CBCS SCHEME

USN 21CV33

Third Semester B.E. Degree Examination, Jan./Feb. 2023
Strength of Materials

Time: 3 hrs. Max. Marks: 100

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. Missing data, if any, may be suitably assumed.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Explain elastic constants. **(08 Marks)**

b. A compound bar ABC 1.5m long is made up of two parts of aluminium and steel and the cross section area of alluminium bar is twice that of steel bar. The bar is subjected to an axial load of 300kN. If the elongations of aluminium and steel parts are equal, then determine the lengths of two parts of the compound bar. Take modulus of elasticity of steel as E_s = 200 GPa and aluminium is $\frac{1}{3}$rd of E_s.

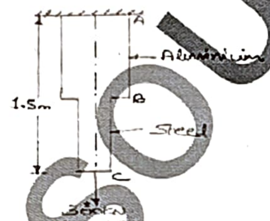

Fig.Q1(b) **(12 Marks)**

OR

2 a. Explain the relationship between Young's modulus and bulk modulus. **(04 Marks)**

b. Prove that the sum of the normal stresses on any two perpendicular planes in a general two dimensional stress system is $(\sigma_x + \sigma_y)$. **(08 Marks)**

c. A steel bar is 16m long at a temperature 24°C find the free expansion of the bar. If the temperature is raised to 62°C. Take E = 200GPa, $\alpha = 12 \times 10^{-6}$/°C. Find thermal stresses produced when : i) free expansion of bar is completely prevented ii) Bar is permitted to expand by 3.3mm only. **(08 Marks)**

Module-2

3 a. Define terms, point of inflexion and point of contraflexure. **(04 Marks)**

b. For a cantilever beam with UDL of 'ω' kN/m throughout the span 'ℓ'm, plot the shearforce and bending moment diagram and indicate the maximum values on the diagram. **(06 Marks)**

c. Draw SFD and BMD for a beam shown in Fig.Q3(c). Locate the point of contraflexure and maximum bending moment.

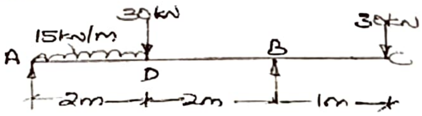

Fig.Q3(c) **(10 Marks)**

OR

4 a. Establish a relationship between SF, BM and intensity of loading. (04 Marks)

b. Draw SFD and BMD for a cantilever beam shown in Fig.Q4(b).

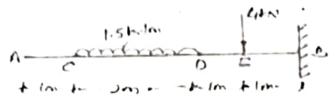

Fig.Q4(b) (06 Marks)

c. A beam AB is loaded as shown in Fig.Q4(c). Plot SFD and BMD.

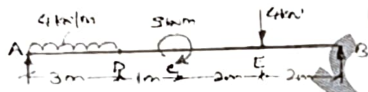

Fig.Q4(c) (10 Marks)

Module-3

5 a. List the assumptions made in simple theory of bending. (04 Marks)

b. Establish a relationship between moment and radius of curvature. (06 Marks)

c. A 1m long cantilever beam with T – section is subjected to a point load of 10kN at its free end the size of the flange is 140 × 10mm and overall depth of section is 150mm. Thickness of web is 10mm. Determine the maximum tensile stress and maximum compressive stress induced in the section and draw bending stress distribution. (10 Marks)

OR

6 a. Derive an expression to determine shear stress for a triangular section. (08 Marks)

b. The unsymmetrical I – section shown in Fig.Q6(b) is subjected to a shear force of 40kN. Draw the shear stress variation diagram across the depth.

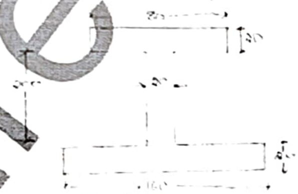

Fig.Q6(b) (12 Marks)

Module-4

7 a. Explain the concept of pure torsion and list the assumption in developing the theory of pure torsion (05 Marks)

b. The diameter of water pipeline is 750mm. It has to withstand a water head of 60m. Find the thickness of seamless pipe if principal stress is 20N/mm^2. Take unit weight of water as 9810N/m^3. (05 Marks)

c. A thick cylindrical pipe with outside diameter and internal diameter 200mm is subjected to an internal fluid pressure of 14N/mm^2. Determine the maximum hoop stress developed in the cross section. Sketch the variation of hoop stress across the thickness of pipe. What is the percentage error, if the maximum hoop stress is found from the equation of these pipes? (10 Marks)

OR

8 a. Derive a torsional equation with a neat sketch. **(08 Marks)**

b. A hollow propeller shaft of a fishing boat is to transport 3750KW @ 240rpm if the internal diameter is 0.8 times the external diameter and if the maximum shear stress developed is to be limited to $160 N/mm^2$. Determine the size of the shaft. **(12 Marks)**

Module-5

9 a. List the various assumptions to derive the expression for buckling load for long column. **(04 Marks)**

b. Derive an expression to determine buckling load for column when one end is fixed other end is hinged. **(06 Marks)**

c. A hollow cast iron column whose outside diameter is 200mm and thickness is 20mm is 4.5m long and is fixed at both ends. Calculate safe load by Rankine's formula using factor of safety 2.5. Find ratio of Euler's to Rankine's rule. Take modulus of elasticity as $1 \times 10^5 N/mm^2$; Rankine's constant $\frac{1}{1600}$ for both ends fixed case and $f_c = 550 N/mm^2$. **(10 Marks)**

OR

10 a. Derive a deferential equation for deflection using standard notation using neat sketch **(08 Marks)**

b. An overhanging beam ABC supported at A and B is loaded as shown in Fig.Q10(b). Determine the deflection at free end C and the maximum deflection between A and B. Take $E = 200 kN/mm^2$, $I = 45 \times 10^6 mm^4$.

Fig.Q10(b) **(12 Marks)**

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KLS VDIT, HALIYAL.

DEPARTMENT OF CIVIL ENGINEERING.

III SEMESTER BE DEGREE EXAMINATION JAN/FEB 2023

STRENGTH OF MATERIALS (21CV33).

QUESTION PAPER SOLUTION.

FACULTY NAME: Prof. HARSHAVARDHAN.V.S ACADEMIC YEAR: 2022-23.

