frac{\pi}{16}$, by reduction formula | 1 |
| | $\underline{\underline{I = \pi/16}}$ | 1 |
| | | 7 |

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DEPT : ECE

| Question Number | Solution | Marks Allocated |
|---|---|---|
| 7c. | By the definition of Beta & Gamma functions | |
| | $\beta(m,n) = 2\int_{\theta=0}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta \rightarrow (1)$ | 1 |
| | $\Gamma n = 2\int_0^\infty e^{-x^2} x^{2n-1} dx \rightarrow (2)$ | |
| | $\Gamma m = 2\int_0^\infty e^{-y^2} y^{2m-1} dy \rightarrow (3)$ | |
| | $\Gamma m+n = 2\int_0^\infty e^{-r^2} r^{2(m+n)-1} dr \rightarrow (4)$ | 1 |
| | Now, $\Gamma m . \Gamma n = 4\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} x^{2n-1} y^{2m-1} dx\,dy \rightarrow (5)$ | 1 |
| | putting, $x = r\cos\theta$, $y = r\sin\theta$ we have $x^2+y^2 = r^2$. Also, $dx\,dy = r\,dr\,d\theta$ $r$ varies from 0 to $\infty$, $\theta$ varies from 0 to $\pi/2$. | |
| | Eqn (5) can be written in the form $\Gamma m \Gamma n = 4\int_{r=0}^\infty \int_{\theta=0}^{\pi/2} e^{-r^2} (r\cos\theta)^{2n-1} (r\sin\theta)^{2m-1} r\,dr\,d\theta$ | 1 |
| | $= 4\int_{r=0}^\infty \int_{\theta=0}^{\pi/2} e^{-r^2} r^{2m+2n-1} \sin^{2m-1}\theta \cos^{2n-1}\theta \, dr\,d\theta$ | 1 |
| | $= \left[2\int_{r=0}^\infty e^{-r^2} r^{2(m+n)-1} dr\right]\left[2\int_{\theta=0}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta\right]$ | 1 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $\therefore \Gamma m \cdot \Gamma n = \Gamma m+n \cdot \beta(m,n)$, by using equations ① and ④, | |
| | Thus, $\beta(m,n) = \dfrac{\Gamma m \, \Gamma n}{\Gamma m+n}$ | 1 |
| | | 7 |
| | OR | |
| 8a. | $I = \int_{y=0}^{a} \int_{x=0}^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\, dx\, dy$ | |
| | $x = \sqrt{a^2-y^2}$ or $x^2+y^2=a^2$ is a circle with centre origin & radius 'a'. Since, y varies from 0 to a, the region of integration is the first quadrant of the circle. | 1 |
| | $x = r\cos\theta$, $y = r\sin\theta$ $\therefore x^2+y^2=r^2$ <br> $r^2 = a^2 \Rightarrow r = a$. <br> Since, $x=0, y=0$ gives $r=0$. Hence $r$ varies from 0 to a. <br> Also, $dx\,dy = r\,dr\,d\theta$ | 1 |
| | $I = \int_{r=0}^{a} \int_{\theta=0}^{\pi/2} r\sin\theta \cdot r \cdot r\, dr\, d\theta$ <br> $= \int_{r=0}^{a} \int_{\theta=0}^{\pi/2} r^3 \sin\theta\, dr\, d\theta$ | 2 |
| | $I = \int_{r=0}^{a} r^3 \left[-\cos\theta\right]_{\theta=0}^{\pi/2} dr = \left[\dfrac{r^4}{4}\right]_{r=0}^{a} = \dfrac{a^4}{4}$ | 2 |
| | | 6 |

