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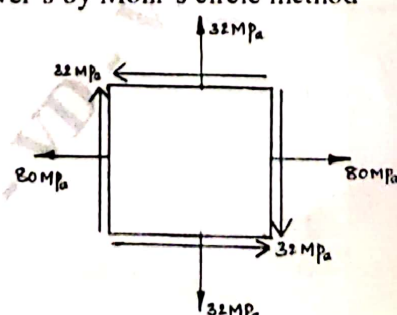
Third Semester B.E./B.Tech. Degree Examination, June/July 2024 Mechanics of Materials

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.

2. M : Marks, L: Bloom's level, C: Course outcomes.

		Module - 1	M	L	C
Q.1	a.	Define the following with necessary equations: (i) Normal stress (ii) Shear stress (iii) Poisson's ratio (iv) Young's modulus (v) Thermal stress	10	L1	CO1
	b.	The tensile test was conducted on a mild steel bar. The following was obtained from the test: Diameter of steel bar = 16 mm ; Gauge length of the bar = 80 mm ; Load at proportionality limit = 72 kN ; Extension at a load of 60 kN = 0.115 mm ; Load at failure = 80 kN ; Final gauge length of bar = 104 mm ; Diameter of the bar at failure = 12 mm Determine: (i) Young's modulus (ii) Proportionality limit (iii) True breaking stress (iv) Percentage elongation (v) Percentage decrease in area	10	L3	CO1
OR					
Q.2	a.	Write the relation between the following with usual notations and meaning: (i) Modulus of elasticity and bulk modulus (ii) Modulus of elasticity and modulus of rigidity (iii) Modulus of elasticity, modulus of rigidity and bulk modulus	06	L1	CO1
	b.	Define the following: (i) Gradual load (ii) Sudden load (iii) Impact load (iv) Shock load	04	L1	CO1
	c.	Rails laid such that there is no stress in them at 24°C. If the rails are 32 m long, determine: (i) The stress in the rails at 80°C, when there is no allowance for expansion. (ii) The stress in the rails at 80°C, when there is an expansion allowance of 8 mm per rail (iii) The expansion allowance for no stress in the rails at 80°C. Take $\alpha = 11 \times 10^{-6}/^{\circ}\text{C}$, $E = 205 \text{ GPa}$.	10	L3	CO1
Module - 2					
Q.3	a.	Derive the expression for normal stress and shear stress on a plane inclined at ' θ ' angle to the vertical axis in a biaxial stress system with shear stress.	10	L2	CO2
	b.	For the two-dimensional stressed element, shown in Fig.Q3(b), determine the value of: (i) Maximum and minimum principal stress (ii) Principal planes (iii) Maximum shear stress and its plane Verify the answer's by Mohr's circle method	10	L3	CO2
			Fig.Q3(b)		
1 of 3					

OR

Q.4	a.	Derive an expression for circumferential stress and longitudinal stress for a thin cylinder subjected to an internal pressure 'P'.	10	L2	CO2
	b.	A thick cylinder of internal diameter 160 mm is subjected to an internal fluid pressure of 40 N/mm ² . If the allowable stress in the material is 120 N/mm ² , find the required wall thickness of the cylinder.	10	L3	CO2

Module - 3

Q.5	a.	Draw the shear force and bending moment diagrams for the cantilever shown in Fig.Q5(a).	10	L4	CO3
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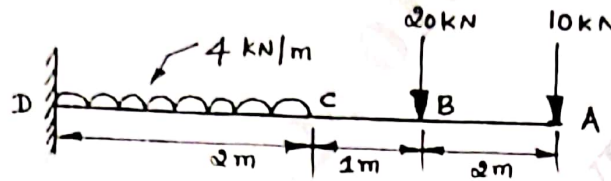


Fig.Q5(a)

	b.	Draw the bending moment and shear force diagram for the overhanging beam shown in Fig.Q5(b). Clearly indicate the point of contraflexure.	10	L4	CO3
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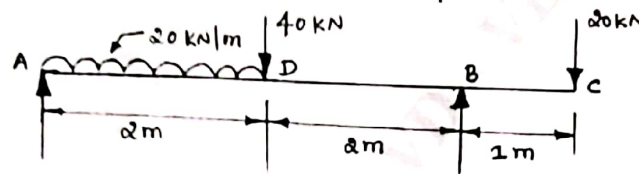


Fig.Q5(b)

OR

Q.6		A simply supported beam of 7m span with overhangs rests on supports which are 4m apart. The left end overhang is 2 m. The beam carries loads of 30 kN and 20 kN on the left and the right ends respectively apart from a uniformly distributed load of 25 kN/m between the supporting points. Draw the shear force and bending moment diagrams. Locate point of contraflexure if any.	20	L4	CO3
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Module - 4

Q.7	a.	Derive the bending equation in the form of $\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$.	10	L2	CO4
	b.	A square beam 20 mm × 20 mm in section and 2 m long is supported at the ends. The beam fails when a point load of 400 N is applied at the centre of the beam. What uniformly distributed load per metre length will break a cantilever of the same material 40 mm wide, 60 mm deep and 3 m long?	10	L3	CO4

OR

Q.8	a.	Derive an expression for section modulus of solid rectangular and circular sections.	10	L2	CO4
	b.	Fig.Q8(b) shows the cross-section of a beam which is subjected to a shear force of 20 kN. Draw the shear stress distribution across the depth making values at salient points.	10	L3	CO4

