CBCS SCHEME

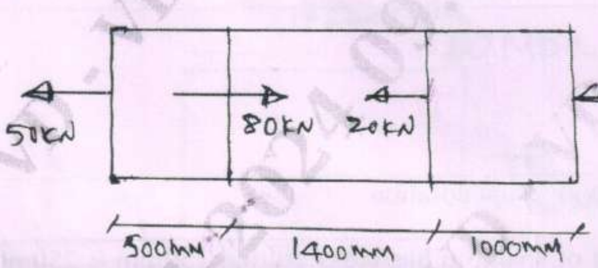
USN

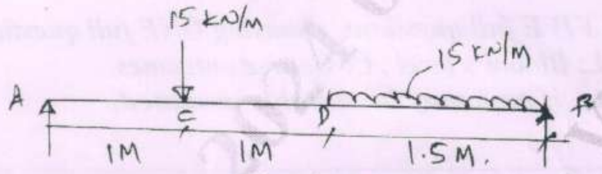
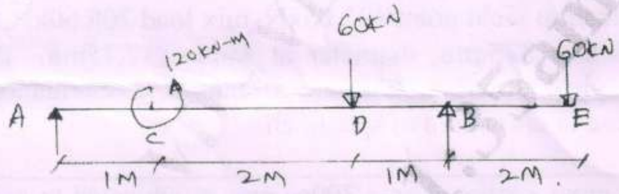
BCV301

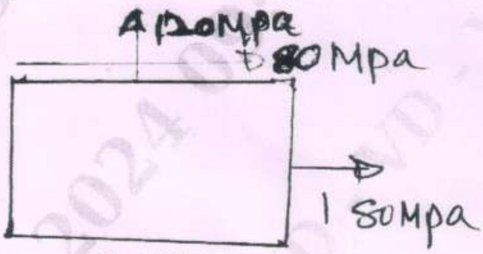
Third Semester B.E/B.Tech. Degree Examination, Dec.2023/Jan.2024
Strength of Materials

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, choosing ONE full question from each module.***
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. Missing data, if any, may be suitably assumed.

		Module – 1	M	L	C
1	a.	Define the following terms : i) Poisson's ratio ii) Modulus of rigidity iii) Impact load iv) Volumetric strain.	4	L1	CO1
	b.	The following data refers to mild steel tested on a lab. Diameter of the specimen 25mm, length of the specimen 300mm, Extension under a load of 15kN is 0.045mm, load at yield point 127.65kN, mix load 208.60kN, length of the specimen at failure 375mm, diameter at failure 17.75mm. Determine young's modulus, yield strength, ultimate strength, % elongation of the specimen, % decrease in c/s at area of specimen.	10	L3	CO1
	c.	A brass bar having cross-sectional area 300sq.mm is subjected to axial forces as shown in the Fig.Q1(c). Determine the total elongation of the bar taking E = 84GPa.  Fig.Q1(c)	6	L3	CO1
		OR			
2	a.	A steel rod 20mm diameter, length 6m is connected at the ends to a pair of walls at a temperature of 120°C. Find the pull exerted on the wall if the temperature falls to 40°C when : i) Supports don't yield ii) Supports yield by 1.1mm, take E = 200GPa. $\alpha = 12 \times 10^{-6}/°C$.	6	L3	CO1
	b.	Derive the relationship between modulus of elasticity, modulus of rigidity and Poisson's ratio.	6	L3	CO1
	c.	A load of 270kN is acting on a short RCC column of size $(200 \times 200)mm^2$. The column is reinforced with 10 bars of 12mm diameter. Determine the loads and the corresponding stresses on steel and concrete. Take $E_s = 16.5E_C$.	8	L3	CO1

		Module – 2			
3	a.	Define Hogging Bending moment and sagging bending moment.	4	L2	CO2
	b.	Derive the relationship between loading, shear force and bending moment.	6	L3	CO2
	c.	A simply supported beam is subjected to point load of 15kN together with a udl of 15kN/m as shown in the Fig.Q3(c). Draw SFD and BMD. Find also point of low shear and the corresponding bending moment.  Fig.Q3(c)	10	L3	CO2
		OR			
4	a.	Show that maximum bending moment for a simply supported beam carrying udl of intensity w/unit length is $\frac{w\ell^2}{8}$.	6	L2	CO2
	b.	Draw SFD and BMD for an overhanging beam carrying forces as shown in the Fig.Q4(b).  Fig.Q4(b)	14	L3	CO2
		Module – 3			
5	a.	Define : i) Modulus of rupture ii) Section modulus iii) Flexural rigidity.	6	L1	CO3
	b.	Derive bending equation with usual notation.	6	L3	CO3
	c.	A simply supported beam of span 5m has cross section 150mm × 250mm. If the permissible stress is $10 N/mm^2$. Find : i) Maximum intensity of UDL it can carry ii) Maximum concentrated kind P applied 2m from one end.	8	L3	CO3
		OR			
6	a.	List the assumptions made pure torsion.	6	L1	CO3
	b.	Drive torque – equation with usual notation.	4	L3	CO3
	c.	A solid shaft has to transmit 250KW power at 100rpm. If the shear stress not to exceed 75MPa what should be the diameter of the shaft. If this shaft is to be replaced by a hallow shaft whose internal diameter, is 0.6 times the external diameter determine the size of the shaft. Also determine the saving on the weight of the material. Assume max shear stress remain the same for both the shifts.	10	L3	CO3

		Module – 4			
7	a.	Define slope, deflection and curvature.	6	L1	CO4
	b.	Derive moment–curvature equation.	6	L3	CO4
	c.	A girder of uniform section and constant depth is simply supported over a span of 3m. If the point load at the mid span is 30kN and $I_{XX} = 15.614 \times 10^{-6} m^4$, calculate : i) Central deflection ii) The slopes at the ends if he beam. Take $E = 200GN/m^2$.	8	L3	CO4
		OR			
8	a.	Differentiate between long columns and short columns.	4	L1	CO4
	b.	Derive Euler's Buckling load for long columns whose ends are hinged.	6	L3	CO4
	c.	A hallow tube 6m length of external diameter 16mm and thickness 10mm is subjected to minimum crippling load. Find Euler's load for this column when : i) Both ends fixed ii) One end fixed and other end hinged. Take E = 200GPa.	10	L3	CO4
		Module – 5			
9	a.	Derive principle planes and principle stresses.	4	L1	CO5
	b.	Differentiate between thin cylinders and thick cylinders.	4	L1	CO5
	c.	The state of stress at a point on a strained material is 120Mpa and is an as shown in the Fig.Q9(c). Determine : i) The direction of principal planes ii) The magnitude of principal stresses iii) The magnitude of maximum shear stress and its directions. Sketch the stresses and planes.  Fig.Q9(c)	12	L3	CO5
		OR			
10	a.	Drive Lame's equation with usual notation.	8	L3	CO5
	b.	A shell 3.25m long; 1m in diameter is subjected to internal fluid pressure of 1MPa, if the thickness of the shell is 10mm find : i) Hoop-stress ii) Longitudinal stress iii) Maximum shear stress iv) Change in diameter and length v) Voltmeter strain and hence measure in volume Take $E = 2 \times 10^3 MPa$; $\frac{1}{m} = 0.30$.	12	L3	CO5

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KLS VDIT, HALIYAL

DEPARTMENT OF CIVIL ENGINEERING

III SEMESTER B E DEGREE EXAMINATION JAN 2024

STRENGTH OF MATERIALS (BCV301)

QUESTION PAPER SOLUTION.

