10.07.2023

Model Question Paper-I with effect from 2022 (CBCS Scheme)

USN | | | | | | | | | | |

Second Semester B.E Degree Examination
Mathematics-II for Computer Science Engineering-BMATS201

TIME: 03 Hours **Max. Marks: 100**

Note:
1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **module.**
2. VTU Formula Hand Book is permitted.
3. M: Marks, L: Bloom's level, C: Course outcomes.

		Module -1	**M**	**L**	**C**
Q.01	a	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$	7	L2	CO1
	b	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$ by changing into polar coordinates.	7	L2	CO1
	c	Show that $\beta(m,n) = \frac{\gamma(m)\gamma(n)}{\gamma(m+n)}$	6	L2	CO1
		OR			
Q.02	a	Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ by changing the order of integration	7	L2	CO1
	b	Using double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	7	L3	CO1
	c	Write the codes to find the volume of the tetrahedron bounded by the planes $x = 0,\ y = 0$ *and* $z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ using Mathematical tools.	6	L3	CO5
		Module-2			
Q. 03	a	Find $\nabla\phi$, if $\phi = x^3 + y^3 + z^3 - 3xyz$ at the point $(1, -1, 2)$	7	L2	CO2
	b	If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $div\, \vec{F}$ and $curl\, \vec{F}$	7	L2	CO2
	c	Express the vector $\vec{A} = z\hat{\imath} - 2x\hat{\jmath} + y\hat{k}$ in cylindrical coordinates.	6	L2	CO2
		OR			
Q.04	a	Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.	7	L2	CO2
	b	Show that the spherical coordinate system is orthogonal	7	L3	CO2
	c	Using the Mathematical tools, write the codes to find the gradient of	6	L3	CO5

		$\emptyset = x^2yz$.			
		Module-3			
Q. 05	a	Prove that the subset $W = \{(x, y, z) \mid x - 3y + 4z = 0\}$ of the vector space R^3 is a subspace of R^3.	7	L3	CO3
	b	Determine whether the matrix $\begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ in the vector space M_{22} of 2×2 matrices.	7	L2	CO3
	c	Find the matrix of the linear transformation $T: V_2(R) \rightarrow V_3(R)$ such that $T(-1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$.	6	L2	CO3
		OR			
Q. 06	a	Show that the set $S = \{(1, 2, 4), (1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is linearly dependent.	7	L2	CO3
	b	Let P_n be the vector space of real polynomial functions of degree $\leq n$. Show that the transformation $T: P_2 \rightarrow P_1$ defined by $T(ax^2 + bx + c) = (a + b)x + c$ is linear.	7	L2	CO3
	c	Verify the Rank-nullity theorem for the linear transformation $T: V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (y - x, y - z)$.	6	L2	CO3
		Module-4			
Q. 07	a	Find the real root of the equation $3x = \cos x + 1$ correct to three decimal places using Newton's Raphson method.	7	L2	CO4
	b	Find y at $x = 5$ if $y(1) = -3,\ y(3) = 9, y(4) = 30, y(6) = 132$ using Lagrange's interpolation formula.	7	L2	CO4
	c	Evaluate $\int_0^1 \frac{1}{1+x} dx$ by taking 7 ordinates and by using Simpson's 3/8 rule.	6	L3	CO4
		OR			
Q. 08	a	Find an approximate root of the equation $x^3 - 3x + 4 = 0$ using the method of false position, correct to three decimal places which lie between -3 and -2. (Carry out three iterations).	7	L2	CO4

	b	Using Newton's appropriate interpolation formula, find the values of y at $x = 8$ and at $x = 22$ from the following table: <table><tr><td>x</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr><tr><td>y</td><td>7</td><td>11</td><td>14</td><td>18</td><td>24</td><td>32</td></tr></table>	7	L3	CO4
	c	Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx$ by using the Trapezoidal rule by taking 7 ordinates.	6	L2	CO4
Module-5					
Q. 09	a	Solve $y'(x) = 3x + \frac{y}{2}$, $y(0) = 1$ then find $y(0.2)$ with $h = 0.2$ using modified Euler's method.	7	L2	CO4
	b	Apply Runge-Kutta method of fourth order to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$.	7	L2	CO4
	c	Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1, y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$ compute $y(0.4)$ using Milne's method.	6	L2	CO4
OR					
Q. 10	a	Employ Taylor's series method to obtain approx. value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$.	7	L2	CO4
	b	Using the Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2$.	7	L2	CO4
	c	Write the Mathematical tool codes to solve the differential equation $\frac{dy}{dx} - 2y = 3e^x$ with $y(0) = 0$ using the Taylors series method at $x = 0.1(0.1)0.3$.	6	L3	CO5

