CBCS SCHEME

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BME403

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Fluid Mechanics

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define the following properties of fluids and write their SI units. i) Density ii) Specific weight iii) Specific volume iv) Kinematic viscosity.	8	L1	CO1
	b.	If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which 'u' is the velocity in meter per second at a distance 'y' meter above the plate, Determine the shear stress at y = 0 and y = 0.15m. Take dynamic viscosity of fluid as 8.63 poises.	6	L3	CO1
	c.	Define capillarity. Derive an expression for capillary rise.	6	L2	CO1
		OR			
Q.2	a.	State and prove Pascal's law.	6	L2	CO2
	b.	Define the following and indicate their relative position on a chart: i) Absolute pressure ii) Gauge pressure iii) Vacuum pressure iv) Atmospheric pressure.	6	L1	CO2
	c.	The right limb of a simple u-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.	8	L3	CO2
		Module – 2			
Q.3	a.	Define the following types of fluid flows: i) Steady and unsteady flow ii) Uniform and non-uniform flow iii) Compressible and incompressible flow.	6	L1	CO2
	b.	Derive the continuity equation in three dimensional Cartesian co-ordinates for a steady, incompressible fluid flow.	8	L2	CO2
	c.	Explain stream function and velocity potential function.	6	L2	CO2

		OR			
Q.4	**a.**	Derive Hagen-Poiseuille's equation for laminar flow through a circular pipe.	**10**	**L2**	**CO2**
	b.	A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. Calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank in 30 seconds. Assume laminar flow.	**6**	**L3**	**CO2**
	c.	Define Reynolds number. Explain its significance in fluid flow.	**4**	**L2**	**CO2**
		Module – 3			
Q.5	**a.**	Derive Euler's equation of motion along a stream line. Deduce Bernoulli's equation from Euler's equation. State the assumptions made.	**10**	**L2**	**CO3**
	b.	A pipeline carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position 'A' to 500 mm diameter at a position 'B' which is 4 m at a higher level. If the pressures at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively and the discharge is 200 lit/s, determine the loss of head and direction of flow.	**10**	**L3**	**CO3**
		OR			
Q.6	**a.**	Derive Darcy – Weisbach equation for loss of head due to friction in pipe.	**10**	**L2**	**CO3**
	b.	A horizontal pipe line 40 m long is connected to a water tank at one end and discharge freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur, determine the rate of flow. Take f = 0.01 for both sections of pipe.	**10**	**L3**	**CO3**
		Module – 4			
Q.7	**a.**	Explain the following terms: i) Drag ii) Lift iii) Friction drag iv) Pressure drag.	**8**	**L2**	**CO4**
	b.	Briefly explain what is meant by boundary layer and hence define the following: i) Boundary layer thickness ii) Displacement thickness.	**6**	**L2**	**CO4**
	c.	State and explain Buckingham's π theorem.	**6**	**L2**	**CO4**
		OR			
Q.8	**a.**	What is similitude? Explain the different types of similitude.	**7**	**L2**	**CO4**
	b.	Explain the dimentional homogeneity with examples.	**3**	**L2**	**CO4**

	c.	The frictional torque (T) of a disc of diameter (D) rotating at a speed (N) in a fluid of viscosity (μ) and density (ρ) in a turbulent flow is given by $T = D5N^2\rho\phi\left[\frac{\mu}{D^2N\rho}\right]$. Prove this by Buckingham's - π theorem.	**10**	**L3**	**CO4**
		Module – 5			
Q.9	**a.**	Define Mach number. Explain the significance of Mach number in compressible fluid flow.	**6**	**L2**	**CO5**
	b.	Derive an expression for velocity of sound wave in a fluid.	**8**	**L2**	**CO5**
	c.	Find the velocity of bullet fired in standard air if Mach angle is 30°. Take R = 287.14 J/kg K and γ = 1.4 for air and temperature of air is 15°C.	**6**	**L3**	**CO5**
		OR			
Q.10	**a.**	An air plane is flying at an altitude of 15 km where the temperature is -50°C. The speed of plane corresponds to Mach number 1.6. Assume γ = 1.4 and R = 287 J/kg K for air. Find speed of plane and Mach angle.	**8**	**L3**	**CO5**
	b.	Define: i) Mach Number ii) Sub-Sonic flow iii) Sonic flow iv) Super-Sonic flow	**4**	**L1**	**CO5**
	c.	Mention the advantages and disadvantages of CFD.	**8**	**L2**	**CO5**

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Fluid Mechanics QP solution

Module – I.

Q1 (a)

i) Density of a fluid is defined as the ratio of the mass of a fluid to its volume.

It is denoted by symbol ρ (rho).

$$\boxed{\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}}$$ unit in SI Kg/m^3.

ii) Specific weight of a fluid is the ratio between the weight of a fluid to its volume.

It is denoted by symbol (w).

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{\text{mass} \times g}{\text{Volume}}$$

$$\boxed{w = \rho \times g} \quad \frac{N}{m^3}.$$

iii) Specific volume: is defined as the volume of fluid occupied by a unit mass. It is denoted by V_s.

$$\therefore V_s = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{mass of fluid}}{\text{Volume of fluid}}}$$

$$\boxed{V_s = \frac{1}{\rho}} \quad \frac{m^3}{Kg}.$$

iv) Kinematic Viscosity (ν): It is defined as the ratio of dynamic viscosity and density of fluid.

It is denoted by symbol "ν" called nu

$$\boxed{\nu = \frac{\mu}{\rho}}$$ Its unit is $\frac{m^2}{s}$.

Q 1
(b)

Given data: $u = \frac{2}{3}y - y^2$

$y = 0.15$ m.

$\mu = 8.63$ poise $= \frac{8.63}{10} \frac{Ns}{m^2}$

$$u = \frac{2}{3}y - y^2$$

$$\therefore \; du = \frac{2}{3} - 2y$$

At $y = 0.15$, $\frac{du}{dy} = \frac{2}{3} - 2(0.15)$

$$= 0.66 - 0.3 = \underline{0.36}$$

Viscosity $\mu = \frac{8.63}{10} \frac{Ns}{m^2}$

W.k.t $\tau = \mu . \frac{du}{dy} = \frac{8.63}{10} \times 0.36$

$$\therefore \boxed{\tau = 0.310 \frac{N}{m^2}}$$

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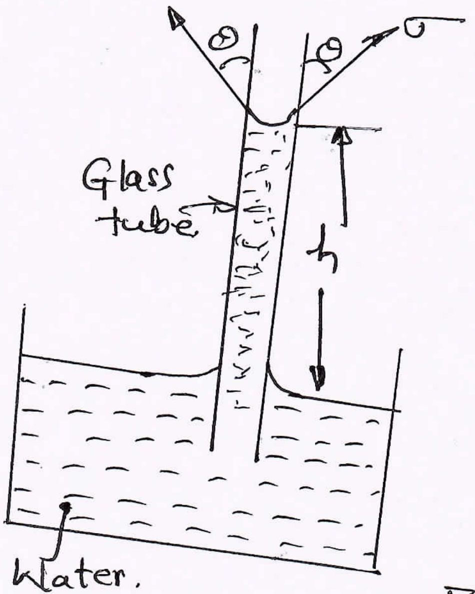
Q 1
(c)

The phenomenon of rise and fall of liquid level in the capillary tube is known as capillarity.

Expression for capillarity rise of a liquid.



