KLS Vishwanathrao Deshpande Institute of Technology
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(Approved by AICTE, New Delhi, Affiliated to VTU, Belagavi)
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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	DR. MEENAL M. KALIWAL
Course Name	:	MATHEMATICS-II FOR EE ENGINEERING STREAM
Course Code	:	BMATE201
Year of Question Paper	:	JUNE / JULY 2023
Date of Submission	:	01-04-2024

Faculty Member

HoD

Head of the Department
KLS V.D.I.T. HALIYAL (U.K.)

Dean (Acad.)
10/04/2024.

CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐ **BMATE201**

Second Semester B.E./B.Tech. Degree Examination, June/July 2023

Mathematics – II for EEE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	**M**	**L**	**C**
Q.1	**a.**	Find the angle between the direction to the normals to the surface $x^2yz=1$ at the point (-1, 1, 1) and (1, -1, -1).	**7**	**L1**	**CO1**
	b.	If $\vec{F}=\text{grad}(xy^3z^2)$ find div $\vec{F}$ and curl $\vec{F}$ at the point (1, -1, 1).	**7**	**L2**	**CO1**
	c.	Find the divergence and curl of the vector $\vec{V}=(xyz)\hat{i}+(3x^2y)\hat{j}+(xz^2-y^2z)\hat{k}$ at the point (2, -1, 1)	**6**	**L1**	**CO1**
		OR			
Q.2	**a.**	Using Green's theorem evaluate $\int_C (y-\sin x)dx+\cos x dy$ where C is the plane triangle enclosed by the lines $y=0$, $x=\frac{\pi}{2}$ and $y=\frac{2}{\pi}x$.	**7**	**L3**	**CO1**
	b.	Use stoke's theorem for vector $\vec{F}=(x^2+y^2)\hat{i}-2xy\hat{j}$ taken around the rectangle bounded by x = 0, x = a, y = 0, y = b.	**7**	**L3**	**CO1**
	c.	Using modern mathematical tools, write the code to find the gradient of $\phi=x^2y+2xz-4$.	**6**	**L3**	**CO5**
		Module – 2			
Q.3	**a.**	Define a subspace. Show that the intersection of any two subspaces of a vector space V is also a subspace of V.	**7**	**L2**	**CO2**
	b.	Show that the set B = {(1, 1, 0) (1, 0, 1) (0, 1, 1)} is a basis of the vector space $V_3(R)$.	**7**	**L3**	**CO2**
	c.	Prove that $T:R^2\to R^3$ be defined by T(a, b) = (a+b, a-b, b) is a linear transform.	**6**	**L3**	**CO2**
		OR			
Q.4	**a.**	Show that the set S = {(1, 0, 1) (1, 1, 0) (−1,0,−1) } is linearly dependent in $V_3(R)$.	**7**	**L3**	**CO2**
	b.	Let $T:R^3\to R^3$ defined by T(x, y, z) = (x+y, x-y, 2x+z). Find the rank and nullity of T and verify rank of T + nullity of T = dim(R^3).	**7**	**L2**	**CO2**
	c.	Using the modern mathematical tool, write the code to find the dimension of subspace spanned by the vectors (1, 2, 3) (2, 3, 1) and (3, 1, 2)	**6**	**L3**	**CO5**

<table>
<tr><td colspan="6">Module – 3</td></tr>
<tr><td>Q.5</td><td>a.</td><td>Find the Laplace transform of,
(i) $e^{-4t}(2\cos 6t - 3\sin 5t)$
(ii) $\dfrac{\cos 2t - \cos 3t}{t}$</td><td>7</td><td>L1</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Find the Laplace transform of a square wave function,
$f(t) = \begin{cases} E & 0 \le t \le \frac{T}{2} \\ -E & \frac{T}{2} \le t \le T \end{cases}$
Show that $L\{f(t)\} = \frac{E}{S}\tanh\left(\frac{ST}{4}\right)$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Express $f(t) = \begin{cases} \sin t & 0 < t < \pi \\ \sin 2t & \pi < t < 2\pi \\ \sin 3t & t \ge 2\pi \end{cases}$ in terms of unit step function and hence find $L\{f(t)\}$.</td><td>6</td><td>L3</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.6</td><td>a.</td><td>Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)(s+3)}\right]$.</td><td>7</td><td>L1</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ using convolution theorem.</td><td>7</td><td>L3</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Solve by Laplace transform method $y'' + 4y' + 4y = e^{-t}$ with $y(0) = y'(0) = 0$</td><td>6</td><td>L3</td><td>CO3</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>Q.7</td><td>a.</td><td>Find the real root of the equation $x \log_{10} x = 1.2$ by using the Regula-Falsi method between 2 and 3 (three iterations).</td><td>7</td><td>L1</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$, find $\sin 52^\circ$ using Newton's forward interpolation formula.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Using Lagrange's interpolation formulae to find f(5) from the following data:<table><tr><td>x</td><td>1</td><td>3</td><td>4</td><td>6</td><td>9</td></tr><tr><td>f(x)</td><td>-3</td><td>9</td><td>30</td><td>132</td><td>156</td></tr></table></td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>Find the real root of the equation, $x \tan x + 1 = 0$ which is near to $x = \pi$ by using Newton-Raphson method.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Using Newton's divided difference formulae and find f(4) given the data :<table><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr><tr><td>f(x)</td><td>-4</td><td>2</td><td>14</td><td>158</td></tr></table></td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Evaluate $\int_0^{0.6} e^{-x^2}\,dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking seven ordinates.</td><td>6</td><td>L5</td><td>CO4</td></tr>
</table>

Module – 5

<table>
<tr><td>Q.9</td><td>a.</td><td>Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ by using Taylor's method upto 4th degree terms and find the value of $y(1.1)$.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Using the Runge-Kutta method of order 4 find y at x = 0.1 given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ and $h = 0.1$</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Apply Milne's predictor corrector method, find $y(0.4)$ from $\frac{dy}{dx} = 2e^x y$
<table>
<tr><td>x</td><td>0</td><td>0.1</td><td>0.2</td><td>0.3</td></tr>
<tr><td>y</td><td>2.4</td><td>2.473</td><td>3.129</td><td>4.059</td></tr>
</table></td><td>6</td><td>L2</td><td>CO4</td></tr>
</table>

