CBCS SCHEME

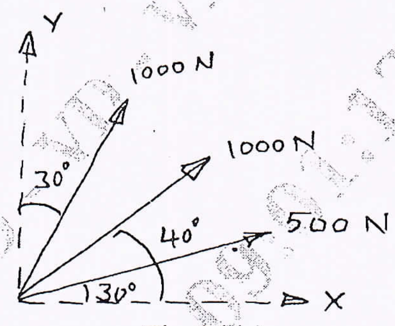
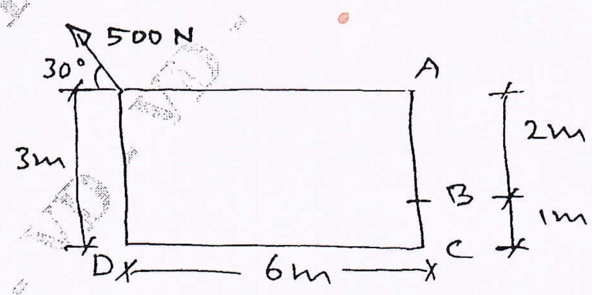
USN ☐☐☐☐☐☐☐☐☐☐ **BCIVC103/203**

First/Second Semester B.E/B.Tech. Degree Examination, Dec.2024/Jan.2025

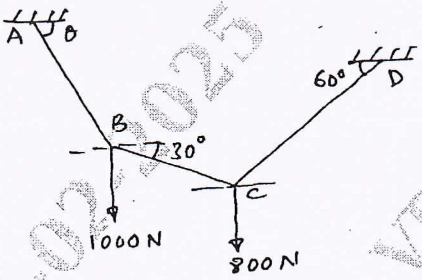
Engineering Mechanics

Time: 3 hrs. Max. Marks:100

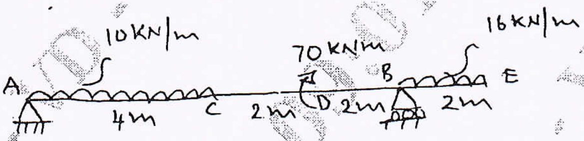
Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	**M**	**L**	**C**
1	a.	Write a note on principle of transmissibility of forces and its limitations.	6	L1	CO1
	b.	What is a force? What are its characteristics?	6	L1	CO1
	c.	Two forces acting on a body are 500 N and 1000 N as shown in Fig.Q1(c). Determine the third force F such that the resultant of all the three forces is 1000 N, directed at 40° to the x-axis.  Fig.Q1(c)	8	L3	CO1
		OR			
2	a.	What is a couple? List its characteristics.	6	L1	CO1
	b.	State and prove Varignon's theorem.	6	L2	CO1
	c.	Find the moment of 500 N force about the points A, B, C and D as shown in Fig.Q2(c).  Fig.Q2(c)	8	L3	CO1

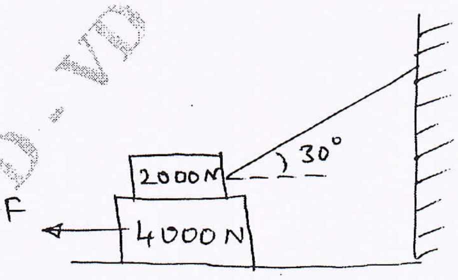
Module – 2

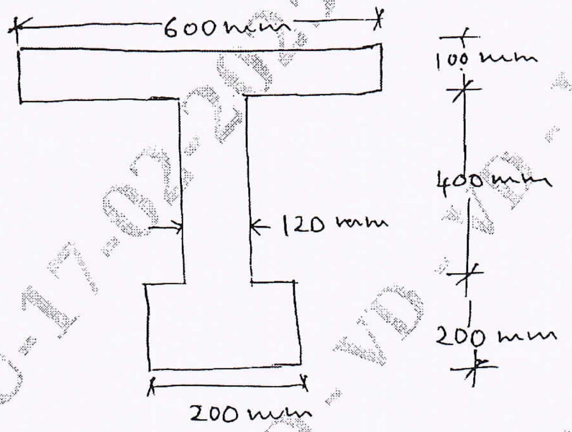
3	a.	Explain with a neat sketch, the different types of supports.	6	L2	CO2
	b.	State and prove Lami's theorem.	6	L2	CO2
	c.	Calculate the tension in the strings. Also calculate 'θ' in Fig.Q3(c).  Fig.Q3(c)	8	L3	CO2

