Modified

CBCS SCHEME

USN | | | | | | | | | | |

BMATS201

Second Semester B.E./B.Tech. Degree Supplementary Examination, June/July 2024

Mathematics – II for CSE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Evaluate $\int_{-1}^{+1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$.	7	L2	CO1
	b.	Evaluate $\int_{0}^{a}\int_{x/a}^{\sqrt{x/a}}(x^2+y^2)\,dy\,dx$ by changing the order of integration.	7	L3	CO1
	c.	Show that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	6	L2	CO1
		OR			
Q.2	a.	Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}}(x^2+y^2)\,dy\,dx$ by changing into polar co-ordinates.	7	L3	CO1
	b.	Find the area between the parabolas $x^2=y$ and $y^2=x$ using double integration.	7	L3	CO1
	c.	Using mathematical number's, write a code to find the area of an ellipse $\dfrac{x^2}{a^2}+\dfrac{x^2}{b^2}=1$ by double integration.	6	L3	CO5
		Module – 2			
Q.3	a.	Find the directional derivative of $\phi=4xz^3-3x^2y^2z$ at (2, -1, 2) in the direction of the vector $2\hat{i}-3\hat{j}+6\hat{k}$.	7	L2	CO2
	b.	If $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$ find div $\vec{F}$ and curl $\vec{F}$.	7	L2	CO2
	c.	Prove that spherical co-ordinate system is orthogonal.	6	L3	CO2
		OR			
Q.4	a.	Find the angle betwen the normals to the surface $xy=z^2$ at the points (4, 1, 2) and (3, 3, -3).	7	L2	CO2
	b.	If $\vec{F}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$, find a, b, c such that curl $\vec{F}=0$.	7	L2	CO2
	c.	Using mathematical tool write a code to find the curl of $\vec{F}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$.	6	L3	CO5
		Module – 3			
Q.5	a.	Prove that the set $w=\left\{(x,y,z)\middle/ x-3y+4z=0\right\}$ is a subspace of $V_3(R)$.	7	L2	CO3

	b.	Express the matrix $M=\begin{bmatrix}4&7\\7&9\end{bmatrix}$ as a linear combination of the matrices, $P=\begin{bmatrix}1&1\\1&1\end{bmatrix}, Q=\begin{bmatrix}1&2\\3&4\end{bmatrix}, R=\begin{bmatrix}1&1\\4&5\end{bmatrix}$	7	L2	CO3
	c.	Find the basis and dimension of subspace spanned by the vectors, $\{(1,-2,3),(1,-3,4)(-1,1,-2)\}$ of $V_3(R)$.	6	L3	CO3
		OR			
Q.6	a.	Find the matrix of linear transformation, $T:R^2 \to R^2$ defined by, $T(x,y)=(2x+3y,4x-5y)$ with respect to the basis, $B_1=\{(1,2),(2,5)\}$ of R^2.	7	L2	CO3
	b.	The transformation $G:R^3 \to R^3$ is defined as $G(x,y,z)=(x+2y-z, y+z, x+2y-2z)$. Find the basis and dimension of Im(G).	7	L2	CO3
	c.	If $f(t)=t+2$, $g(t)=3t-2$, $h(t)=t^2-2t-3$ and $\langle f,g\rangle=\int_0^1 f(t)g(t)dt$, find (i) $\langle f,g\rangle$ (ii) $\langle f,h\rangle$ (iii) $\lVert f\rVert$ and $\lVert g\rVert$	6	L3	CO3
		Module – 4			
Q.7	a.	Find the real root of the equation $x^3-2x-5=0$, correct to three decimal places using Regula-Falsi method. Carry out three iteration.	7	L2	CO4
	b.	Using Newton's forward interpolation formula find y at x = 5 from the following table: x: 0, 2, 4, 6, 8, 10 y: 0, 4, 56, 204, 496, 980	7	L2	CO4
	c.	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}^{th}$ rule taking six equal intervals.	6	L3	CO4
		OR			
Q.8	a.	Find the real root of the equation, $xe^x-2=0$, correct to three decimal places using Newton-Raphson method. Carry out three iterations.	7	L2	CO4
	b.	Using Lagrange's interpolation formula find f(4) given, x : 0, 2, 3, 6 f(x) : -4, 2, 14, 158	7	L2	CO4
	c.	Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by trapezoidal rule considering six equal intervals.	6	L3	CO4
		Module – 5			
Q.9	a.	Employ Taylor's series method to obtain approximate value of y at x = 0.1 for the differential equation, $\frac{dy}{dx}=2y+3e^x$, $y(0)=0$	7	L2	CO4
	b.	Apply Runge-Kutta method of 4^{th} order to find an approximate value of y at x = 0.2, given that $\frac{dy}{dx}=\frac{y-x}{y+x}$, y(0) = 1.	7	L2	CO4
	c.	Apply Milne's method to find y(1.4) given $\frac{dy}{dx}=x^2+\frac{y}{2}$ and the following data : $y(1)=2$, $y(1.1)=2.2156$, $y(1.2)=2.4649$, $y(1.3)=2.7514$.	6	L3	CO5