FACULTY NAME: Prof. HARSHAVARDHAN. V. S ACADEMIC YEAR: 2023-24

Q1.a. Poisson's ratio is defined as the ratio of lateral strain to linear strain.

$$\mu = \frac{\text{Lateral Strain}}{\text{Linear Strain}}$$

Modulus of Rigidity is defined as the ratio of. of shearing stress to shearing strain within elastic limit.

$$G = \frac{\text{Shear stress}(\tau)}{\text{Shear Strain}(\phi)}$$

Impact Load is defined as the force provided by a sudden load on a structural part instead of a constant & sustained force applied over a long period.

Volumetric Strain is defined as ratio of change in volume to the initial volume.

$$e_v = \frac{\text{Change in Volume}}{\text{Initial Volume}}$$

b. GIVEN, $d_o = 25mm$ $L = 300mm$ $P = 15kN$ $\Delta L = 0.045mm$

$P_{YIELD} = 127.65 kN$ $P_{max} = 208.6 kN$ $L_{FINAL} = 375mm$ $d_f = 17.75mm$

To FIND: $E = ?$ $\sigma_y = ?$ $\sigma_{max} = ?$ $\% \Delta L = ?$ $\% \Delta A = ?$

i) To FIND 'E'

- $A = (\pi/4) \times d^2 = (\pi/4) \times 25^2 = 490.87 mm^2$.

$\sigma = P/A = 15\times10^3 / 490.87 = 30.56 N/mm^2$.

$\epsilon = \Delta L / L = 0.045/300 = 1.5\times10^{-4}$

$E = \sigma / \epsilon = 30.56/(1.5\times10^{-4}) = \underline{203.73\times10^3 N/mm^2}$.

ii) To find 'σ_y' 'Yield stress'

$$\sigma_y = P_y/A = \frac{127.65\times10^3\,N}{490.87\,mm^2} = 260\ N/mm^2.$$

iii) Ultimate stress.

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{208.6\times10^3\,N}{490.87\,mm^2} = 424.96\,N/mm^2.$$

iv) Percentage elongation.

$$\%\Delta L = \frac{\text{Elongation}}{\text{Initial length}}\times100\% = \frac{\text{Final length} - \text{Initial length}}{\text{Initial length}}\times100\% = \frac{375-300}{300}\times100\%$$

$$= 25\%$$

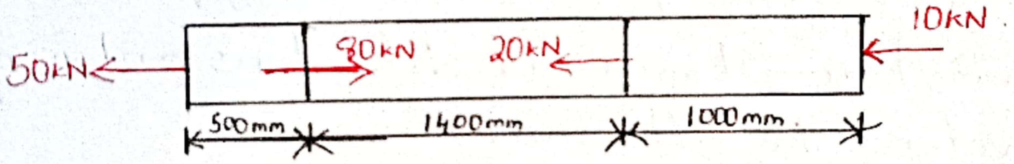
v) Percentage reduction in area

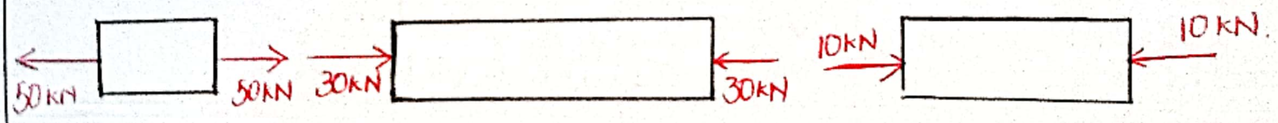
$$\%\Delta A = \frac{\text{Final area} - \text{Initial area}}{\text{Initial area}} = \frac{247.45-490.87}{490.87}\times100\% = -49.58\%$$

$$\text{Final area} = \frac{\pi}{4}\times d_f^2 = \frac{\pi}{4}\times17.75^2 = 247.45\,mm^2.$$

C

Given $A = 300\,mm^2$ $E = 84\,GPa = 84\times10^3\,N/mm^2$.





$P_1 = 50\,kN$.	$P_2 = 30\,kN$	$P_3 = 10\,kN$.
$L_1 = 500\,mm$	$L_2 = 1400\,mm$	$L_3 = 1000\,mm$.
$A_1 = 300\,mm^2$.	$A_2 = 300\,mm^2$.	$A_3 = 300\,mm^2$.
$E_1 = 84\times10^3\,N/mm^2$	$E_2 = 84\times10^3\,N/mm^2$.	$E_3 = 84\times10^3\,N/mm^2$.
$\Delta_1 = \frac{P_1 L_1}{A_1 E_1}$	$\Delta_2 = \frac{P_2 L_2}{A_2 E_2}$	$\Delta_3 = \frac{P_3 L_3}{A_3 E_3}$.
$= \frac{50\times10^3\times500}{300\times84\times10^3}$	$= \frac{30\times10^3\times1400}{300\times84\times10^3}$	$= \frac{30\times10^3\times1000}{300\times84\times10^3}$
$= 0.99\,mm$.	$= 1.67\,mm$.	$= 1.19\,mm$.

$$\Delta = +\Delta_1 - \Delta_2 - \Delta_3 = 0.99 - 1.67 - 1.19 = -1.87\,mm$$

Q2.a. GIVEN, $d = 20mm$ $L = 6m$ $t_1 = 120°C$ $t_2 = 40°C$.

$E = 200GPa = 200\times10^3 N/mm^2$ $\alpha = 12\times10^{-6}/°C$

$t = t_2 - t_1 = 40°C - 120°C = -80°C$., $A = \frac{\pi}{4}\times20^2 = 314.159 mm^2$.

i) FREE EXPANSION IS COMPLETELY PREVENTED (SUPPORTS DONT YIELD).

Free Expansion, $\Delta = \alpha L t = (12\times10^{-6})\times6000mm\times(80) = 5.76mm$.

$$\sigma_t = \frac{\Delta E}{L} = \frac{5.76\cancel{mm}\times200\times10^3 N/mm^2}{6000\cancel{mm}} = 192 N/mm^2.$$

PULL, $P = \sigma\times A = 192 N/mm^2 \times 314.159 mm^2 = \underline{60.32\times10^3 N}$.

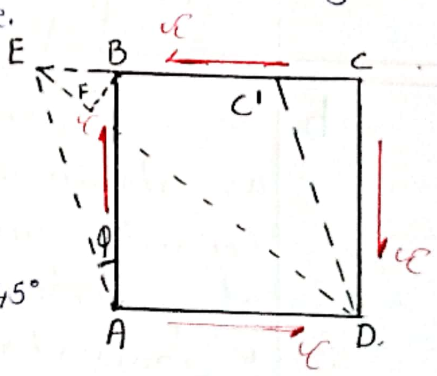
ii) SUPPORTS YIELD BY 1.1mm. ($\delta = 1.1mm$)

$\Delta = \alpha L t - \delta = 5.76 - 1.1 = 4.66mm$.

$$\sigma_t = \frac{\Delta E}{L} = \frac{4.66mm\times200\times10^3 N/mm^2}{6000mm} = 155.33 N/mm^2.$$

PULL, $P = \sigma\times A = 155.33 N/mm^2 \times 314.159 mm^2 = \underline{48.8\times10^3 N}$.

b. Consider a square element ABCD of sides 'a' subjected to pure shear 'τ' as shown in the figure.