OR

<table>
<tr><td>Q.10</td><td>a.</td><td>Solve by using modified Euler's method $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y(0) = 1$ at $x = 0.2$.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Using the Runge-Kutta method of 4th order find $y(0.2)$ given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Using modern mathematical tools write the code to find the solution of $\frac{dy}{dx} = x - y^2$ at $y(0.1)$. Given that $y(0) = 1$ by Runge Kutta 4th order.</td><td>6</td><td>L2</td><td>CO5</td></tr>
</table>

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AY: 2023-24

Department: Electronics & Communication Engineering
Subject with Sub. Code: Mathematics-II for Electrical and Electronics Engineering Stream (BMATE201)
Semester / Branch / Division: II / ECE & EEE **Name of Faculty: Dr. Meenal Kaliwal**

Q.No.	Solution and Scheme	Marks
	Module - 1	
Q. No. 1a.	The given surface is $\phi(x,y,z) = x^2yz - 1$ At any point (x,y,z) of this surface, the normal is along the vector $\nabla\phi = 2xyz\,i + x^2z\,j + x^2y\,k$	1
	Therefore, at the point $P(-1,1,1)$, the normal is along the vector $a = [\nabla\phi]_P = -2i+j+k$	1
	and at the point $Q(1,-1,-1)$, the normal is along the vector $b = [\nabla\phi]_Q = 2i-j-k$	2
	If, θ is the angle between the directions of these normals, we have $\cos\theta = \dfrac{a.b}{\lvert a\rvert\,\lvert b\rvert} = \dfrac{-6}{\sqrt{6}\sqrt{6}} = -1.$	2
	This gives $\theta = \pi$ as the required angle. Thus, at the given points the normals to the given surface are in opposite directions.	1
		7

Faculty: Dr. Meenal M. Kaliwal (Mmf)

Head of the Department
Dept. of Electronic & Communication Engg.
KLS VDIT, HALIYAL (U.K.)

Q.No.	Solution and Scheme	Marks
1b.	Let, $\phi = xy^3z^2$	
	$\therefore \vec{F} = \nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$	
	$= y^3z^2 i + 3xy^2z^2 j + 2xy^3z k$	2
	div $\vec{F} = \nabla.\vec{F}$	
	$= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right).(y^3z^2 i + 3xy^2z^2 j + 2xy^3z k)$	
	$= \frac{\partial}{\partial x}(y^3z^2) + \frac{\partial}{\partial y}(3xy^2z^2) + \frac{\partial}{\partial z}(2xy^3z)$	
	$= 0 + 6xyz^2 + 2xy^3 = 2xy(3z^2+y^2)$	3
	$\therefore$ div $\vec{F}$ at $(1,-1,1) = -2(3+1) = -8$	
	Also, curl $\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^3z^2 & 3xy^2z^2 & 2xy^3z \end{vmatrix}$	1
	$= i(6xy^2z - 6xy^2z) - j(2y^3z - 2y^3z) + k(3y^2z^2 - 3y^2z^2) = \vec{0}$	1
	Thus, the required div $\vec{F} = -8$ and curl $\vec{F} = 0$	7
1c.	Given that, $\vec{F} = xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$	
	$\therefore$ div $\vec{F} = \nabla.\vec{F}$	
	$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial y}\right).[xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}]$	1
	$= \frac{\partial}{\partial x}(xyz) + \frac{\partial}{\partial y}(3x^2y) + \frac{\partial}{\partial z}(xz^2 - y^2z)$	1

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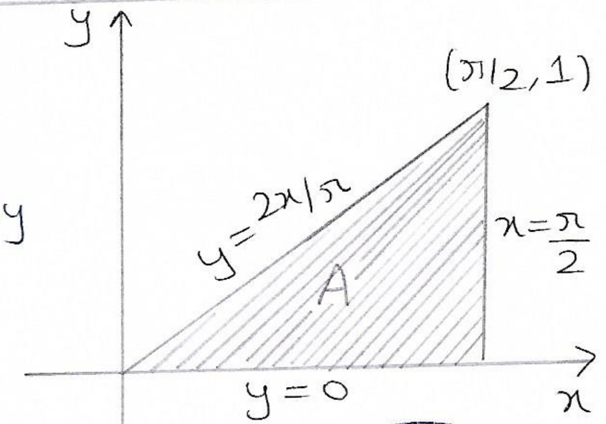
Faculty: Dr. Meenal M. Kaliwal (Mvf)

Q.No.	Solution and Scheme	Marks
	$= yz + 3x^2 + (2xz - y^2)$	
	At, $(2,-1,1)$, div $\vec{F} = -1 + 12 + 3 = 14$	1
	and curl $\vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$	
	$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & 3x^2y & (xz^2 - y^2z) \end{vmatrix}$	1
	$= \hat{i}\,[-2yz - 0] - \hat{j}\,[(z^2 - 0) - (xy)] + \hat{k}\,[6xy - xz]$	1
	At, $(2,-1,1)$ curl $\vec{F} \underline{\underline{=}} 2\hat{i} - 3j - 14\hat{k}$	1
		6
	OR	
2a.	By using Green's Theorem 	
	$\int_C (y - \sin x)\,dx + \cos x\,dy$	
	$= \iint_A \left[\frac{\partial}{\partial x}(\cos x) - \frac{\partial}{\partial y}(y - \sin x)\right] dx\,dy$	1
	$= -\iint_A (1 + \sin x)\,dx\,dy$	
	$= -\int_{x=0}^{\pi/2} \int_{y=0}^{2x/\pi} (1 + \sin x)\,dy\,dx$	1

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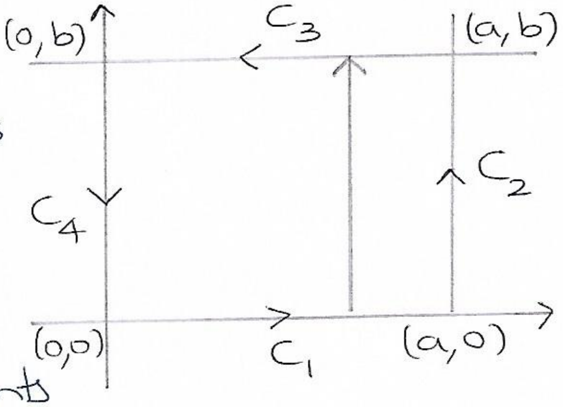
Faculty : Dr. Meenal M. Kaliwal (Mmy)