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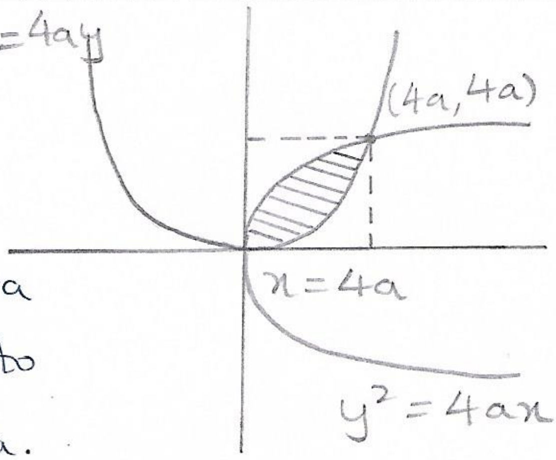
DEPT : ECE

| Question Number | Solution | Marks Allocated |
|---|---|---|
| 8b. |   | |
| | Area $= \iint_R dx\,dy$ | |
| | $x$ varies from 0 to $4a$ | |
| | $y$ varies from $2\sqrt{ax}$ to $x^2/4a$. | |
| | Area $= \int_{x=0}^{4a} \int_{y=x^2/4a}^{2\sqrt{ax}} dy\,dx$ | 1 |
| | $= \int_{x=0}^{4a} [y]_{y=x^2/4a}^{2\sqrt{ax}}\,dx$ | 1 |
| | $= \int_{x=0}^{4a} \left[2\sqrt{ax} - \frac{x^2}{4a}\right] dx$ | 1 |
| | $= \int_{x=0}^{4a} \left[2a^{1/2}x^{1/2} - x^2/4a\right] dx$ | 1 |
| | $= \left[2a^{1/2}\frac{x^{3/2}}{3/2} - \frac{1}{4a}\times\frac{x^3}{3}\right]_{x=0}^{4a}$ | 1 |
| | $= 2a^{1/2}\frac{(4a)^{3/2}}{3/2} - \frac{(4a)^3}{12a}$ | 1 |
| | $= \frac{4a^2(8)}{3} - \frac{64a^3}{12a} = \frac{32}{3}(a^2) - \frac{16a^2}{3}$ | 1 |
| | $\underline{\underline{I = \frac{16a^2}{3}}}$ | 1 |
| | | 7 |

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DEPT: ECE

| Question Number | Solution | Marks Allocated |
|---|---|---|
| 8c. | Let, $I_1 = \int_0^{\pi/2} \frac{d\theta}{\sin\theta} = \int_0^{\pi/2} \sin^{-1/2}\theta \, d\theta$ <br> $= \int_0^{\pi/2} \sin^{-1/2} \cos^0 \theta \, d\theta$ <br> and $I_2 = \int_0^{\pi/2} \sqrt{\sin\theta} \, d\theta = \int_0^{\pi/2} \sin^{1/2}\theta \, d\theta$ | 1 |
| | $= \int_0^{\pi/2} \sin^{1/2}\theta \cos^0\theta \, d\theta$ | |
| | $\int_0^{\pi/2} \sin^p\theta \cos^q\theta \, d\theta = \frac{1}{2} \beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$ | 1 |
| | $I_1 = \frac{1}{2}\beta\left(\frac{-1/2+1}{2}, \frac{0+1}{2}\right) = \frac{1}{2}\beta(1/4, 1/2)$ | 1 |
| | $I_2 = \frac{1}{2}\beta\left(\frac{1/2+1}{2}, \frac{0+1}{2}\right) = \frac{1}{2}\beta(3/4, 1/2)$ | 1 |
| | $\therefore I_1 \times I_2 = \frac{1}{4}\beta(1/4, 1/2) \cdot \beta(3/4, 1/2)$ | |
| | $= \frac{1}{4} \frac{\Gamma(1/4)\,\Gamma(1/2)}{\Gamma(3/4)} \cdot \frac{\Gamma(3/4) \cdot \Gamma(1/2)}{\Gamma(5/4)}$ | 1 |
| | $= \frac{1}{4}\Gamma(1/4)\sqrt{\pi} \times \frac{\sqrt{\pi}}{1/4 \cdot \Gamma(1/4)}$ | 1 |
| | $\therefore I_1 \times I_2 = \underline{\underline{\pi}}$ | 1 |
| | | 7 |