AEC'D is the deformed shape due to shear 'τ'

Drop a $\perp^r$ BF to diagonal DE from B.

Strain in Diagonal BD $= \frac{DE-BD}{BD} = \frac{DE-DF}{AB\sqrt2} = \frac{EF}{a\sqrt2}$ $\left\{\because BD = AB\sqrt2\right\}$

Since Deformation angle is small we can assume $\angle BEF = 45°$

$\therefore \cos45° = EF/BE \Rightarrow EF = BE\cos45°$.

Considering Δ^{le} AEB, $\tan\phi = BE/AB \Rightarrow BE = AB\tan\phi = a\tan\phi$

$\therefore EF = (a\tan\phi)\cos45$

$\therefore$ Strain in Diagonal BD $= \frac{a\tan\phi\cos45}{a\sqrt2} = \frac{\tan\phi(1/\sqrt2)}{\sqrt2} = \frac{\tan\phi}{2} \approx \frac{\phi}{2} = \frac{(\sigma/G)}{2} = \frac{1}{2}\frac{\sigma}{G}$.

WKT in pure shear, a tensile stress 'σ' of equal magnitude is acting along $\rightarrow$ ① diagonal BD & compressive stress 'σ' at 90° to it. These 2 stresses cause tensile strain along diagonal BD.

Strain along BD = LINEAR STRAIN due to TENSILE STRESS + LATERAL STRESS due to COMP STRESS

$= \epsilon + \mu\epsilon$ $\left[\because \mu = \text{LATERAL STRAIN}/\text{LINEAR STRAIN}\right]$

$= \sigma/E + \mu\sigma/E = \sigma/E(1+\mu) \longrightarrow$ ②

From ① and ② $\frac{1}{2}\frac{\cancel{\sigma}}{G} = \frac{\cancel{\sigma}}{E}(1+\mu) \Rightarrow E = 2G(1+\mu)$

c. GIVEN, $P = 270\ kN$ $B = D = 200mm$ $E_s = 16.5E_c$.

TOTAL AREA $= B \times D = 200 \times 200 = 40000mm^2 = A_{TOT}$

AREA OF STEEL, $A_s = \left(\frac{\pi}{4} \times 12^2\right) \times 10\ NOS = 1130.90mm^2$.

$\therefore$ AREA OF CONCRETE $= A_c = A_{TOT} - A_s = 40000 - 1130.9 = 38869.1mm^2$.

$P_s + P_c = 270\ kN \longrightarrow (1)$

$\Delta_s = \Delta_c \Rightarrow \frac{P_s \cancel{L_s}}{A_s E_s} = \frac{P_c \cancel{L_c}}{A_c E_c} \Rightarrow P_s = \frac{A_s E_s P_c}{A_c E_c} = \frac{1130.9 \times 16.5 \cancel{E_c}}{38869.1 \times \cancel{E_c}} P_c \Rightarrow P_s = 0.48P_c$

$\therefore 0.48P_c + P_c = 270kN \Rightarrow 1.48P_c = 270kN \Rightarrow P_c = 182.43kN$

$\therefore P_s = 0.48P_c = 0.48 \times 182.43 = 87.57\ kN$.

$\sigma_c = \frac{P_c}{A_c} = \frac{182.43 \times 10^3}{38869.1} = \underline{4.69\ N/mm^2}$ $\sigma_s = \frac{P_s}{A_s} = \frac{87.57 \times 10^3}{1130.90} = \underline{77.43\ N/mm^2}$

Q3.a. HOGGING BENDING MOMENT: A BM causing concavity ~~downwards~~ will be taken as negative & called hogging BM.

HAGGING BM

SAGGING BENDING MOMENT: A BM causing concavity upwards will be taken as positive & called sagging BM.

SAGGING BM

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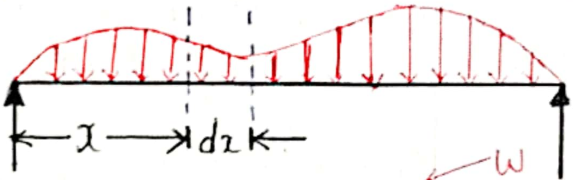
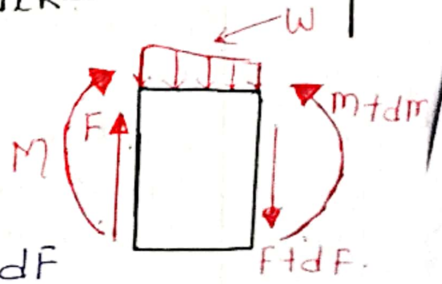
b. Consider the beam AB subjected to general loading as shown in figure.



Consider an element of length 'dx' at a distance 'x' from left support & draw its FBD. For small portion intensity may be taken as constant.



$\Sigma F_v = 0 \Rightarrow F - (F + dF) - w\,dx = 0$

$\Rightarrow \cancel{F} - \cancel{F} - dF - w\,dx = 0 \Rightarrow -dF = -w\,dx \Rightarrow w = \frac{-dF}{dx}$

$\Sigma M_{RIGHT\ FACE} = 0$

$\Rightarrow M + F(dx) - [w(dx)]\frac{dx}{2} - (M + dM) + (F + dF) \times 0 = 0$.

$\Rightarrow \cancel{M} + F(dx) - \cancel{w\,dx^2/2}^{\,0} - \cancel{M} - dM = 0 \Rightarrow F(dx) = dM$.

$\therefore F = dM$

C

SUPPORT REACTION

$\Sigma M_A = 0$

$\Rightarrow 15\times1 + (15\times1.5)\times(2.75) - R_B\times3.5 = 0$

$\Rightarrow R_B = 21.96\ kN.$

$\Sigma F_v = 0$

$\Rightarrow R_A - 15 - 15\times1.5 + R_B = 0$

$\Rightarrow R_A - 15 - 22.5 + 21.96 = 0 \Rightarrow R_A = 12.54\ kN.$

SHEAR FORCE.