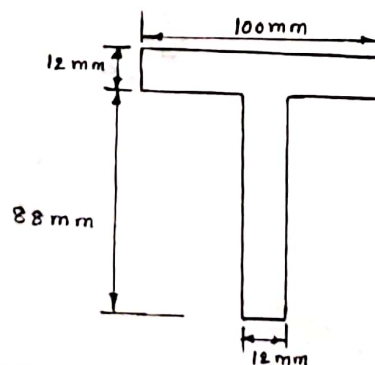


Fig.Q8(b)

Module - 5

Q.9	a.	Define the following with necessary equations: (i) Torque (ii) Polar modulus (iii) Torsional rigidity	06	L1	CO5
	b.	State the assumptions made in theory of torsion.	04	L1	CO5
	c.	Derive torsion equation in the form of $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$.	10	L2	CO5
OR					
Q.10	a.	Define the following: (i) Column (ii) Buckling load (iii) Slenderness ratio (iv) Long column (v) Short column	10	L1	CO5
	b.	Derive an expression for Euler buckling load when both ends of the column are fixed.	10	L2	CO5



Q.No	Solution and Scheme	Mks
Q.1(a)	<p style="text-align: center;"><u>Module - 1</u></p> <p>(i) Normal stress: The stress perpendicular to the plane</p> <p>(ii) Shear stress: The stress parallel to the plane</p> <p>(iii) Poisson's ratio: Ratio of lateral strain to longitudinal strain</p> <p>(iv) Young's modulus: The ratio of stress to strain within elastic limit</p> <p>(v) Thermal stress: Stress generated due to variation of temperature</p> <p>(b) Given! $d = 16 \text{ mm}$, $l = 80 \text{ mm}$, Load at proportional limit = 72 kN, $d_l \text{ at } 60 \text{ kN} = 0.115 \text{ mm}$ Load at failure = 80 kN, $l_f = 104 \text{ mm}$, $d_f = 12 \text{ mm}$</p> <p><u>Soln</u>: (i) $E = \frac{\sigma}{\epsilon}$, $\sigma = \frac{60 \times 10^3}{\frac{\pi}{4} (16)^2} = 298.41 \text{ N/mm}^2$</p> $= \frac{298.41}{1.43 \times 10^{-3}} \epsilon = \frac{d_l}{l} = \frac{0.115}{80} = 1.43 \times 10^{-3}$ $= 207.589 \times 10^3 \text{ N/mm}^2$ <p>(ii) Proportional limit</p> $\sigma = \frac{72 \times 10^3}{\frac{\pi}{4} (16)^2} = 358.09 \text{ N/mm}^2$	10

Q1 b. (iii) True breaking stress $\sigma = \frac{80 \times 10^3}{\frac{\pi}{4}(12)^2} = 707.35 \text{ N/mm}^2$

(iv) % elongation $= \frac{l - l_0}{l_0} \times 100 = \frac{104 - 80}{80} \times 100 = 30\%$

(v) % decrease in area $= \frac{A - A_f}{A_f} \times 100 = \frac{16^2 - 12^2}{12^2} \times 100 = 43.75\%$ 10

Q2 a. (i) Modulus of elasticity & Bulk modulus
OR
 $E = 3K(1 - 2\mu)$

(ii) E & $G \Rightarrow E = 2G(1 + \mu)$ 06

(iii) E, G & $K \Rightarrow E = \frac{9KG}{G + 3K}$

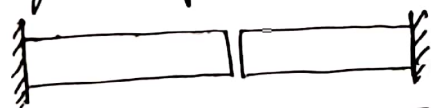
b. (i) Gradual load: Load which increases from zero to its full value over time.

(ii) Sudden load: Load which is applied suddenly

(iii) Impact load: Load which falls through some height, load with certain velocity. 04

(iv) Shock load: Shock load is measured in terms of energy.

C



Given:

$T_1 = 24^\circ\text{C}$ $l = 32 \text{ m} = 32 \times 10^3 \text{ mm}$

$\alpha = 11 \times 10^{-6} / ^\circ\text{C}$ $E = 205 \text{ GPa}$

(i) $\sigma = \alpha t E = 11 \times 10^{-6} \times (80 - 24) \times 205 \times 10^3 = 126.28 \text{ N/mm}^2$

(ii) When expansion allowance is 8 mm at 80°C

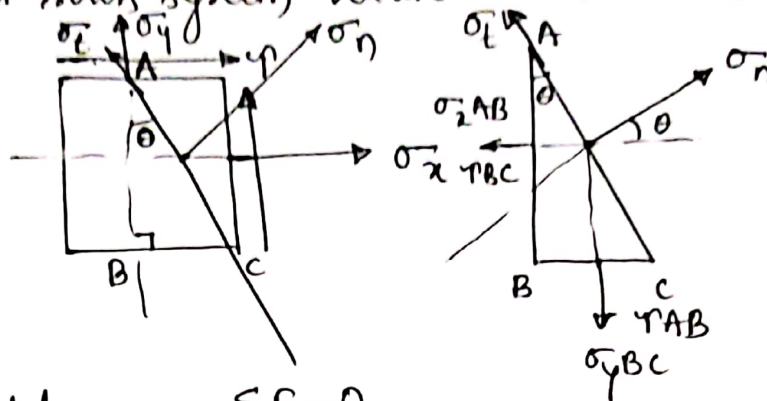
$\Delta = \alpha l \Delta t - 8 = \frac{\sigma l}{E} \Rightarrow \sigma = \frac{E}{l} (\alpha l \Delta t - 8)$

