Model Question Paper –II with effect from 2022

USN | | | | | | | | | | |

First Semester B. E Degree examination

Mathematics-1 for Computer Science Stream (22MATS11)

Time: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

		Module-1	Marks
Q. 01	a	With usual notation prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$	6
	b	Find the angle between the curves$r = \frac{a}{1+cos\theta}$ and $r = \frac{b}{1-cos\theta}$	7
	c	Find the radius of curvature of the curve $y = x^3(x-a)$ at the point $(a, 0)$	7
		OR	
Q. 02	a	Show that the curves $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$ cuts each other orthogonally	6
	b	Find the pedal equation of the curve $r(1 - cos\theta) = 2a$	7
	c	Find the radius of curvature for the curve $y^2 = \frac{a^2(a-x)}{x}$, where the curve meets the x-axis.	7
		Module-2	
Q. 03	a	Expand $\log(1 + sinx)$up to the term containing x^4 using Maclaurin's series.	6
	b	If $u = \log(tanx + tany + tanz)$ show that $sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z} = 2$.	7
	c	Find the extreme values of the function $f(x, y) = x^2 + y^2 + 6x - 12$.	7
		OR	
Q. 04	a	Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$	6
	b	If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$	7
	c	If $x = rsin\theta cos\varphi,\ y = rsin\theta sin\varphi,\ z = rcos\theta$ find the value of $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$.	7
		Module-3	
Q. 05	a	Solve $\frac{dy}{dx} + \frac{y}{x} = y^2x$	6
	b	Find the orthogonal trajectories of $r = a(1 + cos\theta)$ where a is parameter.	7
	c	Solve $p^2 + 2pycotx - y^2 = 0$.	7
		OR	

Q. 06	a	Solve $y(2xy+1)dx - xdy = 0$	6
	b	Find the orthogonal trajectories of the family $r^n sin n\theta = a^n$.	7
	c	Find the general solution of the equation $(px-y)(py+x) = 2p$ by reducing into Clairaut's form by taking the substitution $X = x^2,\ Y = y^2$	7
		Module-4	
Q. 07	a	(i) Find the remainder when 2^{23} is divided by 47. (ii) Find the last digit in 7^{118}.	6
	b	Find the solutions of the linear congruence $11x \equiv 4 \pmod{25}$.	7
	c	Encrypt the message **STOP** using RSA with key (2537, 13) using the prime numbers 43 and 59.	7
		OR	
Q. 08	a	Using Fermat's Little Theorem, show that $8^{30} - 1$ is divisible by 31.	6
	b	Solve the system of linear congruence $x \equiv 3(mod5), \quad y \equiv 2(mod6), \quad z \equiv 4(mod7)$ using Remainder Theorem.	7
	c	(i) Find the remainder when $175 \times 113 \times 53$ is divided by 11. (ii) Solve $x^3 + 5x + 1 \equiv 0(mod27)$.	7
		Module-5	
Q. 09	a	Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$	6
	b	Solve the system of equations by using Gauss-Jordan method: $x+y+z=9$ $2x+y-z=0$ $2x+5y+7z=52$	7
	c	Using power method, find the largest eigenvalue and corresponding eigenvector of the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	7
		OR	
Q. 10	a	Solve the following system of equation by Gauss-Seidel method: $20x+y-2z=17$ $3x+20y-z=-18$ $2x-3y+20z=25$	6
	b	Test for consistency $x-2y+3z=2$, $3x-y+4z=4$, $2x+y-2z=5$ and hence solve	7
	c	Solve the system of equations by Gauss elimination method $2x+y+4z=12,\ 4x+11y-z=33,\ 8x-3y+2z=20$	7

Table showing the Blooms Taxonomy Level, Course outcome and Program outcome				
Question		Blooms Taxonomy level attached	Course outcome	Program outcome
Q.1	a)	L1	CO 01	PO 01
	b)	L2	CO 01	PO 01
	c)	L3	CO 01	PO 02
Q. 2	a)	L1	CO 01	PO 01
	b)	L2	CO 01	PO 01
	c)	L3	CO 01	PO 02
Q. 3	a)	L2	CO 02	PO 01
	b)	L2	CO 02	PO 01
	c)	L3	CO 02	PO 03
Q. 4	a)	L2	CO 02	PO 01
	b)	L2	CO 02	PO 01
	c)	L3	CO 02	PO 02
Q. 5	a)	L2	CO 03	PO 02
	b)	L3	CO 03	PO 03
	c)	L2	CO 03	PO 01
Q. 6	a)	L2	CO 03	PO 02
	b)	L3	CO 03	PO 03
	c)	L2	CO 03	PO 01
Q. 7	a)	L2	CO 04	PO 01
	b)	L2	CO 04	PO 01
	c)	L2	CO 04	PO 02
Q. 8	a)	L2	CO 04	PO 01
	b)	L2	CO 04	PO 01
	c)	L2	CO 04	PO 02
Q. 9	a)	L2	CO 05	PO 01
	b)	L3	CO 05	PO 01
	c)	L3	CO 05	PO 02
Q. 10	a)	L2	CO 05	PO 01
	b)	L3	CO 05	PO 02
	c)	L3	CO 05	PO 01