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	Model qp Solution and Scheme for award of marks	

AY: 2022-23

Department: Mathematics
Subject with Sub. Code: Mathematics-II for Computer Science and Engineering Stream(BMATS201)
Semester/Division: II/D,E
Name of Faculty: Dr. Satish P. Hande

Q.No.	Solution and Scheme	Marks
1 a)	Let $I=\int_{x=0}^{a}\int_{0=y}^{x}\int_{z=0}^{x+y} e^{x+y+z}\,dz\,dy\,dx$	1M
	$=\int_{0}^{a}\int_{y=0}^{x} e^{x+y}\left[e^{z}\right]_{0}^{x+y}dy\,dx=\int_{0}^{a}\int_{0}^{x} e^{x+y}\left[e^{x+y}-1\right]dy\,dx$	2M
	$=\int_{x=0}^{a}\int_{y=0}^{x}\left[e^{2x+2y}-e^{x+y}\right]dy\,dx$	1M
	$=\int_{x=0}^{a}\left\{e^{2x}\left(\frac{e^{2y}}{2}\right)_{0}^{x}-e^{x}\left(e^{y}\right)_{0}^{x}\right\}dx$	
	$=\int_{x=0}^{a}\left(\frac{e^{4x}}{2}-\frac{3}{2}e^{2x}+e^{x}\right)dx=\left[\frac{e^{4a}}{8}-\frac{3e^{2a}}{4}+e^{a}\right]-\left[\frac{1}{8}-\frac{3}{4}+1\right]$	2m
	Thus $\boxed{I=\frac{1}{8}\left[e^{4a}-6e^{2a}+8e^{a}-3\right]}$	1M
b)	Put $x=r\cos\theta$ $y=r\sin\theta$ $x^2+y^2=r^2$ $dx\,dy=r\,dr\,d\theta$	1M
	$I=\int_{0}^{\infty}\int_{0}^{\infty}e^{-(x^2+y^2)}dx\,dy=\int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty}e^{-r^2}r\,dr\,d\theta$	2M
	put $r^2=t$ $r\,dr=\frac{dt}{2}$ t varies from 0 to ∞	1M
	$I=\int_{\theta=0}^{\pi/2}\int_{t=0}^{\infty}e^{-t}\frac{dt}{2}d\theta$	
	$=\frac{1}{2}\int_{\theta=0}^{\pi/2}\left[-e^{-t}\right]_{0}^{\infty}d\theta=\frac{1}{2}\int_{\theta=0}^{\pi/2}d\theta$	2M
	$\therefore \boxed{I=\pi/4}$	1M

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Q.No.	Solution and Scheme	Marks

c) By the definition of Beta & Gamma function

$$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \, \cos^{2n-1}\theta \, d\theta$$

$$\Gamma(m) = 2\int_0^{\infty} e^{-x^2} x^{2n-1} dx \; ; \quad \Gamma(m) = 2\int_0^{\infty} e^{-y^2} y^{2m-1} dy$$ 1M

$$\Gamma(m+n) = 2\int_0^{\infty} e^{-r^2} r^{2(m+n)-1} dr$$

$$\Gamma(m)\Gamma(n) = 4\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} x^{2n-1} y^{2m-1} dx\, dy$$ 2M

Changing to polar coordinates we get.

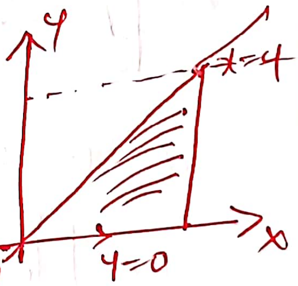
$$\Gamma(m)\Gamma(n) = 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2} r^{2m+2n-1} \sin^{2m-1}\theta \cos^{2n-1}\theta \, dr\, d\theta$$

$$= \left[2\int_{r=0}^{\infty} e^{-r^2} r^{2(m+n)-1} dr\right]\left[2\int_{\theta=0}^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta\right]$$ 2M

$$= \Gamma(m+n)\,\beta(m,n)$$

$$\therefore \boxed{\beta(m,n) = \frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}}$$ 1M

2a) Let $I = \int_{y=0}^{a}\int_{\cancel{y=x}\; x=y}^{a} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$