$\sigma = 205 \times 10^3 (11 \times 10^{-6} \times 32 \times 10^3 \times (80 - 24) - 8) = 75.03 \text{ N/mm}^2$

(iii) Expansion allowance for no stress at 80°C

$= \alpha l \Delta t = 11 \times 10^{-6} \times 32 \times 10^3 \times (80 - 24) = 19.71 \text{ mm}$ 10

Q3a. Expression for normal stress & shear stress on a plane inclined at θ angle to vertical axis in a biaxial stress system with shear stress



For Equilibrium $\sum F_n = 0$

$$\sigma_n AC = \sigma_x AB \cos\theta + \tau BC \cos\theta + \sigma_y BC \sin\theta + \tau AB \sin\theta$$

$$\sigma_n = \sigma_x \frac{AB}{AC} \cos\theta + \tau \frac{BC}{AC} \cos\theta + \sigma_y \frac{BC}{AC} \sin\theta + \tau \frac{AB}{AC} \sin\theta$$

$$\sigma_n = \sigma_x \cos^2\theta + \tau 2 \sin\theta \cos\theta + \sigma_y \sin^2\theta$$

$$\sigma_n = \sigma_x \left(\frac{1 + \cos 2\theta}{2} \right) + \sigma_y \left(\frac{1 - \cos 2\theta}{2} \right) + \tau \sin 2\theta$$

$$\boxed{\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau \sin 2\theta}$$

Similarly $\sum F_t = 0$.

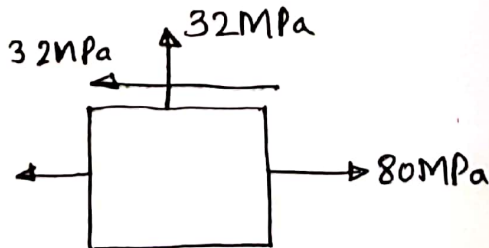
$$\sigma_t AC = -\sigma_x AB \sin\theta - \tau BC \sin\theta + \sigma_y BC \cos\theta + \tau AB \cos\theta$$

$$\sigma_t = -\sigma_x \frac{AB}{AC} \sin\theta - \tau \frac{BC}{AC} + \sigma_y \frac{BC}{AC} \cos\theta + \tau \frac{AB}{AC} \cos\theta$$

$$\sigma_t = -(\sigma_x - \sigma_y) \sin\theta \cos\theta + \tau (\cos^2\theta - \sin^2\theta)$$

$$\boxed{\sigma_t = \left(\frac{-\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau \cos 2\theta}$$

b.



$$\sigma_x = 80 \text{ MPa}$$

$$\sigma_y = 32 \text{ MPa}$$

$$\tau = -32 \text{ MPa}$$

10

Q 3.6. (i) Maximum & Minimum Principal stresses

$$\sigma_{n_{1,2}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \left(\frac{80+32}{2}\right) \pm \sqrt{\left(\frac{80-32}{2}\right)^2 + (-32)^2}$$

$$= 56 \pm \sqrt{(24)^2 + (32)^2} = 56 \pm 40$$

$$\sigma_{n_1} = 96 \text{ MPa} \quad \sigma_{n_2} = 14 \text{ MPa}.$$

(ii) Principal planes.

$$2\theta_p = \tan^{-1} \left\{ \frac{\tau}{\frac{\sigma_x - \sigma_y}{2}} \right\} = \tan^{-1} \left\{ \frac{-32}{56} \right\} = -29.74^\circ$$

$$\theta_{p_1} = -14.87 \quad \theta_{p_2} = -14.87 + 90 = 75.12^\circ$$

(iii) Maximum shear stress & its plane.

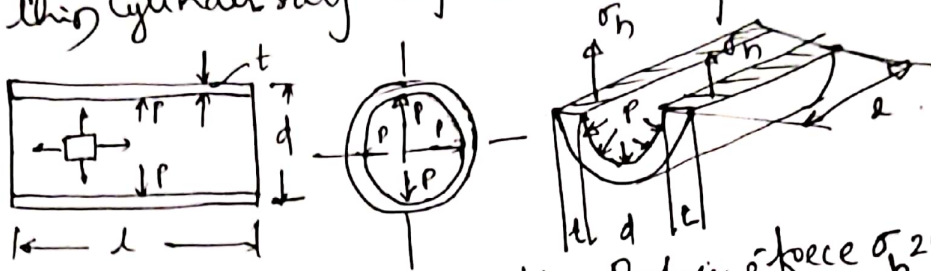
$$\text{Max shear stress } \sigma_{t_{\text{max}}} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{80-32}{2}\right)^2 + (-32)^2}$$

$$= \pm 40 \text{ MPa}.$$

$$2\theta_s = \tan^{-1} \left\{ -\frac{(\sigma_x - \sigma_y)}{\tau} \right\} = \tan^{-1} \left\{ \frac{-56}{-32} \right\} = 60.25^\circ$$

$$\theta_{s_1} = 30.12^\circ \quad \theta_{s_2} = 120.12^\circ$$

Q.4a. ^{OR} Expression for Circumferential & longitudinal stress for a thin cylinder subjected to internal pressure P