OR					
Q.10	a.	Using modified Euler's method, find $y(0.1)$ given $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. Carry out 3 iterations.	7	L2	CO4
	b.	Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0.4) = 1$ at $x = 0.5$.	7	L2	CO4
	c.	Using mathematical tool, write a code to solve the differential equation $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 0$, using Taylor's series method at $x = 0.1$.	6	L3	CO5

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	Scheme of solution	Marks
1 a)	Let $I = \int_{z=-1}^{1}\int_{x=0}^{z}\int_{y=x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$	1m
	$= \int_{z=-1}^{1}\int_{x=0}^{z}\left[xy+\frac{y^2}{2}+yz\right]_{x-z}^{x+z}dx\,dz$	2m
	$= \int_{z=-1}^{1}\int_{x=0}^{z}(4xz+2z^2)\,dx\,dz$	2m
	$= \int_{z=-1}^{1}\left[z(2x^2)+(2z^2)x\right]_{x=0}^{z} = \int_{-1}^{1}4z^3dz$	1m
	$= \left[z^4\right]_{-1}^{1}$	
	$\boxed{I=0}$	1m (7m)
b)	Let $I = \int_{x=0}^{a}\int_{y=x/a}^{\sqrt{x/a}}(x^2+y^2)\,dy\,dx$	1m
	Here x varies from $x=0$ to $x=a$ & y varies from $y=x/a$ to $y=\sqrt{x/a}$	
	After changing the order of Integration	1m
	$I = \int_{y=0}^{1}\int_{x=ay^2}^{ay}(x^2+y^2)\,dx\,dy$	
	$= \int_{y=0}^{1}\left[\frac{x^3}{3}+xy^2\right]_{ay^2}^{ay}dy.$	2m
	$= \int_{y=0}^{1}\left[\left\{\frac{a^3y^3}{3}-\frac{a^3y^6}{3}\right\}+\left\{ay^3-ay^4\right\}\right]dy$	
	$= \int_{y=0}^{1}\left[\frac{a^3}{3}(y^3-y^6)+a(y^3-y^4)\right]dy$	2m
	$= \frac{a^3}{3}\left[\frac{y^4}{4}-\frac{y^7}{7}\right]_0^1+a\left[\frac{y^4}{4}-\frac{y^5}{5}\right]_0^1$	
	$= \frac{a^3}{3}\left[\left(\frac{1}{4}-\frac{1}{7}\right)-0\right]+a\left[\left(\frac{1}{4}-\frac{1}{5}\right)-0\right]$	
	$I = \frac{a^3}{28}+\frac{a}{20}$	1m (7m)