OR

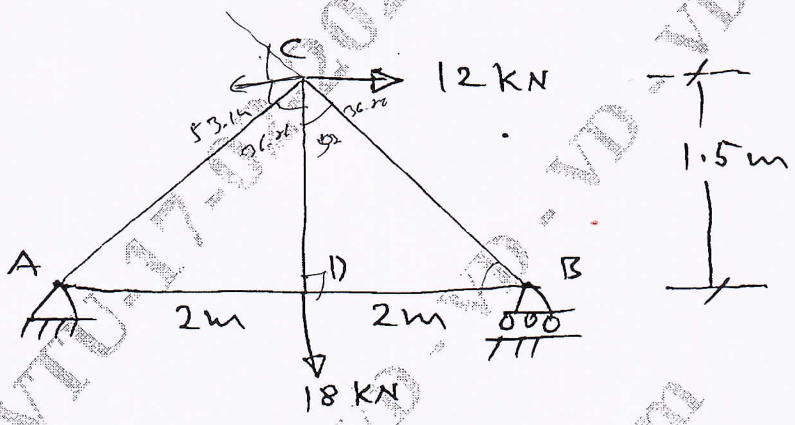
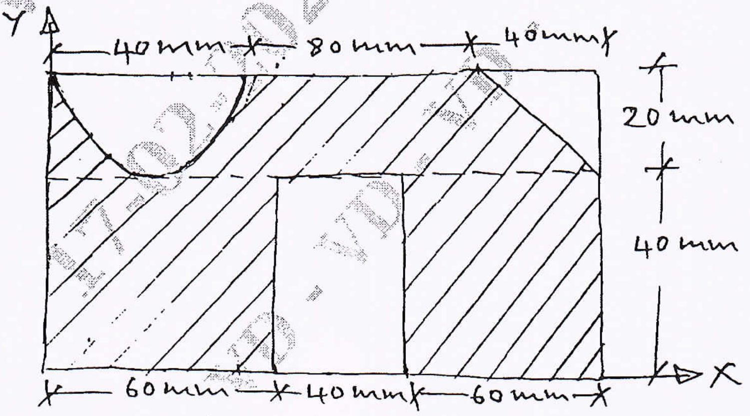
4	a.	Explain the types of loading on the beams.	6	L2	CO2
	b.	Write short notes on the following with examples : i) Determinate beams ii) Indeterminate beams.	6	L1	CO2
	c.	Find support reactions for the beam shown in Fig.Q4(c).  Fig.Q4(c)	8	L3	CO2

Module – 3

5	a.	What are the assumptions made in the analysis of trusses?	4	L1	CO3
	b.	State the laws of static friction.	4	L2	CO3
	c.	A block weighting 4000 N is resting on horizontal surface supports another block of 2000 N as shown in Fig.Q5(c). Find the horizontal force F just to move the block to the left. Take coefficient of friction for all surfaces of contact to be 0.2.  Fig.Q5(c)	12	L3	CO3