* Surface tension in Vertically upward direction is

$$= \sigma \cos\theta \times \pi d \quad \text{—(i)}$$

* Weight of liquid in the tube (above the liquid level) in downward direction is

$$= \rho g \times \frac{\pi}{4} d^2 \times h \quad \text{—(ii)}$$

For equilibrium, eq^n (i) = (ii)

$$\therefore \sigma \cos\theta \times \pi D = \rho g \times \frac{\pi}{4} d^2 \times h.$$

$$\therefore \boxed{h = \frac{4\sigma\cos\theta}{\rho g d}}$$

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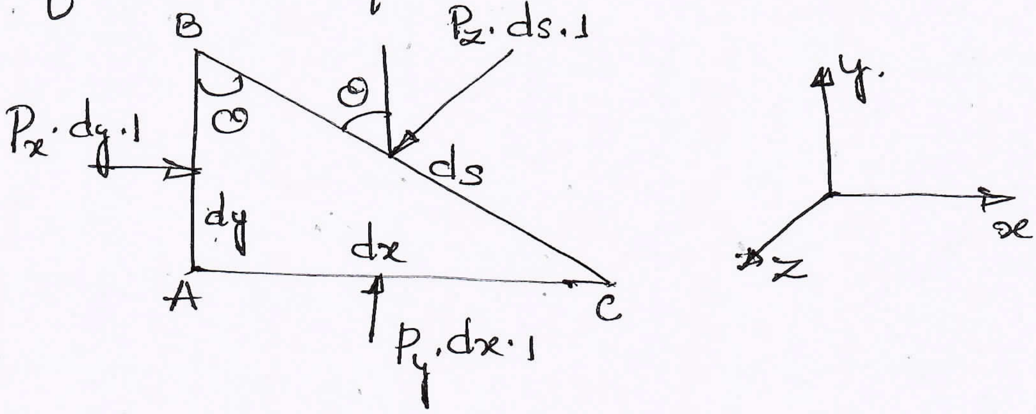
Q 2
(a) Pascal's law states that "the pressure or intensity of pressure at a point in a static fluid is equal in all directions."



consider an arbitrary fluid element of weight
Let the width of the fluid element perpendicular to the plane of paper is unity and P_x, P_y and P_z are pressure intensity acting on the face AB, AC and BC.

The force acting on fluid element

on the face AB $= P_x \times$ Area of the face AB

$= P_x \times dy \times 1$

||ly on the face AC $= P_y \times dx \times 1$

and on the face BC $= P_z \times ds \times 1$

Weight of fluid element = mass of element $\times g$

Resolving the forces in x-direction, we have

$P_x \times dy \times 1 - P_z (ds \times 1) \sin(90° - \theta) = 0$

or $P_x \times dy \times 1 - P_z \cdot ds \times 1 \cos\theta = 0$

But $ds \cos\theta = AB = y$

$\therefore P_x \times \cancel{dy} \times 1 - P_z \times \cancel{dy} \times 1 = 0$

$\therefore \boxed{P_x = P_z}$ — (i)

||ly

$P_y \cdot \cancel{dx} - P_z \times \cancel{dx} = 0$

$\therefore \boxed{P_y = P_z}$ — (ii)

from eq^n (i) & (ii)

$\boxed{P_x = P_y = P_z}$

2 (b)

(i) Absolute pressure: is defined as the pressure which is measured with reference to the absolute vacuum pressure.

(ii) Gauge pressure: is defined as the pressure which is measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken as datum.

(iii) Vacuum pressure: is defined as the pressure below the atmospheric pressure.

(iv) Atmospheric pressure: is the force exerted by the air in Earth's atmosphere on a surface.

Q 2
C

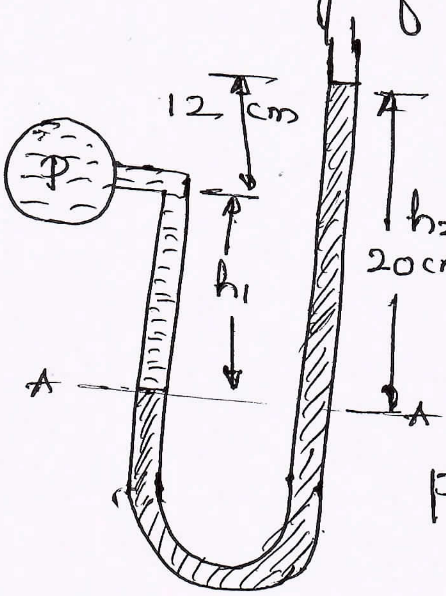
Given data

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ density of fluid $\rho_1 = S_1 \times 1000 = 900\ kg/m^3$.

Sp. gr. of mercury $S_2 = 13.6$.

$\therefore$ density of mercury, $\rho_2 = 13600\ kg/m^3$.



$h_2 = 20\ cm = 0.2\ m$.

$h_1 = 20 - 12 = 8\ cm = 0.08\ m$

Equating the pressure above A–A

We get,

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 0.2$$

$$p = 26683 - 706$$

$$\therefore \boxed{p = 25977\ N/m^2}$$

Module - 2.

3 (a) (i) Steady flow and unsteady flow.

Steady flow is defined as that type of flow in which the fluid characteristics like Velocity, pressure, density etc. at a point do not change with time.

Thus, $\left(\frac{\partial v}{\partial t}\right) = 0, \left(\frac{\partial P}{\partial t}\right) = 0, \left(\frac{\partial \rho}{\partial t}\right) = 0.$

Unsteady flow is defined as that type of flow, in which the velocity, pressure of density at a point changes w.r.t. to time.

Thus, $\frac{\partial v}{\partial t} \neq 0, \left(\frac{\partial P}{\partial t}\right) \neq 0$ etc.

(ii) Uniform and Non-uniform flow.

Uniform flow: is defined as that type of flow which the Velocity at any given time does not changes w.r.t. space.

$$\left(\frac{\partial v}{\partial s}\right)_{t = \text{constant}} = 0.$$

Non-uniform flow: is defined as that type of flow which the velocity at any given time changes w.r.t. space.

$$\left(\frac{\partial v}{\partial s}\right)_{t=c} \neq 0$$

(iii) <u>compressible and incompresible</u>

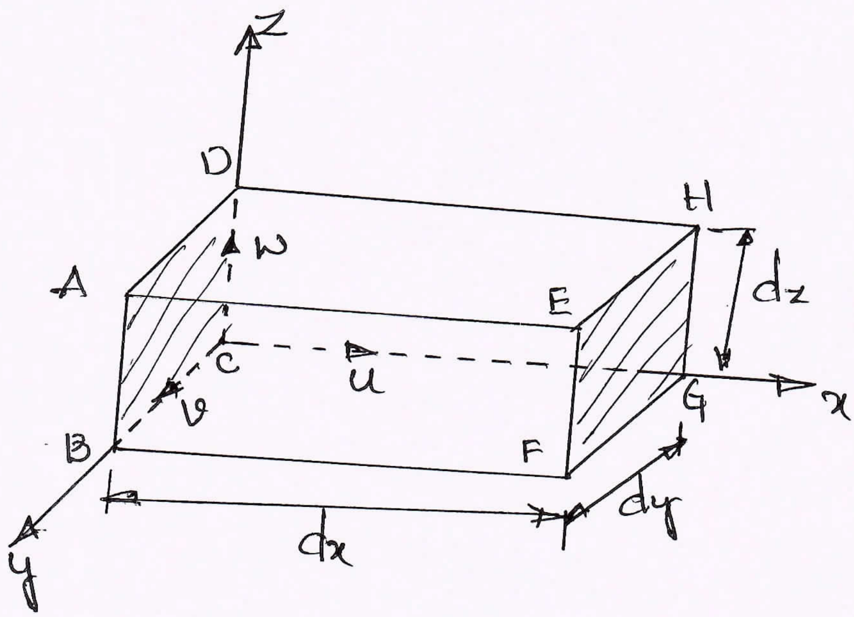
compressible flows : is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ).

$$\rho \neq \text{constant.}$$

Incompressible flow : is that type of flow in which the density is constant for the fluid flow.

$$\rho = \text{constant.}$$

3(b)



Consider a fluid element of length dx, dy and dz in direction of x, y and z. Let u, v and w are the inlet velocity components in x, y and z direction respectively.