$\Sigma F_v = 0$ Bursting force = $pdl =$ Restoring force $\sigma_h 2(tl)$

$$\therefore \sigma_h = \frac{pd}{2t}$$

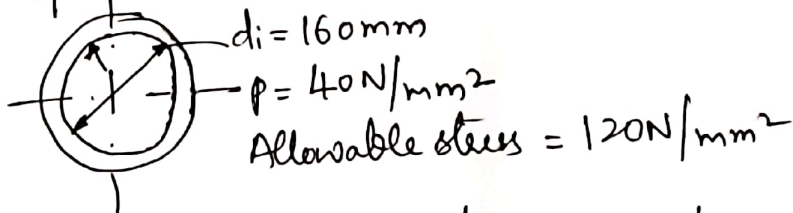
Circumferential hoop stress

$\Sigma F_v = 0$ Bursting force = $pdl =$ Restoring force = $\sigma_c (2tl)$

$$\sigma_c = \frac{pd}{2t}$$

40

b. Given!



$$p = \frac{b}{r^2} - a \Rightarrow 40 = \frac{b}{80^2} - a \quad \text{--- (i)}$$

Hoop stress is max at inner radii

$$120 = \frac{b}{80^2} + a \quad \text{--- (ii)}$$

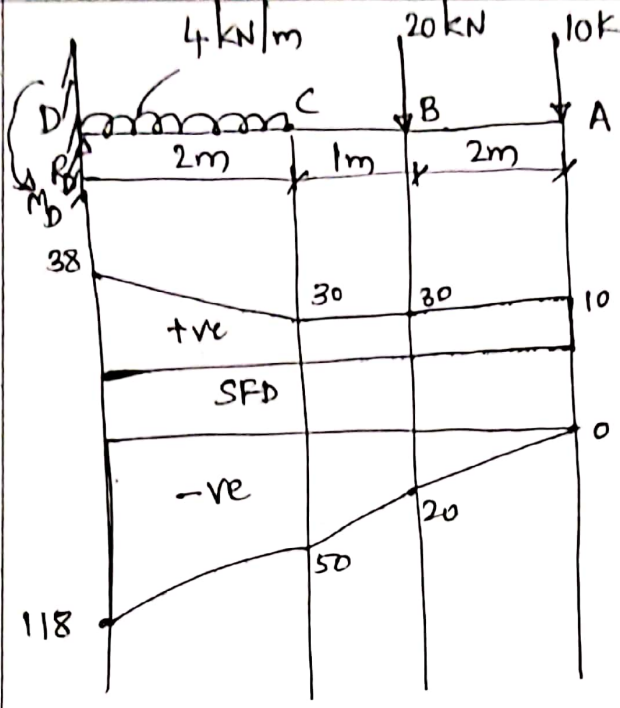
From ① & ② $b = 512000$ & $a = \frac{512000}{80^2} - 40 = 40$

Let external radius r_o since external p_r is zero we

$$\text{get } 0 = \frac{512000}{r_o^2} - 40 \Rightarrow r_o^2 = \frac{512000}{40} = 12800$$

$$r_o = 113.137 \text{ mm} \quad \text{Thickness of cylinder} = 113.137 - 80 = 33.137 \text{ mm}$$

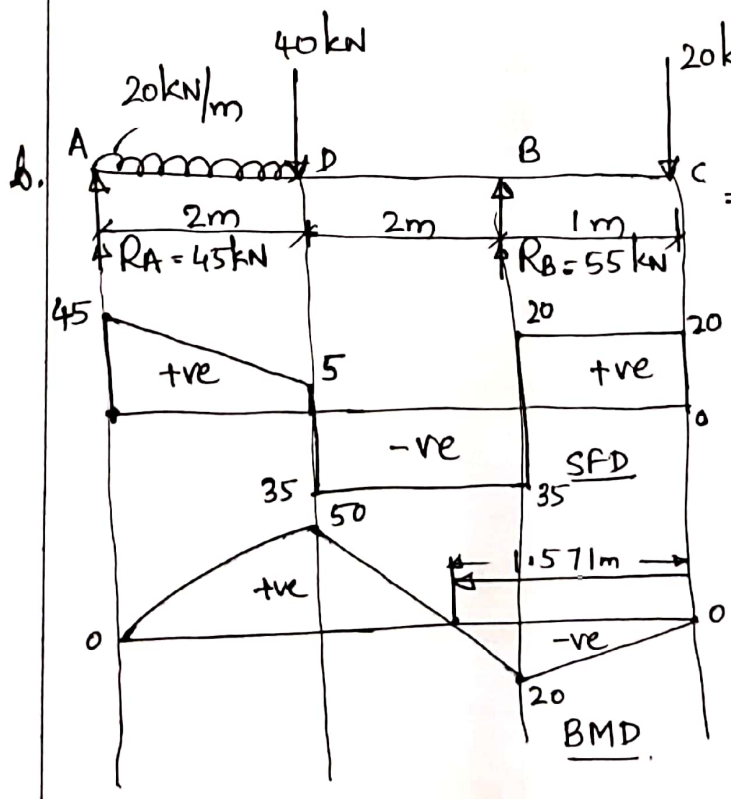
Q.5a



SFD
 At A = +10 kN
 AB = 10 kN
 At B = 10 + 20 = 30 kN
 BC = 30 kN.
 At D = +38 kN.