Q.No.	Solution and Scheme	Marks
	$= -\int_{0}^{\pi/2} \left\{ (1+\sin x)\, y \Big|_{y=0}^{2x/\pi} \right\} dx$	1
	$= -\int_{0}^{\pi/2} \frac{2x}{\pi} (1+\sin x)\, dx$	1
	$= -\frac{2}{\pi} \left[\frac{x^2}{2} - x\cos x + \sin x \right]_{0}^{\pi/2}$	1
	$= -\frac{2}{\pi} \left[\frac{\pi^2}{8} + 1 \right] = -\left(\frac{\pi}{4} + \frac{2}{\pi} \right)$	2
		7
2b.	To prove $\oint_{C} F\cdot dr = \iint_{S} \text{curl} F\cdot n\, ds$  The region is on xy-plane, therefore, $n = k$ ($\because z=0$, represents xy-plane and unit normal to xy plane is k)	
	$\oint_{C} F\cdot dr = \sum_{i=1}^{4} \int_{C} (x^2+y^2)\, dx - 2xy\, dy \; \hookrightarrow (1)$	1
	Equation to $C_1 : y = 0$ ($\because dy = 0$) $C_2 : x = a$ ($\because dx = 0$) $C_3 : y = b$ ($\because dy = 0$) $C_4 : x = 0$ ($\because dx = 0$) Thus, $\int_{C_1} (x^2+y^2)\, dx - 2xy\, dy = \int_{x=0}^{a} x^2\, dx$	
	$= \frac{x^3}{3} \Big|_{x=0}^{a} = \frac{a^3}{3}$	1

Q.No.	Solution and Scheme	Marks
	$\int_{C_2} (x^2+y^2)\,dx - 2xy\,dy = \int_{y=0}^{b} -2ay\,dy = -ab^2$	
	$\int_{C_3} (x^2+y^2)\,dx - 2xy\,dy = \int_{x=a}^{0} (x^2+b^2)\,dx$	
	$= \left(\frac{x^3}{3} + b^2x\right)_{x=a}^{0} = -ab^2 - \frac{a^3}{3}$	1
	$\int_{C_4} (x^2+y^2)\,dx - 2xy\,dy = \int_{b}^{0} 0 = 0$	1
	Substituting these in eqn ①,	
	$\oint_C F \cdot dr = \frac{a^3}{3} - ab^2 - ab^2 - \frac{a^3}{3} = -2ab^2 \rightarrow (2)$	1
	Further,	
	$\operatorname{curl} \vec{F} \cdot \hat{n} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2+y^2 & -2xy & 0 \end{vmatrix} \cdot k$	
	$= (0i - j0 + k[-2y - 2y] \cdot k$	
	$= -4y$	
	Thus, $\iint_S \operatorname{curl} \vec{F} \cdot n\,ds = \int_{x=-a}^{a} \int_{y=0}^{b} (-4y)\,dx\,dy$	
	$= \int_{-a}^{a} dx \int_{0}^{b} (-4y)\,dy = -2ab^2 \longrightarrow (3)$	2
	From (2) & (3), the Stoke's theorem is verified.	
		7

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Faculty: Dr. Mrunal M. Kaliwal (Mrf)

Q.No.	Solution and Scheme	Marks
2c.	`from sympy.vector import *`	1
	`from sympy import symbols`	
	`N = CoordSys3D('N')`	1
	`x, y, z = symbols('x y z')`	
	`A = N.x ** 2 * N.y + 2 * N.x * N.z - 4`	1
	`delop = Del ( )`	1
	`display (delop(A))`	1
	`grad A = gradient (A)`	
	`print (f"\nGradient of {A} is \n"}`	1
	`display (grad A)`	6
	Module-2	
3a.	Definition of Subspace : Let W be a non-empty subset of V, where V is a vector space over a field F. Then, W is said to be a vector sub space of V(F) if W is itself a vector space over F with respect to the same operations defined on V.	2
	Let V(F) be a vector space over F and W_1, W_2 be two subspace of V(F). Then we have to show that $W_1 \cap W_2$ is a subspace of V(F).	1
	Let, $\alpha, \beta \in W_1 \cap W_2 \Rightarrow \alpha, \beta \in W_1$ & $\alpha, \beta \in W_2$	1
	Since, W_1 and W_2 are subspaces of V, so we have $a, b \in W$ and $\alpha, \beta \in W_1 \Rightarrow \alpha a + \beta b \in W_1$ $\hookrightarrow$ ①	1
	and $a, b \in F$ and $\alpha, \beta \in W_2 \Rightarrow a\alpha + b\beta \in W_2 \rightarrow$ ②	1
	From ① & ②,	

Q.No.	Solution and Scheme	Marks
	if $a, b \in F$ and $\alpha, \beta \in W_1 \cap W_2 \Rightarrow a\alpha + b\beta \in W_1 \cap W_2$	1
	Hence, $W_1 \cap W_2$ is a subspace of V.	7
b.	First we shall show that B is linearly independent. $\Rightarrow a(1,1,0) + b(1,0,1) + c(0,1,1) = 0$	1
	$\Rightarrow (a+b, a+c, b+c) = 0$ $\left.\begin{array}{l} a+b=0 \\ a+c=0 \\ b+c=0 \end{array}\right\} \longrightarrow (1)$	1
	The coefficient matrix of the system (1) is, $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$	1
	$\Rightarrow \lvert A \rvert = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1(-1) + (-1)(1-0) + 0$	1
	$\lvert A \rvert = -2 \neq 0$ $\Rightarrow$ Rank of $A = 3$, which is equal to the number of unknowns. Therefore, the system (1) has only zero solution i.e. $a=0$, $b=0$, $c=0$.	2
	$\therefore B = \{(1,1,0), (1,0,1), (1,1,0)\}$ is linearly independent. $\therefore$ B is a basis of the vector space $V_3(R)$.	1
		7