Q.No	Solution and Scheme	Marks

Mass of fluid entering the face ABCD per second

$= \rho \times$ Velocity in x-direction $\times$ Area of ABCD

$= \rho \times u \times (dy \times dz)$.

Then, Mass of fluid leaving the face EFGH per second

$$= \rho u\, dy\, dz + \frac{\partial}{\partial x}(\rho u\, dy\, dz)\, dx.$$

$\therefore$ Gain of mass in x-directions

= Mass through ABCD − Mass through EFGH per sec.

$$= \rho u \cancel{dy}\, dz - \rho u \cancel{dy}\, dz - \frac{\partial}{\partial x}(\rho u\, dy\, dz)\, dx.$$

$$= -\frac{\partial}{\partial x}(\rho u)\, dx\, dy\, dz$$ $\because$ $dy\, dz$ is constant.

Similarly, the net gain of mass in y-directions

$$= -\frac{\partial}{\partial y}(\rho v)\, dx\, dy\, dz$$

and in z-direction $= -\dfrac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz$

$\therefore$ Net gain of mass

$$= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\, dy\, dz \qquad (1)$$

Mass of fluid in the element is $\rho\, dx\, dy\, dz$ and its rate of increase with time is.

$$\frac{\partial}{\partial t}(\rho \cdot dx \cdot dy \cdot dz) \quad \text{or} \quad \frac{\partial \rho}{\partial t}\, dx \cdot dy \cdot dz \qquad (2)$$

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Q.No	Solution and Scheme	Marks

Equating the two expressions,

$$\text{or} \quad -\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]\cancel{dx\,dy\,dz} = \frac{\partial \rho}{\partial t}\cancel{dx\,dy\,dz}$$

$$\text{or} \quad \boxed{\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0} \quad (3)$$

Equation (3) is the continuity equation in Cartesian co-ordinates in its general form.

This equation is applicable to:

i) steady and unsteady flow.

ii) Uniform and non-Uniform flow,

iii) compressible and incompressible fluid.

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3) c

Stream function : It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction.

It is denoted by ψ and defined only for two-dimensional flow.

$$\psi = f(x, y) \text{ such that}$$

$$\frac{\partial \psi}{\partial x} = v \quad \text{and} \quad \frac{\partial \psi}{\partial y} = -u.$$

Velocity potential function: It is defined as a scalar function of space and time such that its negative derivative with respect to that its negative derivative with respect to any direction gives the fluid velocity in that direction.

It is defined by ϕ (phi).

$$u = -\frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \phi}{\partial y}$$

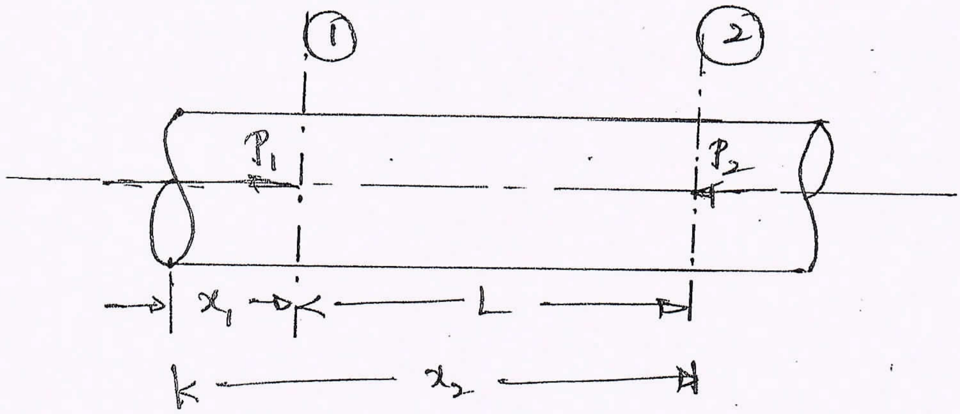
$$w = -\frac{\partial \phi}{\partial z}$$

For 3D flow. $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

for 2D flow: $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Module-3.

4
Q(9)



Marks
10

We have Average Velocity

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

or $$\left(-\frac{\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

Integrating the above equation w.r.t. x

We get, $$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\,dx.$$

$$\therefore -[P_1 - P_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2]$$

or $$(P_1 - P_2) = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$ | $x_2 - x_1 = L$

$$= \frac{8\mu\bar{u}}{R^2}L$$ | $R = \frac{D}{2}$.

$$(P_1 - P_2) = \frac{8\mu\bar{u}L}{(D/2)^2} = \frac{32\mu\bar{u}L}{D^2}$$

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$\therefore$ Loss of pressure head $= \frac{P_1 - P_2}{\rho g} = h_f$.

$$\therefore \boxed{h_f = \frac{32\mu\bar{u}L}{\rho g D^2}}$$

Hence it is Hagen Poiseuille formula

(4)
(b)

Given data :

$$\mu = 0.97 \text{ poise} = \frac{0.97}{10} = 0.097 \ Ns/m^2$$

Relative density $= 0.9$

$\therefore \ \rho_{oil} = 0.9 \times 1000 = 900 \ kg/m^3$

Dia. of pipe, $D = 100 \ mm = 0.1 \ m.$

$L = 10 \ m.$

Mass of oil collected, $M = 100 kg.$

in time, $t = 30$ seconds

The difference of pressure

$$P_1 - P_2 = \frac{32 \mu \bar{u} L}{D^2}, \quad \text{where, } \bar{u} = \frac{Q}{\text{Area}},$$

Now, mass of oil/sec $= \frac{100}{30} \frac{kg}{s}$

$= \rho_{oil} \times Q = 900 \times Q$

$$\therefore \ \frac{100}{30} = 900 \times Q$$

$$\therefore \ Q = \frac{100}{30} \times \frac{1}{900} = 0.0037 \ m^3/s.$$

$$\therefore \ \bar{u} = \frac{Q}{\text{Area}} = \frac{0.0037}{\frac{\pi}{4} D^2} = \frac{0.0037}{\frac{\pi}{4}(0.1)^2}$$