BMD
 At A = 0 kN-m.
 At B = -20 kN-m
 At C = -10x3 - 20x1 = -50 kN-m
 At D = -10x5 - 20x3 - 8x1 = -118 kN-m.

10



$M_B @ B$
 $= R_A \times 4 - 20 \times 2 \times 3 - 40 \times 2 + 20 \times 1 = 0$
 $R_A = 45 \text{ kN}$
 $R_B = 40 + 40 + 20 - 45 = 55 \text{ kN}$

SFD
 At C = +20 kN
 At B = +20 kN
 At D = +20 - 55 = -35 kN
 BD = -35 kN
 At A = -35 + 40 = +5 kN
 DA = 5 + 40 = 45 kN

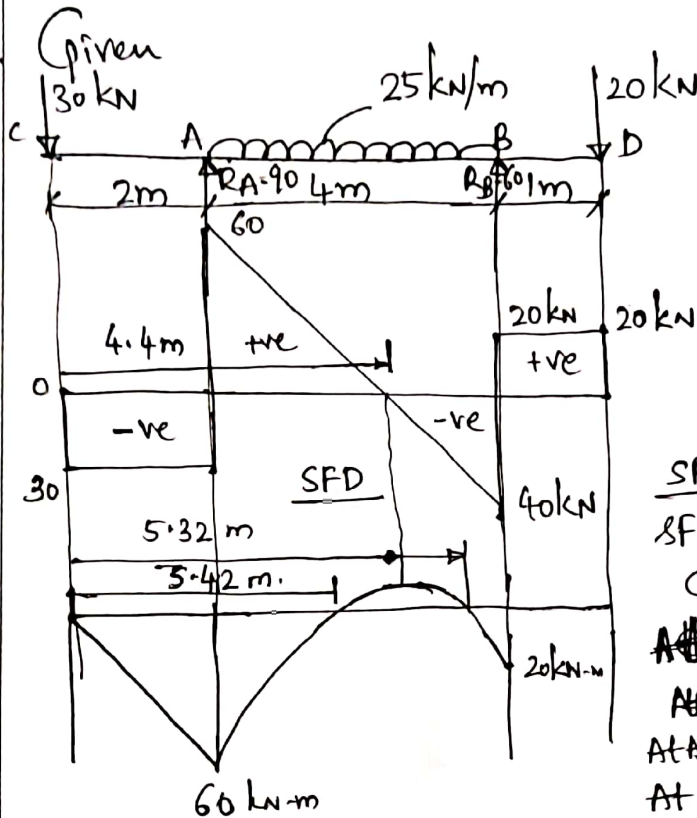
BMD
 At C = 0
 At B = -20x1 = -20 kN-m
 At D = -20x3 + 55x2 = +50 kN-m
 At A = 0

10

Point of Contraflexure
 $35x - 55 - 20x + 55(x-1) \Rightarrow 35x - 55 = 0 \Rightarrow x = 1.571 \text{ m}$

Q.6

OR



Taking moment @ B

$$R_A \times 4 - 30 \times 6 - 25 \times 4 \times 2 + 20 \times 1 = 0$$

$$R_A = 90 \text{ kN}$$

$$R_B = 30 + 20 + 25 \times 4 - 90 = 60 \text{ kN}$$

SFD

SF at C = -30 kN

CA = -30 kN

AA = -30 + 90 = +60 kN

AB = -30 + 90 - (25 \times 4) = -40 kN

At B = -40 + 60 = +20 kN

BD = +20 kN

At D = 20 - 20 = 0

Shear force is zero at

$$-30 + 90 - 25(x-2) = 0$$

$$60 + 50 - 25x = 0 \Rightarrow x = 4.4 \text{ m}$$

BMD

BM at C $M_C = 0$

A $M_A = -30 \times 2 = -60 \text{ kN-m}$

B $M_B = -30 \times 6 + 90 \times 4 - 100 \times 2 = -20 \text{ kN-m}$

D $M_D = 0$

At $x = 4.4 \text{ m}$ $M_{4.4} = -30 \times 4.4 + 90(4.4-2) - \frac{25(4.4-2)^2}{2}$

$= 5.5 \text{ kN-m}$

Point of Contraflexure points

$$-30x + 90(x-2) - \frac{25(x-2)^2}{2} = 0$$

$$-30x + 90(x-2) - 12.5(x^2 - 4x + 4) = 0$$

$$-30x + 90x - 180 - 12.5x^2 + 50x - 50 = 0$$

$$12.5x^2 - 110x + 230 = 0 \quad x_{1,2} = \frac{110 \pm \sqrt{110^2 - 4 \times 12.5 \times 230}}{2 \times 12.5}$$