Scheme of solution | Marks

c) By the definition of Beta & Gamma functions

$$B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$$ 1m

$$\Gamma(n) = 2\int_0^{\infty} e^{-x^2} x^{2n-1} dx \; ; \; \Gamma(m) = 2\int_0^{\infty} e^{-y^2} y^{2m-1} dy$$

$$\Gamma(m+n) = 2\int_0^{\infty} e^{-r^2} r^{2(m+n)-1} dr$$ 1m

Consider $\Gamma(m)\Gamma(n) = 4\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} x^{2n-1} \cdot y^{2m-1} dx\,dy$

putting $x = r\cos\theta$, $y = r\sin\theta \Rightarrow x^2+y^2 = r^2$ 1m

$dx\,dy = r\,dr\,d\theta$, r varies from $r=0$ to ∞ & 1m

θ varies from $\theta = 0$ to $\pi/2$

$$\therefore \Gamma(m)\Gamma(n) = 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2} r^{2m+2n-1} \sin^{2m-1}\theta \cos^{2n-1}\theta \, dr\,d\theta$$ 1m

$$= \left[2\int_{r=0}^{\infty} e^{-r^2} r^{2(m+n)-1} dr\right]\left[2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta\right]$$ 1m

$$\therefore \Gamma(m)\Gamma(n) = \Gamma(m+n) \cdot \beta(m,n)$$

Thus $B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (6m)

2 a) Let $I = \int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dy\,dx$ 1m

put $x = r\cos\theta$, $y = r\sin\theta$, $x^2+y^2 = r^2$, $dx\,dy = r\,dr\,d\theta$ 1m

$\therefore$ r varies from 0 to 1 & θ varies from 0 to $\pi/2$ 1m

$$I = \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} r^3 dr\,d\theta = \int_{\theta=0}^{\pi/2} \left(\frac{r^4}{4}\right)_0^1 d\theta.$$ 2m

$$= \frac{1}{4}\int_0^{\pi/2} d\theta = \frac{1}{4}\left[\theta\right]_0^{\pi/2}$$ 1m

$$I = \frac{\pi}{8}$$ 1m

(7m)

Scheme of solution

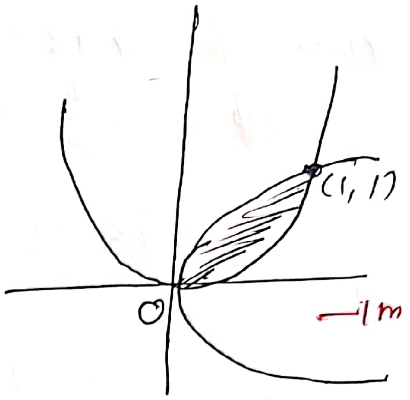
Marks

2 b) Given $x^2 = y$ and $y^2 = x$

$\Rightarrow x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0$

$\therefore x = 0$, or 1 —— 1m

$\therefore$ The pt of intersection are $(0, 0)$ and $(1, 1)$



$\therefore A = \int_{x=0}^{1} \int_{y=x^2}^{\sqrt{x}} dy\, dx$ —— 1m

$= \int_{x=0}^{1} [y]_{x^2}^{\sqrt{x}} dx = \int_0^1 [\sqrt{x} - x^2]\, dx.$ 1m

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right]_0^1 = [(2/3 - 1/3) - 0]$ 2m

$\therefore A = \frac{1}{3}$ 1m

(7m)

c)

```
from sympy import *
x = symbol('x')
y = symbol('y')
# a = symbol('a')
# b = symbol('b')
a = 4
b = 6
W3 = 4 * integrate(1, (y, 0, (b/a) * sqrt(a**2 - x**2)), (x, 0, a))
print (W3)
```

1m 2m 1m 2m

(6m)

Scheme of solution | Marks

3 a) Given $\phi = 4xz^3 - 3x^2y^2z$, $\vec{a} = 2i - 3j + 6k$.