Here x varies from $x=0$ to $x=a$

& y varies from $y=x$ to $y=a$

after changing the order of integration

y varies from $y=0$ to $y=x$ & 1M

x varies from $x=0$ to $x=a$ 1M

$$I = \int_{x=0}^{a}\int_{y=0}^{x} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx. = \int_{y=0}^{a}\int_{x=0}^{y} \frac{x}{\sqrt{x^2+y^2}}\, dx\, dy$$

$$= \int_{y=0}^{a} \frac{1}{2}\left[\log(x^2+y^2)\right]_0^y$$

Put $x^2+y^2 = t$

$2x\,dx = dt$

$x\,dx = dt/2$

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Q.No.	Solution and Scheme	Marks
	$I = \int_{y=0}^{a} \int_{x=0}^{y} \frac{dt/2}{\sqrt{t}}\, dy$	
	$= \int_{y=0}^{a} \left[\sqrt{x^2+y^2}\right]_{x=0}^{y} dy$	
	$= \int_{y=0}^{a} \left[\sqrt{2}y - y\right] dy = (\sqrt{2}-1)\left[\frac{y^2}{2}\right]_0^a$	
	$\boxed{I = \frac{a[\sqrt{2}-1]}{2}}$	
a)	By changing the order of integration	
	$I = \int_{x=0}^{a} \int_{y=0}^{x} x \cdot \frac{1}{x^2+y^2}\, dy\, dx$	1M
	$= \int_{x=0}^{a} x \cdot \frac{1}{x} \left[\tan^{-1}(y/x)\right]_{y=0}^{x} dx$	2M
	$= \int_{x=0}^{a} \left[\tan^{-1}1 - \tan^{-1}0\right] dx$	1M
	$= \int_0^a \pi/4\, dx = \frac{\pi}{4}[x]_0^a$	1M
	$\boxed{I = \frac{\pi a}{4}}$	
b)	Area $= \iint_R dx\,dy = 4 \times$ Area in the first quadrant	1M
	$A = 4 \int_{x=0}^{a} \int_{y=0}^{(b/a)\sqrt{a^2-x^2}} dy\, dx = 4 \int_0^a [y]_0^{b/a\sqrt{a^2-x^2}} dx$	2M
	$= 4 \int_0^a \frac{b}{a}\sqrt{a^2-x^2}\, dx = \frac{4b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}(x/a)\right]_0^a$	2M
	$= \frac{4b}{a}\left[0 + \frac{a^2}{2}\left[\sin^{-1}1 - \sin^{-1}0\right]\right] = \pi ab$	2M

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Q.No.	Solution and Scheme	Marks

c)	#Reading length of major axis
	major = flot(input ("Enter length of major axis:"))
	# Reading length of minor axix.
	minor= float (input ("Enter length of minor axis :"))
	# calculating area of Ellipse
	area = 3.141592 * major * minor
	# Display result
	print("Area of an ellipse = " area)
		2M
		
		2M
		
		1M
		1M

3a)	Given $\phi = x^3+y^3+z^3-3xyz$ $\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$ $= (3x^2-3yz)i+(3y^2-3xz)j+(3z^2-3xy)k.$ at the pt (1, -1, 2) $\nabla\phi_{(1,-1,2)} = (3+6)i+(3-2)j+(12+3)k.$ $= 9\hat{i}+\hat{j}+15k.$	2M 1M 4M
b)	Given $\vec{F} = \nabla(x^3+y^3+z^3-3xyz)$ $= \underbrace{(3x^2-3yz)}_{F_1}i+\underbrace{(3y^2-3xz)}_{F_2}j+\underbrace{(3z^2-3xy)}_{F_3}k$ $\operatorname{div}\vec{F} = \frac{\partial F_1}{\partial x}+\frac{\partial F_2}{\partial y}+\frac{\partial F_3}{\partial z} = 6x+6y+6z = 6(x+y+z)$ $\operatorname{curl}\vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2-3yz & 3y^2-3xz & 3z^2-3xy \end{vmatrix}$	2M 1M 2M