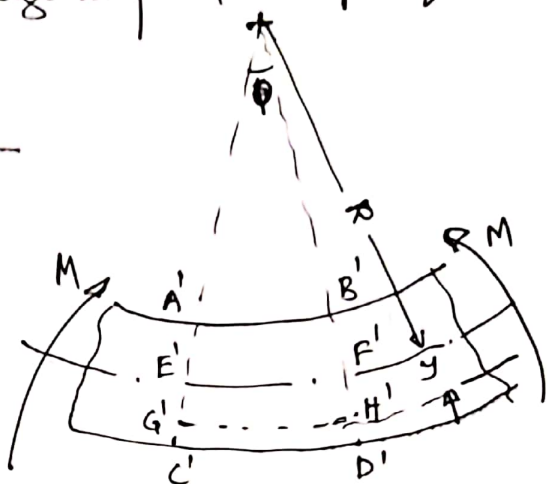
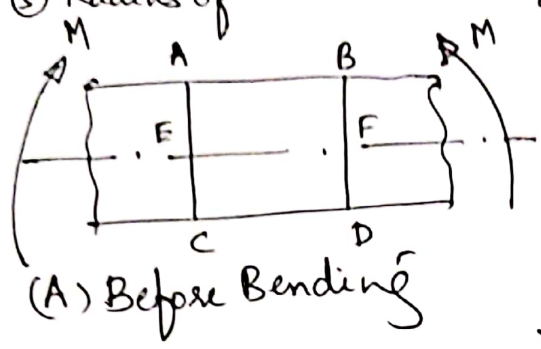
$$x = 5.38 \text{ m} \quad \& \quad x = 3.42 \text{ m}$$

20M

Q.7a Derivation of Bending Equation. $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$.

Assumptions

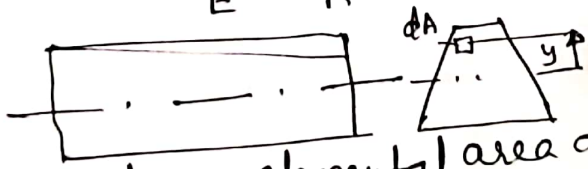
- ① Stresses are within elastic limit
- ② Plane sections remain plane even after bending.
- ③ Radius of Curvature is large compared to depth of beam



Strain in the layer GH = $\frac{G'H' - GH}{GH} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R} = \epsilon$ — (1)

If σ_b is bending stress & E then $\epsilon = \frac{\sigma_b}{E}$ — (2)

Hence $\frac{\sigma_b}{E} = \frac{y}{R}$ — (3) or $\sigma_b = \frac{E}{R} y$ $\sigma_b \propto y$. $\frac{E}{R} = \frac{\sigma_b}{y}$ — (3)



Consider an elemental area dA at distance y from NA

Stress σ on this element $\sigma = \frac{E}{R} y$

Force on element = $\sigma dA = \frac{E}{R} y dA$

Moment of this resisting force @ NA = $\frac{E}{R} y dA \cdot y = \frac{E}{R} y^2 dA$.

Total Moment of resistance of whole area = $\sum \frac{E}{R} y^2 dA$

= $\frac{E}{R} \sum y^2 dA = \frac{E}{R} I$ ($\because I = \sum y^2 dA$)

For Equilibrium Moment of resistance = Applied.

$\therefore M = \frac{E}{R} I \Rightarrow \frac{M}{I} = \frac{E}{R}$ — (4)

From (3) & (4)

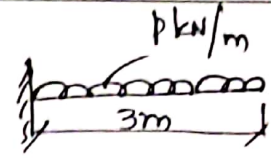
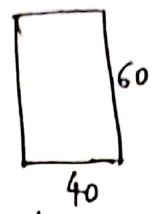
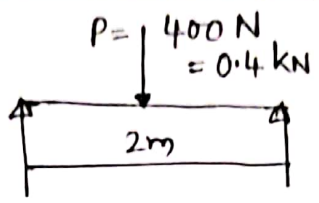
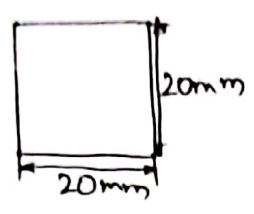
$\frac{M}{I} = \frac{E}{R} = \frac{\sigma_b}{y}$

Q.No.

Solution and Scheme

Mks

Q7b.



(i)

$$\text{Max BM} = \frac{Pl}{4} = \frac{0.4 \times 2}{4} = 0.2 \text{ kN-m}$$

Max B stress occurs at extreme fibers i.e. at the top flange. $\sigma_{\text{max}} = \frac{M_{\text{max}} \times y_{\text{max}}}{I}$

$$Z = \frac{bd^2}{6} = \frac{0.2 \times 0.2^2}{6} = 1.33 \times 10^{-3} \text{ m}^3$$

$$M = \sigma_{\text{max}} Z \Rightarrow \sigma_{\text{max}} = \frac{M}{Z} = \frac{0.2 \text{ kNm}}{1.33 \times 10^{-3}} = 150 \text{ kPa}$$