$\bar{u} = 0.471 \ m/s.$

radius at 'r' at $u = \bar{u} = 0.75 \ m/s.$

$$\therefore\ 0.75 = 1.5\left[1-\left(\frac{r}{D/2}\right)^2\right]$$

$$= 1.5\left[1-\left(\frac{r}{0.2/2}\right)^2\right]$$

$$\therefore\ \frac{0.75}{1.5} = 1-\left(\frac{r}{0.1}\right)^2$$

$$\therefore\ \left(\frac{r}{0.1}\right)^2 = 1-\frac{0.75}{1.5} = 1-\frac{1}{2} = \frac{1}{2}$$

$$\therefore\ \frac{r}{0.1} = \sqrt{\frac{1}{2}} = \sqrt{0.5}$$

$$\therefore\ r = 0.1 \times \sqrt{0.5} = 0.1 \times 0.707$$

$$r = 0.0707\ m.$$

$$\boxed{r = 70.7\ mm}$$

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4) C Reynolds number is a dimensionless quanity used in fluid mechanics to predict the type of flow pattern in a fluid.

$$Re = \frac{\rho V L}{\mu}$$

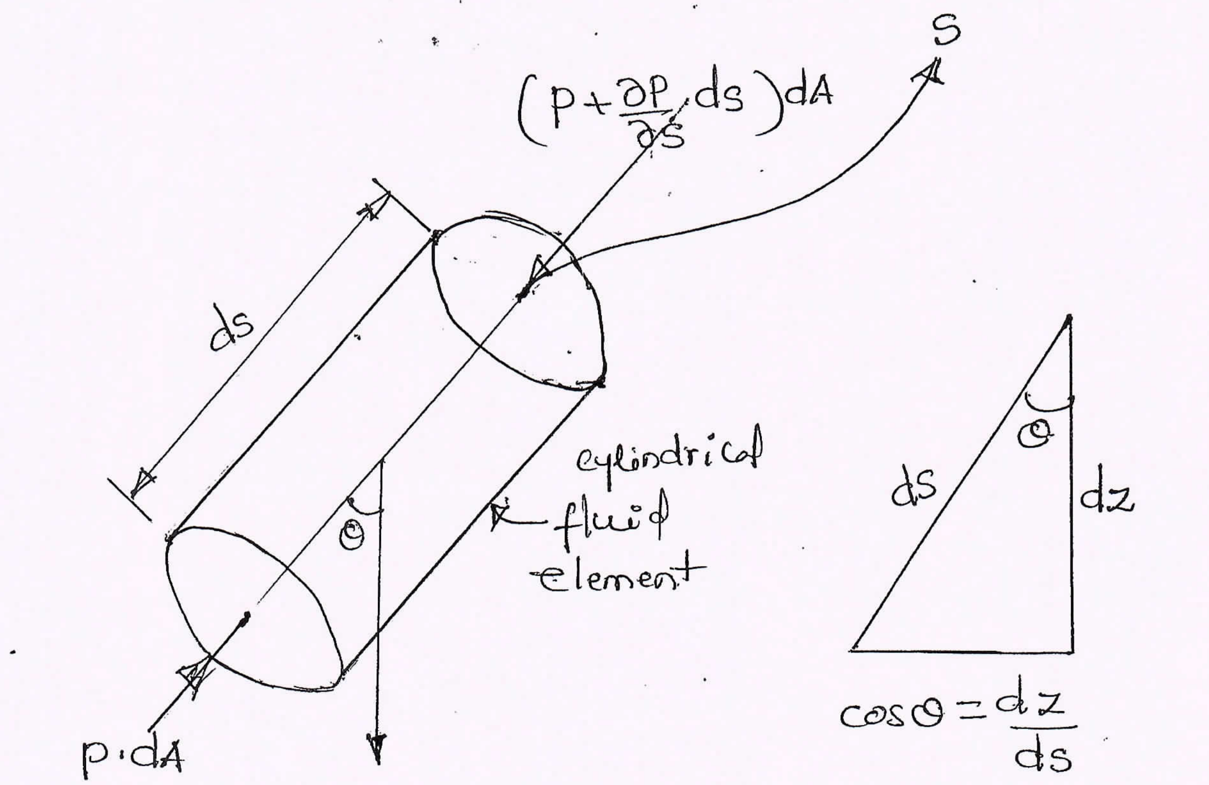
Significance: The Reynolds number is crucial because it helps to predict wheather a fluid flow will be Laminar or Turbulent flow.

If $Re < 2000$ then flow is laminar

If $Re > 4000$ then flow is Turbulent.

Q.No	Question, solution and scheme.	Marks
5a	Derive an expression for Euler's equation of motion along a stream line and deduce Bernoulli's equation	10

Ans



$\cos\theta = \frac{dz}{ds}$

consider a stream-line in which flow is taking place in S-direction. consider a cylindrical element of cross-section dA and length ds.

The forces acting on the cylindrical elements are

1. pressure force in the direction of flow $= p.dA$.
2. pressure force in the opposite to the direction of flow $= (p+\frac{\partial p}{\partial s}.ds)dA$
3. Weight of element $= \rho g\, dA\, ds$

let, 'θ' is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element X acceleration in the direction S.

$$pdA - \left(p + \frac{\partial p}{\partial s} \cdot ds\right) dA - \rho g \, dA \, ds \cos\theta = \rho \, dA \, ds \times a_s \quad \text{——} (1)$$

Where a_s is the acceleration in the direction of s.

Now, $a_s = \frac{dv}{dt}$, where v is a function of s & t

$$= \frac{\partial v}{\partial s} \cdot \frac{\partial s}{\partial t} + \frac{\partial v}{\partial t} = \frac{v \partial v}{\partial s} + \frac{\partial v}{\partial t} \qquad \because \frac{ds}{dt} = v$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore a_s = \frac{v \partial v}{\partial s}.$$

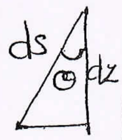
Substituting the value of a_s in equation (1)

$$-\frac{\partial p}{\partial s} ds \, dA - \rho g \, dA \, ds \cos\theta = \rho \, dA \, ds \times v \frac{\partial v}{\partial s} \quad \text{——} (2)$$

Dividing by $\rho \, ds \, dA$ to equation (2) | $\rho \partial s \, dA$ is constant.

$$-\frac{\partial p}{\rho \partial s} - g\cos\theta = v \frac{\partial v}{\partial s}$$

or $$\frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0.$$ | $\cos\theta = \frac{dz}{ds}$



$$\therefore \frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{dz}{ds} + v \frac{\partial v}{\partial s} = 0$$

or $$\frac{\partial p}{\rho} + g dz + v dv = 0 \quad \text{——} (3)$$

Equation (3) is known as Euler's equation of motion.

Bernoulli's equation is obtained by integrating the Euler's equation of motion.

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}.$$

If flow is incompressible, ρ is constant and

$$\therefore \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}.$$

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$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{constant.}$$

or $$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{constant}} \quad (4)$$

Equation (4) is a Benoulli's equation in which

$\frac{P}{\rho g}$ = pressure energy per unit weight of fluid.

$\frac{V^2}{2g}$ = kinetic head.

z = potential head.