Q.No.	Solution and Scheme	Marks
	$= i[-3x+3x] - j[-3y+3y] + k[-3z+3z]$ $\text{curl } \vec{F} = 0$	2M
c)	Given $\vec{A} = z\hat{i} - 2x\hat{j} + y\hat{k}$ The required cylindrical coordinates are (ρ, ϕ, z) can be obtained by in matrix form	1M
	$\begin{bmatrix} \rho \\ \phi \\ z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z \\ -2x \\ y \end{bmatrix}$	2M
	$\rho = z\cos\phi - 2x\sin\phi$ $\phi = -z\sin\phi - 2x\cos\phi$ $z = y$	3M
4. a)	Given $\phi = x^2yz + 4xz^2$ $\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$	1M
	$= (2xyz + 4z^2)\, i + (x^2z)\, j + (x^2y + 8xz)\, k$	2M
	at $(1, -2, 1)$ $\nabla\phi_{(1,-2,1)} = (-4+4)i + (1)j + (-2+8)k$ $= j + 6k$	2M
	D.D $= \nabla\phi \cdot \hat{a} = (j+6k)\cdot(2i - j - 2k) = 0 - 1 - 12$ $= -13$	2M

Q.No.	Solution and Scheme	Marks
b)	1) Cartesian coordinate system	
	Here $u = x,\ v = y,\ w = z$ $\quad \therefore \vec{r} = xi + yj + zk$	1M
	$\frac{\partial \vec{r}}{\partial u} = \frac{\partial \vec{r}}{\partial x} = \frac{\partial}{\partial x}(xi + yj + zk) = i$	
	$\frac{\partial \vec{r}}{\partial v} = \frac{\partial \vec{r}}{\partial y} = j,\quad \frac{\partial \vec{r}}{\partial w} = k$	2M
	Scale factor	
	$h_1 = \left\lvert \frac{\partial \vec{r}}{\partial u} \right\rvert = \lvert i \rvert = 1,\quad h_2 = \left\lvert \frac{\partial \vec{r}}{\partial v} \right\rvert = \lvert \vec{j} \rvert = 1$	
	lly $\left\lvert \frac{\partial \vec{r}}{\partial w} \right\rvert = \lvert \vec{k} \rvert = 1$	1M
	Therefore the base vectors are	
	$\hat{e}_1 = \frac{1}{h_1} \cdot \frac{\partial \vec{r}}{\partial u} = \hat{i},\quad \hat{e}_2 = \frac{1}{h_2} \frac{\partial \vec{r}}{\partial v} = \hat{j},\quad \hat{e}_3 = \hat{k}$	1M
	Further $\hat{e}_1 \cdot \hat{e}_2 = \hat{i} \cdot \hat{j} = 0,\quad \hat{e}_2 \cdot \hat{e}_3 = 0,\quad \hat{e}_3 \cdot \hat{e}_1 = 0$	
	$\hat{e}_1 \times \hat{e}_2 = i \times j = \hat{k}$, lly $\hat{e}_2 \times \hat{e}_3 = \hat{e}_1,\ \hat{e}_3 \times \hat{e}_1 = \hat{e}_2$	2M
	$\therefore$ spherical coordinate system is orthogonal.	
c)	import scipy as sp	
	import numdifftools as nd	1M
	def h(x): return sp.array(x[0][y [0]]**2, 2*x[1][1]**2*y[1][2][0] *, 3*y[2]*z[1]])	2M
	def curl(f,x): jac = nd.Jacobian[f][x] return sp.array[jac [2,1] -Jac[1,2], Jac[0,2] - jac[2,0], jac[1,0]-Jac[0,1]])	2M
	X = sp.array[[1, 2, 3]] curl[h, x]	1M

Q.No.	Solution and Scheme	Marks

5 a) Given $W = \{(x, y, z) \mid x - 3y + 4z = 0\}$ — 1M

Let $u = (x_1, y_1, z_1)$, $v = (x_2, y_2, z_2) \in W$

$\therefore x_1 - 3y_1 + 4z_1 = 0 \Rightarrow ax_1 - 3ay_1 + 4az_1 = 0$

$x_2 - 3y_2 + 4z_2 = 0 \qquad bx_2 - 3by_2 + 4bz_2 = 0$ — 2M

$\forall a, b \in F$ — 1M

$au + bv = (ax_1 + bx_2) - 3ay_1 - 3by_2 + 4az_1 + 4bz_2$

$= (ax_1 - 3ay_1 + 4az_1) + (bx_2 - 3by_2 + 4bz_2)$

$= a(x_1 - 3y_1 + 4z_1) + b(x_2 - 3y_2 + 4z_2)$ — 2M

$= a \times 0 + b \times 0 = 0$

$\therefore au + bv \in R^3$. — 2M

$\therefore W$ is subspace of R^3.

b) $\begin{bmatrix} -1 & 7 \\ 8 & 1 \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + b\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix} + c\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ — 2M

$= \begin{bmatrix} a & 0 \\ 2a & a \end{bmatrix} + \begin{bmatrix} 2b & -3b \\ 0 & 2b \end{bmatrix} + \begin{bmatrix} 0 & c \\ 2c & 0 \end{bmatrix}$