$$\nabla\phi = \left(\frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k\right)$$

$$= (4z^3 - 6xy^2z)i + (0 - 6x^2yz) + (12xz^2 - 3x^2y^2)k$$

at $(2, -1, 2)$

$$\nabla\phi_{(2,-1,2)} = 8i + 48j + 84k.$$

$$\nabla\phi \cdot \hat{a} = 8i + 48j + 84k \cdot \frac{(2i - 3j + 6k)}{\sqrt{4+9+36}}$$

$$= \frac{(16 - 144 + 504)}{\sqrt{49}} = \frac{376}{7}$$

b) Given $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$

$$\vec{F} = (3x^2 - 3yz)i + (3y^2 - 3xz)j + (3z^2 - 3xy)k.$$

$$\therefore \operatorname{div}\vec{F} = 6x + 6y + 6z$$

$$\operatorname{curl}\vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2 - 3yz & 3y^2 - 3xz & 3z^2 - 3xy \end{vmatrix}$$

$$= i[(-3x) - (-3x)] - j[-3y + 3y] + k[-3z + 3z]$$

$$\operatorname{curl}\vec{F} = 0$$

c) $\vec{r} = r\sin\theta\cos\phi\, i + r\sin\theta\sin\phi\, j + r\cos\theta\, k$

$\hat{e}_r = \sin\theta\cos\phi\, i + \sin\theta\sin\phi\, j + \cos\theta\, k$

$\hat{e}_\theta = \cos\theta\cos\phi\, i + \cos\theta\sin\phi\, j + (-\sin\theta)k.$

$\hat{e}_\phi = -\sin\phi\, i + \cos\phi\, j + 0.k.$

$\therefore \hat{e}_r \cdot \hat{e}_\phi = 0, \quad \hat{e}_\theta \cdot \hat{e}_\phi = 0, \quad \hat{e}_\phi \cdot \hat{e}_r = 0$

$\therefore$ Spherical coordinate system is orthogonal.

Scheme of solution — Marks

4 a)

$\nabla\phi = yi + xj - 2zk$ — 1m

$\nabla\phi_1 = i + 4j - 4k$ — 1m

$\nabla\phi_2 = 3(i + j + 2k)$ — 1m

$\cos\theta = \dfrac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$ — 1M

— 3m

$\cos\theta = \dfrac{-1}{\sqrt{22}}$ — (7M)

b)

Given $\vec{F} = (x+y+az)i + (bx+2y-z)j + (x+cy+2z)k$ — 1M

$$\text{curl}\,\vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+y+az & bx+2y-z & x+cy+2z \end{vmatrix} = 0$$ — 1M

$= i(c+1) + (-j)(1 - a) + k(b-1) = 0i + 0j + 0k.$ — 2M

$c+1 = 0, \quad 1-a = 0 \quad b-1 = 0$ — 1m

$\Rightarrow a = 1, \; b = 1, \; c = -1$ — 2M

— (7M)

c)

```
from sympy.vector import *
from sympy import symbols
N= CoordSys3D('N')
x, y, z = symbols('x. y z')
A = N.x***2 + N.y***3 + N.z***3
delop = Del( )
curl A = delop.cross(A)
display (curl A)
Print (f"\n curl of {A} is \n")
display (curl (A) )
```

Marks: 1M, 1m, 1M, 1M, 1m, 1m — (6M)