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6

KLS Vishwanathrao Deshpande Institute of Technology, Haliyal - 581 329

Doc. No.: VDIT/ACAD/AR/05b	Rev.No.:01
Page 1	Rev. Dt: 25/03/2021

Model Question Paper- II Solution and Scheme for award of marks

AY: 2022-23

Subject with Sub. Code: Mathematics-I for Computer Science & Engineering Stream (22MATS11)

Name of Faculty: Prof. Dr. Satish P. Hande **Semester: I**

Q.No.	Solution and Scheme	Marks
1 a)	We know that	
	$p = r\sin\phi \;;\; p^2 = r^2\sin^2\phi$	1M
	$\frac{1}{p^2} = \frac{1}{r^2\sin^2\phi} = \frac{1}{r^2}\operatorname{cosec}^2\phi$	1M
	$\frac{1}{p^2} = \frac{1}{r^2}\left[1+\cot^2\phi\right]$	1M
	$= \frac{1}{r^2}\left[1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right]$	2M
	$\therefore \frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$	1M
b)	Given	
	$r(1+\cos\theta) = a \;;\; r(1-\cos\theta) = b$	
	Taking logarithm on both sides	
	$\log r + \log(1+\cos\theta) = \log a \;;\; \log r + \log(1-\cos\theta) = \log b$	1M
	differentiating w.r.t θ on both sides we get	
	$\frac{1}{r}\frac{dr}{d\theta} + \frac{-\sin\theta}{1+\cos\theta} = 0 \;;\; \frac{1}{r}\frac{dr}{d\theta} + \frac{\sin\theta}{1-\cos\theta} = 0$	1M
	$\cot\phi_1 = \frac{\sin\theta}{1+\cos\theta} \;;\; \cot\phi_2 = \frac{-\sin\theta}{1-\cos\theta}$	2M
	$\cot\phi_1 = \frac{2\sin\theta/2\cdot\cos\theta/2}{2\cos^2\theta/2} \;;\; \cot\phi_2 = \frac{-2\sin\theta/2\cdot\cos\theta/2}{2\sin^2\theta/2}$	1M
	$\cot\phi_1 = \tan\theta/2 \;;\; \cot\phi_2 = -\cot\theta/2$	
	$\cot\phi_1 = \cot(\pi/2-\theta/2) \;;\; \phi_2 = -\theta/2$	1M
	$\phi_1 = \pi/2-\theta/2$	
	$\therefore$ The angle between the curves is	
	$\lvert\phi_1-\phi_2\rvert = \left\lvert\frac{\pi}{2}-\frac{\theta}{2}-\left(-\frac{\theta}{2}\right)\right\rvert = \pi/2$	1M

Q.No.	Solution and Scheme	Marks
1. c)	Given $y = x^4 - ax^3$	
	diff w.r.t x we get	1M
	$\frac{dy}{dx} = 4x^3 - 3ax^2$	1M
	$\frac{d^2y}{dx^2} = 12x^2 - 6ax$	
	At $(a, 0)$	
	$\frac{dy}{dx} = 4a^3 - 3a^3 = a^3$	2M
	$\frac{d^2y}{dx^2} = 12a^2 - 6a^2 = 6a^2$	1M
	$\rho = \frac{[1+(y_1^2)]^{3/2}}{y_2} = \frac{[1+(a^3)^2]^{3/2}}{6a^2} = \frac{(1+a^6)^{3/2}}{6a^2}$	2M
2 a)	$r = a(1+\cos\theta)$; $r = a(1-\cos\theta)$	
	Taking logarithm on both sides	
	$\log r = \log a + \log(1+\cos\theta)$; $\log r = \log a + \log(1-\cos\theta)$	1M
	diff w.r.t θ we get	
	$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{-\sin\theta}{1+\cos\theta}$; $\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\sin\theta}{1-\cos\theta}$	2M
	$\cot\phi_1 = -\tan\theta/2$; $\cot\phi_2 = \cot(\theta/2)$	
	$= \cot(\pi/2 + \theta/2)$; $\cot\phi_2 = \cot(\theta/2)$	
	$\phi_1 = \frac{\pi}{2} + \frac{\theta}{2}$; $\phi_2 = \theta/2$	2M
	$\therefore$ The angle between two curves is	
	$\lvert\phi_1 - \phi_2\rvert = \lvert\frac{\pi}{2} + \frac{\theta}{2} - \frac{\theta}{2}\rvert = \frac{\pi}{2}$	1M
b)	$r(1-\cos\theta) = 2a$	
	Taking logarithm on both sides	
	$\log r + \log(1-\cos\theta) = \log 2a$	1M
	diff w.r.t θ we get	
	$\frac{1}{r}\frac{dr}{d\theta} + \frac{\sin\theta}{1-\cos\theta} = 0$	1M