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Faculty: Dr. Meenal M. Kaliwal

Q.No.	Solution and Scheme	Marks
3c.	Obviously, T is a mapping from $V_2(R)$ into	
	$V_3(R)$, because $(a+b, a-b, b) \in V_3(R)\ \forall$	1
	(a, b)	
	For each $\alpha, \beta \in F$ and $(a_1, b_1), (a_2, b_2) \in V_2(R)$	
	$T[\alpha(a_1, b_1) + \beta(a_2, b_2)]$	
	$= T(\alpha a_1 + \beta a_2, \alpha b_1 + \beta b_2)$	1
	$= [(\alpha a_1 + \beta a_2) + (\alpha b_1 + \beta b_2),$	2
	$(\alpha a_1 + \beta a_2) - (\alpha b_1 + \beta b_2), \alpha b_1 + \beta b_2]$	
	$= \alpha(a_1 + b_1, a_1 - b_1, b_1) + \beta(a_2 + b_2, a_2 - b_2, b_2)$	1
	$= \alpha T(a_1, b_1) + \beta(a_2, b_2)$	1
	Hence, $T: R^2 \to \underline{\underline{R_3}}$ is a linear transformation	6
	OR	
4a.	Let, $a, b, c \in R$ such that	
	$a(1,0,1) + b(1,1,0) + c(-1,0,-1) = (0,0,0)$	
	$\Rightarrow \left.\begin{array}{r} a+b-c=0 \\ b=0 \\ a-c=0 \end{array}\right\} \longrightarrow$ ①	1
	Here equation ① represents a system of homogenous equations. The coefficient matrix of this system is given by,	
	$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \Rightarrow \lvert A \rvert = \begin{vmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix}$	2
	$A = 1(-1-0) - 1(0) - 1(0-1) = -2 \neq 0$	1

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Faculty: Dr. Meenal M. Kaliwal (Mkj)

Q.No.	Solution and Scheme	Marks
	$\Rightarrow \text{Rank}(A) = 3$	1
	which is equal to the number of unknowns a, b, c. Therefore, the system (1) has only zero solution. i.e. $a=0,\ b=0,\ c=0$.	2
	Hence, the vectors $(1,0,1)$, $(1,1,0)$ & $(-1,0,-1)$ are linearly independent.	7
4b.	$T(x,y,z) = (x+y,\ x-y,\ 2x+z)$	
	$\text{Ker}(T) = \{(x,y,z) \in \mathbb{R} : T(x,y,z) = (0,0,0)\}$	
	$= \{(x,y,z) \in \mathbb{R} : (x+y,\ x-y,\ 2x+z) = (0,0,0)\}$	
	$= \{(x,y,z) \in \mathbb{R} : x+y=0,\ x-y=0,\ 2x+z=0\}$	1
	$= \{(x,y,z) \in \mathbb{R} : x=-y,\ x=y,\ z=-2y\}$	
	$= \{(-y,\ y,\ -2y) \in \mathbb{R} : y \in \mathbb{R}\}$	
	$= \{y(-1,1,-2) \in \mathbb{R} : y \in \mathbb{R}\}$	2
	$\therefore \dim(\text{Ker}(T)) = 1$	
	Let, $(a,b,c) \in \text{Range}(T)$. Then, $T(x,y,z) = (a,b,c)$ for some $(x,y,z) \in \mathbb{R}$.	
	Therefore, $x+y=a,\ x-y=b,\ 2x+z=c$	
	It follows that,	
	$x+y = 2x+z-(x-y) = c-b$	1
	i.e. $a = c-b$	
	Then, $\text{Range}(T) = \{(a,b,c) \in \mathbb{R} : T(x,y,z) = (a,b,c)\}$	
	$= \{(c-b,\ b,\ c) : b, c \in \mathbb{R}\}$	1

Faculty: Dr. Mrunal M. Kaliwal (Maj)

Q.No.	Solution and Scheme	Marks
	Consequently, $\dim(\text{Range}(T)) = 2$	1
	$\therefore \dim(\text{Ker}(T)) + \dim(\text{range}(T)) = 1+2 = 3$	1
	$= \dim(\mathbb{R}^3)$	
	Thus, the rank nullity theorem is verified.	7
4c.	import numpy as np	1
	import sympy as sp	
	V = np.array([[1,2,3],[2,3,1],[3,1,2]])	2
	basis = np.linalg.matrix_rank(V)	1
	dimension = V.shape[0]	1
	print("Basis of the matrix", basis)	1
	print("Dimension of the matrix", dimension)	
		6

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGG ★ KLS VDIT HALIYAL

Faculty: Dr. M. M. Kaliwal (Mmy)

Q.No.	Solution and Scheme	Marks
	Module - 3	
5a.	(i) $L[e^{-4t}(2\cos 6t - 3\sin 5t)]$	
	Let, $f(t) = 2\cos 6t - 3\sin 5t$	
	$L[f(t)] = 2.\frac{s}{s^2+36} - 3.\frac{5}{s^2+25}$	1
	$= \frac{2s}{s^2+36} - \frac{15}{s^2+25}$	
	Now, $L[e^{-4t}f(t)] = \left\{\frac{2s}{s^2+36} - \frac{15}{s^2+25}\right\}_{s\to s+4}$	1
	$= \frac{2(s+4)}{(s+4)^2+36} - \frac{15}{(s+4)^2+25}$	
	$= \frac{2s+8}{s^2+8s+52} - \frac{15}{s^2+8s+41}$	1
	(ii) $f(t) = \frac{\cos 2t - \cos 3t}{t}$	
	$L[f(t)] = \int_s^\infty L(\cos 2t - \cos 3t)\, ds$	1
	$= \int_s^\infty \left[\frac{s}{s^2+4} - \frac{s}{s^2+9}\right] ds$	1
	$= \left[\frac{1}{2}\log(s^2+4) - \frac{1}{2}\log(s^2+9)\right]_s^\infty$	
	$= \log\sqrt{\frac{s^2+4}{s^2+9}} = \log\sqrt{\frac{1+(4\|s^2)}{1+(9\|s^2)}}\Bigg\|_{s=\infty}$	1
	$- \log\sqrt{s^2+4\|s^2+9}$	
	$= \log 1 - \log\sqrt{s^2+4\|s^2+9} = \log\sqrt{s^2+9/s^2+4}$	1
		7