Q1.a. MODULUS OF ELASTICITY (E).

is defined as the ratio of LINEAR STRESS to LINEAR STRAIN within elastic limit

$$E = \frac{\text{LINEAR STRESS } (\sigma)}{\text{LINEAR STRAIN } (\epsilon)} = \frac{\sigma}{\epsilon} \quad N/mm^2.$$

MODULUS OF RIGIDITY (G).

is defined as the ratio of shearing stress to SHEARING STRAIN within the elastic limit.

$$G = \frac{\text{SHEAR STRESS } (\tau)}{\text{SHEAR STRAIN } (\phi)} = \frac{\tau}{\phi}.$$

BULK MODULUS (K)

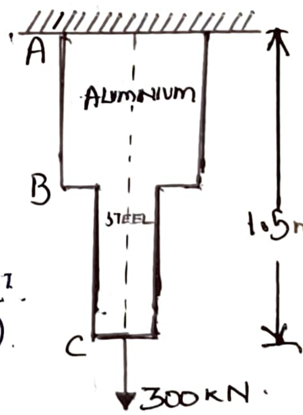
When a body is subjected to identical stresses 'σ' in 3 mutually $\perp^r$ directions it undergoes changes in 3 directions without undergoing distortion of shape. Then the bulk modulus is defined as the ratio of direct stress (σ) to the volumetric strain (e_v).

$$K = \frac{\text{DIRECT STRESS } (\sigma)}{\text{VOLUMETRIC STRAIN } (e_v)} = \frac{\sigma}{e_v}.$$

Q1.b. GIVEN, $L_{AL} + L_{ST} = 1.5m$ $\quad A_{AL} = 2A_{ST}$.

$P = 300kN$. $\Delta L_{AL} = \Delta L_{ST}$ $\quad E_{ST} = 200GPa$ $\quad E_{AL} = \frac{200}{3} GPa$.

To FIND: $L_{AL} = ?$ $\quad L_{ST} = ?$



$\therefore \Delta_{AL} = \Delta_{ST}$:

$$\Rightarrow \frac{P_{AL} L_{AL}}{A_{AL} E_{AL}} = \frac{P_{ST} L_{ST}}{A_{ST} E_{ST}} \Rightarrow \frac{(300kN) L_{AL}}{(2A_{ST})(\frac{200}{3})} = \frac{(300kN) L_{ST}}{A_{ST}(200)}$$

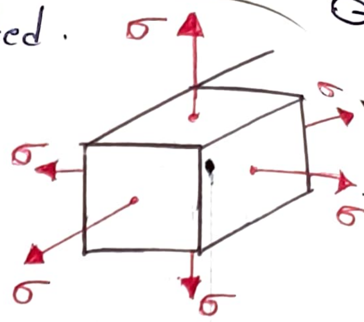
$\Rightarrow 3L_{AL} = L_{ST}$.

We have $L_{AL} + L_{ST} = 1.5$ $\Rightarrow L_{AL} + (3L_{AL}) = 1.5m$ $\Rightarrow 4L_{AL} = 1.5m$

$\therefore L_{AL} = 1.5/4 = 0.375m$

$L_{ST} = 1.5 - 0.375 = 1.125m$.

Q2.a. Consider a cubic element subjected to stress 'σ' in 3 mutually $\perp^r$ directions.



Now, the stress 'σ' in X-dir causes

i) tensile strain $e = \sigma/E$ in X-dir.

ii) Compressive strain, $= \mu e = \mu\sigma/E$ in Y-dir

iii) Compressive strain $= \mu e = \mu\sigma/E$ in Z-dir.

$$\therefore e_x = \frac{\sigma}{E} - \frac{\mu\sigma}{E} - \frac{\mu\sigma}{E} = \frac{\sigma}{E}(1-2\mu).$$

Similarly $e_y = e_z = \frac{\sigma}{E}(1-2\mu)$.

Volumetric strain, $e_v = e_x + e_y + e_z$.

$$\Rightarrow e_v = 3 \times \frac{\sigma}{E}(1-2\mu). \quad \Rightarrow \frac{\cancel{\sigma}}{K} = 3 \times \frac{\cancel{\sigma}}{E}(1-2\mu).$$

$$\Rightarrow E = 3K(1-2\mu).$$

Q2.b. We have, for a plane with inclination 'θ'

$$\sigma_{n_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau \sin 2\theta \longrightarrow ①$$

Where σ_x & σ_y are stresses acting in X & Y dir respectively

Consider another plane at an inclination '$\theta + 90$'

$$\sigma_{n_2} = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2(\theta+90) + \tau \sin 2(\theta+90).$$

$$\Rightarrow \sigma_{n_2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}\cos(2\theta+180) + \tau \sin(2\theta+180^\circ).$$

$$\Rightarrow \sigma_{n_2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2}(+\cos 2\theta) + \tau(-\sin 2\theta).$$

$$\Rightarrow \sigma_{n_2} = \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta - \tau \sin 2\theta \longrightarrow ②$$

① + ②

$$\sigma_{n_1} + \sigma_{n_2} = 2 \times \frac{\sigma_x + \sigma_y}{2} + 0 + 0.$$

$$\therefore \boxed{\sigma_{n_1} + \sigma_{n_2} = \sigma_x + \sigma_y}$$

Q2.c. GIVEN, $L = 16m$ $t_1 = 24°C$ $t_2 = 62°C$ $E = 200 GPa$.
$\alpha = 12\times10^{-6}/°C$ $\delta = 3.3mm$.
TO FIND: $\Delta = ?$ i) σ_t = ? when free expansion is prevented
ii) σ_t = ? when partially expansion is allowed

Free expansion of Bar $= \alpha t L = 12\times10^{-6}\times(62-24)\times16000 mm$
$= 7.296 mm$.

i) when free expansion is prevented.

Thermal stress, $\sigma_t = E\alpha t$
$= 200\times10^3 N/mm^2 \times 12\times10^{-6}\times(62-24)$
$= 91.2 N/mm^2$

ii) when partial expansion of 3.3mm is allowed.

Thermal stress, $\sigma_t = \alpha L t - \delta$.
$= 12\times10^{-6}\times16000mm\times(62-24) - 3.3$.
$= 3.996 N/mm^2$.

Q3.a. POINT OF INFLEXION.

It is a point on elastic curve (deflected shape of beam) where beam changes its curve from concave to convex or vice versa.

POINT OF CONTRAFLEXURE.

It is a point where the bending moment changes its sign. Therefore at such points bending moment is zero.

Q3.b. Consider a beam of length 'L' as shown in figure subjected to UDL 'w' per meter.

$\Sigma F_V = 0 \Rightarrow R_A - w(L) = 0 \therefore R_A = wL$.

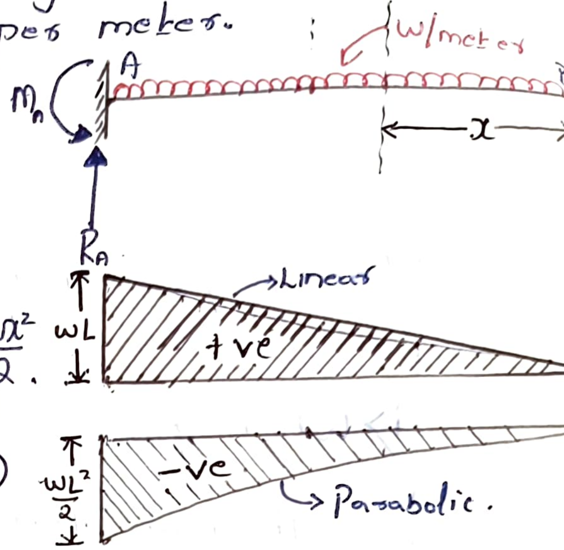
$\Sigma M_B = 0 \Rightarrow -M_A + R_A(L) - (wL)(L/2) = 0$.