Q.No.	Solution and Scheme	Marks
	$\frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin\theta}{1-\cos\theta} = \frac{-2\sin\theta/2 \cdot \cos\theta/2}{2\sin^2\theta/2}$	1M
	$\cot\phi = -\cot\theta/2$	
	$\boxed{\phi = -\theta/2}$	1M
	$\therefore\ p = r\sin\phi = r\sin(-\theta/2)$	
	$p = -r\sin\theta/2$	1M
	From the given eqn	
	$1-\cos\theta = 2a/r$	
	$2\sin^2\theta/2 = 2a/r$	
	$\sin\theta/2 = \sqrt{2a/r}$	
	$p = -r\sqrt{\frac{2a}{r}}$	
	$\Rightarrow p^2 = ar$	2M
c)	$y^2 = \frac{a^2(a-x)}{x} = \frac{a^3}{x} - a^2$	
	differentiating w.r.t x.	
	$\cancel{2y}\, y_1 = \frac{-a^3}{2yx^2}$	1M
	At the point when curve meets the x-axis $y=0$	
	$\Rightarrow a^2(a-x) = 0 \Rightarrow x = a$	
	$\therefore$ The pt. of intersection is $(a,0)$	1M
	at $(a,0)$	
	$y_1 = \infty$	1M
	$\therefore\ \frac{dx}{dy} = x_1 = \frac{-2}{a^3}(x^2y)$ $\quad x_1 = 0$ at $(a,0)$	
	$x_2 = \frac{-2x^2}{a^3} \Rightarrow x_2 = -2/a$ at $(a,0)$	2M
	$\therefore\ \rho = \frac{[1+x_1^2]^{3/2}}{x_2}$	1M
	$= \left\| \frac{(1+0^2)^{3/2}}{-2/a} \right\| = \left\| \frac{-a}{2} \right\|$	
	$\therefore\ \rho = \frac{a}{2}$	1M

Dr. Satish P. Hande, Dept. of Mathematics
KLS Vishwanathrao Deshpande Institute of Technology

3 a) Let $y = \log(1+\sin x)$ $\quad y(0) = \log(1) = 0$

$$y_1 = \frac{1}{1+\sin x} \cdot \cos x$$

$(1+\sin x)\, y_1 = \cos x$ 1M

$(1+\sin 0)\, y_1(0) = \cos 0 \Rightarrow \boxed{y_1(0) = 1}$

$(1+\sin x) y_2 + y_1 \cos x = -\sin x \Rightarrow \boxed{y_2(0) = -1}$ 1M

$(1+\sin x) y_3 + y_2 \cos x + y_2 \cos x - y_1 \sin x = -\cos x.$

$(1+\sin x) y_3 + 2y_2 \cos x - y_1 \sin x = -\cos x$

$\Rightarrow y_3(0) - 2\cos(0) - \sin 0 = -\cos 0.$

$\boxed{y_3(0) = 1}$ 1M

$(1+\sin x) y_4 + y_3 \cos x + 2y_3 \cos x - 2y_2 \sin x - y_1 \cos x - y_2 \sin x = \sin x.$

$y_4(0) = -3 + 1 = -2 \Rightarrow \boxed{y_4(0) = -2}$ 1M

$\therefore$ By Maclaurin's Series Expansion

$$y = y(0) + x\, y_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0) + \frac{x^4}{4!} y_4(0) + \cdots$$

$$\therefore y = 0 + x(1) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(1) + \frac{x^4}{4!}(-2) + \cdots$$

$$\log(1+\sin x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \cdots$$ 2M

b) Given $u = \log(\tan x + \tan y + \tan z) \longrightarrow (1)$

diff (1) partially w.r.t x, y, z.