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGG. * KLS VDIT HALIYAL

Faculty: Dr. Meenal M. Kaliwal (Mvf)

Q.No.	Solution and Scheme	Marks
5b.	$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$	1
	$= \frac{1}{1-e^{-sT}}\left\{\int_0^{T/2} e^{-st}E\,dt + \int_{T/2}^{T} e^{-st}(-E)\,dt\right\}$	
	$= \frac{E}{1-e^{-sT}}\left\{ \left.\frac{e^{-st}}{-s}\right\|_{t=0}^{T/2} + \left(\frac{e^{-st}}{+s}\right)_{t=T/2}^{T}\right\}$	1
	$= \frac{E}{1-e^{-sT}}\left\{-\frac{1}{s}\left[e^{-sT/2}-e^{0}\right]+\frac{1}{s}\left[e^{-sT}-e^{-sT/2}\right]\right\}$	
	$= \frac{E}{s(1-e^{-sT})}\left\{-e^{-sT/2}+1+e^{-sT}-e^{-sT/2}\right\}$	
	$= \frac{E}{s(1-e^{-sT})}\left(1-2e^{-sT/2}+e^{-sT}\right)$	
	$= \frac{E\left(1-e^{-sT/2}\right)^2}{s\left(1-e^{-sT}\right)} = \frac{E\left(1-e^{-sT/2}\right)^2}{s\left(1-e^{-sT/2}\right)\left(1+e^{-sT/2}\right)}$	2
	$= \frac{E\left(1-e^{-sT/2}\right)}{s\left(1+e^{-sT/2}\right)}$	
	Multiplying both the numerator & denominator by $e^{sT/4}$	
	$L[f(t)] = \frac{E\left(e^{sT/4}-e^{-sT/4}\right)}{s\left(e^{sT/4}+e^{-sT/4}\right)} = \frac{E.2\sinh(sT/4)}{s.2\cosh(sT/4)}$	2
	Thus, $L[f(t)] = \underline{\underline{\frac{E}{s}\tanh\left(\frac{sT}{4}\right)}}$	1
		7

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING KLE's IT HALIYAL

Faculty: Dr. Meenal M. Kaliwal

Q.No.	Solution and Scheme	Marks
5c.	$f(t) = \sin t + [\sin 2t - \sin t]u(t-\pi)$ $+ [\sin 3t - \sin 2t]\, u(t-2\pi)$ Now, $L[f(t)] = L[\sin t] + L\{[\sin 2t - \sin t]\, u(t-\pi)\}$ $+ L\{[\sin 3t - \sin 2t]\, u(t-2\pi)\}$ $\hookrightarrow (1)$	1
	Consider, $L\{[\sin 2t - \sin t]\, u(t-\pi)\}$ Let, $F(t-\pi) = \sin 2t - \sin t$ $\Rightarrow F(t) = \sin 2(t+\pi) - \sin(t+\pi)$ $F(t) = \sin(2\pi+2t) - \sin(\pi+t)$ $F(t) = \sin 2t + \sin t$ $\bar{F}(s) = L[f(t)] = \frac{2}{s^2+4} + \frac{1}{s^2+1}$	1
	But, $L[F(t-\pi)\, u(t-\pi)] = e^{-\pi s}\, \bar{F}(s)$	1
	$L[[\sin 2t - \sin t]\, u(t-\pi)] = e^{-\pi s}\left\{\frac{2}{s^2+4} + \frac{1}{s^2+1}\right\}$	1
	Also, let $G(t-2\pi) = \sin 3t - \sin 2t$ $\Rightarrow G(t) = \sin 3(t+2\pi) - \sin 2(t+2\pi)$ $G(t) = \sin 3t - \sin 2t$ $\bar{G}(s) = \frac{3}{s^2+9} - \frac{2}{s^2+4}$	1
	$L\{G(t-2\pi)\, u(t-2\pi)] = e^{-2\pi s}\, \bar{G}(s)$ $L\{[\sin 3t - \sin 2t]\, u(t-2\pi)\}$ $= e^{-2\pi s}\left[\frac{3}{s^2+9} - \frac{2}{s^2+4}\right]$	1
		6

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGG. KLS VDIT HALIYAL

Faculty: Dr. Meenal M. Kaliwal (Mey)

Q.No.	Solution and Scheme	Marks
	Thus, $L[f(t)] = \frac{1}{s^2+1} + e^{-\pi s}\left[\frac{2}{s^2+4} + \frac{1}{s^2+1}\right]$ $+ e^{-2\pi s}\left[\frac{3}{s^2+9} - \frac{2}{s^2+4}\right]$	
	OR	
6a.	$\frac{1}{s(s+1)(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{s+3}$	
	Multiplying by $s(s+1)(s+2)(s+3)$	
	$1 = A(s+1)(s+2)(s+3) + B(s+2)(s+3)s$ $+ Cs(s+1)(s+2) + Ds(s+1)(s+2)$	2
	put : $s=0$: $1 = A(6) \Rightarrow A = 1/6$ $s=-1$: $1 = B(-2) \Rightarrow B = -1/2$ $s=-2$: $1 = C(2) \Rightarrow C = 1/2$ $s=-3$: $1 = D(-6) \Rightarrow D = -1/6$	3
	Now, $L^{-1}\left[\frac{1}{s(s+1)(s+2)(s+3)}\right]$ $= \frac{1}{6} L^{-1}\left[\frac{1}{s}\right] - \frac{1}{2} L^{-1}\left[\frac{1}{s+1}\right] + \frac{1}{2} L^{-1}\left[\frac{1}{s+2}\right]$ $- \frac{1}{6} L^{-1}\left[\frac{1}{s+3}\right]$	1
	$= \frac{1}{6} - \frac{1}{2}e^{-t} + \frac{1}{2}e^{-2t} - \frac{1}{6}e^{-3t}$	1
	$=$	7