		OR			
8	a.	Determine the centriod of a semi circular lamina of radius 'R' by the method of integration.	**8**	**L3**	**CO4**
	b.	Determine the moment of inertia of a pre-stressed concrete beam section shown in Fig.Q8(b), about horizontal and vertical axis passing through centriod.  Fig.Q8(b)	**12**	**L3**	**CO4**
		Module – 5			
9	a.	Derive the equations of motion.	**6**	**L2**	**CO5**
	b.	What is super elevation? Why is it necessary?	**4**	**L1**	**CO5**
	c.	A ball is dropped from the top of a tower 30 high. At the same instant another ball is thrown upward from the ground with an initial velocity of 15 m/s. When and where do they cross?	**10**	**L3**	**CO5**
		OR			
10	a.	State and explain D'Alembert's principle.	**4**	**L2**	**CO5**
	b.	Define the following with a neat sketch : i) Angle of projection ii) Horizontal range iii) Time of flight.	**4**	**L1**	**CO5**
	c.	A cricket ball is thrown by a player from a height of 2 m above the ground at an angle of 30° to the horizontal with a velocity 20 m/s is caught by another fieldsman at a height of 1 m from the ground. Find the distance between the two players.	**12**	**L3**	**CO5**

* * * * *

		OR			
6	a.	Explain different types of trusses.	**4**	**L2**	**CO3**
	b.	Explain : i) Angle of friction ii) Cone of friction.	**4**	**L2**	**CO3**
	c.	Analyse the frame and tabulate the member forces for the frame shown in Fig.Q6(c).  Fig.Q6(c)	**12**	**L3**	**CO3**
		Module – 4			
7	a.	State and prove parallel axis theorem.	**8**	**L2**	**CO4**
	b.	Locate centriod of the shaded area shown in the Fig.Q7(b).  Fig.Q7(b)	**12**	**L3**	**CO4**

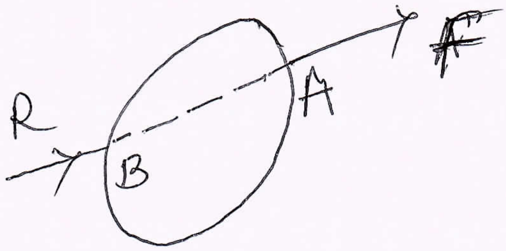
DEC. 2024 JAN 2025 QP SCHEME & SOLUTION
ENGINEERING MECHANICS (BCIVC103/203)

Q.1 a Write a note on Principle of transmissibility of forces and it's limitations. (06M)

Ans: Principle of transmissibility of forces.

Statement: It states that "the state of rest or uniform motion of the body is unultered if a force acting on a body is replaced by another force of same magnitude and in the same direction but along the line of action of replaced force". — — — — — — 03M

Consider a rigid body as shown below in fig. Let F be the force acting at point A.



As per this principle the state of rest or uniform motion is unultered if F is replaced by R only if the magnitude of R is equal to F & is replaced along the line of action of F as shown above in figure. — — — — — 02M

Limitation:

This principle is only applicable for rigid bodies as here we won't consider the internal changes like stress & strain of the bodies. — — — — — 01M

Total 06M

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Q.1 b What is a force? What are its Charecteristics. 06M

Ans: With the help of Newton's I law of motion force can be defined as "an external agent which alters the state of rest or uniform motion of the bodies.

With the help of Newton's II Law of motion force can be defined as "the rate of change of momentum of a body".

$$\text{Force} \propto \text{rate of change of momentum.}$$

$$\propto \text{rate of change of mass and}$$

$$\text{'' '' '' velocity.}$$

$$F = m a$$

as rate of change of mass is constant & the rate of change of velocity is accelaration (a)

∴ Force can be defined as the product of mass and accelaration of the body.

-------- 04M.

Charecteristics of force:

1) As force have both magnitude and direction it is a vector quantity

2) Force is measured in terms of Killogram force (kg-f) in MKS system & is measured in terms of N, KN in SI system of measurements.

--- 02M

Total 06M.

Q.1 c) ② Two forces acting on a body are 500N and 1000N as shown in fig Q1(c). Determine the third force F such that the resultant of three forces is 1000N directed at 40° to the x-axis.