$$\frac{\partial u}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y + \tan z}$$ 1M

$$\frac{\partial u}{\partial y} = \frac{\sec^2 y}{\tan x + \tan y + \tan z}$$ 1M

$$\frac{\partial u}{\partial z} = \frac{\sec^2 z}{\tan x + \tan y + \tan z}$$ 1M

Dr. Satish P. Hande, Dept. of Mathematics
KLS Vishwanathrao Deshpande Institute of Technology

Q.No.	Solution and Scheme	Marks
	$\sin 2x \frac{\partial u}{\partial x} = \sin 2x \times \frac{\sec^2 x}{\tan x + \tan y + \tan z}$	1M
	$= \frac{2\tan x}{\tan x + \tan y + \tan z}$	
	Similary $\sin 2y \frac{\partial u}{\partial y} = \frac{2\tan y}{\tan x + \tan y + \tan z}$	1M
	& $\sin 2z \frac{\partial u}{\partial z} = \frac{2\tan z}{\tan x + \tan y + \tan z}$	1M
	$\therefore \sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = \frac{2(\tan x + \tan y + \tan z)}{\tan x + \tan y + \tan z}$	
	ie $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$	1M
3 c)	Given $f(x,y) = x^2 + y^2 + 6x - 12$	
	$f_x = 2x + 6 = 0 \Rightarrow 2(x+3) = 0 \Rightarrow x = -3$	2M
	$f_y = 2y = 0 \Rightarrow y = 0$	1M
	$\therefore$ The stationary pt is	
	$r = f_{xx} = 2 > 0$	
	$s = f_{xy} = 0$	
	$t = f_{yy} = 2$	2M
	$rt - s^2 = 4 > 0$	
	$\because r > 0$ and $rt - s^2 > 0$	
	$\therefore f(x,y)$ is minimum at $(-3, 0)$	
	$\therefore$ Extrem value is $f(x,y) = f(-3,0) = (-3)^2 + 0^2 + 6\times(-3) - 12$	2M
	$= 9 - 18 - 12$	
	$= -21$	
4 a)	Let $k = \lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$ ---- 1^{∞} form	1M
	Taking logarithm on both sides	

Q.No.	Solution and Scheme	Marks

$$\log k = \lim_{x\to 0} \frac{\log\left(\frac{a^x+b^x+c^x}{3}\right)}{x} \quad \text{.... } \left(\tfrac{0}{0} \text{ form}\right)$$ 1M

Applying L-Hospital rule we get

$$\log k = \lim_{x\to 0} \frac{\frac{3}{a^x+b^x+c^x}\left[a^x\log a+b^x\log b+c^x\log c\right]}{1}$$ 2M

$$= \frac{a^0\log a+b^0\log b+c^0\log c}{a^0+b^0+c^0}$$

$$= \frac{1}{3}\log(abc) = \log(abc)^{1/3}$$ 2M

$$\therefore k=(abc)^{1/3}$$

b)

Let $p=\frac{x}{y}$, $q=\frac{y}{z}$, $r=\frac{z}{x}$

$\therefore u=f(p,q,r) \longrightarrow (1)$

diff (1) partially wrt x, y, z

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial x}+\frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial x}+\frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x}$$ 1M

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial y}+\frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial y}+\frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial y}$$ 1M

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial z}+\frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial z}+\frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial z}$$ 1M

$$\frac{\partial u}{\partial x} = \frac{1}{y}\frac{\partial u}{\partial p}+0\cdot\frac{\partial u}{\partial q}+\left(-\frac{z}{x^2}\right)\frac{\partial u}{\partial r}$$

$$\frac{\partial u}{\partial y} = \left(\frac{-x}{y^2}\right)\frac{\partial u}{\partial p}+\frac{1}{z}\frac{\partial u}{\partial q}+0\cdot\frac{\partial u}{\partial r}$$

$$\frac{\partial u}{\partial z} = 0\cdot\frac{\partial u}{\partial p}+\frac{-y}{z^2}\frac{\partial u}{\partial q}+\frac{1}{x}\cdot\frac{\partial u}{\partial r}$$ 3M

$$x\frac{\partial u}{\partial x}+y\cdot\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z} = \frac{x}{y}\frac{\partial u}{dp}-\frac{z}{x}\frac{\partial u}{\partial r}-\frac{x}{y}\frac{\partial u}{\partial p}+\frac{y}{z}\frac{\partial u}{\partial q}-\frac{y}{z}\frac{\partial u}{\partial q}+\frac{z}{x}\frac{\partial u}{\partial r}$$