$F_A = 0 \quad F_A = 0 + 12.54 = 12.54\ kN.$

$F_C = 12.54\ kN \quad F_C = 12.54 - 15 = -2.46\ kN$

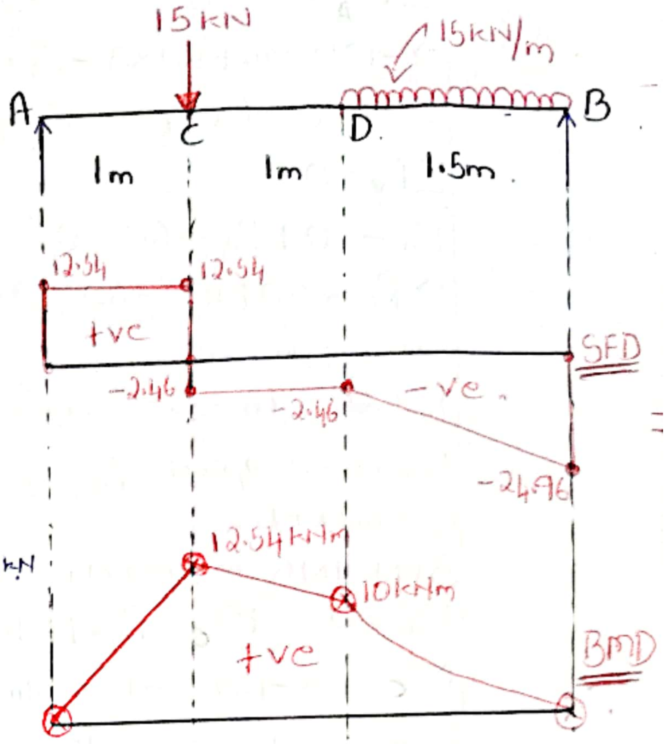
$F_D = 12.54 - 15 = -2.46\ kN \quad F_B = -R_B = -24.96\ kN$

BENDING MOMENT.

$M_A = M_B = 0.$

$M_C = R_A\times1 = 12.54\ kNm$

$M_D = R_A\times2 - 15\times1 = 10\ kNm.$



Q4.a. Consider a simply supported beam AB of span 'L' subjected to a UDL 'w'

As loading is symmetrical

$$R_A = R_B = \frac{\text{TOTAL LOAD}}{2} = \frac{wL}{2}.$$

At section X, $M_x = R_A x - wx\left(\frac{x}{2}\right)$

$$\Rightarrow M_x = \frac{wL}{2}x - \frac{wx^2}{2} \longrightarrow (1).$$

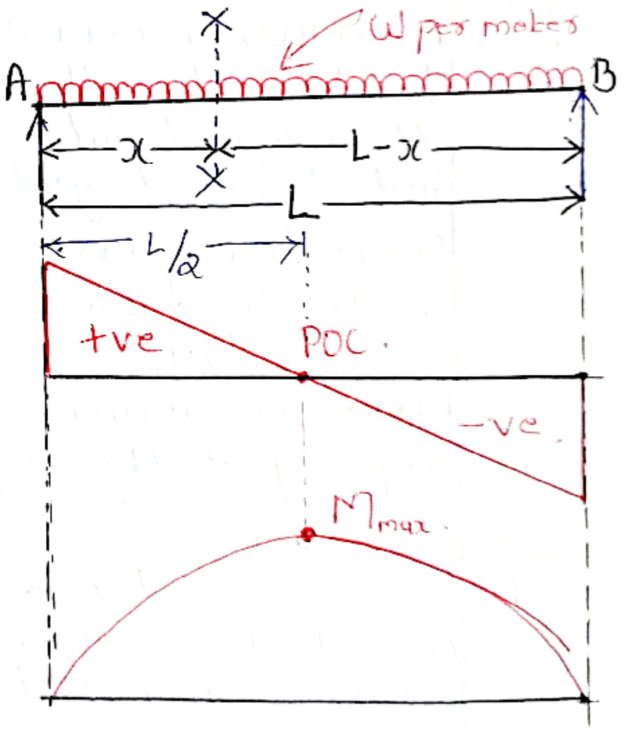
$$V_x = R_A - wx = \frac{wL}{2} - wx$$

For $M = M_{max}$, $V_x = 0$.

$$\Rightarrow \frac{wL}{2} - wx = 0 \Rightarrow x = L/2$$

$$\therefore M_{max} = (M_x \text{ at } x = L/2) = \frac{wL}{2}\left(\frac{L}{2}\right) - \frac{w}{2}\left(L/2\right)^2$$

$$\Rightarrow M_{max} = \frac{wL^2}{4} - \frac{wL^2}{2\times4} = \frac{wL^2}{4} - \frac{wL^2}{8} \quad \therefore \boxed{M_{max} = \frac{wL^2}{8}.}$$



b SUPPORT REACTION

$\sum M_A = 0$

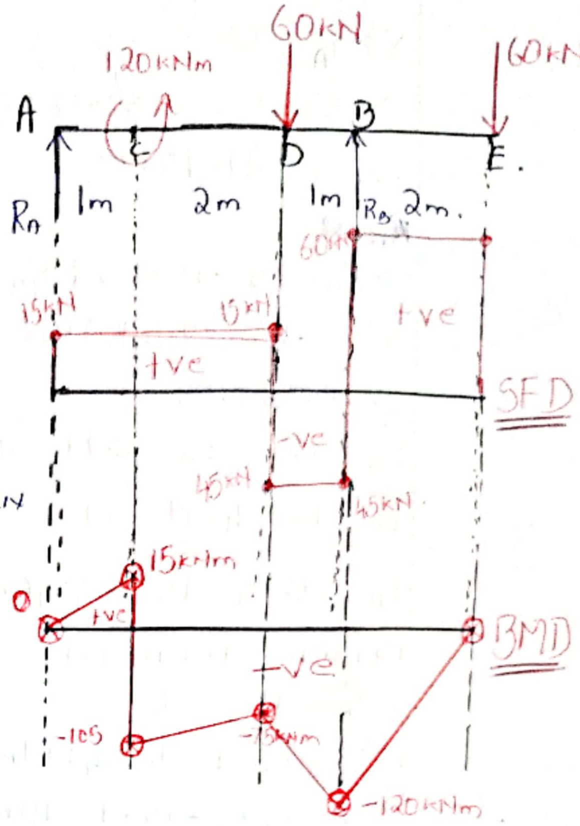
$\Rightarrow -120\,kNm + 60\times3 - R_B\times4 + 60\times6 = 0$

$\Rightarrow 420 = R_B \times 4 \quad \therefore R_B = 105\,kN.$

$\sum F_V = 0$

$R_A - 60 + R_B - 60 = 0.$

$\Rightarrow R_A - 60 + 105 - 60 = 0. \Rightarrow R_A = 15\,kN.$



SHEAR FORCE.

$F_A = 0 \quad F_A = 15\,kN \quad F_C = 15\,kN \quad F_D = 15\,kN$

$F_D = 15 - 60 = -45\,kN \quad F_B = -45\,kN \quad F_B = -45 + 105 = 60\,kN$

$F_E = 60\,kN.$

BENDING MOMENT

$M_A = 0 \quad M_C = R_A\times1 = 15\times1 = 15\,kNm$

$M_C = 15 - 120 = -105\,kNm.$

$M_D = R_B\times1 - 60\times3 = 105\times1 - 180 = -75$

$M_B = -60\times2 = -120\,kNm \quad M_E = 0.$

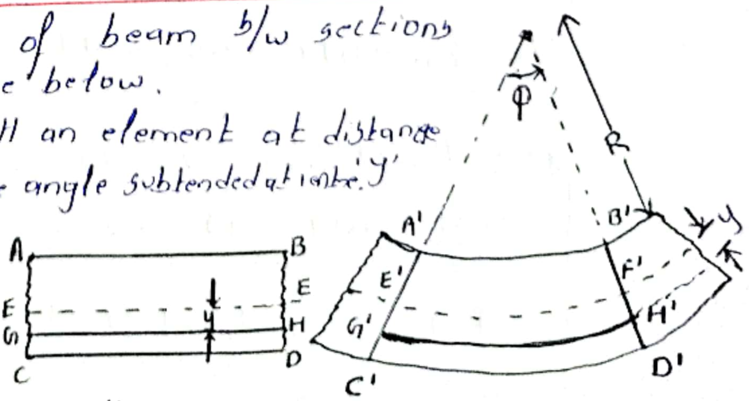
Q5.a. MODULUS OF RUPTURE: is also known as flexural strength, bending strength or rupture strength. It is a material property, defined as the stress in a material just before it yields in a flexural test. $[\sigma = My/I]$.