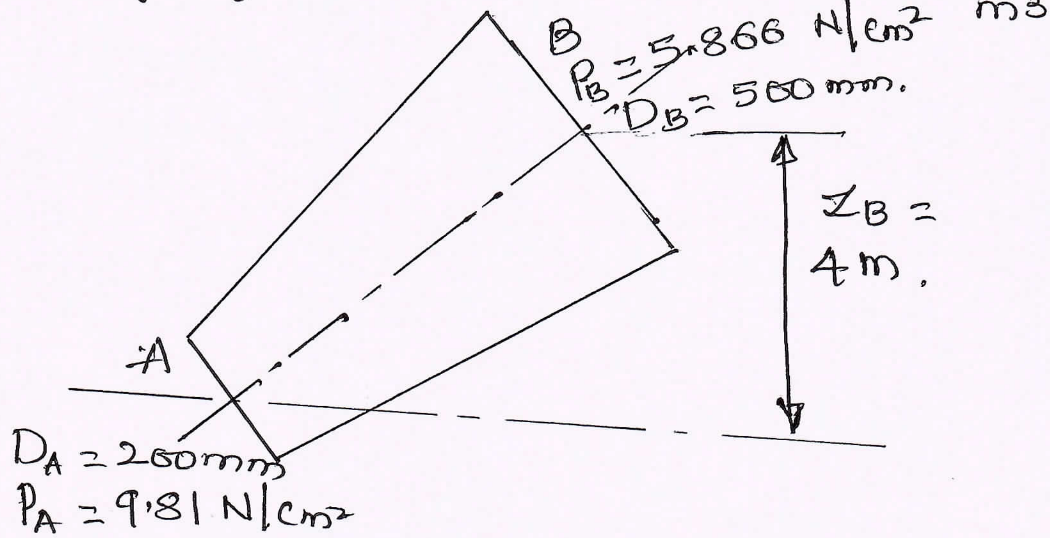
(5) b.

Given data

Discharge, $Q = 200$ lit/s $= 0.2\ m^3/s$.

Sp. gravity of oil $= 0.87$

$\therefore$ density of oil $= 0.87 \times 1000 = 870\ \frac{kg}{m^3}$



At section A, $D_A = 200\ mm = 0.2\ m.$

$$A_A = \frac{\pi}{4}(D_A)^2 = \frac{\pi}{4}(0.2)^2$$

$$= 0.0314\ m^2$$

$$P_A = 9.81 \times 10^4\ N/m^2.$$

$$Z_A = 0.$$

$$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.369\ m/s$$

At Section B,

$$D_B = 500\ mm = 0.50\ m.$$

$$A_B = \frac{\pi}{4}(D_B)^2 = 0.1963\ m^2$$

$$P_B = 5.886 \times 10^4\ N/m^2$$

$$Z_B = 4\ m.$$

$$V_B = \frac{Q}{Area} = \frac{0.2}{0.1963} = \underline{\underline{1.018}}\ m/s.$$

Total Energy at A

$$E_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$$

$$= \frac{9.81\times10^4}{870\times9.81} + \frac{(6.369)^2}{2\times9.81} + 0$$

$$E_A = \underline{\underline{13.557\ m.}}$$

Total Energy at 'B'.

$$E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B.$$

$$= \frac{5.886\times10^4}{870\times9.81} + \frac{(1.018)^2}{2\times9.81} + 4$$

$$\therefore E_B = \underline{\underline{10.948\ m}}$$

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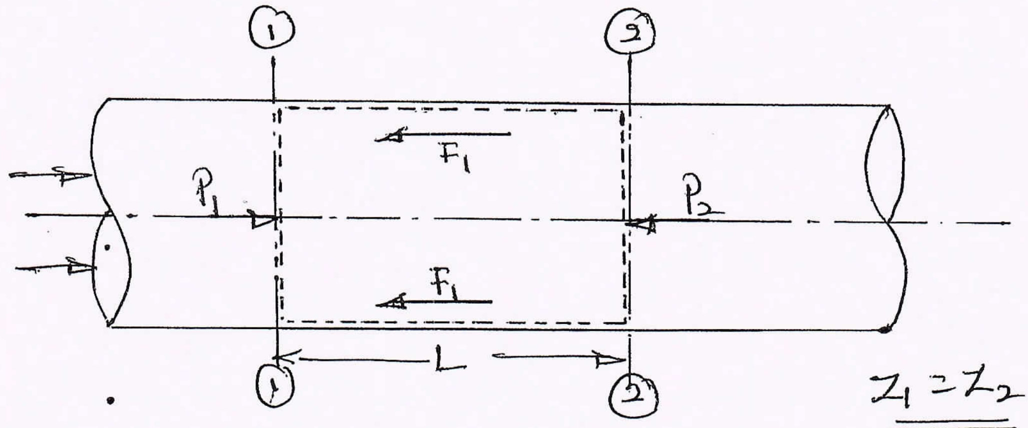
i) Direction of flow. As E_A is more than E_B and hence flow is taking place from $\boxed{A \text{ to } B}$

ii) Loss of head (h_L)

$$h_L = E_A - E_B = 13.557 - 10.948$$

$$\therefore \boxed{h_L = 2.609\ m}\ \text{Ans}$$

Q.No	Solution and Scheme	Marks
6(a)		10



$Z_1 = Z_2$

Consider a uniform horizontal pipe, having steady flow. let 1-1 and 2-2 are two section of pipe.

Let, P_1 = pressure intensity at section 1-1
P_2 = pressure intensity at section 2-2
L = length of the pipe between section 1-1 & 2-2
d = diameter of pipe.
f' = frictional resistance/unit wetted area/velocity
h_f = Loss of head due to friction.
V_1 = Velocity at section 1-1
V_2 = Velocity at section 2-2.

Applying Bernoulli's equation between 1-1 and 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

But, $Z_1 = Z_2$ as pipe is horizontal.
$V_1 = V_2$ as dia. of pipe is same at 1-1 & 2-2

$$\therefore \frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

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or $$h_f = \left(\frac{P_1 - P_2}{\rho g}\right) \quad \text{—— (i)}$$

(Chandrakanth M T)

Q.No	Solution and Scheme	Marks

Now,

frictional resistance = frictional resistance per unit wetted area per unit Velocity × Wetted area × V^2

or $$F_1 = f' \times \pi d L \times V^2 \quad \text{——(ii)}$$ ∥ ∵ Wetted area $= \pi d \times L$

The forces acting on the fluid between section ①-① & ②-② are, (i) pressure force at section ①-① $= P_1 A$

(ii) pressure force at section ②-② $= P_2 A$

(iii) frictional force F_1.

Resolving all forces in the horizontal direction, we have

$$P_1 A - P_2 A - F_1 = 0.$$

or $$(P_1 - P_2)A = F_1 = f' \pi d L V^2$$

or $$(P_1 - P_2) = \frac{f' \pi d L V^2}{A}$$

But, $(P_1 - P_2) = \rho g h_f$ ∥ πd = perimeter, $\pi d = P$

$$\therefore \rho g h_f = \frac{f' P L V^2}{A}$$

or $$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \text{——(iii)}$$ ∥ $\frac{P}{A} = \frac{\text{Wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = 4/d$

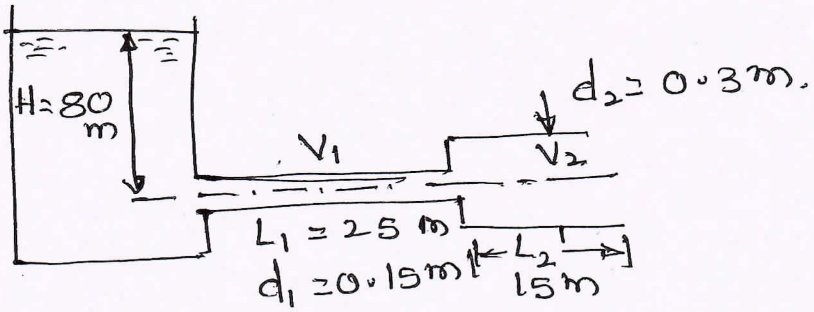
$$\therefore h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2$$

$$\therefore \boxed{h_f = \frac{4 f L V^2}{2 g d}} \quad \text{——(iv)}$$

Equation (iv) is known as Darcy-Weisbach eqⁿ.