$$\therefore x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$$ 1M

Q.No.	Solution and Scheme	Marks
4c)	$x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$	
	$J = \dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = \begin{vmatrix} \partial x/\partial r & \partial x/\partial\theta & \partial x/\partial\phi \\ \partial y/\partial r & \partial y/\partial\theta & \partial y/\partial\phi \\ \partial z/\partial r & \partial z/\partial\theta & \partial z/\partial\phi \end{vmatrix}$	1M
	$J = \begin{vmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{vmatrix}$	2M
	On Expanding by the last ~~two~~ row	
	$J = \cos\theta\{r^2\sin\theta\cos\theta\cos^2\phi + r^2\sin\theta\cos\theta\sin^2\phi\}$	1M
	$+ r\sin\theta\{r\sin^2\theta\cos^2\phi + r\sin^2\theta\sin^2\phi\}$	1M
	$J = r^2\sin\theta\cos^2\theta + r^2\sin^3\theta = r^2\sin\theta.$	2M
	$J = r^2\sin\theta$	
5 a)	Given $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2x$	
	dividing by y^2	1M
	$y^{-2}\dfrac{dy}{dx} + y^{-1}\cdot\dfrac{1}{x} = x$	
	put $y^{-1} = t$ $\quad (-1)y^{-2}\cdot\dfrac{dy}{dx} = \dfrac{dt}{dx} \Rightarrow y^{-2}\dfrac{dy}{dx} = -\dfrac{dt}{dx}$	
	$-\dfrac{dt}{dx} + \dfrac{t}{x} = x$	2M
	$\dfrac{dt}{dx} - \dfrac{t}{x} = -x$	
	$P = -\dfrac{1}{x}\quad Q = -x$	1M
	$I.F = e^{\int P.dx} = e^{\int(-\frac{1}{x})dx} = e^{-\log x} = \dfrac{1}{x}$	
	$\therefore$ soln is $t\cdot\left(\dfrac{1}{x}\right) = \int -x\left(\dfrac{1}{x}\right)dx + c$	1M

Q.No.	Solution and Scheme	Marks
	$\frac{t}{x} = -x + C$	
	$\frac{y^{-1}}{x} = -x + C$	
	$\frac{1}{xy} + x = C$ is the required soln.	1M
b)	We have $r = a(1+\cos\theta)$	
	$\log r = \log a + \log(1+\cos\theta)$ diff. w.r.t θ we get	
	$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{-\sin\theta}{1+\cos\theta}$ replacing $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$	2M
	$\frac{1}{r}\left[-r^2\frac{d\theta}{dr}\right] = \frac{-\sin\theta}{1+\cos\theta}$	1M
	$r\frac{d\theta}{dr} = \frac{\sin\theta}{1+\cos\theta}$ or $\frac{1+\cos\theta}{\sin\theta} = \frac{dr}{r}$	1M
	Integrating.	
	$\int\frac{dr}{r} - \int\frac{1+\cos\theta}{\sin\theta}\,d\theta = C$	
	$\log r - \log[\operatorname{cosec}\theta - \cot\theta] - \log\sin\theta = \log b$	2M
	$\log r - \log[\sin\theta\{\operatorname{cosec}\theta - \cot\theta\}] = \log b$	
	$\Rightarrow r = b(1-\cos\theta)$ is the required O.T.	1M
c)	Given $p^2 + 2py\cot x - y^2 = 0$.	
	$p = \frac{-2y\cot x \pm 2y\operatorname{cosec}x}{2}$	2M
	$p = y(-\cot x + \operatorname{cosec}x)$ or $p = y(-\cot x - \operatorname{cosec}x)$	
	$\frac{dy}{y} = (\operatorname{cosec}x - \cot x)\,dx$ or $\frac{-dy}{y} = (\cot x + \operatorname{cosec}x)\,dx$	2M
	Integrating	
	$\log y = \log\tan(x/2) - \log\sin x + k$ or $\log y + \log\sin x + \log\tan(x/2) = k$	
	or $\log y = \log\left[\frac{c\tan(x/2)}{\sin x}\right]$ $\quad$ $\log[y\sin x\tan(x/2)] = \log c$	
	$y = \frac{c\tan(x/2)}{\sin x}$ $\quad$ $y \cdot 2\sin(x/2)\cos(x/2)\frac{\sin x/2}{\cos x/2} = C$	2M
	ie $y(1+\cos x) = c$ $\quad$ or $y(1-\cos x) = C$	
	$\therefore$ general soln is $[y(1+\cos x) - c][y(1-\cos x) - c] = 0$	1M