$\begin{bmatrix} -1 & 7 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} a + 2b & -3b + c \\ 2a + 2c & a + 2b + 0 \end{bmatrix}$ — 3M

$\left.\begin{array}{l} a + 2b = -1 \\ -3b + c = 7 \\ 2a + 2c = 8 \\ a + 2b = -1 \end{array}\right\}$ $\qquad$ $\begin{array}{l} a + 2b + 0.c = -1 \\ 0.a - 3b + c = 7 \\ 2a + 0.b + 2.c = 8 \end{array}$ — 1M

Solving

$\therefore \boxed{a = 3, \; b = -2, \; c = 1}$ — 1M

$\therefore$ The given matrix is a linear combination of given matrices.

c) The ordered standard basis of R^2 & R^3 are — 1M

$\beta = \{(1, 0)\ (0, 1)\}$ — 1M

$\beta' = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

Given $T(1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$ — 1M

$(-1, 0, 2) = -1(1, 0, 0) + 0(0, 1, 0) + 2(0, 0, 1)$ — 1M

$(1, 2, 1) = 1(1, 0, 0) + 2(0, 1, 0) + 1(0, 0, 1)$ — 1M

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$\therefore$ The matrix of linear transformation is

$$\begin{bmatrix} -1 & 1 \\ 0 & 2 \\ 2 & 1 \end{bmatrix}$$ 1M

6] a) Consider the linear Combination

$au + bv + cw + dx = 0.$ 1M

Where $u = (1,2,4)$, $v = (1,0,0)$, $w = (0,1,0)$, $x = (0,0,1)$

$$a\begin{bmatrix}1\\2\\4\end{bmatrix} + b\begin{bmatrix}1\\0\\0\end{bmatrix} + c\begin{bmatrix}0\\1\\0\end{bmatrix} + d\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$ 2M

$$A = \begin{bmatrix} 1 & \cancel{0}1 & 0 & 0 \\ 2 & \cancel{0} & 1 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -4 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix}$$ which gives

$$\left.\begin{aligned} a+b &= 0 \\ -2b+c &= 0 \\ -2c+d &= 0 \end{aligned}\right\} \qquad \begin{aligned} a+b+0.c+0.d &= 0 \\ 0.a-2b+c+0.d &= 0 \\ 0.a+0.b-2c+d &= 0 \end{aligned}$$ 3M

$\therefore$ we have 3 equation in 4 unknowns.

$\rho[A] = 3 <$ number of unknowns $= 4$. 1M

$\therefore$ Set 'S' is linearly dependent.

b) Given $T: P_2 \to P_1$ defined by $T(ax^2 + bx + c) = (a+b)x + c$. Let $u = (x_1, y_1, z_1)$, $v = (x_2, y_2, z_2)$

- Let $T(u) = ax_1^2 + bx_1 + c$, $T(v) = ax_2^2 + bx_2 + c$ 2M

$T(ax_1^2 + bx_1 + c) = (a+b)x_1 + c$

$T(ax_2^2 + bx_2 + c) = (a+b)x_2 + c.$

(5)

Q.No.	Solution and Scheme	Marks
	$T(u+v) = T[ax_1^2+bx_1+c+ax_2^2+bx_2+c]$	
	$= T[ax_1^2+\cancel{b}\, ax_2^2 + bx_1 + bx_2, c+c]$.	
	$= \cancel{T[a}\ \cancel{x}[(a+b)x_1+c]+[(a+b)x_2+c]$.	3m
	$= T(u)+T(v)$	
	$\cancel{T(ru) = r}$ $r\in R$	
	$T(ru) = T[arx_1^2+brx+cr]$	
	$= (ar+br)x+cr$.	
	$= r[(a+b)x+c]$	2m
	$= rT(u)$	
	$\therefore T$ is linear transformatn.	
c)	The ordered basis of R^3 is	
	$\{(1,0,0),(0,1,0),(0,0,1)\}$	
	$T(1,0,0) = (-1,0)$	
	$T(0,1,0) = (1,1)$	2m
	$T(0,0,1) = (0-0,0-1) = (0,-1)$	
	$\therefore$ The vectors $(-1,0),(1,1)$ & $(0,-1)$ are L.I.	
	$\forall x,y\in R$.	1M
	$x(-1,0)+y(1,1)+z(0,-1) = (0,0)$	
	$[-x+y, y-z] = (0,0,0)$	2M
	$y-x=0,\ y-z=0$	
	$x=y,\ y=z$.	
	$\therefore$ ~~$x=0, y=0, z=0$~~	
	The vectors are not linearly independent	1M
	$\therefore$ The theorem does not hold good.	