Q5(b)

Solution.



$f = 0.05$.

By applying Bernoulli's theorem

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{All losses.}$$

$$0 + 0 + 8 = 0 + \frac{V_2^2}{2g} + h_i + h_{f_1} + h_e + h_{f_2} \quad \text{— (2)}$$

where,

$$h_i = 0.5\frac{V_1^2}{2g}$$

$$h_{f_1} = \frac{4fL_1V_1^2}{d_1 \times 2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$h_{f_2} = \frac{4fL_2V_2^2}{d_2 \times 2g}$$

Head Losses.

from continuity equation.

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2V_2}{A_1} = \frac{\cancel{\frac{\pi}{4}}(d_2)^2V_2}{\cancel{\frac{\pi}{4}}(d_1)^2} = \left(\frac{d_2}{d_1}\right)^2 V_2$$

$$V_1 = \left(\frac{0.3}{0.15}\right)^2 \times V_2 = 4V_2$$

Substitute V_1 in different head losses.

$$h_i = \frac{0.5V_1^2}{2g} = \frac{0.5(4V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

$$h_{f_1} = \frac{4 \times 0.01 \times 25 \times (4V_2)^2}{0.15 \times 2 \times g} = 106.67\frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f_2} = \frac{4 \times 0.1 \times 15 \times V_2^2}{0.3 \times 2g} = \frac{4 \times 0.1 \times 15}{0.3} \times \frac{V_2^2}{2g}$$

$$= 2 \times \frac{V_2^2}{2g}.$$

Substitute the values of these losses in eq^n (1)

$$8 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2 \times \frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}\left[1 + 8 + 106.67 + 9 + 2\right] = 126.67\frac{V_2^2}{2g}$$

$$\therefore V_2 = \sqrt{\frac{8 \times 2 \times g}{126.67}} = \sqrt{\frac{8 \times 2 \times 9.81}{126.67}}$$

$$\therefore \boxed{V_2 = 1.113 \text{ m/s}}$$

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$\therefore$ Rate of flow,

$$Q = A_2V_2 = \frac{\pi}{4}(0.3)^2 \times 1.113$$

Ans

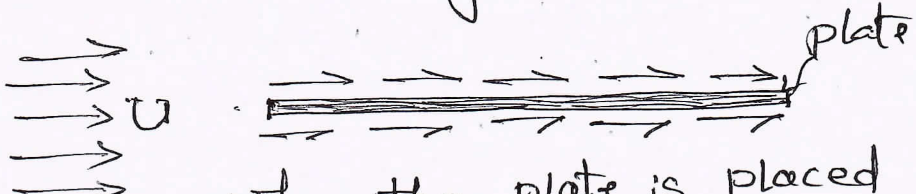
$$\boxed{Q = 0.0786 \text{ m}^3/\text{s}}$$

Module- 4.

7(a) (i) Drag: The component of the total force in the direction of motion is called 'drag'. Thus drag is the force exerted by the fluid in the direction of motion.

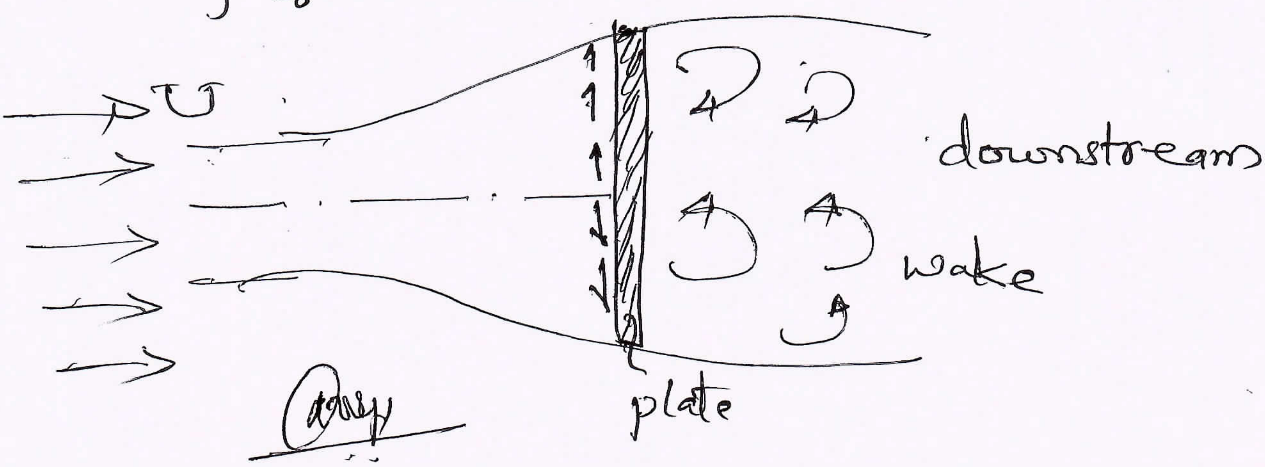
(ii) Lift: The component of the total force in the direction perpendicular to the direction of motion is known as '<u>lift</u>'

(iii) Friction Drag:



When the plate is placed parallel to the direction of the flow, in this case $\cos\theta$ which is the angle made by pressure with the direction motion, will be 90°. Thus the term $\int P\cos\theta . dA$ will be zero and hence total drag will be equal to frictional drag.

(iv) Pressure drag: If the plate is placed perpendicular to the flow, the angle θ will be zero. Hence the term $\int \tau_o \sin\theta . dA$ will become zero.



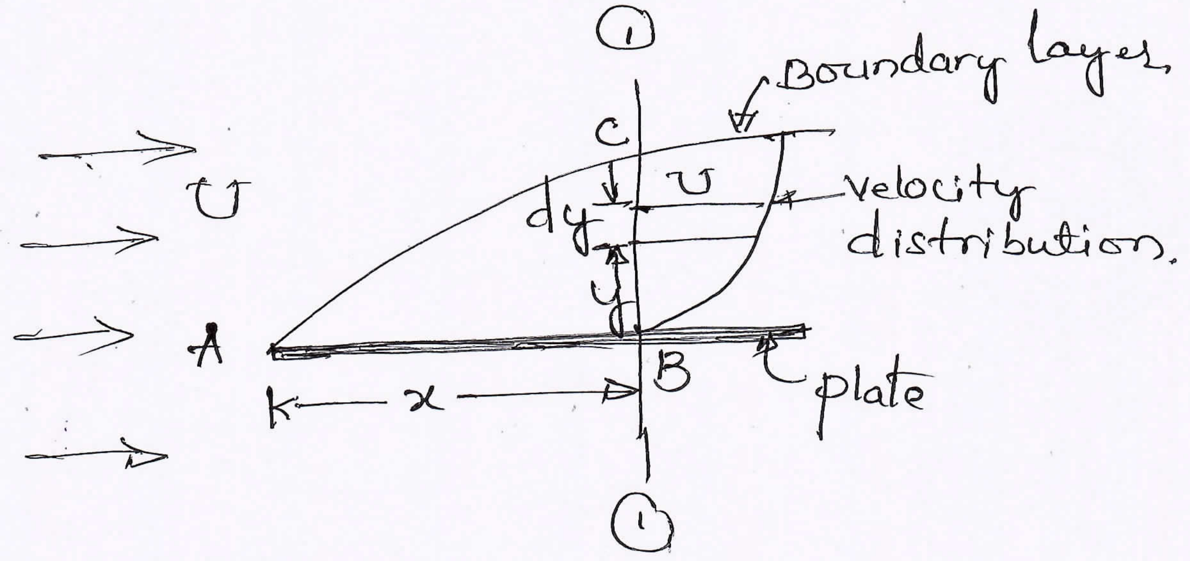
7(b)

(i) Boundary Layer Thickness (δ)

It is defined as the distance from the boundary of the solid body measured in the direction 'y' to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream.