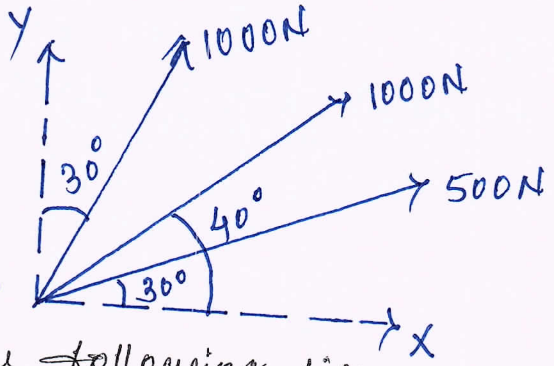
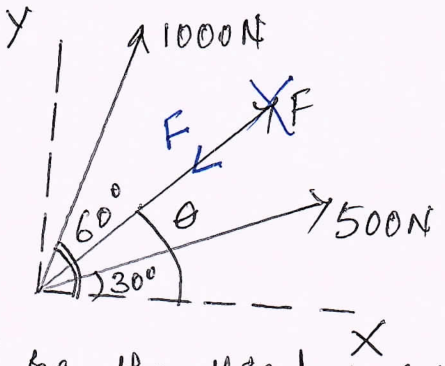


Fig. Q1(c) ———— 08M

Soln

Consider the following fig.



Let F be the third force & θ be its inclination with respect to x-axis as shown above in fig.

Given $R = 1000N$

$\alpha = 40°$

$\Rightarrow \Sigma F_x = R\cos\alpha = 1000\cos 40° = 766.04N$

$\Sigma F_y = R\sin\alpha = 1000\sin 40° = 642.79N$ ———— 02M

$766.04 = 500\cos 30 + F\cos\theta + 1000\cos 60$

$\Rightarrow F\cos\theta = -166.97$ — ① ———— 02M

$642.79 = 500\sin 30° + F\sin\theta + 1000\sin 60°$

$\Rightarrow F\sin\theta = -473.23$ — ② ———— 02M

② ÷ ①

$$\frac{F\sin\theta}{F\cos\theta} = \frac{473.23}{166.97}$$ ———— 02M

$\Rightarrow \tan\theta = 2.83 \Rightarrow \theta = 70.54°$

sub in ① or ②

$F = -501.18N$

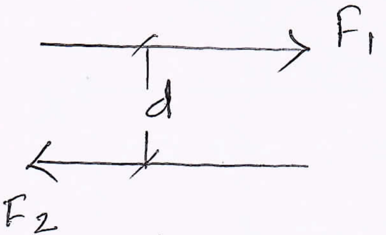
Total 08M.

③

OR

Q.2 a) What is a Couple ? List its charectoristics. (06M)

Ans: * Two forces which are equal in magnitude parallel to each other separated by a defenite distance, and in opposit direction are said to form a couple.



Two parallel forces F_1 & F_2 separated by distance d ($\perp^{er}$) if and only if

$F_1 = F_2$ - - - - - - 03M

* Charecteristics of couple:

i) Couple consists of two parallel force which are equal in magnitude and opposite in direction.

ii) The moment of a couple about any point in the plane is equal to the product of magnitude of force and the perpendicular distance between two parallel forces.

iii) The translatory effect of a couple on the body is nill or zero. - - - - 03M

Total 06M.

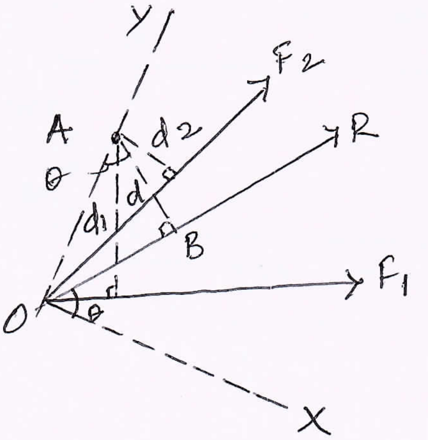
Q.2 b) State and prove Varignon's theorem. (06M)

Ans: Varignon's theorem: This principle or theorem is also called as Varignon's principle of moments or simply called as "principle of moments.