Q.No.	Solution and Scheme	Marks
7 a)	Let $f(x) = \cos x + 1 - 3x$ $f(0) = 270$, $f(1) = -1.46 < 0$ $\cancel{f(0)}\ f(0.6) = \cancel{0.1919}$ $= 0.02533$	1M
	Let $x_0 = 0.6$ $f'(x) = -\sin x - 3$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.6 - \frac{f(0.6)}{f'(0.6)}$	–
	$= 0.6 + \frac{[\cos(0.6) + 1 - 3\times 0.6]}{3 + \sin(0.6)}$	2m
	$x_1 = \cancel{0.6102}\ 0.6071$ $x_2 = 0.60\cancel{02}71 + \frac{f(0.60\cancel{02}71)}{f'(0.\cancel{6102})\ 0.6071}$	2m
	$\cancel{x_2 = 0.6233}$ $x_2 = 0.6071$ $\therefore$ The real root of the equation correct to three decimal places is $\boxed{x = 0.6071}$	2m
b)	Given $x_0 = 1$ $x_1 = 3$, $x_2 = 4$, $x_3 = 6$ $y_0 = -3$ $y_1 = 9$, $y_2 = 30$, $y_3 = 132$ By the Lagranges Interpolation form $y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1$ $+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$	2m
	At $x = 5$ $y(5) = \frac{2\times 1\times(-1)}{(-2)(-3)(-5)} \times (-3) + \frac{(4)(1)(-1)}{(2)(-1)(-3)} \times 9 +$	2m
	$+ \frac{(4)(2)(-1)}{(3)(1)(-2)} \times 30 + \frac{(4)(2)(1)}{(5)(3)(2)} \times 132$	2m

Q.No.	Solution and Scheme	Marks

$y(5) = \frac{1}{5} - 6 + 40 + \frac{17.6}{5}$

$\boxed{y(5) = 69.4}$ — 1M

c) By Simpson's 3/8 rule

$I = \frac{3h}{8}\left[y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + 2y_6 + \cdots\right]$ — 1M

$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6} = 0.16$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6
x	0	1/6	2/6	3/6	4/6	5/6	6/6
$y = 1/1+x$	1 ~~0.8571~~	0.8571	0.75	0.66	0.6	0.5454	0.5

2M

$I = \frac{3 \times 1/6}{8}\left[1 + 3\times 0.8571 + 3\times 0.75 + 2\times 0.66 + 3\times 0.6 + 3\times 0.5454 + 0.5\right]$ — 2M

$\int_0^1 \frac{dx}{1+x} = \frac{1}{16} \times 11.07$

$\therefore \boxed{\int_0^1 \frac{dx}{1+x} = 0.6918}$ — 1M

8 a) Let $f(x) = x^3 - 3x + 4$ $\quad a = -3,\ b = -2$

$f(-3) = -14 < 0$, $f(-2) = 6 > 0$

$\therefore$ The root lies in the interval $(-3, -2)$

$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(-3)(6) - (-2)(-14)}{6 + 14}$

$x_1 = \frac{-18 - 28}{20} = \frac{46}{20} \Rightarrow \boxed{x_1 = \cancel{2.3}\ -2.3}$

$f(-2.3) = -1.267 < 0$

$\therefore$ The root lies in $(-2.3, -2)$. — 1M

Q.No.	Solution and Scheme	Marks
	$x_2 = \dfrac{(-2.3\times 6)-(-2)\times(-1.267)}{6+\cancel{0}\,1.267} = \dfrac{16.334}{7.267}$	
	$x_2 = -2.2476$	1M
	$f(x_2) = -0.6114 < 0.$ $\therefore$ The new interval is $(-2.2476, -2)$	2M
	$x_3 = -2.2247$ $f(x_3) = -0.03331 < 0$ $\therefore$ The new interval is $(-2.2247, -2)$	2M
	$x_4 = -2.2128$ $f(x_4) = -0.1965 < 0$	
	$x_5 = -2.2060$ The real root is $\boxed{x = -2.2060}$	1M
b)	Preparing the difference table.	3M

x	y	I D	II D	III D	IV D	V D
0	7	4	-1	2	-1	0
5	11	3	1	1	-1	
10	14	4	2	0		
15	18	6	2			
20	24	8				
25	32					

From the difference table.