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DEPT : ECE

| Question Number | Solution | Marks Allocated |
|---|---|---|
| | Module-5 | |
| 9a. | Let, $A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$ | |
| | $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$ & $R_4 \to R_4 - 4R_1$ | |
| | $A \sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & -4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & -5 & 7 \end{bmatrix}$ | 2 |
| | $R_3 \Leftrightarrow R_4$ | |
| | $A \sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & -4 & 3 \\ 0 & -1 & -5 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | 2 |
| | $R_3 \to R_3 - R_2$ | |
| | $A \sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & -1 & -4 & 3 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | 2 |
| | $\therefore$ The rank of the matrix A is, $\rho[A] = 4$ | |
| | | 6 |

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DEPT : ECE

| Question Number | Solution | Marks Allocated |
|---|---|---|
| 9b. | Let, the augmented matrix be | |
| | $[A:B] = \left[\begin{array}{ccc\|c} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{array}\right]$ | |
| | $R_1 \Leftrightarrow R_3 \sim \left[\begin{array}{ccc\|c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{array}\right]$ | 1 |
| | $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$ | |
| | $[A:B] \sim \left[\begin{array}{ccc\|c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & -7 & -17 & -22 \end{array}\right]$ | 2 |
| | $R_3 \to 10R_3 - 7R_2$ | |
| | $[A:B] \sim \left[\begin{array}{ccc\|c} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & 0 & -2 & 188 \end{array}\right]$ | 1 |
| | By back substitution method | |
| | $-2z = 188 \Rightarrow z = -94$ | |
| | $-10y - 24z = -30$ | |
| | $-10y - 24(-94) = -30$ | 3 |
| | $\Rightarrow -10y = -2286 \Rightarrow y = 228.6$ | |
| | $x + 4y + 9z = 16$ | |
| | $x + 4(228.6) + 9(-94) = 16$ | |
| | $x = -52.4$ | |
| | $\therefore x = -52.4, y = 228.6, z = -94$ | 7 |

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DEPT : ECE

| Question Number | Solution | Marks Allocated |
|---|---|---|
| 9c. | The given system of equations are diagonally dominant,<br>$x = \frac{1}{5}[12-2y-z]$<br>$y = \frac{1}{4}[15-x-2z]$<br>$z = \frac{1}{5}[20-x-2y]$ | 1 |
| | $x^{(0)} = 1, \; y^{(0)} = 0, \; z^{(0)} = 3$<br>First iteration:<br>$x^{(1)} = \frac{1}{5}[12-2(0)-3] = 1.8$<br>$y^{(1)} = \frac{1}{4}[15-1.8-2(3)] = 1.8$<br>$z^{(1)} = \frac{1}{5}[20-1.8-2(1.8)] = 2.92$ | 1 |
| | Second iteration:<br>$x^{(2)} = \frac{1}{5}[12-2(1.8)-2.92] = 1.096$<br>$y^{(2)} = \frac{1}{4}[15-1.096-2(2.92)] = 2.016$<br>$z^{(2)} = \frac{1}{5}[20-1.096-2(2.016)] = 2.9744$ | 1 |
| | Third iteration:<br>$x^{(3)} = \frac{1}{5}[12-2(2.016)-2.9744] = 0.99872$<br>$y^{(3)} = \frac{1}{4}[15-0.99872-2(2.9744)] = 2.01312$<br>$z^{(3)} = \frac{1}{5}[20-0.99872-2(2.01312)] = 2.995$ | 2 |