It states that " the algebraic sum of moments of coplanar forces about a moment center in a plane is equal to the moment of

Ⓐ

their resultant force about the same Moment center. — — — — 02M



Consider two forces F_1 & F_2 which are acting at point O & R be their resultant as shown in figure.

Let d_1, d_2 & d be the moment arms of F_1, F_2 & R respectively about point A. then as per statement

$$R\cdot d = F_1 d_1 + F_2 d_2$$

Let A be any moment centre now join OA, Consider it as Y-axis & draw X-axis as shown above in fig. Let θ be inclination of R with respect to x-axis.

By knowing $\angle OAB = \theta$

In △le OAB

$$\cos\theta = \frac{d}{OA} \Rightarrow d = OA\cos\theta.$$

Moment of R about A = $R\cdot d$

$\Rightarrow \quad R\cdot d = R\,OA\cos\theta \Rightarrow R\cdot d = OA\,R\cos\theta$

$\Rightarrow \quad R\cdot d = OA\,R_x$ (R_x is x component of R) $\underline{\qquad}$ 1

Similarly we can prove

$F_1\cdot d_1 = OA\,F_{1x}$ - - - ②

$F_2\cdot d_2 = OA\,F_{2x}$ - - - ③

② + ③

$F_1 d_1 + F_2 d_2 = OA\cdot F_1 x + OA\,F_2\cdot x$

$F_1 d_1 + F_2 d_2 = OA(F_1 x + F_2 x)$

$F_1 d_1 + F_2 d_2 = OA\,R_x \Rightarrow \boxed{R\cdot d = F_1 d_1 + F_2 d_2}$ — 04M

Total 06M

thus Proved

Q.2 c) Find the moment of 500N force about the points A, B, C & D as shown in fig. Q2 (c) (08M)

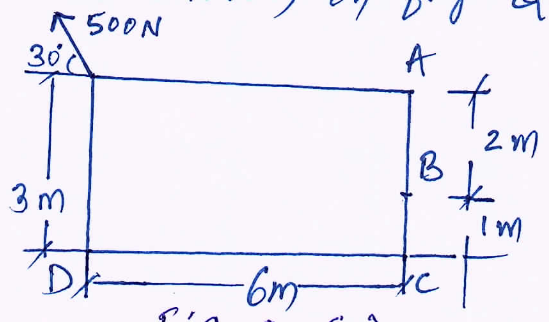
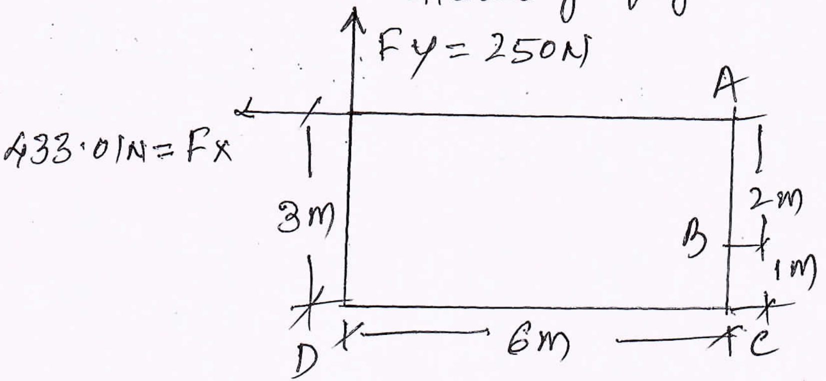


Fig. Q2 (c)

Soln Consider the following fig.



Let Fx & Fy be the components of 500N along x & y axis & let M_A, M_B, M_C & M_D be the moments of 500N about points A, B, C & D respectively.