$\Delta y_0 = 4,\quad \Delta^2 y_0 = -1 \quad \Delta^3 y_0 = 2,\quad \Delta^4 y_0 = -1$

$\nabla y_5 = 8,\quad \nabla^2 y_5 = 2,\quad \nabla^3 y_5 = 0,\quad \nabla^4 y_5 = -1$

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Q.No.	Solution and Scheme	Marks

To find y at $x=8$

$x_p = 8 = x_0 + ph \Rightarrow p = \frac{x_p - x_0}{h} = \frac{8-0}{5} = 1.6$

By NFIF

$y(p) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$

$y(8) = 7 + (1.6\times 4) + \frac{(1.6)(0.6)}{2!}\times 2 + \frac{(-0.38)}{6}\times 2 + \frac{0.532}{24}(-1)$ 2m

~~y(8) = 11.21~~

$\boxed{y(8) = 12.77}$

To find y at $x=22$

$x_p = 8 = x_n + ph \Rightarrow p = \frac{x_p - x_n}{h} = \frac{22-25}{5} = \frac{-3}{5} = -0.6$

By NBIF   2m

$y(p) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$

$y(22) = 32 + (-0.6)\times 8 + (-0.24) + 0 + 0.0336$

$\boxed{y(22) = 26.99}$

c) $h = \frac{b-a}{n} = \frac{\pi/2 - c}{6} = \frac{\pi^c}{12} = 15^\circ$ 1m

x°	0	15	30	45	60	75	90
$\cos x$	1	0.9828	0.9306	0.8409	0.7071	0.5087	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

2m

By trapezoidal rule

$I = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$

$= \frac{15}{2}\left[1 + 2(0.9828 + 0.9306 + 0.8409 + 0.7071 + 0.5087) + 0\right]$ 2m

$\boxed{\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta = 1.1702}$ 1m

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(8)

Q.No.	Solution and Scheme	Marks
9 a)	Given $\frac{dy}{dx} = 3x+\frac{y}{2}$ $x_0=0$ $y_0=1$ $x_1=0.2$ $h=0.2$	1M
	$y_1^{(0)} = y_0 + h\,f(x_0, y_0)$ $= 1+0.2\left[3x_0+\frac{y_0}{2}\right] = 1+0.2\left[3\times 0+\frac{1}{2}\right]$ $\boxed{y_1^{(0)} = 1.1}$	2M
	$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f\{x_1, y_1^{(0)}\}\right]$ $= 1+\frac{0.2}{2}\left[0.5+\left\{3\times 0.2+\frac{1.1}{2}\right\}\right]$ $= 1+0.1\left[0.5+\left\{0.6+\frac{1.1}{2}\right\}\right]$	2M
	$y_1^{(1)} = 1.165$ $y_1^{(2)} = 1.1682$ $y_1^{(3)} = 1.1684$ $y_1^{(4)} = 1.1684$ as y_3 & y_4 are identical $\therefore\ \boxed{y = 1.1684}$	2m
b)	Given $\frac{dy}{dx} = x+y = f(x,y)$ $x_0=0,$ $y_0=1$ $x_1=0.2.$	1M
	Let $h=0.2$	1M
	$k_1 = h\,f(x_0,y_0) = 0.2\times f(0,1) = 0.2[0+1] = 0.2$	1M
	$k_2 = h\,f\left[x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right] = 0.2f[0.1, 1.1] = 0.24$	1M
	$k_3 = h\,f\left[x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right] = 0.2f[0.1, 1.12] = 0.244$	1m
	$k_4 = h\,f[x_0+h, y_0+k_3] = 0.2f[0.2, 1.244] = 0.2888$	
	$k = \frac{1}{6}[k_1+2k_2+2k_3+k_4] = 0.2428$	
	$y_1 = y_0 + k$ $y(0.2) = 1+0.2428$ $\boxed{y(0.2) = 1.2428}$	2M