$\Rightarrow wL(L) - wL^2/2 = M_A \therefore M_A = wL^2/2$.

At s/c x $V_x = wx$ & $M_x = -wx(x/2) = -\frac{wx^2}{2}$.

At $x=0$ $V_x = 0$ and $M_x = 0$ (free end)

At $x = L$ $V_x = wL$ & $M_x = -\frac{wL^2}{2}$ (FIXED END)



Q3.c. STEP 1: SUPPORT REACTION

$\Sigma F_v = 0 \Rightarrow R_A - (15\times2) - 30 + R_B - 30 = 0$.

$\Rightarrow R_A - 90 + R_B = 0 \quad \therefore R_A + R_B = 90 \longrightarrow (1)$

$\Sigma M_A = 0 \Rightarrow R_A \times 0 - (15\times2)\times1 - 30\times2 + R_B\times4 - 30\times5 = 0$.

$\Rightarrow -30 - 60 + 4R_B - 150 = 0 \quad \Rightarrow 4R_B = 240\,kN$.

$\therefore R_B = 60\,kN \quad \therefore R_A = 90 - 60 = 30\,kN$.

STEP 2: SHEAR FORCE.

$F_A = 0$, $F_A = R_A = 30\,kN$.

$F_D = R_A - 15\times2 = 30 - 30 = 0$.

$F_D = 0 - 30 = -30\,kN$.

$F_B = R_A - 15\times2 - 30 = -30\,kN$ $\quad F_B = -30\,kN + 60\,kN = +30\,kN$.

$F_C = +30\,kN$ $\quad F_C = 0$

STEP 3: BENDING MOMENT.

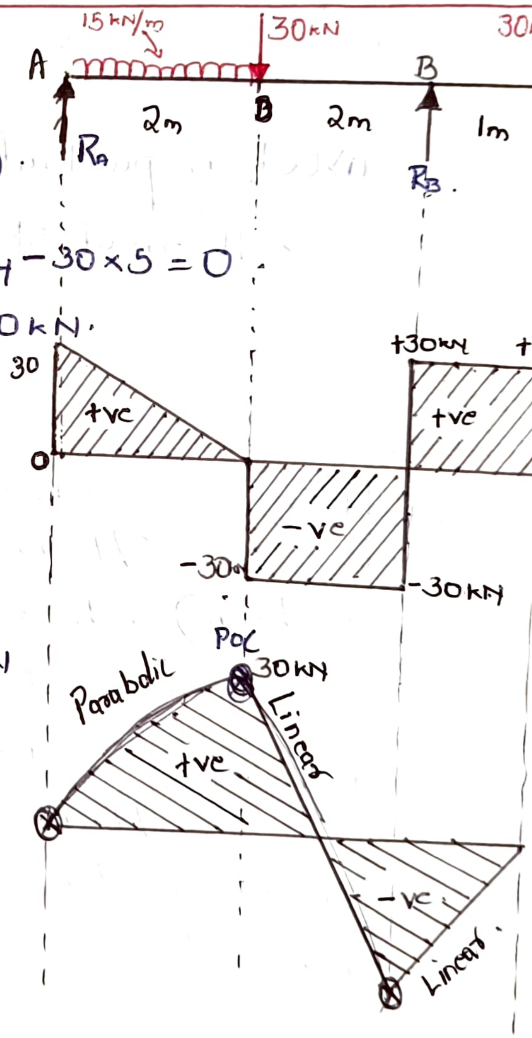
$M_A = 0$

$M_D = R_A\times2 - (15\times2)\times1 = 30\times2 - 30 = 30\,kNm$.

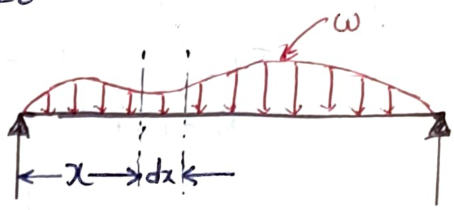
$M_B = -30\times1 = -30\,kNm$.

$M_C = 0$.

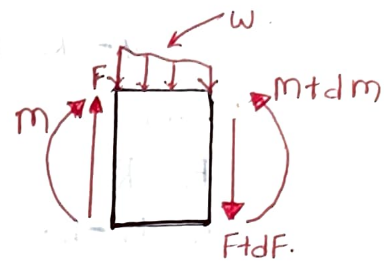
POINT OF CONTRAFLEXURE: POINT D



Q4.a. Consider a beam AB subjected to a general loading as shown in figure. Consider an element of length dx at distance 'x' from left support & draw its FBD.



For such a small element, intensity of loading may be taken as constant.



Considering the equilibrium of forces we get

$\Sigma F_v = 0.$

$\Rightarrow (F) - (F + dF) - w(dx) = 0 \quad \Rightarrow \cancel{F} - \cancel{F} - dF - w(dx) = 0.$

$\Rightarrow -dF = w(dx) \qquad \therefore \boxed{\dfrac{dF}{dx} = -w}$

Now taking the moment equilibrium of element. about the right face, we get.

$M + F(dx) - [w(dx)]\dfrac{dx}{2} - (m + dm) + (F + dF) \times 0 = 0.$

$\Rightarrow \cancel{M} + F(dx) - \underbrace{\dfrac{w(dx)^2}{2}}_{\to 0} - \cancel{M} - dM = 0.$ (Neglecting higher order term).

$\Rightarrow F(dx) - dM = 0 \qquad \Rightarrow F(dx) = dM \qquad \therefore \boxed{\dfrac{dM}{dx} = F}$

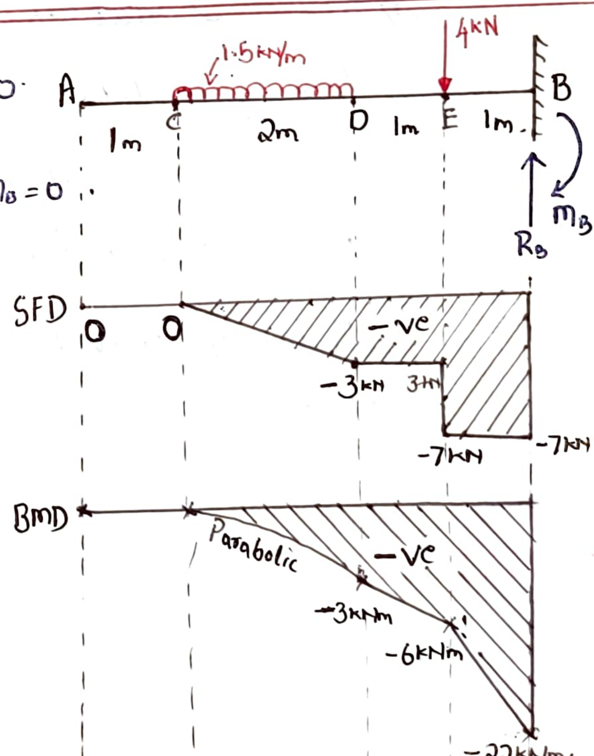
Q4.b. STEP 1: REACTIONS

$\Sigma F_v = 0 \Rightarrow -1.5 \times 2 - 4 + R_B = 0 \quad \Rightarrow -3 - 4 + R_B = 0$

$\Rightarrow R_B = +7\,kN.$

$\Sigma M_A = 0 \Rightarrow (1.5 \times 2) \times 2 + 4 \times 4 - M_B = 0 \Rightarrow 22 - M_B = 0$

$\Rightarrow M_B = 22\,kNm.$



STEP 2: SHEAR FORCE.