$F_x = 500 \cos 30° = 433 \cdot 01 N$

$F_y = 500 \sin 30° = 250 N$ ——— 02

$M_A = +250 \times 6 = +1500 N \cdot m$ ——— 01

$M_B = -433 \cdot 01 \times 2 + 250 \times 6 = +633 \cdot 98 N \cdot m$ — 02

$M_C = -433 \cdot 01 \times 3 + 250 \times 6 = +200 \cdot 97 N \cdot m$ — 02

$M_D = -433 \cdot 01 \times 3 = -1299 \cdot 03 N \cdot m$ ——— 01

Total: 08M.

Module – 02

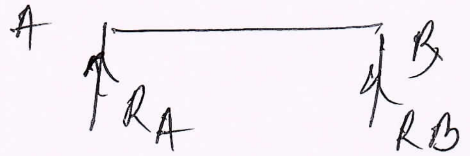
Q.3 a) Explain with neat sketch, the different types of supports. (06M)

Ans:- Following are the types of supports.

i) Simple support
ii) Roller support
iii) Hinged or pinned support
iv) Fixed support. ———— 02M

i) Simple support: It is a kind of support over which the ends of ~~of~~ the support ~~sut~~ beams simply rest over rigid supports & there is no restriction for the ~~a~~ rotation of ends & there will be development of normal reaction at the support.

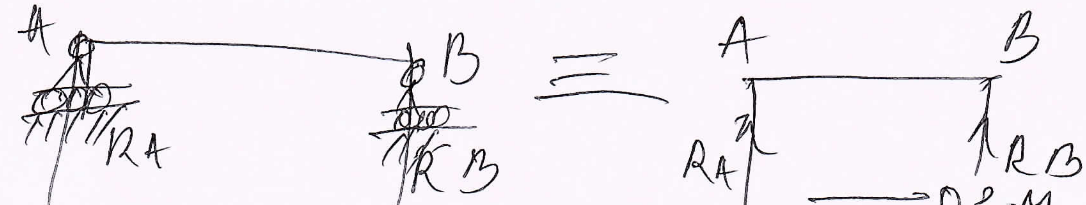
Example:- Simply supported beam.



Here RA & RB are ⊥$^{\text{lr}}$ to the centre line of beam AB. ———— 02M

ii) Roller support: In this support the ends of the beam are free to move in any direction or roll over the beam. There will not be any restriction for the movement. The reaction developed ~~way to~~ will be normal to the the axis of the beam. The notation used to represent the roller support is  -

Ex: Beam supported with rollers on both the ends.



———— 02M

Total 06M.

Q.3.b) State and prove Lami's theorem.(06M)

Ans: Lami's theorem:

Statement: It states that ~~the~~ if a body is in equilibrium under the action of three coplanar concurrent forces then each force is proportional to the sine of the angle between other two forces. ——— 02M.

Proof: consider three forces F_1, F_2 & F_3 as shown in fig.

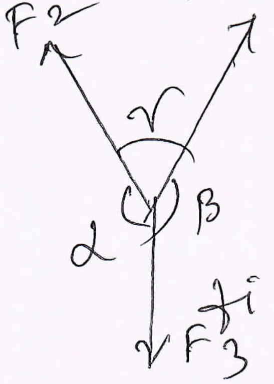


fig 1

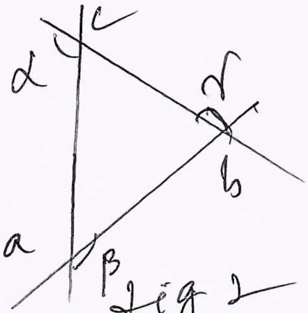


fig 2

Draw three forces F_1, F_2 & F_3 one after the other in direction and magnitude starting from point A. Since the body is in equilibrium the resultant should be zero, which means the last point of force diagram should coincide with A. Thus, it results in a Δ^{le} of forces abca fig 2. Now the external angles at a, b, c are equal to β, γ & α. since ab, bc, & ca are // to F_1, F_2 & F_3 respectively. ——— 02

In Δ^{le} of forces abc.