(ii) Displacement thickness (δ^*)

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the direction in flow rate on account of boundary layer formation.



7) C Buckingham's π – theorem states that

"If there are n-variables (independent and dependent variables) in a physical phenomenon and if these variables contains m fundamental dimensions (M, L, T), Then the variables are arranged into (n−m) dimensionless terms. Each term is called π – term"

Let, $X_1, X_2, X_3 \ldots\ldots X_n$ are variables.

Let, X_1 be the dependent variable and $X_2, X_3 \ldots\ldots X_n$ are the independent variables on which X_1 depends.

Then, $X_1 = f(X_2, X_3, \ldots\ldots X_n)$

Then No. of π-terms $= (n-m)$

Hence, $f(\pi_1, \pi_2, \ldots\ldots \pi_{n-m}) = 0.$

Now,
$$\pi_1 = X_2^{a_1} \cdot X_3^{b_1} \cdot X_4^{c_1} \cdot X_1$$
$$\pi_2 = X_2^{a_2} \cdot X_3^{b_2} \cdot X_4^{c_2} \cdot X_5$$
$$\vdots$$
$$\pi_{n-m} = X_2^{a_{n-m}} \cdot X_3^{b_{n-m}} \cdot X_4^{c_{n-m}} \ldots\ldots X_n$$

$$\pi_1 = \phi(\pi_2, \pi_3 \ldots\ldots \pi_{n-m})$$

or
$$\pi_2 = \phi_1(\pi_1, \pi_3 \ldots\ldots \pi_{n-m})$$

Q8 (a)

Similitude - types of similarities.

Simititude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties or model and prototype are completely similar.

Three types of similarities must exist between the model and prototype. They are
(i) Geometric similarities (ii) Kinematic similarity
(iii) Dynamic similarity.

Geometric similarity: The geometric similarity is said to exist between the model and the prototype. The ratio of all corresponding linear dimension in the model and prototype are equal.

let, L_m = length of model, b_m = Breadth of model
D_m = Diameter of model, A_m = Area of model
$\forall_m$ = Volume of model.

and $L_p, b_p, D_p, A_p, \forall_p$ = Corresponding value of prototype.

For geometric similarity b/w model & prototype, we must have the relation.

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r$$

$L_r \rightarrow$ is called scale rati

For area's ratio and volume's ratio the relation should be as given below:

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2$$

and

$$\frac{\forall_p}{\forall_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3$$

8 (b)

Dimensional Homogeneity

Dimensional homogeneity means the dimensions of each terms in an equation on both sides equal. thus if the dimensions of each term on both sides of an equation are the same the equation is known as dimensionally homogeneous equation. the power of fundamental dimensions (i.e, L, M, T) on both sides of the equation will be identical for a dimensionally homogeneous equation. Such equations are independent of the system units.

Let us consider the equation, $V = \sqrt{2gH}$

Dimension of L·H·S $\quad V = \frac{L}{T} = LT^{-1}$

Dimension of R·H·S. $\quad \sqrt{2gH} = \sqrt{\frac{L}{T^2} \times L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} = LT^{-1}$

Dimension of LHS = Dimension of RHS = LT^{-1}

∴ Equation $V = \sqrt{2gH}$ is dimensionally homogeneous. so it can be used in any systems of units.

Methods of Dimensional Analysis.

If the number of variable involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods:

1. Rayleigh's method and
2. Buckingham's π- theorem.

(signature)

Q8, (c) The frictional torque T of a disc of diameter D rotating at a speed N in a fluid of viscosity μ and density ρ in a turbulent flow is given by.

$$T = D^5 N^2 \rho \phi\left(\frac{\mu}{D^2 N \rho}\right)$$

Use Buckingham π – theorem.

Solution: The functional relationship between the dependent and independent variables is given by.

$$T = f(D, N, \rho, \mu)$$

or $f(T, D, N, \rho, \mu) = 0$ or constant. —— (I)

Dimension of different variables are,

Variables	T	D	N	ρ	μ
Dimensions	$[ML^2T^{-2}]$	$[L]$	$[T^{-1}]$	$[ML^{-3}]$	$[ML^{-1}T^{-1}]$

· Total number of variables $n = 5$

Number of primary dimensions, $m = 3$.

$\therefore$ Number of π - terms $= n - m = 5 - 3 = 2$

$\therefore f(\pi_1, \pi_2) = 0$

The repeating variables are selected such that,

i) Geometric property (D)

ii) flow property (N)

iii) Fluid property (ρ)

— As per Buckingham π – theorem the π terms can be written as

$$\pi_1 = D^{a_1} N^{b_1} \rho^{c_1} T \quad \text{—— (II)}$$

$$\pi_2 = D^{a_2} N^{b_2} \rho^{c_2} \mu. \quad \text{—— (III)}$$

1. Consider equation (II) and substitute the dimensions,

$$M^0L^0T^0 = [L]^{a_1}[T^{-1}]^{b_1}[ML^{-3}]^{c_1}[ML^{2}T^{-2}]$$

Equating the power of MLT on both sides.

For M : $0 = c_1 + 1$ $\quad \therefore \boxed{c_1 = -1}$

For L : $0 = a_1 - 3c_1 + 2 = a_1 - 3(-1) + 2 = a_1 = -5$

$\therefore \boxed{a_1 = -5}$

For T : $0 = -b_1 - 2$ $\quad \therefore \boxed{b_1 = -2}$

Substitute values of a_1, b_1 and c_1 in equation -(II)

$$\therefore \pi_1 = D^{-5} N^{-2} \rho^{-1} T$$

$$\therefore \boxed{\pi_1 = \frac{T}{D^5 N^2 \rho}}$$

Consider equation (III) and Substitute the dimensions.

$$M^0L^0T^0 = [L]^{a_2}[T^{-1}]^{b_2}[ML^{-3}]^{c_2}[ML^{-1}T^{-1}]$$

Equating the power of MLT on both sides.