$F_A = 0, \quad F_C = 0, \quad F_D = -1.5 \times 2 = -3\,kN.$

$F_E = -1.5 \times 2 = -3\,kN \quad F_E = -3 - 4 = -7\,kN.$

$F_B = -7 + 7 = 0.$

STEP 3: BENDING MOMENT.

$M_A = 0, \quad M_C = 0$

$M_D = -1.5 \times 2 \times 1 = -3\,kNm.$

$M_E = -1.5 \times 2 \times 2 = -6\,kNm.$

Q4.c. STEP 1: SUPPORT REACTION.

$\Sigma F_v = 0 \Rightarrow R_A - 4\times3 - 4 + R_B = 0.$

$\Rightarrow R_A - 16 + R_B = 0 \quad \Rightarrow R_A + R_B = 16$

$\Sigma M_A = 0 \Rightarrow R_A\times0 + 4\times3\times1.5 + 9 + 4\times6 - R_B\times7 = 0$

$\Rightarrow 51 - 7R_B = 0 \quad \therefore R_B = 7.3\,kN \quad \therefore R_A = 8.7\,kN$

STEP 2: SHEAR FORCE.

$F_A = R_A = 8.7\,kN.$

$F_D = R_A - 4\times3 = 8.7 - 12 = -3.3\,kN.$

$F_C = -3.3\,kN.$

$F_E = -3.3\,kN, \quad F_E = -3.3 - 4 = -7.3\,kN.$

$F_B = -7.3\,kN \quad F_B = -7.3 + 7.3 = 0.$

STEP 3: BENDING ~~MOMENT~~ MOMENT.

$M_A = 0, \; M_B = 0.$

$M_D = R_A\times3 - 4\times3\times1.5 = 8.1\,kNm.$

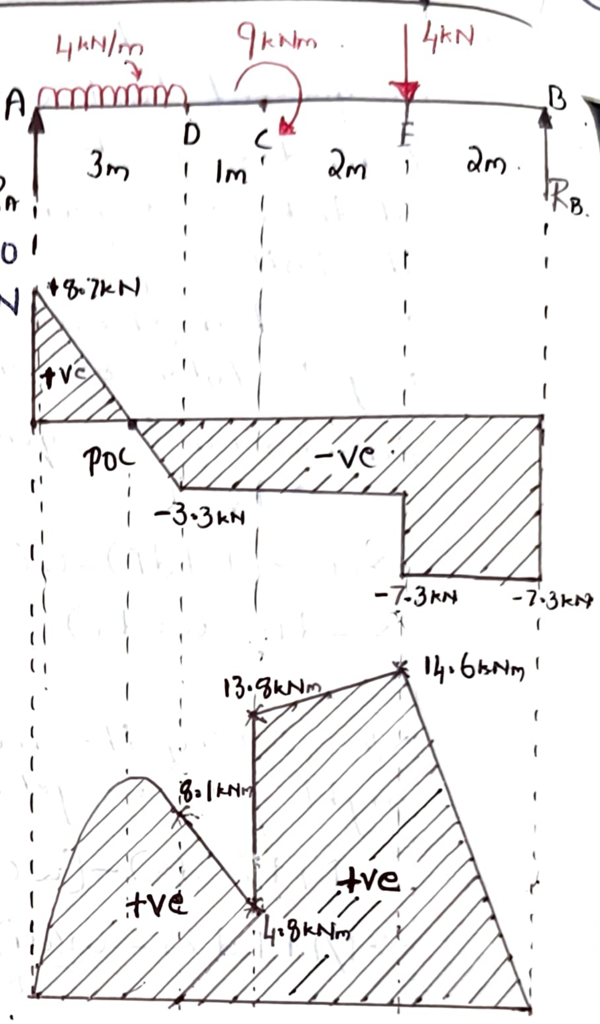
$M_C = R_A\times4 - 4\times3\times2.5 = 4.8\,kNm.$

$M_C = 4.8 + 9 = 13.8\,kNm.$

$M_E = R_B\times2 = 14.6\,kNm.$

SFD

BMD



Q5.a. The assumptions made in simple theory of bending are

1) The material is homogeneous and isotropic.

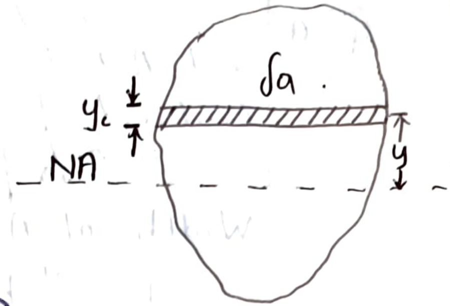
2) Young's modulus is same in tension & compression.

3) Stresses are within elastic limit.

4) Plane section remains plane even after bending.

5) Radius of curvature is large compared to the depth of beam.

Q5.b. Consider an element of area 'δa' at a distance 'y' from NA.



Stress on the element is given by

$$\sigma = \frac{E}{R} y.$$

Force on element, $F = \sigma(\delta a) = \frac{E}{R} y (\delta a)$.

Moment of Resistance of this c/s area $= F \times y = \frac{E}{R} y^2 (\delta a)$.

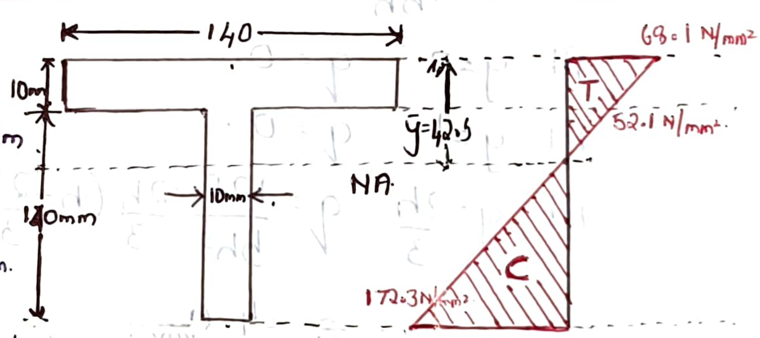
Moment of Resistance for whole s/c, $M' = \Sigma \frac{E}{R} y^2 (\delta a) = \frac{E}{R} \Sigma y^2 (\delta a)$.

From definition, $I = (\delta a) \times y \times y = (\delta a) y^2$.

$\therefore M' = \frac{E}{R}(I)$.