$ab = F_1$, $bc = F_2$, $ca = F_3$

Applying sine rule:

$$\frac{ab}{\sin(180-\alpha)} = \frac{bc}{\sin(180-\beta} = \frac{ca}{\sin(180-\gamma)}$$

⑧

$$\Rightarrow \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$ — 02

thus proved. Total 06M.

Q.3 c. Calculate the tension in the strings. Also Calculate θ in fig Q3(c). (08M)

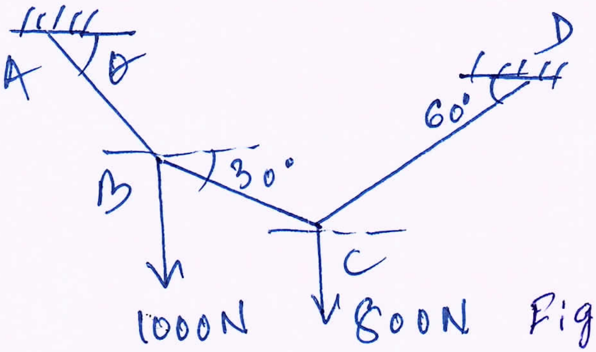
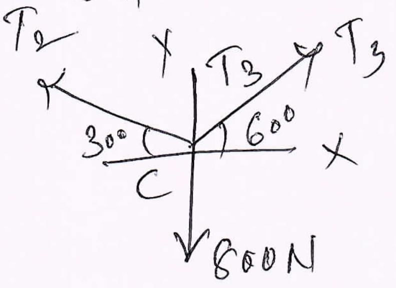
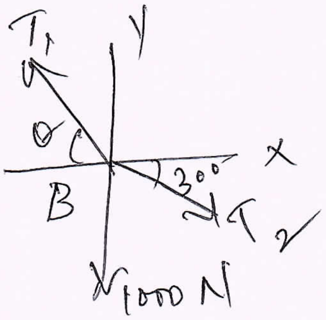


Fig. Q3(c)

Soln Let T_1, T_2 & T_3 be the tensions in the string segments AB, BC & CD respectively. FBDs for points C & B are as shown below.





Applying Lami's thm to point C

$$\frac{T_2}{\sin(90+60)} = \frac{T_3}{\sin(90+30)} = \frac{800}{\sin 90}.$$

$$\Rightarrow T_2 = \frac{800 \sin 150}{\sin 90} = 400 N$$

$$T_3 = \frac{800 \sin 120}{\sin 90} = 692.82 N$$ 04M

Applying Lami's them to point B.

$$\frac{T_1}{\sin 60} = \frac{400}{\sin(90+\theta)} = \frac{1000}{\sin(30+90+90-\theta)}$$

OR

Applying equations of equilibrium. to point B

$\Sigma Fx = 0 \Rightarrow +400\cos 30 - T_1\cos\theta = 0$

$\Rightarrow T_1\cos\theta = 400\cos 30$ ---- ①

$\Sigma Fy = 0 \Rightarrow -400\sin 30 + T_1\sin\theta - 1000 = 0$

$\Rightarrow T_1\sin\theta = 1000 + 400\sin 30$ --- ②

② ÷ ① ———— 02M

$$\frac{T_1\sin\theta}{T_1\cos\theta} = \frac{1000+400\sin 30}{400\cos 30}.$$

$\Rightarrow \tan\theta = 3.4641$

$\Rightarrow \theta = \tan^{-1} 3.4641$

$\Rightarrow \theta = 73.90°$ ----

Sub in ① or ②

$$T_1 = \frac{400\cos 30}{\cos 73.90}$$

$T_1 = 1249.15N$ ———— 02M

Total 08M.