SECTION MODULUS: is defined as the ratio of moment of inertia to distance between neutral axis and farthest fibre.

$$Z = I/y_{max}.$$

FLEPXURAL RIGIDITY: The term 'EI' is known as the flexural rigidity of the section.
Flexural Rigidity = EI.

b. Consider a portion of beam b/w sections AC and BD as shown in figure below. Let EF be the neutral axis & GH an element at distance. Let 'R' be radius of curvature & 'φ' the angle subtended at centre 'y'



AFTER BENDING.

Since EF is neutral axis, $EF = E'F' = R\phi$

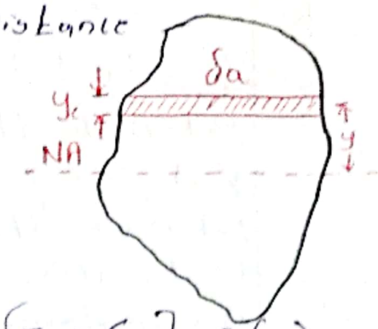
Strain in layer GH $= \dfrac{G'H' - GH}{GH} = \epsilon$

$GH = EF = R\phi$ and $G'H' = (R+y)\phi = R\phi + y\phi$.

$\therefore \epsilon = \dfrac{(R\phi + y\phi) - R\phi}{R\phi} = \dfrac{y\phi}{R\phi} \Rightarrow \epsilon = \dfrac{y}{R} \Rightarrow \dfrac{\sigma}{E} = \dfrac{y}{R}$

$$\therefore \frac{\sigma}{y} = \frac{E}{R}$$

Consider an element of area 'δa' at a distance 'y' from Neutral Axis.



Stress on element, $\sigma = \frac{E}{R} y$

Force on the element, $F = \sigma \cdot \delta a = \frac{E}{R} y (\delta a)$

Moment of Resistance of this c/s area $= F \times y = \frac{E}{R} y^2 (\delta a)$

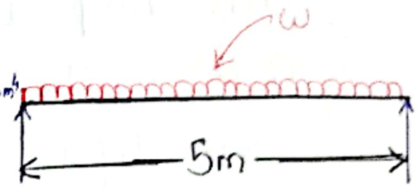
Moment of Resistance of whole c/s, $M' = \Sigma \frac{E}{R} y^2 (\delta a) = \frac{E}{R} \left[\Sigma y^2 \delta a\right] = \frac{E}{R}(I)$

$\therefore M' = \frac{E}{R} I \Rightarrow \frac{M}{I} = \frac{E}{R} \quad \therefore \boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$.

C GIVEN, $L = 5m$ $B = 150mm$ $D = 250mm$ $\sigma_{max} = 10 N/mm^2$.

TO FIND: $W_{max} = ?$ $P = ?$ 2m from end.

Moment of Inertia, $I = \frac{BD^3}{12} = \frac{150 \times 250^3}{12} = 195.31 \times 10^6 mm^4$



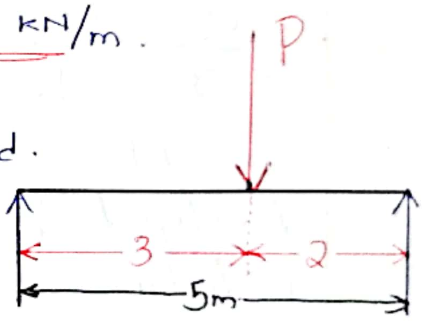
$y_{max} = 250/2 = 125$

i) SUBJECTED TO UDL.

$M_{max} = \frac{wL^2}{8} = \frac{w \times 5000^2}{8} = 3.125 \times 10^6 w$.

$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \frac{3.125 \times 10^6 w}{195.131 \times 10^6} = \frac{10}{125} \Rightarrow w = 5 kN/m$.

ii) SUBJECTED TO POINT LOAD 'P' at 2m from 1 end.



$M_{max} = \frac{Pab}{L} = \frac{P \times 3000 \times 2000}{5000} = 1200P$

$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \frac{1200 P}{195.131 \times 10^6} = \frac{10}{125} \Rightarrow P = 13 \times 10^3 N$
$= 13 kN$.

Q6.a. The assumptions made for pure torsion are

1) The material is homogeneous & isotropic.

2) Stress is proportional to strain i.e all the stresses are within the elastic limit.

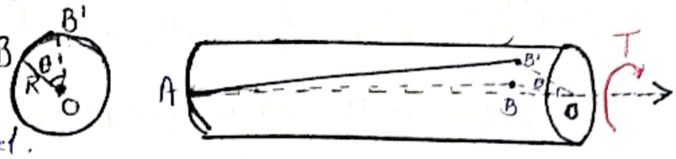
3) Radial lines remain radial or straight after torsion.

4) Torsion is uniform along the shaft. (i.e all normal c/s which are at same axial distance suffer equal relative rotation)

5) Cross-sections which are plane before applying the torsion remain plane even after application of torsion. i.e no warping takes place.

b Consider a shaft of length 'L' radius 'R' fixed at one end & subjected to a torque 'T' at the other end.

Let 'O' be the centre of O^{lar} section & B be a point on the surface.



Let AB be the line on the shaft parllel to its axis.

Due to torque 'T' applied let B move to B'.

If 'φ' is the shear strain $\angle \phi = BOB'$ and θ is the ∠le of twist then

$$BB' = R\theta .$$

$$\tan\phi = \frac{BB'}{AB} \Rightarrow \phi = \frac{BB'}{AB} \quad (\text{for small angles } \phi \simeq \tan\phi)$$

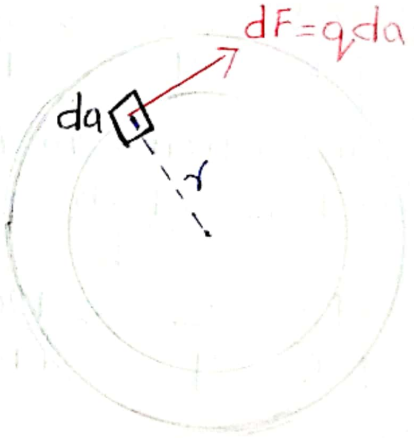
$$\Rightarrow BB' = AB\,\phi = L\phi \qquad \therefore R\theta = L\phi .$$

If 'τ' is the shear stress and G is modulus of Rigidity then.

$$R\theta = L\left(\frac{\tau_s}{G}\right) \Rightarrow \frac{\tau_s}{R} = \frac{G\theta}{L} \longrightarrow ①$$

If any point 'B' at distance 'r' from centre is considered then

$$\frac{\tau}{r} = \frac{G\theta}{L} \qquad \therefore \frac{\tau_s}{R} = \frac{\tau}{r} \longrightarrow ②$$



Now consider torsional resistance developed by an elemental area 'δa' at a distance 'r' from the centre.