For equilibrium, MOR m' = applied moment M. $\therefore M = \frac{E}{R} I$.

Q5.c. For a cantilever beam subjected to point load at free end, $M_{max} = P \times L = 10 \times 1 = 10\,kNm$



$A = 140 \times 10 + 140 \times 10 = 2800\,mm^2$.

$$\bar{y} = \frac{(140 \times 10) \times 5 + (140 \times 10)(10 + 70)}{2800} = 42.5\,mm.$$

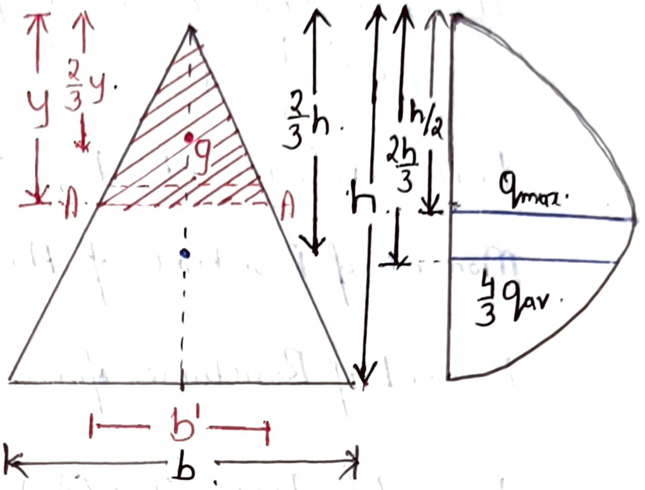
$$I = \frac{140 \times 10^3}{12} + (140 \times 10)(42.5 - 5)^2 + \frac{10 \times 140^3}{12} + (140 \times 10)(80 - 42.5)^2 = 6.24 \times 10^6\,mm^4.$$

$$\sigma_t = \frac{M}{I} y_t = \frac{10 \times 10^6\,Nmm}{6.24 \times 10^6\,mm^4} \times 42.5 = 68.1\,N/mm^2.$$

$$\sigma_{junction} = \frac{M}{I} y = \frac{10 \times 10^6\,Nmm}{6.24 \times 10^6\,mm^4} \times (42.5 - 10) = 52.1\,N/mm^2.$$

$$\sigma_b = \frac{M}{I} y_b = \frac{10 \times 10^6}{6.24 \times 10^6} \times (150 - 42.5) = 172.3\,N/mm^2.$$

Q6.a. Consider the isosceles Δ^{lar} s/c as shown in figure. Let A-A be the s/c at a distance 'y' from top fibre. Centroid of the area above A-A is at $2y/3$ & centroid of c/s is at $2h/3$ from top fibre.



Width at A-A is.

$$b' = \frac{y}{h} b.$$

Moment of Inertia of the section about the centroid is

$$I = \frac{1}{36} bh^3.$$

Shear stress at A-A is

$$q = \frac{F}{b'I} a\bar{y}$$

$$\Rightarrow q = \frac{F}{b' \times \frac{1}{36} bh^3} \times \frac{1}{2} b'y \left[\frac{2h}{3} - \frac{2y}{3}\right] = \frac{36}{bh^3} \cdot \frac{1}{3} y(h-y).$$

$$\therefore q = \frac{12F}{bh^3} y(h-y).$$

At $y=0$, $q=0$.

At $y=h$ $q=0$.

At $y=\frac{2h}{3}$ $q = \frac{12F}{bh^3} \cdot \frac{2h}{3}\left(h - \frac{2h}{3}\right) = \frac{8F}{bh^2} \cdot \frac{1}{3} h = \frac{4}{3} \cdot \frac{F}{\frac{1}{2}bh}.$

$$q = \frac{4}{3} q_{av}.$$

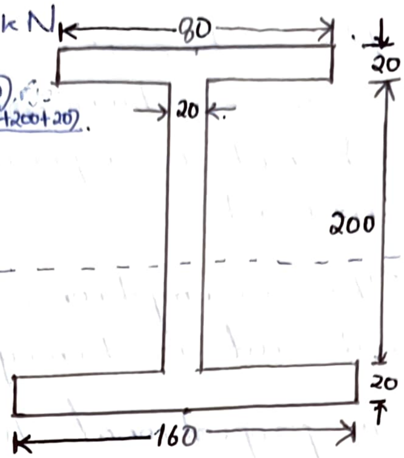
For q_{max}, $\frac{dq}{dy} = 0$.

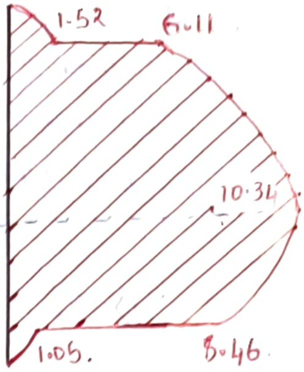
$$\Rightarrow \frac{12F}{bh^3} \cdot (h - 2y) = 0. \quad \Rightarrow h - 2y = 0 \quad \therefore y = \frac{h}{2}.$$

$$\therefore q_{max} = \frac{12F}{bh^3} \frac{h}{2}\left(h - \frac{h}{2}\right) = \frac{6F}{bh^2} \cdot \frac{h}{2} = \frac{3F}{bh} = \frac{1.5F}{\frac{1}{2}bh}.$$

$$\therefore q_{max} = 1.5\, q_{av}.$$

Q6.b. Given $F = 40\,kN$





$$\bar{y} = \frac{(80\times20)\times10 + (200\times20)\times(100+20) + (160\times20)\times(10+200+20)}{(80\times20)+(200\times20)+(160\times20)}.$$

$\bar{y} = 140\,mm$ from top.

$I = I_1 + I_2 + I_3$.

$$I_1 = \frac{80\times20^3}{12} + (80\times20)(140-10)^2$$
$$= 27.1\times10^6\,mm^4$$

$$I_2 = \frac{20\times200^3}{12} + (20\times200)(140-120)^2 = 14.93\times10^6\,mm^4.$$

$$I_3 = \frac{160\times20^3}{12} + (160\times20)(140-230)^2 = 26.03\times10^6\,mm^4.$$

$$\therefore I = 27.1\times10^6 + 14.93\times10^6 + 26.03\times10^6$$
$$= 68.06\times10^6\,mm^4$$

$$q = \frac{F}{bI}(a\bar{y}).$$

At top fibre, $q = 0$

At junction of web & flange, $q = \dfrac{40\times10^3}{80\times(68.06\times10^6)}\times(80\times20)(140-10)$.
$= 1.52\,N/mm^2$.

$$q = \frac{40\times10^3}{20\times68.06\times10^6}(80\times20)(140-10) = 6.11\,N/mm^2.$$

At NA, $q = \dfrac{40\times10^3}{20\times(68.06\times10^6)}[(80\times20)(140-10)+(120\times20)(120/2)] = 10.34\,N/mm^2$.