Scheme of solution

		Marks
5 a)	Let $u_1 = (x_1, y_1, z_1)$ and $u_2 = (x_2, y_2, z_2)$	1m
	such that	
	$x_1 - 3y_1 + 4z_1 = 0$ & $x_2 - 3y_2 + 4z_2 = 0$	1m
	Consider	
	$c_1u_1 + c_2u_2 = c_1(x_1, y_1, z_1) + c_2(x_2, y_2, z_2)$	
	$= (c_1x, c_1y_1, c_1z_1) + (c_2x_2, c_2y_2, c_3z_2)$	2m
	$= c_1(x_1 - 3y_1 + 4z_1) + c_2(x_2 - 3y_2 + 4z_2)$	1m
	$= c_1 \cdot 0 + c_2 \cdot 0 = 0$	
	$\therefore c_1u_1 + c_2u_2 = 0$	2m
	$\therefore W$ is a subspace of $V_3(R)$	7m
b)	Let $M = ap + bQ + cR$.	
	ie $\begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix} = a\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + c\begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}$	1m
	$\therefore [A\|B] = \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 1 & 2 & 1 & 7 \\ 1 & 3 & 4 & 7 \\ 1 & 4 & 5 & 9 \end{array}\right]$	2m
	$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, R_4 \to R_4 - R_1$	
	$\sim \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & 3 & 3 \\ 0 & 3 & 4 & 5 \end{array}\right]$ $R_3 \to R_3 - 2R_2$, $R_4 \to R_4 - 3R_2$	2m
	$\sim \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 4 & -4 \end{array}\right]$	
	which gives $\boxed{a_1 = 2,\ a_2 = 3,\ a_3 = -1}$	2m
		7m

Scheme of solution | Marks

6 a) ~~Given $T(x, y) = (2x+3y, 4x-5y)$~~
~~$B_1 = \{(1,2), (2,5)\}$~~
~~$T(1, 2) = [2$~~

5 c a) Let $u_1 = (1,-2, 3)$, $u_2 = (1, -3, 4)$, $u_3 = (-1, 1, -2)$ | 1m

Let $A = [u_1, u_2, u_3] = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -3 & 1 \\ 3 & 4 & -2 \end{bmatrix}$

$R_2 \to R_2 + 2R_1$, $R_3 \to R_3 - 3R_1$ | 1m

$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ $R_3 \to R_3 + R_2$ | 2m

$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ | 1M

$\therefore u_1, u_2, u_3$ are linearly dependent

$\therefore$ Basis of a subspace are $(1, 1, -1), (0, -1, -1)$ | 1m

6m

---

6 a) Given $T(x, y) = (2x+3y, 4x-5y)$

$B_1 = \{(1, 2)\ (2, 5)\}$ | 1m

$T(1, 2) = (8, -6)$, $T(2, 5) = (19, -17)$ | 2M

W.K.T. $T(\alpha_1) = a_{11}\alpha_1 + a_{12}\alpha_2$

$T(\alpha_2) = a_{21}\alpha_1 + a_{22}\alpha_2$

ie $T(1, 2) = a_{11}(1, 2) + a_{12}(2, 5)$

$T(2, 5) = a_{21}(1, 2) + a_{22}(2, 5)$ | 2m

Solving

$a_{11} = 52$, $a_{12} = -22$, $a_{22} = -55$, $a_{21} = 129$ | 1m

$\therefore A = \begin{bmatrix} 52 & 129 \\ -22 & -55 \end{bmatrix}$ | 1m

7m

Scheme of solution | Marks

6 b) Given $G(x,y,z) = \{(x+2y-z), y+z, x+2y-2z\}$

Let e_1, e_2, e_3 be the standard basis of G. 1m

$\therefore G(1,0,0) = (1,0,1)$

$G(0,1,0) = (2,1,2)$

$G(0,0,1) = (-1, 1, -2)$ 2M

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix} R_3 \to R_3 - R_1$$ 1m

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$ 1m

Basis of $Im(G) = \{(1,2,-1)\ (0,1,1), (0,0,-1)\}$

$Dim(Im(G)) = 3$ 2m

7m

c) $\langle f, g\rangle = \int_0^1 (3t^2+4t-4)\,dt$

$= (t^3+2t^2-4t)_0^1 = (-1-0) = -1$ 2m

$\langle f, h\rangle = \int_0^1 (t^3-7t-6)\,dt$

$= \left(\frac{t^4}{4} - 7\frac{t^2}{2} - 6t\right)_0^1$

$= \frac{1}{4} - \frac{7}{2} - 6 = \frac{2-28-48}{8} = \frac{-74}{8} = \frac{-37}{4}$ 3M