Q.No.	Solution and Scheme	Marks
6 a)	Given $y(2xy+1)dx - xdy = 0$	
	$M = 2xy^2 + y$; $N = -x$	
	$\frac{\partial M}{\partial y} = 4xy+1$ $\quad \frac{\partial N}{\partial x} = -1 \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$	1M
	$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \cancel{2x-1}\ 4xy+1+1 = 4xy+2 = 2(2xy+1)$	
	$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2}{y(2xy+1)} \times (2xy+1) = \frac{2}{y}$	2M
	$IF = e^{-\int g(y)\,dy} = e^{-\int 2/y\,dy} = e^{-2\log y} = 1/y^2$	1M
	From the given d.e by M. by $1/y^2$	
	$\left(2x + \frac{1}{y}\right)dx - \frac{x}{y^2}dy$	1M
	$M = 2x + \frac{1}{y}$ $\quad N = -x/y^2$	
	$\frac{\partial M}{\partial y} = \frac{-1}{y^2}$ $\quad \frac{\partial N}{\partial x} = -1/y^2 \Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	1M
	soln is $x^2 + \frac{x}{y} = c$	
b)	Given $r^n \sin n\theta = a^n$ Taking logarithm on both sides	
	$n\log r + \log\sin n\theta = n\log a$ differentiating w.r.t θ	1M
	$\frac{n}{r}\frac{dr}{d\theta} + \frac{n\cos n\theta}{\sin n\theta} = 0$	1M
	$\frac{1}{r}\frac{dr}{d\theta} = -\cot n\theta$	
	$\frac{1}{r}\left(-r^2\frac{d\theta}{dr}\right) = -\cot n\theta$	2M
	$r\frac{d\theta}{dr} = \cot n\theta$	
	$\frac{d\theta}{\cot n\theta} = \frac{dr}{r} \Rightarrow \tan n\theta\, d\theta = \frac{dr}{r}$ Integrating	
	$-\frac{\log\cos n\theta}{n} = \log r + \log c$	2M
	$\log r^n + \log\cos n\theta + \log c$	
	$\log(r^n \cos n\theta) = \log C_1$	
	$r^n \cos n\theta = C$	
	is the required O.T.	1M

Q.No.	Solution and Scheme	Marks
6)	Given $(px-y)(py+x)=2p$. Also $X=x^2,\ Y=y^2$ $\frac{dX}{dx}=2x \quad \frac{dY}{dy}=2y$	1M
	$P=\frac{dY}{dX}=\frac{dY}{dX}\ \frac{dy}{dx}=\frac{dy}{dY}\cdot\frac{dY}{dX}\cdot\frac{dX}{dx}=\frac{1}{2y}\cdot P\cdot 2x.$ $p=\frac{x}{y}P$ put $p=\frac{x}{y}P=\sqrt{\frac{X}{Y}}P$	1M
	$\left[\sqrt{\frac{X}{Y}}P\sqrt{X}-\sqrt{Y}\right]\left[\sqrt{\frac{X}{Y}}P\cdot\sqrt{Y}+\sqrt{X}\right]=2\sqrt{\frac{X}{Y}}P.$ $\left[\frac{XP-Y}{\sqrt{Y}}\right]\sqrt{X}\left[P+1\right]=2\sqrt{\frac{X}{Y}}P.$	2M
	$[XP-Y][P+1]=2P.$ $[XP-Y]=\frac{2P}{P+1}.$ $Y=XP-\frac{2P}{P+1}$ is a clairauts eqn	2M
	∴ Soln is $Y=cX-\frac{2c}{c+1}$ ie $y^2=cx^2-\frac{2c}{c+1}$ is the required soln.	1M
7 a)	$2^7\equiv 34 \pmod{47}$	1M
	$(2^7)^3\equiv(34)^3 \pmod{47}$ $2^{21}\equiv 12 \pmod{47}$	2M
	$(2^{21})(2^2)\equiv 12(2)^2 \pmod{47}$ $2^{23}\equiv 48 \pmod{47}$.	2M
	$2^{23}\equiv 1 \pmod{47}$ ∴ The remainder when 2^{23} is divided by 47 is 1	1M