At junction of web & flange. ($\bar{y}$ from bottom $= 240-140 = 100\,mm$).

$$q = \frac{40\times10^3}{20\times(68.06\times10^6)}(160\times20)(100-10) = 8.46\,N/mm^2.$$

$$q = \frac{40\times10^3}{160\times(68.06\times10^6)}(160\times20)(100-10) = 1.05\,N/mm^2.$$

Q7.a. A member is said to be in pure torsion when its cross-section are subjected to only torsional moments & not accompanied by axial forces or bending moment.

The assumptions of pure torsion theory are.

1) The material is homogenous & isotropic.

2) Stress is proportional to strain i.e all stresses are within the elastic limit.

3) Radial lines remain radial or straight after torsion.

4) Torsion is uniform along the shaft.

Q7.b. Given, $d = 750mm$ $h = 60m$ $f_1 = 20 N/mm^2$.

$\omega = 9810 \ N/m^3$.

To find, $t = ?$

Pressure of water $= wh = 9810 N/m^3 \times 60m$.

$= 588600 \ N/m^2$.

$= 588600 \times 10^{-6} \ N/mm^2$.

$= 0.5886 \ N/mm^2$.

Maximum stress, $f_1 = \frac{pd}{2t}$.

$\Rightarrow t = \frac{pd}{2f_1} = \frac{0.5886 N/mm^2 \times 750mm}{2 \times 20 N/mm^2} = 11.03mm$.

provide 12mm thick pipes.

Q7.c. Given, $d_o = 300mm$ $d_i = 200mm$ $p = 14\ N/mm^2$

To Find, f_x distribution, $f_{x\,max.}$, % error.

$r_o = 300/2 = 150mm$ $r_i = 200/2 = 100mm$ $\therefore t = 150-100 = 50mm.$

At inner face, $x = r_i = 100mm$ $p_x = 14\ N/mm^2.$

$p_x = \frac{b}{x^2} - a$ $\Rightarrow 14 = \frac{b}{100^2} - a \rightarrow (1).$

At outer face, $x = r_o = 150mm$ $p_x = 0$

$p_x = \frac{b}{x^2} - a$ $\Rightarrow 0 = \frac{b}{150^2} - a \rightarrow (2).$

$(1) - (2) = 14 - 0 = \left(\frac{b}{100^2} - \frac{b}{150^2}\right) - a - (-a)$. $\Rightarrow 14 = b\left(\frac{1}{100^2} - \frac{1}{150^2}\right) - 0.$

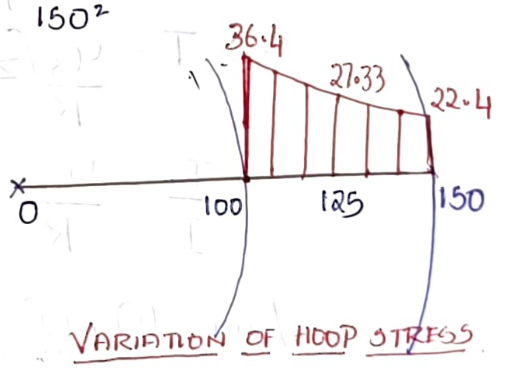
$\Rightarrow 14 = b\,(5.55\times10^{-5})$. $\therefore b = 252000$

from eqn (2) $0 = \frac{252000}{150^2} - a$ $\Rightarrow a = \frac{252000}{150^2} = 11.2.$

At $x = 100mm$ (inner face), $f_x = \frac{b}{x^2} + a = \frac{252000}{100^2} + 11.2 = 36.4\ N/mm^2.$

At $x = 125mm$ (centre), $f_x = \frac{b}{x^2} + a = \frac{252000}{125^2} + 11.2 = 27.33\ N/mm^2$

At $x = 150mm$ (outer face), $f_x = \frac{b}{x^2} + a = \frac{252000}{150^2} + 11.2 = 22.4\ N/mm^2.$



VARIATION OF HOOP STRESS

Hoop stress calculated assuming s/c as thin cylinder

$f = \frac{pd}{2t} = \frac{14\times200}{2\times50} = 28\ N/mm^2.$

% error in estimating max hoop stress

$= \frac{36.4 - 28}{36.4}\times100 = 23.1\%$

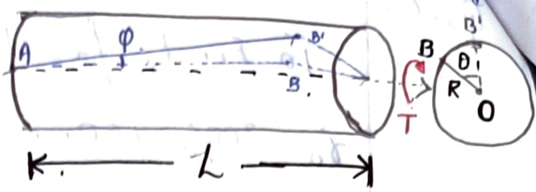
Q 8.a. Consider a shaft of length 'L' radius 'R' fixed at one end & subjected to torque 'T' at other end.

Let O be centre & 'B' be a point on surface.

Let AB be the line on shaft ||el to its axis.

Due to torque 'T' applied let B move to B'

If ϕ is shear strain & θ is angle of twist then.



$BB' = R\theta$.

$\tan\phi = \frac{BB'}{AB} \Rightarrow \phi = \frac{BB'}{AB}$ (for small angles $\phi \approx \tan\phi$) $\Rightarrow BB' = (AB)\phi$.

$\therefore AB(\phi) = R\theta \Rightarrow L\phi = R\theta$.

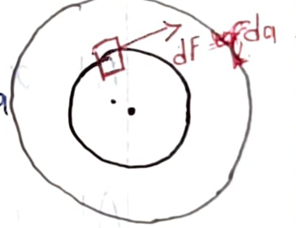
If τ is the shear stress & G is the modulus of rigidity then.

$$R\theta = L\left(\frac{\tau_s}{G}\right) \quad \left[\because \phi = \frac{\tau_s}{G}\right] \quad \therefore \frac{\tau_s}{R} = \frac{G\theta}{L} \rightarrow (1)$$

If any point 'B' at distance 'r' from centre is considered then $\frac{\tau}{r} = \frac{G\theta}{L}$. $\therefore \frac{\tau}{r} = \frac{\tau_s}{R} \rightarrow (2)$.

Now consider torsional resistance developed by an elemental area 'δa' at a distance 'r' from the centre

$\therefore$ Resisting Torsional moment.



$$dT = dF \times r = (\tau\, da)r = \frac{\tau_s}{R} r\, da\, r = \frac{\tau_s r^2 da}{R}$$

Total Resisting moment, $T = \Sigma\left[\frac{\tau_s r^2 da}{R}\right]$.

$$\Rightarrow T = \frac{\tau_s}{R}(\Sigma r^2 da) \Rightarrow T = \frac{\tau_s}{R}(J)$$

$\left[\because J = \Sigma r^2 da\right.$ Polar moment of Inertia$\left.\right]$

$$\Rightarrow \frac{T}{J} = \frac{\tau_s}{R}.$$

From (1) & (2) $\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}}$

Q8.b. Given, $P = 3750\,kW$ $N = 240$ rpm $d_i = 0.8 d_o$.
$\tau_s = 160\,N/mm^2$

To find, $d_i = ?$ $d_o = ?$

We have $P = \frac{2\pi N T}{60}$. $\Rightarrow 3750 \times 10^6 = \frac{2\pi \times 240 \times T}{60}$.