(ii) $Z = \frac{bd^2}{6} = \frac{0.4 \times 0.6^2}{6} = 0.024 \text{ m}^3$

$$M_{\text{max}} = \frac{Pl^2}{4} = \frac{P \times 3^2}{4} = 2.25 \text{ kN-m}$$

$$2.25 P = 150 \times 0.024 = 1.6 \text{ kN/m}$$

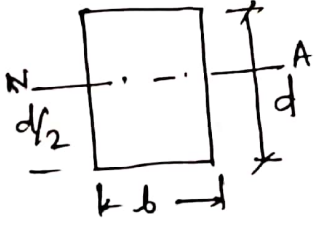
$$P = 1.6 \text{ kN/m}$$

OR

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Q.8

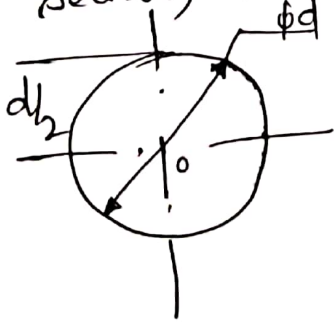
a. Section modulus of Solid rectangular section



$$I = \frac{bd^3}{12} \quad \text{Section modulus} = \frac{I}{y_{\text{max}}}$$

$$= \frac{bd^3}{12} \times \frac{1}{d/2} = \frac{bd^2}{6}$$

Section modulus of Solid circular section

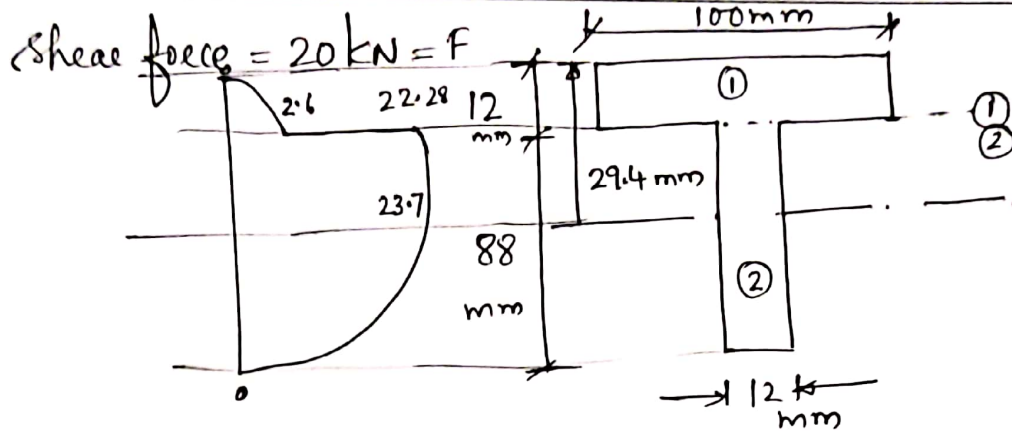


$$I = \frac{\pi d^4}{64} \quad y = \frac{d}{2} \quad Z = \frac{I}{y}$$

$$Z = \frac{\pi d^4}{64} \times \frac{2}{d} = \frac{\pi d^3}{32}$$

10

Q86.



$$\bar{y} = \frac{\sum Ay}{y} = \frac{100 \times 12 \times 6 + 12 \times 88 (44 + 12)}{100 \times 12 + 88 \times 12} = 29.40 \text{ mm}$$

$$I = \sum \frac{bd^3}{12} + A(\bar{y} - y)^2$$

$$= \frac{100 \times 12^3}{12} + 100 (29.4 - 6)^2 + \frac{12 \times 88^3}{12} + 12 \times 88 \times (56 - 29.4)^2$$

$$= 2100127.3 \text{ mm}^4$$

Shear stress at bottom of flange

$$q = \frac{F}{bI} a\bar{y} = \frac{20 \times 10^3 \times 100 \times 12 \times (29.40 - 6)}{100 \times 2100127.3} = 2.675 \frac{\text{N}}{\text{mm}}$$

Shear stress at upper layer of web

$$q = \frac{20 \times 10^3 \times 100 \times 12 \times (29.40 - 6)}{12 \times 2100127.3} = 22.288 \frac{\text{N}}{\text{mm}^2}$$

Shear stress at neutral axis

$$a\bar{y} = 12 \times 100 \times (29.40 - 6) + 12 \times (29.4 - 12) \times \left(\frac{29.4 - 12}{2} \right)$$

$$= 29902.195 \text{ mm}^3$$

$$q_{NA} = \frac{20 \times 10^3 \times 29902.195}{12 \times 2100127.3} = 23.730 \frac{\text{N}}{\text{mm}^2}$$

~~Module~~

10

Q.9 (i) Torque! A force that produces or tends to produce rotation or torsion $\frac{T}{J} = \frac{f_s}{R} = \frac{G\theta}{l}$, $P = \frac{2\pi NT}{60}$

(ii) Polar modulus! Ratio of polar MI to the radius of the shaft $J_s = \frac{J}{R}$.

(iii) Torsional Rigidity! It is defined as how much an object of specified material resists twisting force or torque. $K_t = \frac{T}{\theta} = \frac{GJ}{l}$.