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGG. KLS VDIT HALIYAL

Faculty : Dr. Meenal M. Kaliwal

Q.No.	Solution and Scheme	Marks
6b.	Let $\bar{F}(s) = \frac{1}{s^2+a^2}$; $\bar{g}(s) = \frac{s}{s^2+a^2}$	
	$\Rightarrow f(t) = L^{-1}[\bar{F}(s)] = \frac{\sin at}{a}$	1
	$g(t) = L^{-1}[\bar{g}(s)] = \cos at$	1
	We have convolution theorem,	
	$L^{-1}[\bar{F}(s).\bar{g}(s)] = \int_{u=0}^{t} f(u)\, g(t-u)\, du$	1
	$\therefore L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \int_{u=0}^{t} \frac{\sin au}{a} . \cos(at-au)\, du$	1
	$= \frac{1}{2a}\int_{u=0}^{t} [\sin(au+at-au)+\sin(au-at+au)]\, du$	1
	$= \frac{1}{2a}\int_{u=0}^{t} [\sin at + \sin(2au-at)]\, du$	
	$= \frac{1}{2a}\left\{ \sin at\, [u]_{u=0}^{t} - \left[\frac{\cos(2au-at)}{2a}\right]_{u=0}^{t} \right\}$	1
	$= \frac{1}{2a}\left\{ \sin at\, (t-0) - \frac{1}{2a}(\cos at - \cos at) \right\}$	
	$= \frac{t \sin at}{2a}$	
	Thus, $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \underline{\underline{\frac{t \sin at}{2a}}}$	1
		7

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGG. ★ KLS VDIT HALIYAL

Faculty : Dr. Meenal M. Kaliwal (My)

Q.No.	Solution and Scheme	Marks
6c.	$y''+4y'+4y=e^{-t}$, $y(0)=0=y'(0)$	
	$L[y''(t)]+4L[y'(t)]+4L[y(t)]=L[e^{-t}]$	
	i.e. $\{s^2 L[y(t)]-sy(0)-y'(0)\}+4\{sL(y(t)]-y(0)\}$	1
	$+4L[y(t)]=\frac{1}{s+1}$	
	Using the given initial conditions,	
	$L[y(t)]=\frac{1}{(s+1)(s+2)^2}$	1
	Let, $\frac{1}{(s+1)(s+2)^2}=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{(s+2)^2}$	
	Multiplying with $(s+1)(s+2)^2$, we obtain	
	$1=A(s+2)^2+B(s+1)(s+2)+C(s+1)$	
	putting $s=-1$ we get $A=1$	2
	putting $s=-2$ we get $C=-1$	
	putting $s=0$, we have $B=-1$	
	Hence, $\frac{1}{(s+1)(s+2)^2}=\frac{1}{s+1}-\frac{1}{s+2}-\frac{1}{(s+2)^2}$	
	$\therefore L^{-1}\left[\frac{1}{(s+1)(s+2)^2}\right]=L^{-1}\left[\frac{1}{s+1}\right]-L^{-1}\left[\frac{1}{s+2}\right]$	1
	$-L^{-1}\left[\frac{1}{(s+2)^2}\right]$	
	$y(t)=e^{-t}-e^{-2t}-e^{-2t}L^{-1}[1/s^2]$	
	Thus, $y(t)=e^{-t}-e^{-2t}-t.e^{-2t}$	1
		6

Faculty: Dr. Meenal M. Kaliwal (Mmk)

Q.No.	Solution and Scheme	Marks
	Module-4	
7a.	Let, $f(x) = x \log_{10} x - 1.2$	
	$f(1) = -1.2$, $f(2) = -0.6 < 0$, $f(3) = 0.23 > 0$ The real root lies in $(2, 3)$	1
	1 iteration: $a = 2$, $b = 3$ $f(a) = -0.6$, $f(b) = 0.23$ $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2.26}{0.83} = 2.7229$	2
	2nd iteration: $f(2.7229) = -0.01545$ ∴ the root lies in $(2.7229, 3)$ $x_2 = \frac{2.7229(0.23) - 3(-0.01545)}{0.23 - (-0.01545)}$	2
	$x_2 = 2.7403$ $f(x_2) = 2.7403 \log_{10} (2.7403) - 1.2 = -0.000302$	
	3rd iteration: $a = 2.7403$, $b = 3$ $f(a) = -0.000302$, $f(b) = 0.23$ $x_3 = \frac{2.7403(0.23) - 3(-0.000302)}{0.23 + 0.000302}$	2
	$x_3 = \frac{0.631175}{0.230302} = 2.7406$	
	Thus, the required approximate root is $x \simeq \underline{\underline{2.7406}}$	7

Faculty : Dr. Meenal M. Kaliwal

Q.No. **Solution and Scheme** **Marks**

7b. To find the value of $f(x) = \sin x$ at $x = 52^\circ$ which is in between 50 & 55°. Hence, we will use Newton's Forward interpolation formula. The difference table is as follows.

x	$f(x) = y$	Δy	$\Delta^2 y$	$\Delta^3 y$
45	0.7071			
		0.0589		
50	0.7660		-0.0057	
		0.0532		-0.0007
55	0.8192		-0.0064	
		0.0468		
60	0.8660			

2

Newton's forward interpolation formula,

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + ----$$

where, $r = \frac{x - x_0}{h} = \frac{52-45}{5} = 1.4$ 1

From the table, $\Delta y_0 = 0.0589$, $\Delta^2 y_0 = -0.0057$, $\Delta^3 y_0 = -0,0007$

$$f(52) = 0.7071 + (1.4)(0.0589) + \frac{1.4(1.4-1)}{2} \times (-0.0057) + \frac{1.4(1.4-1)(1.4-2)}{6}(-0.0007)$$ 2

$$= 0.7071 + 0.08246 - 0.003192/2 + 0.0000392$$ 1

$\sin 52^\circ = 0.7880032$ 1

7

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGG. KLS VDIT HALIYAL

Faculty: Dr. Meenal M. Kaliwal (Mof)