$\therefore T = 149.207 \times 10^6\ N\text{-}mm$.

$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}[d_o^4 - (0.8 d_o)^4] = \frac{\pi}{32} d_o^4 [1 - 0.8^4] = 0.0579 d_o^4$.

From Torsion equation, $\frac{T}{J} = \frac{\tau_s}{R}$.

$\Rightarrow \frac{149.207 \times 10^6}{0.0579 d_o^4} = \frac{160}{d_o/2}$. $\Rightarrow \frac{149.207 \times 10^6}{0.0579 \times 160 \times 2} = \frac{d_o^4}{d_o}$.

$\Rightarrow d_o^3 = 8044718.3$ $\therefore d_o = 200.37\,mm$

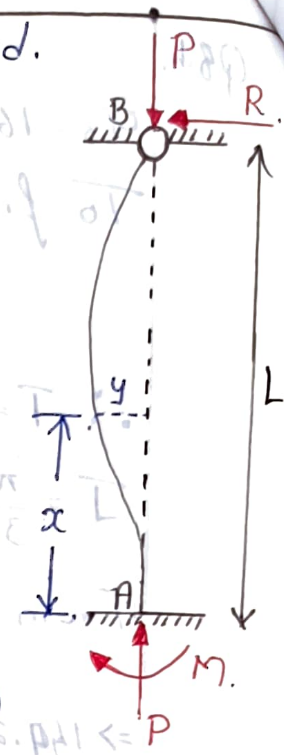
$d_i = 0.8 d = 0.8 \times 200.37\,mm = 160.3\,mm$.

Q9.a. The assumptions made to derive expression for buckling load for long column are

1) Initially, column is perfectly straight & load is axial.
2) The c/s of the column is uniform throughout its length.
3) The column will fail by buckling alone.
4) Self weigth of column is negligible.

Q9.b. Let A be the fixed end & B be hinged. Let M be the BM induced at A & this involves presence of force 'R' at B to maintain equilibrium. Hence we have

$$M_x = EI\frac{d^2y}{dx^2} = -Py + R(L-x).$$



$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{Py}{EI} + \frac{R}{EI}(L-x) \Rightarrow \frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{R}{EI}(L-x).$$

The solution of above differential eqn. is

$$y = C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{R}{P}(L-x). \longrightarrow (1)$$

Diff w.r.t x we get.

$$\frac{dy}{dx} = -C_1\sqrt{\frac{P}{EI}}\sin\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sqrt{\frac{P}{EI}}\cos\left(x\sqrt{\frac{P}{EI}}\right) - \frac{R}{P} \longrightarrow (2).$$

At $x=0$, $y=0$ $\therefore$ (1) becomes.

$$0 = C_1(1) + C_2(0) + (R/P)(L) \quad \therefore C_1 = -RL/P.$$

At $x=0$, $dy/dx = 0$ $\therefore$ (2) becomes.

$$0 = -C_1(\sqrt{P/EI})(0) + C_2(\sqrt{P/EI})(1) - R/P \Rightarrow C_2 = \frac{R}{P}\sqrt{\frac{EI}{P}}.$$

At $x = L$, $y = 0$ (1) becomes.

$$0 = (-RL/P)\cos\left(L\sqrt{\frac{P}{EI}}\right) + \frac{R}{P}\sqrt{\frac{EI}{P}}\sin\left(L\sqrt{\frac{P}{EI}}\right) + \frac{R}{P}(0)$$

$$\Rightarrow \frac{\cancel{R}L}{\cancel{P}}\cos\left(L\sqrt{\frac{P}{EI}}\right) = \frac{\cancel{R}}{\cancel{P}}\sqrt{\frac{EI}{P}}\sin\left(L\sqrt{\frac{P}{EI}}\right) \Rightarrow L\sqrt{\frac{P}{EI}} = \tan\left(L\sqrt{\frac{P}{EI}}\right).$$

Solution for above eqn. is

$$L\sqrt{\frac{P}{EI}} = 4.49\,\text{rad} \Rightarrow L^2\frac{P}{EI} = 20 \simeq 2\pi^2.$$ (Squaring both sides)

$$\boxed{P_E = \frac{2\pi^2 EI}{L^2}}$$

R 9.c.

GIVEN, $d_o = 200mm$ $t = 20mm$ $L = 4.5m = 4500mm$.

$F = 2.5$ $E = 1\times10^5 \ N/mm^2$. $\alpha = 1/1600$ BOTH ENDS FIXED $f_c = 550 N/mm^2$

TO FIND: $P_{cr(Rankine)} = ?$ $P_{cr(Euler)}/P_{cr(Rankine)} = ?$

$d_i = d_o - 2t = 200 - 2\times20 = 160mm$, For fixed end, $L_e = L/2 = 2250mm$

$$\text{Area, } A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(200^2 - 160^2) = 11309.734 \ mm^2.$$

$$\text{Moment of Inertia, } I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(200^4 - 160^4) = 46.36\times10^6 \ mm^4.$$

$$\text{Radius of Gyration, } k = \sqrt{\frac{I}{A}} = \sqrt{\frac{46.36\times10^6}{11309.734}} = 64.03mm.$$

$$\text{Rankine's Critical load, } P_R = \frac{f_c A}{1+\alpha(L_e/k)^2} = \frac{550\times11309.734}{1+(1/1600)(2250/64.03)^2}$$

$$= 3510.91 \ kN.$$

$$\text{Safe Load using Rankine's formula} = \frac{P_R}{FOS} = \frac{3510.91}{2.5}$$

$$= 1404.36 \ kN.$$

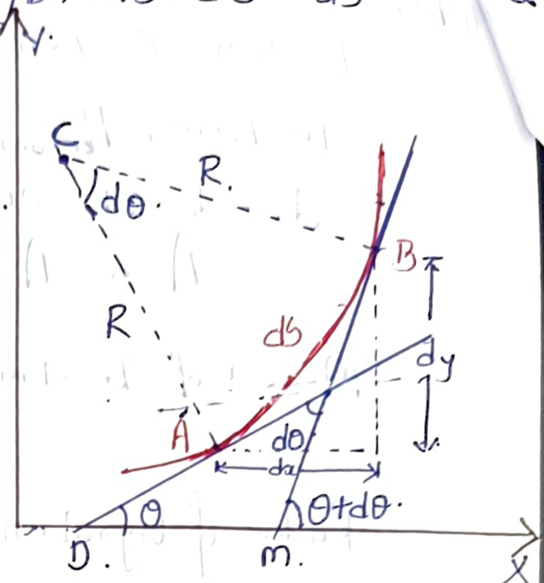
$$\text{Euler's Critical load, } P_E = \frac{4\pi^2 EI}{L^2} = \frac{4\pi^2\times(1\times10^5)\times(46.36\times10^6)}{(4500)^2}$$

$$= 9040.052 \ kN.$$

$$\frac{P_E}{P_R} = \frac{9040.052}{3510.91} = 2.575.$$

(1)

Q10.a. Consider an element of length $AB = ds$ as shown in figure.