∴ Resisting Torsional moment, $dT = dF \times r = (\tau . da)\, r .$

$$\Rightarrow dT = \left(\frac{\tau_s r}{R}\right) da\, r = \frac{\tau_s r^2 da}{R} .$$

Total Resisting Torsional Moment, $T = \Sigma \frac{\tau_s r^2 da}{R} = \frac{\tau_s}{R} \Sigma r^2 da .$

$$\therefore T = \frac{\tau_s}{R}(J) \Rightarrow \frac{T}{J} = \frac{\tau_s}{R} \longrightarrow ③$$

From ①, ② and ③

$$\boxed{\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}}$$

C GIVEN, $P = 250\,kW$ $N = 100\,rpm$ $\tau_{max} = 75\,N/mm^2$ $d_i = 0.6\,d_o$

To FIND i) $d = ?$ ii) $d_i = ?$ $d_o = ?$ considering hollow shaft iii) % saving.

$$T = \frac{60P}{2\pi N} = \frac{60 \times 250 \times 10^6}{2\pi \times 100} = \underline{23.87 \times 10^6\,Nmm}.$$

i) Considering shaft is SOLID.

$r = d/2 = 0.5d$ $J = \frac{\pi}{32} \times d^4 = 0.098d^4$.

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow \frac{23.87 \times 10^6}{0.098d^4} = \frac{75}{0.5d} \Rightarrow \frac{d^4}{d} = \frac{23.87 \times 10^6 \times 0.5}{0.098 \times 75} \Rightarrow d^3 = 1.624 \times 10^6$$

$\therefore \underline{d = 117.54\,mm.}$

ii) Considering shaft is hollow.

$r = d_o/2 = 0.5d_o$ $J = \frac{\pi}{32} \times (d_o^4 - d_i^4) = \frac{\pi}{32}\left[d_o^4 - (0.6d_o)^4\right] = \frac{\pi}{32} d_o^4\left[1 - 0.6^4\right] = 0.085d$

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow \frac{23.87 \times 10^6}{0.085d_o^4} = \frac{75}{0.5d_o} \Rightarrow \frac{d_o^4}{d_o} = \frac{23.87 \times 10^6 \times 0.5}{0.085 \times 75} \Rightarrow d_o^3 = 1.872 \times 10^6$$

$\therefore \underline{d_o = 123.24\,mm}$ $\therefore d_i = 0.6 \times 123.24 = \underline{73.95\,mm}.$

iii) Saving in the weight.

$$A_{SOLID} = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 117.54^2 = 10850.78\,mm^2.$$

$$A_{HOLLOW} = \frac{\pi}{4} \times (d_o^2 - d_i^2) = \frac{\pi}{4}(123.24^2 - 73.95^2) = 7633.67\,mm^2.$$

$$\%\ SAVING = \frac{10850.78 - 7633.67}{10850.78} \times 100\% = \underline{29.64\%}.$$

Q7. a. SLOPE: of a beam at any section in a deflected beam is defined as the angle in radians which the tangent at the section makes with the original axis of beam

DEFLECTION: of a beam at any point on the axis of the beam is the distance b/w its position before and after the loading.

CURVATURE: is defined as the rotation of member per unit length of the member.

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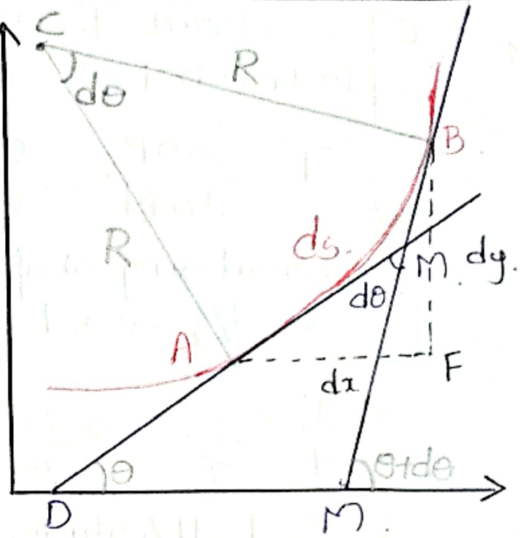
b. Consider an element of length AB = ds as shown in the figure.

Let tangent be drawn at A and B make angles 'θ' and 'θ+dθ' with X-axis & intersect it at 'D' and 'E' respectively.

Let M be the intersection point of these 2 tangents.



$\angle DME + \theta + [180 - (\theta + d\theta)] = 180°$

$\Rightarrow \angle DME + \cancel{\theta} + 1\cancel{80} - \cancel{\theta} - d\theta = 1\cancel{80} \Rightarrow \angle DME = d\theta$.

Segment of Circle AB, $ds = R\,d\theta \Rightarrow \frac{1}{R} = \frac{d\theta}{ds}$.

Treating ABF as Δ^{le}, $\tan\theta = \frac{dx}{dy}$ and $\sec\theta = \frac{ds}{dx}$. $\Rightarrow \frac{dy}{dx} = \tan\theta$.

Differentiating w.r.t x, $\frac{d^2y}{dx^2} = \sec^2\theta \frac{d\theta}{dx} = \sec^2\theta \frac{d\theta}{ds}\frac{ds}{dx} = \sec^2\theta\left(\frac{1}{R}\right)\sec\theta$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{R}\sec^3\theta$. $\Rightarrow \frac{1}{R} = \frac{d^2y/dx^2}{(\sec^3\theta)^{2/2}} = \frac{d^2y/dx^2}{(\sec^2\theta)^{3/2}} = \frac{d^2y/dx^2}{(1+\tan^2\theta)^{3/2}} = \frac{d^2y/dx^2}{\left[1+(dy/dx)^2\right]^{3/2}}$

dy/dx is a small quantity & hence $(dy/dx)^2$ can be neglected.

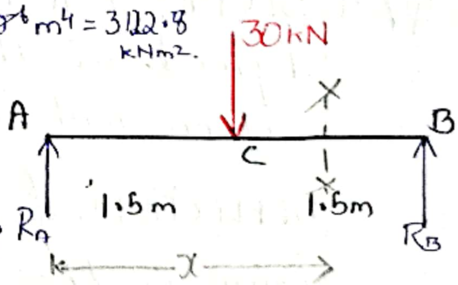
$\therefore \frac{1}{R} = \frac{d^2y}{dx^2}$. [From Bending Eqn. $\frac{M}{I} = \frac{E}{R} \Rightarrow \frac{M}{EI} = \frac{1}{R}$].

$\therefore \frac{M}{EI} = \frac{d^2y}{dx^2} \Rightarrow \boxed{M = EI\frac{d^2y}{dx^2}}$.

---

c GIVEN; $L = 3m$ $I_{xx} = 15.614 \times 10^{-6} m^4$. $E = 200 GN/m^2 = 200 \times 10^3 N/mm^2$. $P = 30kN$

$R_A = R_B = \frac{30}{2} = 15kN$. $EI = 200\times10^6 \frac{kN}{m^2} \times 15.614\times10^{-6} m^4 = 3122.8\ kNm^2$.