$\|f\| = \frac{\sqrt{51}}{3}$ ~~1m~~

$\|g\| = 1$ 1m

6m

Scheme of solution | Marks

7 a) Let $f(x) = x^3 - 2x - 5$

$f(a) = f(2) = 8 - 4 - 5 = -1$

$f(b) = f(3) = 27 - 6 - 5 = 16$ — 1m

$\therefore$ The root lies in the interval $(2, 3)$

$$\therefore x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{2(16) - (3)(-1)}{16 + 1}$$ — 2m

$x_1 = 2.0942$

$f(x_1) = (2.0942)^3 - (2 \times 2.0942) - 5 = -0.0392$ — 1m

$\therefore f(x_1) < 0$

$\therefore$ The root lies in the $(2.0942, 3)$

$x_2 = 2.0945$ — 1m

$x_3 = 2.0945$ — 1m

$\therefore$ The real root upto 3rd iteration is

$x = 2.0945$ — 1m

(7m)

b)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	0	$\underline{4}$	$\underline{48}$	$\underline{48}$	0
2	4	52	96	48	0
4	56	148	144	48	
6	204	292	192		
8	496	484			
10	980				

3m

From the above table $\Delta y_0 = 4$, $\Delta^2 y_0 = 48$, $\Delta^3 y_0 = 48$

Given $x_p = 5 = x_0 + ph \Rightarrow p = \frac{x_p - x_0}{h} = \frac{5 - 0}{2} = 2.5$ — 2m

$\therefore$ By Newton's Forward interpolation formula

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$\boxed{y(5) = 115}$ — 2m

(7m)

Scheme of Solution | Marks

c) Let $f(x) = \frac{1}{1+x^2}$ $n = \frac{b-a}{h} = \frac{1-0}{6} \Rightarrow h = \frac{1}{6}$ | 1m

$\therefore x$	0	1/6	2/6	3/6	4/6	5/6	6/6
y	1	0.973	0.9	0.8	0.692	0.590	0.5

2m

By simpson's $\frac{1}{3}$ rule

$$I = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right]$$ 2m

$\therefore I = 0.785$ 1m

(6m)

8

a) Let $f(x) = xe^x - 2$

$f'(x) = xe^x + e^x = e^x(x+1)$ 1m

$f(0) = -2$, $f(1) = 0.7182$ 1m

$\therefore$ The root line in the interval $(0, 1)$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ Let $x_0 = 1$

$x_1 = 1 - \frac{0.7182}{5.43656} = 0.8678$ 2m

$x_2 = 0.8527$ 2m

$x_3 = 0.853$ 1m

(7m)

b) Given $x_0 = 0$, $x_1 = 2$, $x_2 = 3$, $x_3 = 6$

$y_0 = -4$, $y_1 = 2$, $y_2 = 14$, $y_3 = 158$

By Lagranges interpolation formula

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$ 1m

Scheme of solution | Marks

8 b) Substituting $x=4$ and given values we get — 5m

$f(4)=40$ — 1m

(7m)

c) Given $f(x) = \frac{x}{1+x^2}$

By taking 6 equal intervals $h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$ — 1m

x	0	1/6	2/6	3/6	4/6	5/6	6/6
$f(x)$	0	0.162	0.3	0.4	0.462	0.491	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

2m

By Trapezoidal rule

$$I = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + 2y_5 + y_6\right]$$

$$= \frac{1}{12}\left[0 + 2[0.162 + 0.3 + 0.4 + 0.462 + 0.492] + 0.5\right]$$ — 2m

$\therefore I = 0.346$ — 1m

(6m)

9

a) Given $y_1 = 2y + 3e^x$ $\quad x_0 = 0, \ y_0 = 0$ — 1m

$y_1 = 2y + 3e^x \Rightarrow y_1(0) = 3$

$y_2 = 2y_1 + 3e^x \Rightarrow y_2(0) = 9$

$y_3 = 2y_2 + 3e^x \Rightarrow y_3(0) = 21$

$y_4 = 2y_3 + 3e^x \Rightarrow y_4(0) = 45$

— 3m

By Taylor's series method

$$y = y(x_0) + x\,y_1(0) + \frac{x^2}{2!}y_2(0) + \frac{x^3}{3!}y_3(0) + \frac{x^4}{4!}y_4(0)$$ — 1m