Let tangents be drawn at A and B make angles 'θ' and '$\theta + d\theta$' with X-axis & intersect it at 'D' and 'E' respectively.
Let 'M' be the intersection point of these 2 tangents.

$\angle DME + \theta + [180 - (\theta + d\theta)] = 180$.

$\Rightarrow \angle DME + \cancel{\theta} + \cancel{180} - \cancel{\theta} - d\theta = \cancel{180}$

$\therefore \angle DME = d\theta$.

Segment of Circle AB, $ds = R\,d\theta$.

$\Rightarrow (1/R) = d\theta/ds$.

Treating ABF as Δ^{le}, $\tan\theta = \dfrac{dx}{dy}$ and $\sec\theta = \dfrac{ds}{dx}$.

$\dfrac{dy}{dx} = \tan\theta$. Diff w.r.t x we get.

$$\frac{d^2y}{dx^2} = \sec^2\theta \frac{d\theta}{dx} = \sec^2\theta \frac{d\theta}{ds}\frac{ds}{dx} = \sec^2\theta\left(\frac{1}{R}\right)\sec\theta.$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^3\theta\left(\frac{1}{R}\right) \qquad \Rightarrow \frac{1}{R} = \frac{d^2y/dx^2}{(\sec^3\theta)^{2/2}} = \frac{d^2y/dx^2}{(\sec^2\theta)^{3/2}}.$$

$$\Rightarrow \frac{1}{R} = \frac{d^2y/dx^2}{[1+\tan^2\theta]^{3/2}}. \qquad \Rightarrow \frac{1}{R.} = \frac{d^2y/dx^2}{[1+(dy/dx)^2]^{3/2}}.$$

Terms of higher order in denominator can be neglected.

$$\therefore \frac{1}{R} = \frac{d^2y/dx^2}{(1+0)^{3/2}} \qquad \Rightarrow \frac{1}{R} = \frac{d^2y}{dx^2}.$$

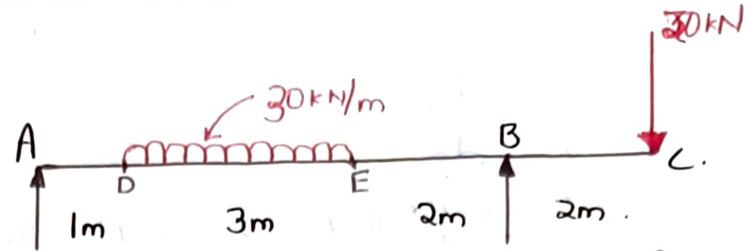
From bending eqn. $\dfrac{M}{I} = \dfrac{E}{R.}$ $\Rightarrow \dfrac{M}{EI} = \dfrac{1}{R.}$

$$\therefore \frac{d^2y}{dx^2} = \frac{M}{EI}. \qquad \therefore \boxed{M = EI\frac{d^2y}{dx^2}.}$$

Q10.b. $\Sigma F_V = 0$

$\Rightarrow V_A + V_B - 30\times3 - 20 = 0.$

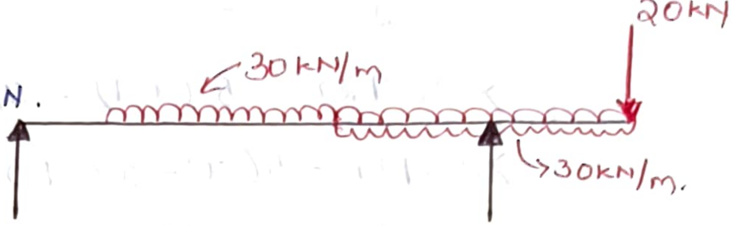
$\Rightarrow V_A + V_B = 110$



$\Sigma M_A = 0.$

$\Rightarrow 30\times3\times2.5 + 20\times8 = V_B\times6.$

$V_B = 64.167\,kN. \quad \therefore V_A = 45.833\,kN.$



Considering beam which is subjected to equivalent load.

$$M_x = 45.83x - \frac{30(x-1)^2}{2} + \frac{30(x-4)^2}{2} + 64.167(x-6).$$

$$EI\frac{d^2y}{dx^2} = 45.83x - 15(x-1)^2 + 15(x-4)^2 + 64.167(x-6).$$

$$EI\frac{dy}{dx} = \frac{45.83x^2}{2} - \frac{15(x-1)^3}{3} + \frac{15(x-4)^3}{3} + \frac{64.167(x-6)^2}{2} + C_1$$

$$EIy = \frac{45.83x^3}{6} - \frac{15(x-1)^4}{12} + \frac{15(x-4)^4}{12} + \frac{64.167(x-6)^3}{6} + C_1x + C_2.$$

At $x = 0$, $y = 0$.

$\Rightarrow 0 = 0 + 0 + C_2 \qquad \Rightarrow C_2 = 0$

At $x = 6m$ $y = 0$.

$$\Rightarrow 0 = \frac{45.83\times6^3}{6} - \frac{15(6-1)^4}{12} + \frac{15(6-4)^4}{12} + C_1(6)$$

$\Rightarrow C_1 = -148.1.$

Deflection at C is given by $(x = 8)$.

$$EIy_c = \frac{45.83\times8^3}{6} - \frac{15\times7^4}{12} + \frac{15\times4^4}{12} + \frac{64.167\times2^3}{6} + (-148.1)\times8.$$

$\Rightarrow 200\times10^3\,N/mm^2 \times 45\times10^6\,mm^4 = 130.33.$

$y_c\,9\times10^{12}\,Nmm^2 = 130.33 \qquad \Rightarrow y_c\,9\times10^{12}\times10^{-9}\,Nm^2 = 130.33$

$\therefore y_c = 1.448\times10^{-3}\,m = 1.448\,mm.$

Assuming that y is maximum in portion DE.

$\therefore \frac{dy}{dx} = 0$.

$\Rightarrow \frac{45.83x^2}{2} - \frac{15(x-1)^3}{3} - 148.1 = 0$

$\Rightarrow 22.92x^2 - 5(x-1)^3 - 148.1 = 0$.

$\Rightarrow 22.92x^2 - 5(x^3 - 3x^2 + 3x - 1) - 148.1 = 0$.

$\Rightarrow 22.92x^2 - 5x^3 + 15x^2 - 15x + 5 - 148.1 = 0$.

$\Rightarrow -5x^3 + 37.92x^2 - 15x - 143.1 = 0$

$\Rightarrow 5x^3 - 37.92x^2 + 15x + 143.1 = 0$.

Solving above cubic eqn. we get

$x = 2.77\,m$.

For DE portion.

$EI\,y_{max} = \frac{45.83(2.77)^3}{2} - \frac{15(2.77-1)^4}{2} - 148.1\times 2.77$

Substituting we get.

$EI\,y_{max} = 490.2$

$F_x = 45.833\times 2.77 - 15(2.77-1)^2$.

$y_{max} = \frac{490.2}{9000} = 0.0545\,m = 54.5\,mm$

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