Q.No.	Solution and Scheme	Marks
7c.	Let, $x_0 = 1, x_1 = 3, x_2 = 4, x_3 = 6, x_4 = 9$ $y_0 = -3, y_1 = 9, y_2 = 30, y_3 = 132, y_4 = 156$ Lagrange's interpolation formula,	
	$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times y_0$ $+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times y_1$ $+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2$ $+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3$ $+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times y_4$	2
	$= \frac{(5-3)(5-4)(5-6)(5-9)}{(1-3)(1-4)(1-6)(1-9)} \times (-3)$ $+ \frac{(5-1)(5-4)(5-6)(5-9)}{(3-1)(3-4)(3-6)(3-9)} \times 9$ $+ \frac{(5-1)(5-3)(5-6)(5-9)}{(4-1)(4-3)(4-6)(4-9)} \times 30$ $+ \frac{(5-1)(5-3)(5-4)(5-9)}{(6-1)(6-3)(6-4)(6-9)} \times 132$ $+ \frac{(5-1)(5-3)(5-4)(5-6)}{(9-1)(9-3)(9-4)(9-6)} \times 156$	2
	$= -0.1 - 4 + 32 + 46.933 - 1.7333$	1
	$f(5) = 73.1$	1
		6

Faculty: Dr. Meenal M. Kaliwal

KLS VDIT HALIYAL

Q.No.	Solution and Scheme	Marks
8a.	Let, $f(x) = x\tan x + 1 = 0$	
	$f(x) = x\frac{\sin x}{\cos x} + 1 = 0$	
	$\Rightarrow x\sin x + \cos x = 0$	
	i.e. $f(x) = x\sin x + \cos x$	1
	$f'(x) = x\cos x + \sin x - \sin x = x\cos x$	
	Newton-Raphson formula is,	
	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	
	$\Rightarrow x_{n+1} = x_n - \frac{x_n\sin x_n + \cos x_n}{x_n\cos x_n}$	
	$= \frac{x_n^2\cos x_n - x_n\sin x_n - \cos x_n}{x_n\cos x_n}$	1
	Given, $x_0 = \pi$	
	I-iteration : $x_1 = \frac{x_0^2\cos x_0 - x_0\sin x_0 - \cos x_0}{x_0\cos x_0}$	
	$x_1 = \frac{(\pi)^2\cos\pi - \pi\sin\pi - \cos\pi}{\pi\cos\pi}$	
	$x_1 = 2.823282767 \simeq 2.8233$	2
	II-iteration : $x_2 = 2.798599894 \simeq 2.7986$	1
	III-iteration : $x_3 = 2.798386062 \simeq 2.7984$	1
	IV-iteration ; $x_4 = 2.798386046 \simeq 2.7984$	1
	Thus the root is $x = \underline{\underline{2.7984}}$	
		7

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGG. ★ KLS VDIT HALIYAL

Faculty : Dr. Meenal M. Kaliwal

Q.No.	Solution and Scheme	Marks
8b.	Form the table of divided differences	

x	$f(x)$	1^{st} D.D
$x_0=0$	$f(x_0)=-4$	$f(x_0,x_1)=\frac{2-(-4)}{2-0}=3$
$x_1=2$	$f(x_1)=2$	$f(x_1,x_2)=\frac{14-2}{3-2}=12$
$x_2=3$	$f(x_2)=14$	$f(x_2,x_3)=\frac{158-14}{6-3}=48$
$x_3=6$	$f(x_3)=158$	

2^{nd} Divided Differences	3^{rd} D.D.
$f(x_0,x_1,x_2)=\frac{12-3}{3-0}=3$	$f(x_0,x_1,x_2,x_3)$ $=\frac{9-3}{6-0}=1$
$f(x_1,x_2,x_3)=\frac{48-12}{6-2}=9$	

3

Newton's divided difference formula is,

$$f(x)=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)+(x-x_0)(x-x_1)(x-x_2)\times f(x_0,x_1,x_2,x_3)+----$$ 1

$$\therefore f(4)=-4+(4-0)3+(4-0)(4-2)3+(4-0)(4-2)(4-3)1$$ 2

$$=-4+12+24+8=40$$

Thus, $\underline{\underline{f(4)=40}}$ 1

7

Faculty: Dr. Meenal M. Kaliwal (Mk)

Q.No.	Solution and Scheme	Marks

8c. Length of each sub-interval $(h) = \frac{0.6-0}{6} = 0.1$

$n=6$.

The values of x and $y = e^{-x^2}$ correct to four decimal places are tabulated.

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$y=e^{-x^2}$	1	0.99	0.9608	0.9139	0.8521	0.7788	0.6977
	y_0	y_1	y_2	y_3	y_4	y_5	$= y_6$

3

Simpson's 1/3rd rule for $n=6$ is given by

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$ 1

$$\int_0^{0.6} e^{-x^2}\,dx = \frac{0.1}{3}\left[(1+0.6977)+4(0.99+0.9139+0.7788)+2(0.9608+0.8521)\right]$$ 1

$$\int_0^{0.6} e^{-x^2}\,dx = 0.5351$$ 1

6

Module - 5

9a. Taylor's series expansion of $y(x)$ is given by

$$y(x) = y(x_0) + (x-x_0)\,y'(x_0) + \frac{(x-x_0)^2}{2!}\,y''(x_0) + \frac{(x-x_0)^3}{3!}\,y'''(x_0) + \frac{(x-x_0)^4}{4!}\,y''''(x_0) + \cdots \longrightarrow ①$$