Q. No.	Solution and Scheme	Marks
7 b)	$7^1 = 7 = 7$ $7^2 = 49 = 9$ $7^3 = 343 = 3$ $7^4 = 2041 = 1$	2M
	$7^5 = 7^4 \cdot 7^1 = 1 \cdot 7^1 = 7$ $7^6 = 7^4 \cdot 7^2 = 1 \cdot 7^2 = 9$ $7^7 = 7^4 \cdot 7^3 = 1 \cdot 3 = 3$ $7^8 = 7^4 \cdot 7^4 \cdot 1 \cdot 1 = 1$	2M
	$\therefore 7^{112} = (7^4)^{28} = 1$	1M
	$7^{118} = 7^{112} \cdot 7^6 = 1 \times 7^6 = 1 \times 9 = 9.$ $\therefore$ The last digit of 7^{118} is 9	2M
c)	$p = 43,\ q = 59$	1M
	$n = 43 \times 59 = 2537$ & $e = 13$	1M
	Note $\gcd[e, (p-1)(q-1)] = \gcd[13, 42 \times 58] = 1$	1M
	STOP $\rightarrow$ 1819 1415	1.M
	$c = M^{13} \bmod 2537$ $= 1819^{13} \bmod 2537 = 2081$ & $1415^{13} \bmod 2537 = 2182.$ Hence encrypted message is 2081, 2182	3M
8 a)	$a = 8,\ p = 31$ 8 is not divisible by 31	1M
	$\therefore$ By Fermat's little theorm	1M
	$a^{p-1} \equiv 1 \pmod p$ $8^{31-1} \equiv 1 \pmod{31}$ $8^{30} \equiv 1 \pmod{31}$	2M
	ie $8^{30} - 1 \equiv 0 \pmod{31}$ ie $8^{30} - 1$ is divisible by 31	2M

Q.No.	Solution and Scheme	Marks
b)	$a_1 = 3, \quad a_2 = 2, \quad a_3 = 4$	
	$m_1 = 5, \quad m_2 = 6, \quad m_3 = 7$	1M
	$m = m_1 \times m_2 \times m_3 = 5 \times 6 \times 7 = 210$	
	$z_1 = m/m_1 = 210/5 = 42$	
	$z_2 = m/m_2 = 210/6 = 35$	
	$z_3 = m/m_3 = 210/7 = 30$	2M
	$y_1 = z_1^{-1} (\bmod m_1) = 42^{-1} (\bmod 5) = 3.$	
	$y_2 = z_2^{-1} (\bmod m_2) = 35^{-1} (\bmod 6) = 1$	
	$y_3 = z_3^{-1} (\bmod m_3) = 30 (\bmod 7) = 4 = 4$	2M
	$w_1 = y_1 z_1 (\bmod m) = 126 (\bmod 210) = 126$	
	$w_2 = y_2 z_2 (\bmod m) = 35 (\bmod 210) = 35$	
	$w_3 = y_3 z_3 (\bmod m) = 120 (\bmod 210) = 120$	2M
	$x = \sum a_i w_i = 3 \times 126 + 2 \times 35 + 4 \times 120 = 928$	
c)	i) $175 \equiv 10 \pmod{11}$ $11 \,)\, 175 \,(\, 15$; 165; 10	
	$113 \equiv 3 \pmod{13}$	
	$53 \equiv 9 \pmod{11}$	1M
	$\therefore 175 \times 113 \times 53 \equiv 10 \times 3 \times 9 \pmod{11}$	
	$175 \times 113 \times 53 \equiv 270 \pmod{11}$	
	$175 \times 113 \times 53 \equiv 6 \pmod{11}$	2M
	$\therefore$ The remainder is 6	

Dr. Satish P. Hande, Dept. of Mathematics
KLS Vishwanathrao Deshpande Institute of Technology

Q.No.	Solution and Scheme	Marks

9 a) Let $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ $R_1 \leftrightarrow R_2$ — 1M

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 6R_1$ — 2M

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$ $R_3 \to 5R_3 - 4R_2$, $R_4 \to 5R_4 - 9R_2$ — 1M

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 33 & 22 \end{bmatrix}$ $R_4 \to R_4 - R_3$ — 1M

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = 3$ — 1m

---

b) The augmented matrix is

$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \\ 2 & 5 & 7 & 52 \end{array}\right]$ — 1M

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 2R_1$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array}\right]$ $R_3 \to R_3 + 3R_2$ — 2M

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right]$ $R_3 \to -\frac{1}{4}R_3$ — 1M

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & 1 & 5 \end{array}\right]$ — 1m

Dr. Satish P. Hande, Dept. of Mathematics
KLS Vishwanathrao Deshpande Institute of Technology

Q.No.	Solution and Scheme	Marks

$R_1 \to R_1 - R_3,\ R_2 \to R_2 + 3R_3$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad R_1 \to R_1 + R_2$$

1M

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Rewriting the system of Eq^n