$M_x = R_A x - 30(x-1.5) \Rightarrow EI\frac{d^2y}{dx^2} = 15x - 30(x-1.5)$

$EI\frac{dy}{dx} = C_1 + \frac{15x^2}{2} - \frac{30(x-1.5)^2}{2} = C_1 + 7.5x^2 - 15(x-1.5)^2 \longrightarrow$

$EIy = C_2 + C_1x + 7.5/3\ x^3 - 15(x-1.5)^3/3 = C_2 + C_1x + 2.5x^3 - 5(x-1.5)^3$

At $x=0$, $y=0$ $\therefore 0 = C_2 + 0 + 0 \Rightarrow 0 = C_2 \Rightarrow C_2 = 0$.

At $x = 3m$ $y = 0$ $\therefore 0 = 0 + C_1\times3 + 2.5\times3^3 - 5(3-1.5)^3 \Rightarrow 0 = 3C_1 - 50.625 \therefore C_1 = 16.875$.

i) CENTRAL DEFLECTION $(x = 1.5)$.

$\Rightarrow EIy = 0 + 16.875\times1.5 + 2.5\times1.5^3 \Rightarrow EIy = 33.75 \Rightarrow y = \frac{33.75}{EI} = \frac{33.75}{3122.8}$

$= 0.0108\ m$.

ii) SLOPE AT END $(x=0)$

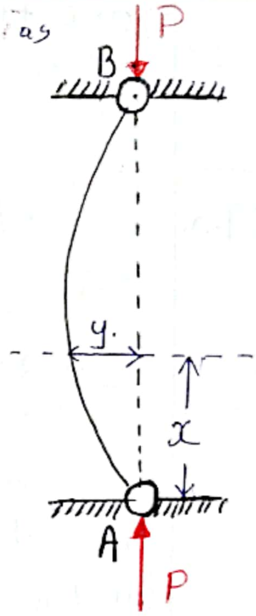
$EI\frac{dy}{dx} = 16.875 + 7.5\times0 \Rightarrow EI\frac{dy}{dx} = \Rightarrow \frac{dy}{dx} = \frac{16.86}{3122.8} = 5.4\times10^{-3}\ rad$.

$= 0.31\ deg$.

Q8.a.

SHORT COLUMN	LONG COLUMN.
1) $L/B \leq 12$	1) $L/B > 12$.
2) Tendency to Buckle is very low.	2) Tendency for Crushing is very low.
3) Column fails mostly due to Crushing	3) Column usually fails by buckling
4) Higher Load carrying Capacity	4) Lower Load Carrying Capacity.

Q8.b. Consider a column with both ends hinged as shown in the figure.



Consider a section at a distance 'x' from 'A' & let 'y' be its deflection from centreline.

We have, $M_x = -Py$ and $M_x = EI\dfrac{d^2y}{dx^2}$.

$$\Rightarrow EI\frac{d^2y}{dx^2} = -Py \quad \Rightarrow \frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

This is a standard differential equation whose solution is

$$y = C_1\cos\left[x\sqrt{\frac{P}{EI}}\right] + C_2\sin\left[x\sqrt{\frac{P}{EI}}\right] \longrightarrow ①.$$

where C_1 and C_2 are integration constants.

At 'A' $x = 0$ and $y = 0$ ∴ substituting in ①.

$$0 = C_1(1) + C_2(0) \quad \therefore C_1 = 0.$$

At 'B' $x = L$ and $y = 0$ ∴ Substituting in ①.

$$0 = 0 + C_2\sin L\sqrt{\frac{P}{EI}} = 0 \Rightarrow C_2\sin L\sqrt{\frac{P}{EI}} = 0.$$

C_2 cannot be zero in which case case $y = 0$ meaning the column has not bent at all.

$$\therefore \sin\left(L\sqrt{\frac{P}{EI}}\right) = 0 \Rightarrow L\sqrt{\frac{P}{EI}} = 0, \pi, 2\pi, 3\pi \ldots\ldots$$

Taking first non zero value i.e π we get.

$$L\sqrt{\frac{P}{EI}} = \pi \Rightarrow \left[L\sqrt{\frac{P}{EI}}\right]^2 = \pi^2 \Rightarrow \frac{L^2P}{EI} = \pi^2 \Rightarrow \boxed{P_c = \frac{\pi^2 EI}{L^2}}$$

C

GIVEN, $L = 6\,m$ $d_o = 160\,mm$ $t = 10\,mm$ $E = 200\,GPa = 200\times10^3\,N/mm^2$

$d_o = 160\,mm$ $d_i = 160 - 2t = 160 - 20 = 140\,mm.$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(160^4 - 140^4) = 13.31\times10^6\,mm^4.$$

i) BOTH ENDS FIXED.

$$P_E = \frac{4\pi^2 EI}{L^2} = \frac{4\pi^2\times200\times10^3\times13.31\times10^6}{6000^2} = 2919.2\,kN.$$

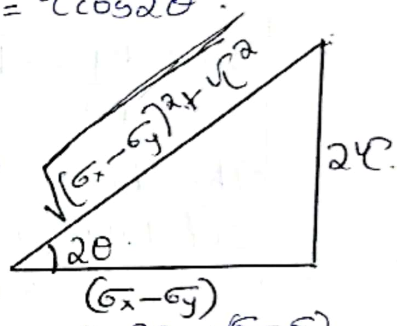
ii) ONE FIXED END & OTHER END HINGED.

$$P_E = \frac{2\pi^2 EI}{L^2} = \frac{2\pi^2\times200\times10^3\times13.31\times10^6}{6000^2} = 1459.6\,kN.$$

Q9.a. In principal planes we have, $\sigma_t = 0$

$$\Rightarrow \sigma_t = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau\cos 2\theta = 0 \Rightarrow \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \tau\cos 2\theta.$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau}{\sigma_x - \sigma_y} \quad \Rightarrow \tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}$$



There are 2 values for 2θ differing by $180°$ for which $\tan 2\theta = 2\tau/(\sigma_x - \sigma_y)$. Let $2\theta_1$ & $2\theta_2$ be the soln.

$$\sin 2\theta_1 = \frac{2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} \quad \cos 2\theta_1 = \frac{\sigma_x-\sigma_y}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} \quad \sin 2\theta_2 = \frac{-2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} \quad \cos 2\theta_2 = \frac{-(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

To get principle stresses σ_1 & σ_2, substitute values of $2\theta_1$ & $2\theta_2$ in 'σ_n' expression.

Substituting $\theta = 2\theta_1$ in σ_n expression we get 'σ_1'

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \frac{(\sigma_x-\sigma_y)}{2}\cos 2\theta_1 + \tau\sin 2\theta_1 = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\left[\frac{\sigma_x-\sigma_y}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}\right] + \tau\left[\frac{2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}\right].$$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \frac{(\sigma_x-\sigma_y)^2}{2\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} + \left[\frac{2\tau^2}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}\right]\frac{2}{2} = \frac{\sigma_x+\sigma_y}{2} + \left[(\sigma_x-\sigma_y)^2+4\tau^2\right]\left[\frac{1}{2\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}\right]$$

$$\boxed{\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

Similarly substituting $\theta = 2\theta_2$ in σ_n expression we get 'σ_2'

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta_2 + \tau\sin 2\theta_2 = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\left[\frac{-(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}\right] + \tau\left[\frac{-2\tau}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}\right].$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \frac{(\sigma_x-\sigma_y)^2}{2\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}} - \left[\frac{2\tau^2}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}\right]\frac{2}{2} = \frac{\sigma_x+\sigma_y}{2} - \left[(\sigma_x-\sigma_y)^2+4\tau^2\right]\frac{1}{2\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

$$\boxed{\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}}$$

b.