For M : $0 = c_2 + 1$ $\quad \therefore \boxed{c_2 = -1}$

For L : $0 = a_2 - 3c_2 + 1$ $\quad \therefore a_2 - 3(-1) + 1 = 0$

$\boxed{a_2 = -2}$

For T : $0 = -b_2 - 1$ $\quad \therefore \boxed{b_2 = -1}$

Substitute values of a_2, b_2 and c_2 in equation -(III)

$$\therefore \pi_2 = D^{-2} N^{-1} \rho^{-1} \mu$$

$$\therefore \boxed{\pi_2 = \frac{\mu}{D^2 N \rho}}$$

Now, put the values of Π_1, Π_2 in equation (I)

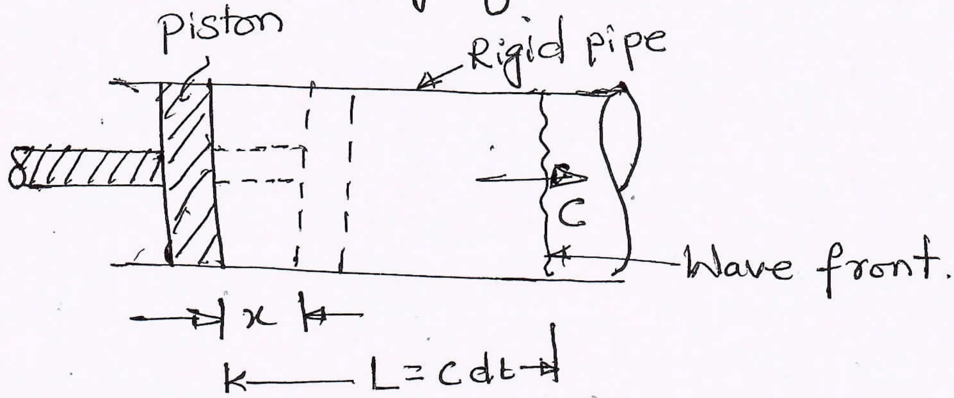
$$\therefore f\left(\frac{T}{D^5 N^2 \rho}, \frac{\mu}{D^2 N \rho}\right) = 0.$$

$$\text{or} \quad \frac{T}{D^5 N^2 \rho} = \phi\left(\frac{\mu}{\rho N D^2}\right)$$

$$\therefore \boxed{T = \rho N^2 D^5 \phi\left(\frac{\mu}{\rho N D^2}\right)}$$

Q 9

(b) Expression for velocity of sound wave in a fluid.



Distance travelled by the pressure waves in time dt

$= \text{Velocity of pressure wave} \times dt$

$= C \times dt$.

Length of tube equal to $(L-x)$, the fluid will be compressed.

let, $p + dp$ = pressure after compression

$\rho + d\rho$ = Density after compression.

Now, mass of fluid for a length 'L' before compression

$= \rho \times A \times L = \rho \times A \times C \times dt$ —— (i)

Mass of fluid after compression for length $(L-x)$

$= (\rho + d\rho) A \times (L - x)$

$= (\rho + d\rho) A \times (Cdt - V \cdot dt)$ —— (ii)

$\because L = Cdt$, $x = V \cdot dt$

From the continuity equation, we have.

equation (i) = (ii)

$\therefore \rho A C dt = (\rho + d\rho) A \times (Cdt - Vdt)$

or $\rho A C dt = (\rho + d\rho) A \times dt (C - V)$

Dividing by $A \times dt$

$\therefore \rho C = (\rho + d\rho)(C - V) = \rho C - \rho V + C d\rho - V d\rho$

$\therefore C d\rho = \cancel{\rho C} - \cancel{\rho C} + \rho V + V d\rho = \rho V + V d\rho$ —— (iii)

But, the velocity of piston V, is very small as compared to the Velocity of the pressure wave C.

Equation (iii) becomes.

$$C\,dp = \rho V \quad \text{——} \quad (iv)$$

Applying the impulse momentum equation.

Net force on the fluid = Rate of change of momentum

or $(p+dp)A - p \times A = $ mass per second.

or $$dp \times A = \frac{\text{Total mass}}{\text{time}}[V-0] = \frac{\rho AL}{dt}[V-0]$$

$$= \frac{\rho AC\cancel{dt}}{\cancel{dt}}[V-0]$$

$$dp \times A = \rho AC[V-0] = \rho ACV$$

or $$dp = \frac{\rho \cancel{A} CV}{\cancel{A}}$$

or $$C = \frac{dp}{\rho V} \quad \text{——} \quad (v)$$

Multiplying equation (iv) and (v)

We get, $$C^2 d\rho = \rho V \times \frac{dp}{\rho V} = dp$$

$$C^2 = \frac{dp}{d\rho}$$

$$\therefore \boxed{C = \sqrt{\frac{dp}{d\rho}}}$$

9 (a).

Mach Number: is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force.

$$\therefore \quad M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{F_i}{F_e}}$$

$$M = \sqrt{\frac{\rho A V^2}{K \times L}} = \sqrt{\frac{\rho \times L^2 \times V^2}{K \times L}} = \sqrt{\frac{V^2}{K/\rho}}$$

$$M = \frac{V}{\sqrt{\frac{K}{\rho}}} = \frac{V}{C} \qquad \Bigg\| \quad \sqrt{\frac{K}{\rho}} = C$$

$$\boxed{M = \frac{V}{C}}$$

For the compressible fluid flow, Mach number is an important non-dimensional parameter. On the basis of the Mach-number, the flow is defined as

(i) Sub-sonic flow, $M < 1$

(ii) Sonic flow, $M = 1$

(iii) Super sonic flow, $M > 1$.

9 G
Sol.

Given data :

Mach angle $\alpha = 30^\circ$

$R = 287.14\ J/kg\text{-}K$

$k = 1.4$

$t = 15^\circ C.$

$\therefore\ T = 15 + 273 = 288^\circ K.$

Velocity of sound is given by

$$C = \sqrt{kRT}$$

$$= \sqrt{1.4 \times 287.14 \times 288}$$

$$C = 340.25\ m/s.$$

Using the relation, $\sin\alpha = \frac{C}{V}$

$$\sin(30^\circ) = \frac{340.25}{V}$$

$$V = \frac{340.25}{\sin 30^\circ}$$

$$\boxed{V = 680.50\ m/s.}$$

10 (a)

Given data:

$z = 15$ km

Temperature, $t = -50^\circ C$

$T = -50 + 273 = 233^\circ C$.

Mach number, $M = 2$

$k = 1.4$

$R = 287$ J/kg-K.

Velocity of sound as

$C = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 223}$

$C = \underline{299.33}$ m/s.

We have, $M = \frac{V}{C}$

or $V = M \times C = 2 \times 299.33$

$V = 598.66$ m/s.

$= \frac{598.66 \times 60 \times 60}{1000}$

$\boxed{V = 2155.17}$ km/hour

Arup

10 (b)

i) Mach number : is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force. It is denoted by letter 'M'.

(ii) Sub-sonic flow : A flow is said, sub-sonic flow if the Mach number is less than 1 which means the velocity of flow is less than the velocity of sound wave. $(V < C)$.

(iii) Sonic flow : If the Mach number equal to 1. This means that when the velocity of flow (V) is equal to the velocity of sound C, the flow is said sonic flow.

(iv) Super-sonic flow : If the Mach number is greater than 1, $(M > 1)$. This means that when velocity of flow V is greater than the velocity of sound wave, the flow is said to be super-sonic flow.

10) c.

Advantages of CFD.

i) Reduced Development costs: CFD simulation are generally less expensive than physical experiments.

(ii) Faster Design cycle: CFD allows

(iii) comprehensive information: CFD simulation can provide detailed information about the flow field, such as velocity, pressure & temp.

(iv) Ability to simulate various conditions: CFD simulation can be used to study a wide range of flow conditions, including that are difficult or dangerous.

(v) Improve understanding of fluid flow: CFD simulation can help engineers and scientists to better understand the underlying physics of fluid flow.

Disadvantages of CFD:

1. Accuracy limitations
2. computational cost.
3. Need for expertise.
4. Validation challenges.
5. Over-reliance on simulations.

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