1M

$x = 1,\ y = 3\ \ z = 5$

c) Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ & $X^{(0)} = [1, 1, 1]'$

$$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$

$$Ax^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 7.34 \\ -2.67 \\ 4.01 \end{bmatrix} = 7.34\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \lambda^{(2)} x^{(2)}$$

$$Ax^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 7.82 \\ -3.63 \\ 4.01 \end{bmatrix} = 7.82\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \lambda^{(3)} x^{(3)}$$

$$Ax^{(3)} = \begin{bmatrix} 7.94 \\ -3.89 \\ 3.99 \end{bmatrix} = \cancel{7.82\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix}} = 7.94\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \lambda^{(4)} x^{(4)}$$

$$Ax^{(4)} = \begin{bmatrix} 7.98 \\ -3.97 \\ 3.99 \end{bmatrix} = 7.98\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(5)} x^{(5)}$$

$$Ax^{(5)} = \begin{bmatrix} 8 \\ -4 \\ 4 \end{bmatrix} = 8\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} \quad \therefore \lambda = 8,\ X = [1, -0.5, 0.5]'$$

Q. No.	Solution and Scheme	Marks
10 a)	The equation are diagonally dominant	
	$x = \frac{1}{20}[17 - y + 2z]$; $y = \frac{1}{20}[-18 - 3x + z]$	1M
	$z = \frac{1}{20}[25 - 2x + 3y]$	
	We start with the trial solution $x=0, y=0, z=0$	
	<u>Ist Iteration:</u> $x^{(1)} = \frac{17}{20} = 0.85$	
	$y^{(1)} = \frac{1}{20}[-18 - 3(0.85)] = -1.0275$	2M
	$z^{(1)} = \frac{1}{20}[25 - 2(0.85) + 3(-1.0275)] = 1.0109$	
	<u>2nd Iteration</u>	1M
	$x^{(2)} = 1.0025$, $y^{(2)} = 0.9998$, $z^{(2)} = 0.9998$	
	<u>Third Iteration</u>	
	$x^{(3)} = 0.99997 \simeq 1$, $y^{(3)} = -1.00005 \simeq -1$ $z^{(3)} = 1.0000022 \simeq 1$	2M
	Thus $x=1, y=-1, z=1$	
b)	The augmented matrix is	
	$[A\|B] = \left[\begin{array}{ccc\|c} 1 & -2 & 3 & 2 \\ 3 & -1 & 4 & 4 \\ 2 & 1 & -2 & 5 \end{array}\right]$ $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$	1M
	$\sim \left[\begin{array}{ccc\|c} 1 & -2 & 3 & 2 \\ 0 & 5 & -5 & -2 \\ 0 & 5 & -8 & 1 \end{array}\right]$ $R_3 \to R_3 - R_2$	2M
	$\sim \left[\begin{array}{ccc\|c} 1 & -2 & 3 & 2 \\ 0 & 5 & -5 & -2 \\ 0 & 0 & -3 & 3 \end{array}\right]$ $\Rightarrow \rho[A] = \rho[A\|B] = 3 = n$	2M
	$\therefore$ The system has unique soln.	
	Rewriting the system of Equation	
	$z = -1$ ~~z=1~~, $5y - 5z = -2$	
	$5y = -2 - 5 = -7 \Rightarrow y = -7/5 = -1.4$	
	$x = 2 + 2y - 3z = 2 + (2 \times -1.4) + 3 = 2.2$	2M
	$x = 2.2, y = -1.4, z = -1$	

Q.No.	Solution and Scheme	Marks
c)	The Augmented matrix is	
	$[A\|B] = \left[\begin{array}{ccc\|c} 2 & 1 & 4 & 12 \\ 4 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right]$	1M
	$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 4R_1$	
	$\sim \left[\begin{array}{ccc\|c} 2 & 1 & 4 & 12 \\ 0 & 9 & -9 & +9 \\ 0 & -7 & -14 & -28 \end{array}\right]$	2M
	$R_2 \to \frac{1}{9} R_2,\ R_3 \to R_3 \times \frac{-1}{7}$	
	$\sim \left[\begin{array}{ccc\|c} 2 & 1 & 4 & 12 \\ 0 & -1 & +1 & -4 \\ 0 & 1 & 2 & 4 \end{array}\right]$	2M
	$R_3 \to R_3 + R_2$	
	$\sim \left[\begin{array}{ccc\|c} 2 & 1 & 4 & 12 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 3 & 3 \end{array}\right]$	1M
	Rewritting the Eqn we get	
	$x=3,\ z=1,\ y=2$	1M

—END—