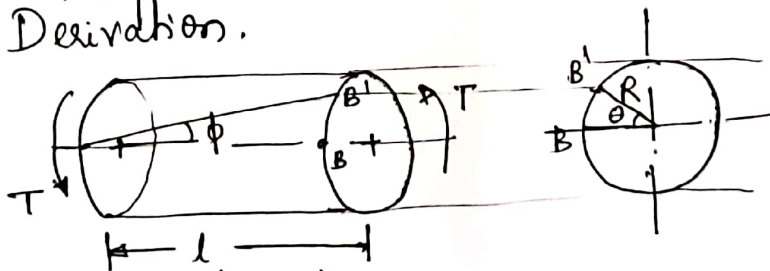
06

b. Assumptions

- ① Material is homogeneous and isotropic
- ② Stresses are within elastic limit
- ③ Crosssections, which are plane before, they remain plane even after the application of torque
- ④ Radial lines remain radial even after applying torque

04

c. Derivation.



$$R\theta = BB' = L\phi$$

If q_s be shear stress & G -modulus of rigidity

$$\phi = \frac{q_s}{G}$$

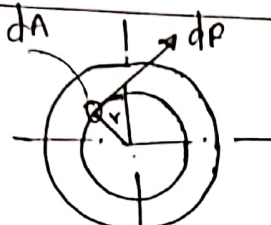
$$R\theta = L \frac{q_s}{G}$$

$$\frac{q_s}{R} = \frac{G\theta}{l} \quad \text{--- (1)}$$

Why if B is at distance r from zero at axis to the maximum value q_s at surface.

$$\frac{q}{r} = \frac{G\theta}{l} \Rightarrow \frac{q_s}{R} = \frac{G\theta}{l} \quad \therefore \frac{q_s}{R} = \frac{q}{r} \quad \text{--- (2)}$$



Q.No	Solution and Scheme	Mks
Q9c	 <p>If q is the shear stress developed in the element resisting force = $dF = q da$. resisting internal moment $dT = dF \cdot r$ $= q r da$</p> <p>From Eqn (2) $q = q_s \frac{r}{R}$ $\therefore dT = q_s \frac{r^2}{R} da$ \therefore Total resisting torsional moment $T = \int q_s \frac{r^2}{R} da = \frac{q_s}{R} \int r^2 da$ $T = \frac{q_s}{R} J_p \quad \because J_p = \int r^2 da$ $\frac{T}{J_p} = \frac{q_s}{R} \quad \text{--- (3)}$ From (1) & (3) <div style="border: 1px solid black; padding: 5px; display: inline-block;">$\frac{T}{J_p} = \frac{q_s}{R} = \frac{G\theta}{L}$</div></p>	10

Q.10

(i) Column: Vertical structural members subjected to compressive load.

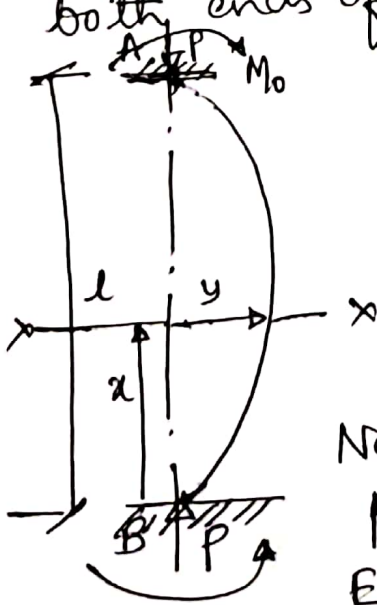
(ii) Buckling load: The load which causes bending of beam

(iii) Slenderness Ratio: The ratio of effective length of column to least moment of inertia of cross section of column.

(iv) Long column: Columns which fail only because of buckling load whose slenderness ratio is more than 100

(v) Short column: Columns which fail only because of compressive force not because of buckling load, whose slenderness ratio is less than 30.

b. Expression for Euler buckling load when both ends of the column are fixed.



Consider column of AB of length 'l' with fixed ends as shown in fig.

Let the end moments developed be M_0

Now the bending moment at any point is given by.

$$EI \frac{d^2y}{dx^2} = M_0 - Py \quad \text{--- (1)}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{EI} \quad \text{--- (2)}$$

The solution of above differential equation

$$y = C_1 \cos \sqrt{\frac{P}{EI}} x + C_2 \sin \sqrt{\frac{P}{EI}} x + \frac{M_0}{P} \quad \text{--- (3)}$$

C_1 & C_2 are evaluated by applying initial conditions

(i) From At $x=0$ $\frac{dy}{dx} = 0$ in below eqn

$$\text{We get } \frac{dy}{dx} = \cancel{C_1} \sqrt{\frac{P}{EI}} \sin \sqrt{\frac{P}{EI}} x + C_2 \cos \sqrt{\frac{P}{EI}} x \sqrt{\frac{P}{EI}}$$

$$C_2 \sqrt{\frac{P}{EI}} = 0 \implies C_2 = 0 \quad \text{--- (4)}$$

(ii) At $x=0$ $y=0$ in eqn. --- (3)

$$0 = C_1 + \frac{M_0}{P} \implies C_1 = -\frac{M_0}{P} \quad \text{--- (5)}$$

Sub (4) & (5) in eqn. 3

$$y = -\frac{M_0}{P} \cos \left(\sqrt{\frac{P}{EI}} x \right) + \frac{M_0}{P}$$

From BCs $y=0$ at $x=l$.

$$0 = -\frac{M_0}{P} \cos \sqrt{\frac{P}{EI}} \cdot l + \frac{M_0}{P}$$

$$\cos \sqrt{\frac{P}{EI}} \cdot l = 1$$

$$\therefore \sqrt{\frac{P}{EI}} \cdot l = \cos^{-1}(1) = 0, 2\pi, 4\pi, \dots$$

$$\sqrt{\frac{P}{EI}} \cdot l = 2\pi$$

$$P = \frac{4\pi^2 EI}{l^2}$$