By data, $x_0 = 0$, $y_0 = 2$, $y' = e^x - y$

Q.No.	Solution and Scheme	Marks
	Consider, $y' = e^x - y$; $y'(0) = e^0 - 2 = -1$	
	$y'' = e^x - y'$; $y''(0) = e^0 - (-1) = 1+1 = 2$	
	$y''' = e^x - y''$; $y'''(0) = e^0 - 2 = 1 - 2 = -1$	3
	$y'''' = e^x - y'''$; $y''''(0) = e^0 - (-1) = 1+1 = 2$	
	To find y(1.1), we shall substitute these values along with $x = 1.1$ in ①,	
	$y(1.1) = 2 + (1.1-0)y'(0) + \frac{(1.1-0)^2}{2}y''(0) + \frac{(1.1-0)^3}{6}\times y'''(0) + \frac{(1.1-0)^4}{24}y''''(0) + ---$	1
	$y(1.1) = 2 + (1.1)(-1) + \frac{(1.1)^2}{2}\times 2 + \frac{(1.1)^3}{6}\times(-1) + \frac{(1.1)^4}{24}\times 2 + ---$	1
	$y(1.1) = 2 - 1.1 + 1.21 - 0.221833 + 0.1220083333$	1
		1
	$y(1.1) = 2.010175003$	7
9 b.	By data, $y' = 3e^x + 2y$, $y(0) = 0 \Rightarrow y_0 = 0$, $x_0 = 0$	
	$h = 0.1$	
	$k_1 = hf(x_0, y_0) = (0.1)f(0,0) = (0.1)[3e^0 + 0] = 0.3$	
	$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.1)f(0.05, 0.15)$	2
	$= 0.1\left[3e^{0.05} + 2(0.15)\right] = 0.3454$	
	$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.1)f(0.05, 0.1727)$	

Q.No.	Solution and Scheme	Marks

$k_3 = (0.1)\left[3e^{0.05} + 2(0.1727)\right] = 0.3499$

$k_4 = f(x_0+h, y_0+k_3)h$

$= (0.1) f(0+0.1, 0+0.3499)$

$= (0.1)\left[3e^{0.1} + 2(0.3499)\right]$ 2

$k_4 = 0.4015$

We have, $y(x_0+h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ 1

$\therefore y(0.1) = 0 + \frac{1}{6}\left[0.3 + 2(0.3454) + 2(0.3499) + 0.4015\right]$ 1

Thus, $y(0.1) \underline{\underline{=}} 0.3487$ 1

7

9c.

x	y	$y' = 2e^x y$
$x_0 = 0$	$y_0 = 2.4$	$y_0' = 2e^0 \times 2.4 = 4.8$
$x_1 = 0.1$	$y_1 = 2.473$	$y_1' = 2e^{0.1}(2.473) = 5.4662$
$x_2 = 0.2$	$y_2 = 3.129$	$y_2' = 2e^{0.2}(3.129) = 7.6435$
$x_3 = 0.3$	$y_3 = 4.059$	$y_3' = 2e^{0.3}(4.059) = 10.9582$

2

Milne's predictor formula,

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$$

$$= 2.4 + \frac{4}{3} \times (0.1)\left[2(5.4662) - 7.6435 + 2(10.9582)\right]$$

$y_4^{(P)} = 5.7607$ 1

Now, $y_4' = 2e^{0.4}(5.7607) \simeq 17.1879$

Q.No.	Solution and Scheme	Marks
	Next, we have Milne's corrector formula,	
	$y_4^{(c)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$	
	$y_4^{(c)} = 3.129 + \frac{0.1}{3}[7.6435+4(10.9582)+17.1879]$	1
	$y_4^{(c)} = 5.4178$	1
	$y_4' = 2e^{0.4}(5.4178) = 16.16481569$	
	$y_4^{(c)} = 3.129 + \frac{0.1}{3}[7.6435+4(10.9582)+16.16482]$	
	$y_4^{(c)} = 5.383704$	
	$\therefore y(0.4) \simeq 5.3837$	1
		6
	OR	
10a.	We need to find $y(0.2)$ by taking $h=0.2$	
	By data, $x_0 = 0$, $y_0 = 1$, $f(x,y) = 3x + y/2$	1
	$f(x_0, y_0) = 0.5$, $x_1 = x_0 + h = 0 + 0.2 = 0.2$	
	$y(x_1) = y(0.2) = ?$	
	Euler's formula : $y_1^{(0)} = y_0 + hf(x_0, y_0)$	
	$y_1^{(0)} = 1 + (0.2)(0.5) = 1.1$	1
	Modified Euler's formula,	
	$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(0)})]$	
	$= 1 + \frac{0.2}{2}\left[0.5 + 3x_1 + \frac{y_1^{(0)}}{2}\right]$	
	$= 1 + 0.1\left[0.5 + 3(0.2) + \frac{1.1}{2}\right]$	

Question Number	Solution	Marks Allocated
	$y_1^{(1)} = 1.165$	2
	The second iterative value is got by replacing $y_1^{(0)}$ by $y_1^{(1)}$.	
	$y_1^{(2)} = 1+0.1\left[0.5+3(0.2)+\frac{1.165}{2}\right]$	1
		1
	$y_1^{(2)} = 1.16825$	1
	Thus, $y(0.2) \underline{\underline{=}} 1.16825$	
		7
b.	By data, $f(x,y) = \frac{y-x}{y+x}$, $x_0=0, y_0=1, h=0.2$	1
	$k_1 = hf(x_0,y_0) = 0.2f(0.1) = 0.2$	1
	$k_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right) = 0.2f(0.1, 1.1)$	
	$= 0.2\left[\frac{1.1-0.1}{1.1+0.1}\right] = 0.1667$	1
	$k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right) = 0.2f(0.1, 1.0835)$	
	$k_3 = 0.1662$	1
	$k_4 = hf(x_0+h, y_0+k_3) = 0.1414$	1
	$y(x_0+h) = y_0+\frac{1}{6}(k_1+2k_2+2k_3+k_4)$	1
	$\therefore y(0.2) \underline{\underline{=}} 1.1679$	1
		7

Q.No. | **Solution** | **Marks**

10 c.

```
from sympy import *
import numpy as np
def RungeKutta (g,x0,h,y0,xn):
    x,y = symbols ('x,y')                              2
    f = lambdify ([x,y],g)
    xt = x0+h
    Y = [y0]
    while xt <= xn:
        k1 = h*f(x0,y0)
        k2 = h*f(x0+h/2, y0+k1/2)                      2
        k3 = h*f(x0+h/2, y0+k2/2)
        k4 = h*f(x0+h, y0+k3)
        y1 = y0+(1/6)*(k1+2*k2+2*k3                    1
                                   +k4)
        Y.append (y1)
        print ('y(%3.3f'%xt,') is %3.3f'
                             %y1)
        x0 = xt
        y0=y1                                          1
        xt = xt+h
    return np.round (Y,2)
    RungeKutta ('x-y*2', 0,0.1,0.1,1)
                                                       6
```

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