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Q.No.	Solution and Scheme	Marks
c)	Given $\frac{dy}{dx} = xy + y^2$	
	x y $y' = xy + y^2$	
	$x_0 = 0$ $y_0 = 1$ $y_0' = 1$	
	$x_1 = 0.1$ $y_1 = 1.1169$ $y_1' = 1.3591$	
	$x_2 = 0.2$ $y_2 = 1.2773$ $y_2' = 1.8869$	1M
	$x_3 = 0.3$ $y_3 = 1.5049$ $y_3' = 2.7161$	
	$x_4 = 0.4$ $y_4 = y(0.4) = ?$	
	By Milnes predictor formula.	
	$y_4^{(P)} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$	
	$= 1 + \frac{4\times 0.1}{3}[2\times 1.3591 - 1.8869 + 2\times 2.7161]$	
	$y_4^{(P)} = 1.8351$	2M
	$y_4' = 4.1016$	
	$y_4^{(C)} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$	
	$= 1.2773 + \frac{0.1}{3}[1.8869 + (2.7161)\times 4 + 4.1016]$	2M
	$y_4^{(C)} = 1.8390$	
	$y_4' = 4.1175$	
	$\boxed{y_4^{(C)} = 1.8395}$	1M

Q.No.	Solution and Scheme	Marks

10 a) Given $\frac{dy}{dx} = 2y+3e^{x}$ $\quad x_0=0,\ y_0=0 \quad h=0.2$

$x_1 = x_0+h = 0.2$

consider

$y_1 = 2y+3e^{x} \Rightarrow y_1(0) = 2\times 0+3e^{0} = 3$ — 1M

$y_2 = 2y_1+3e^{x} \Rightarrow y_2(0) = 2\times 3+3e^{0} = 9$ — 1M

$y_3 = 2y_2+3e^{x} \Rightarrow y_3(0) = 2\times 9+3e^{0} = 21$ — 1M

$y_4 = 2y_3+3e^{x} \Rightarrow y_4(0) = 2\times 21+3e^{x} = 45$ — 1M

By Taylor's series formula.

$y = y(0)+(x-\cancel{x_0})y_1(0)+\frac{(x-\cancel{x_0})^2}{2!}y_2(0)+\frac{(x-0)^3}{3!}y_3(0)+\cdots$ — 1M

$y = 0+x\times 3+\frac{x^2}{2!}(9)+\frac{x^3}{3!}(21)+\frac{x^4}{4!}(45)$

$y = 3x+\frac{9}{2}x^2+\frac{7}{2}x^3+\frac{15}{8}x^4+\cdots$

$y(0.2) = (3\times 0.2)+\frac{9}{2}(0.2)^2+\frac{7}{2}(0.2)^3+\frac{15}{8}(0.2)^4$

$\boxed{y(0.2) = 0.811}$ — 2M

b) Given $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2} = f(x,y)$ $\quad x_0=0,\ y_0=1,\ h=0.2$

$k_1 = hf(x_0, y_0) = 0.2f(0,1) = 0.2$ — 1M

$k_2 = hf\left[x_0+\frac{h}{2},\ y_0+\frac{k_1}{2}\right] = 0.2f(0.1, 1.1) = 0.1967$ — 1M

$k_3 = hf\left[x_0+\frac{h}{2},\ y_0+\frac{k_2}{2}\right] = 0.2f(0.1; 1.0983) = 0.1967$ — 1M

$k_4 = hf[x_0+h,\ y_0+k_3] = 0.2f[0.2, 1.1967] = 0.1891$ — 1M

$k = \frac{1}{6}[k_1+2k_2+2k_3+k_4] = 0.1959$ — 1M

$y_1 = y(0.2) = y_0+k = 1+0.1959$

$\boxed{y(0.2) = 1.1959}$ — 2M

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Q.No.	Solution and Scheme	Marks

c)	from numpy import array
	def taylor(deriv, x, y, xStop, h):
	X=[ ]
	Y=[ ]
	X.append(x)
	Y.append(y)
	While X < x stop :
	D = deriv(X, Y)
	H = 1.0
	for j in range(3):
	H = H*h / (j+1)
	y = y + D[j] * H
	x = x + h
	X.append(x) # Append result to
	Y.append(y) # lists x and y.
	return array(X), array(Y) # convert lists that returns array
	# deriv = user - supplied function
	def derive(x, y):
	D = zeros (4,1)
	D[0] = [2* y[0] + 3* exp(x)]
	D[1] = [4* y[0] + 9* exp(x)]
	D[2] = [8* y(0)] + 21* exp (x)]
	D[3] = [16* y[0] + 45* exp (x)]
	return D
	x = 0.0
	xstop = 0.3
	y = array([0.0])
	h = 0.1
		2M
		
		1M
		
		2M

Q.No.	Solution and Scheme	Marks
	x, y = taylor (derive, x, y, xstop, h) print ("The required value are: at x = %0.2f, y= %0.5f, x= %0.2f, y= %0.5f, x = %0.2f, y= %0.5f" %(x[0], Y[0], x[1] , Y[1], X[2], Y[2]))	1M
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