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DEPT : ECE

| Question Number | Solution | Marks Allocated |
|---|---|---|

Fourth iteration:

$$x^{(4)} = \frac{1}{5}\left[12 - 2(2.01312) - 2.995\right]$$

$$= 0.995752$$

$$y^{(4)} = \frac{1}{4}\left[15 - 0.995752 - 2(2.995)\right]$$

$$= 2.003562$$ 2

$$z^{(4)} = \frac{1}{5}\left[20 - 0.995752 - 2(2.003562)\right]$$

$$z^{(4)} = 2.9994248$$

Thus, $x = 0.9958$, $y = 2.0036$, $z = 2.9994$

7

OR

10a. $$A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

$R_2 \to 2R_2 - R_1$, $R_3 \to 2R_3 - R_1$, $R_4 \to 2R_4 - R_1$

$$A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 0 & 2 & 4 & 7 \\ 0 & 6 & 2 & 13 \\ 0 & 6 & 0 & 7 \end{bmatrix}$$ 2

$R_4 \to R_4 - R_3$ $\therefore A \sim \begin{bmatrix} 4 & 0 & 2 & 1 \\ 0 & 2 & 4 & 7 \\ 0 & 6 & 2 & 13 \\ 0 & 0 & -2 & -6 \end{bmatrix}$ 1

$R_3 \to R_3 - 3R_2$

| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $\therefore A \sim \begin{bmatrix} 4 & 0 & 2 & 1 \\ 0 & 2 & 4 & 7 \\ 0 & 0 & -10 & -8 \\ 0 & 0 & -2 & -6 \end{bmatrix}$ | 1 |
| | $R_4 \to 5R_4 - R_3$ | |
| | $\therefore A \sim \begin{bmatrix} 4 & 0 & 2 & 1 \\ 0 & 2 & 4 & 7 \\ 0 & 0 & -10 & -8 \\ 0 & 0 & 0 & -22 \end{bmatrix}$ | 2 |
| | Thus, the rank of the given matrix is, $\rho[A] = 4$ | 1 |
| | | 7 |
| 10b. | $[A:B] = \left[\begin{array}{ccc:c} 2 & 1 & 3 & 1 \\ 4 & 4 & 7 & 1 \\ 2 & 5 & 9 & 3 \end{array}\right]$ | |
| | $R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - R_1$ | |
| | $[A:B] \sim \left[\begin{array}{ccc:c} 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 4 & 6 & 2 \end{array}\right]$ | 2 |
| | $R_1 \to 2R_1 - R_2$ & $R_3 \to R_3 - 2R_2$ | |
| | $[A:B] \sim \left[\begin{array}{ccc:c} 4 & 0 & 5 & 3 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 4 & 4 \end{array}\right]$ | 2 |
| | $R_3 \to R_3 \cdot \frac{1}{4}$ | |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | $[A:B] \sim \left[\begin{array}{ccc\|c} 4 & 0 & 5 & 3 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$ | |
| | $R_2 \to R_2 - R_3,\ R_1 \to R_1 - 5R_3$ | |
| | $[A:B] \sim \left[\begin{array}{ccc\|c} 4 & 0 & 0 & -2 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$ | 1 |
| | Hence, $4x = -2 \Rightarrow x = -0.5$<br>$2y = -2 \Rightarrow y = -1$<br>$z = 1$ | 2 |
| | $\therefore x = -0.5,\ \underline{\underline{y = -1 \ \& \ z = 1}}$ | 7 |
| 10c. | $x + 2y - z = 1$<br>$2x + y + 4z = 2$<br>$3x + 3y + 4z = 1$ | |
| | A = np.matrix([[1,2,-1],[2,1,4],[3,3,4]]) | 2 |
| | B = np.matrix([[1],[2],[1]])<br>AB = np.concatenate((A,B),axis=1) | |
| | rA = np.linalg.matrix_rank(A)<br>rAB = np.linalg.matrix_rank(AB)<br>n = A.shape[1]<br>if (rA == rAB): | 1 |

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| Question Number | Solution | Marks Allocated |
|---|---|---|
| | if (rA==n): | |
| | print ("The system has unique solution") | 1 |
| | print (np.linalg.solve (A,B)) | 1 |
| | else: | |
| | print ("The system of equations is inconsistent") | 1 |
| | | 6 |

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