THIN CYLINDER	THICK CYLINDER.
1> $t < d/10$.	1> $t > d/10$
2> Radial stresses are negligible.	2> Radial Stresses are not negligible
3> Hoop Stress is uniform across the c/s	3> Hoop Stress is not uniform across the c/s

c. GIVEN, $\sigma_x = 180MPa$ $\sigma_y = 120MPa$ $\tau = 80MPa$.

To FIND: $\theta_1 = ?$ $\theta_2 = ?$ $\sigma_1 = ?$ $\sigma_2 = ?$ $\tau_{max} = ?$ $\theta' = ?$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2} = \frac{180+120}{2} \pm \frac{1}{2}\sqrt{(180-120)^2+4\times80^2}$$

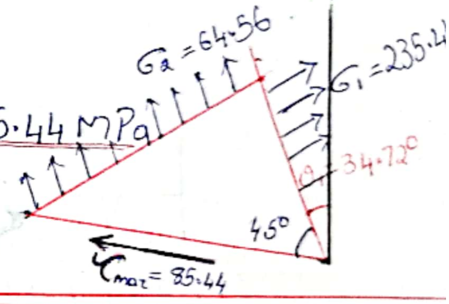
$\sigma_{1,2} = 150 \pm 85.44$ $\therefore$ $\sigma_1 = 235.44 MPa$ $\sigma_2 = 64.56MPa$.

$$\tan 2\theta_1 = \frac{2\tau}{\sigma_x-\sigma_y} = \frac{2\times80}{180-120} = 2.667 \Rightarrow 2\theta_1 = \tan^{-1}(2.667) \Rightarrow 2\theta_1 = 69.44$$

$\theta_1 = 34.72°$ $\theta_2 = \theta_1 + 90° = 124.72°$.

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2} = \frac{1}{2}\sqrt{(180-120)^2+4\times80^2} = 85.44MPa$$

Plane of max shear, $\theta' = \theta_1 + 45° = 79.72°$.

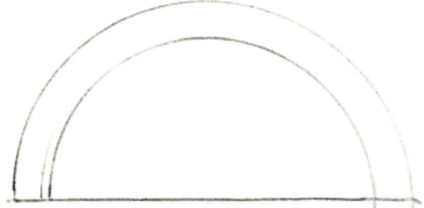


Q10.a Consider a semicircular ring element with internal radius 'x' & thickness 'δx'.

Let $P_x \rightarrow$ internal pressure, $P_x+\delta P_x \rightarrow$ external pressure.

Let L be the length of element. Then Bursting force

$= P_x(2xL) - (P_x+\delta P_x)[2(x+\delta x)L]$



If f_1 is hoop stress, Resisting force $= f_1(2\cdot\delta x\cdot L)$

For equilibrium, $f_1(\cancel{2}\cdot\delta x\cdot\cancel{L}) = P_x(\cancel{2}x\cancel{L}) - (P_x+\delta P_x)[\cancel{2}(x+\delta x)\cancel{L}]$

$\Rightarrow f_1\delta x = \cancel{P_x x} - \cancel{P_x x} - P_x\delta x - x\delta P_x - \underbrace{\delta P_x\delta x}_{\to 0} = -P_x\delta x - x\delta P_x$

$\Rightarrow f_1\delta x + P_x\delta x + xP_x = 0$ $\rightarrow f_1 + P_x + x\,\delta P/\delta x = 0 \rightarrow (1)$.

If $e_2 \rightarrow$ Longitudinal Strain & $f_2 \rightarrow$ Longitudinal Stress. then $e_2 = \frac{f_2}{E} - \frac{\mu f_1}{E} + \frac{\mu P_x}{E}$

According to Lame theory. e_2 is constant $\therefore$ since f_2 is constant $= f_2/E - \mu/E\,(f_1 - P_x)$

$f_1 - P_x$ should be constant. Let $f_1 - P_x = 2a$ be a constant.

$\therefore f_1 = P_x + 2a$.

$\therefore$ (1) becomes $(P_x+2a) + P_x + x(\delta P/\delta x) = 0 \Rightarrow 2P_x + 2a + x(\delta P/\delta x) = 0 \Rightarrow x\frac{\delta P}{\delta x} = -2(P_x+a)$

$\therefore \frac{\delta P}{P_x+a} = \frac{-2\delta x}{x}$ Integrating both sides we get $\log(P_x+a) = -2\log x + C$.

$\log(P_x+a) = -2\log x + \log b \Rightarrow \log(P_x+a) = -\log x^2 + \log b = \log(b/x^2)$ $\therefore P_x + a = b/x^2$ where $C = \log b$

$\boxed{P_x = \frac{b}{x^2} - a}$ and $(P_x+2a) = \frac{b}{x^2} - a + 2a$ $\therefore \boxed{f_1 = \frac{b}{x^2} + a}$. THESE EQN are CALLED LAME'S EQUATION

$p = 1\ N/mm^2$

b. GIVEN $L = 3.25m$ $d = 1m$ $t = 10mm$. $E = 2\times10^3\ N/mm^2$ $\frac{1}{m} = \mu = 0.3$.

TO FIND: $f_1 = ?$ $f_2 = ?$ $q_{max} = ?$ $\delta d = ?$ $\delta L = ?$ $\frac{\delta V}{V} = ?$ $\delta V = ?$

Hoop stress, $f_1 = Pd/2t = 1\times1000/(2\times10) = \underline{50\ N/mm^2}$.

Longitudinal Stress, $f_2 = Pd/4t = 1\times1000/(4\times10) = \underline{25\ N/mm^2}$.

MAX shear stress, $q_{max} = (f_1 - f_2)/2 = (50-25)/2 = \underline{12.5\ N/mm^2}$.

$e_1 = \frac{\delta d}{d} = \frac{1}{E}[f_1 - \mu f_2] \Rightarrow \frac{\delta d}{d} = \frac{1}{2\times10^3}[50 - 0.3\times25] \Rightarrow \frac{\delta d}{1000} = 0.02125 \therefore \underline{\delta d = 21.25mm}$.

$e_2 = \frac{\delta L}{L} = \frac{1}{E}[f_2 - \mu f_1] \Rightarrow \frac{\delta L}{3250} = \frac{1}{2\times10^3}[25 - 0.3\times50] \Rightarrow \delta L = 3250\times(5\times10^{-3}) \therefore \underline{\delta L = 16.25mm}$

$\frac{\delta V}{V} = 2e_1 + e_2 = 2\times0.02125 + 5\times10^{-3} = \underline{0.0475}$

$\delta V = 0.0475\times V = 0.0475\times\frac{\pi}{4}\times1000^2\times3250 = \underline{121.25\times10^6\ mm^3}$.

(HARSHAVARDHAN. V. S)

HEAD
Dept of Civil Engg
KLS V.D.I.T. Haliyal

Dean, Academics
KLS VDIT, HALIYAL