$\therefore y(0.1) = 0.3485$ — 2m

(7m)

Scheme of solution | Marks

b) Given $x_0 = 0$, $y_0 = 1$ Let $h = 0.2$

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

$$k_1 = hf(x_0, y_0) = 0.2 f(0, 1) = 0.2\left[\frac{1-0}{1+0}\right]$$

$$\Rightarrow \boxed{k_1 = 0.2}$$ 1m

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.1667$$ 1m

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.1667$$ 1m

$$k_4 = hf(x_0 + h, y_0 + k_3) = 0.1414$$ 1m

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] = 0.1679$$ 1m

$$\therefore y(0.2) = y_0 + k = 1 + 0.1679$$

$$\therefore y(0.2) = 1.1679$$ 2m

(7m)

c)

x	y	$y' = x^2 + \frac{y}{2}$
1	2	$y_0' = 2$
1.1	2.2156	$y_1' = 2.3178$
1.2	2.4649	$y_2' = 2.6724$
1.3	2.7514	$y_3' = 3.065$

1m

By Milnes predictor formula.

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3'] = 3.0793$$ 2m

By Milnes corrector formula.

$$y_4^{(C)} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4'] = 3.0794$$ 3m

$$y(1.4) = 3.0794$$

(6m)

Scheme of solution | Marks

10 a) Given $f(x,y) = x - y^2$, $x_0 = 0$, $y_0 = 1$ Let $h = 0.1$

By Euler's formula $x_1 = x_0 + h = 0.1$ — 1m

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$= 1 + 0.1\,[0 - (1)^2] = 0.9$ — 1m

By Modified Euler's formula

$y_1^{(1)} = y_0 + \frac{h}{2}\,[f(x_0, y_0) + f(x_1, y_1^{(0)})]$

$= 1 + \frac{0.1}{2}\,[0.9 + \{0.1 - (1.1)^2\}]$

$= 1 + 0.05\,[0.9 + (-1.11)] = 0.9145$ — 2m

$y_1^{(2)} = 0.932$ — 1m

$y_1^{(3)} = 0.932$ — 1m

$y(0.1) = 0.932$ — 1m

(7m)

b) Given $f(x,y) = \frac{1}{x+y}$, $x_0 = 0.4$, $y_0 = 1$, $h = 0.1$ — 1m

$x_1 = x_0 + h = 0.4 + 0.1 = 0.5$

$k_1 = h f(x_0, y_0) = 0.1\, f(0.4, 1) = 0.1 \times \frac{1}{0.4+1}$

$k_1 = 0.0714$ — 1m

$k_2 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = 0.0673$ — 1m

$k_3 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) = 0.0674$ — 1m

$k_4 = h f(x_0 + h, y_0 + k_3) = 0.0638$ — 1m

$K = \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4] = 0.0674$ — 1m

$y_1 = y_0 + k$

$y(0.5) = 1.0674$ — 1m

(7m)

c)

```
from numpy import array
def taylor(deriv, x, x, x stop, h):
    X = [ ]
    Y = [ ]
    X. append(x)
    Y. append(Y)
    while x < x stop :
        D = deriv(x, y)
        H = 1.0
        for j in range(2):
        H = H * h/(j+1)
            Y = y + D [j] * H
            X = X + h
        ~~X~~ . X. append (x)
            Y. append (y)
    def deriv(x, y):
        D = Zeros((4, 1))
        D[0] = [x**2 + y**2]
        D[1] = [2*X + 2 x**x2 + 4Y*x2]
        D[2] = [2 ~~7~~, 4*X + 8Y*2]
```

1m

1m

1m

1m

~~1m~~ 1m

2m

6m

—— END ——

Hande
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