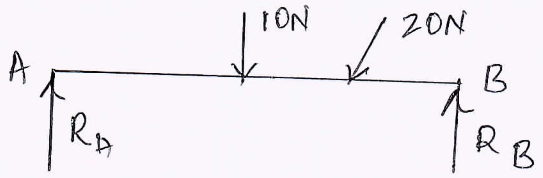
OR.

. 4 a) Explain the types of loading on the beams. (06M)

Ans. Beams can be loaded with following loads.

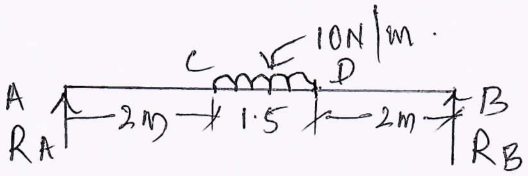
i) Point load
ii) Uniformly distributed load (UDL)
iii) Uniformly Varying load (UVL)
iv) External Moment. ———— 02M

i) **Point load**: These will have point of Contact with beam & these can be shown with line & arrow head. Arrow head shows the direction of the load.



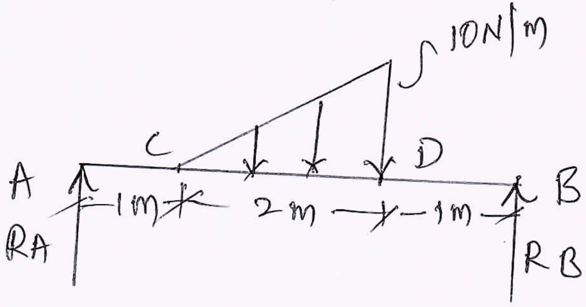
——— 01M

ii) **UDL**: Such load will have unit magnitude for a given length of the beam. The Unit of the load is N/m or KN/m.



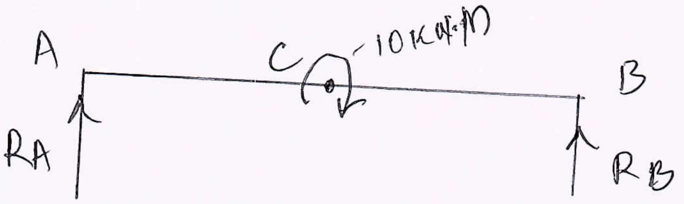
——— 01M

iii) **UVL**: Such load will have certain magnitude at a point & the same will be increased or decreased for a length of the beam. Therefore these will be in the form of triangle or trapizium. The Unit of UVL is N/m or KN/m.



——— 01M

iv) **External Moment**: The beams may also be loaded with external moment & the unit is KN·m or N·m. These may be in clockwise or anticlockwise.



——— 01M

Total 06M.

Q.4 b) Write short notes on the following with examples.

i) Determinate beam.

ii) Indeterminate beam. —— (06M)

Ans:

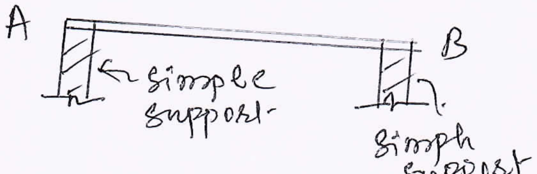
i) Determinate beam:

These are the beams whose equillibrium can be analyzed using the three equations of equilibrium.

i.e $\Sigma Fx = 0$, $\Sigma Fy = 0$ & $\Sigma M = 0$.

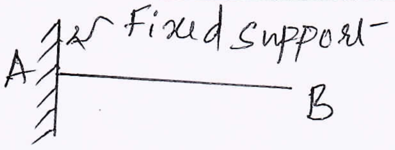
Here the no of unknown forces will be less than or equal to the no of equations of equillibrium.

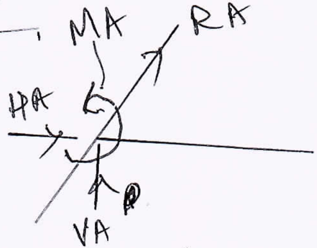
Examples: * Simply supported beam.



Here RA & RB are the normal reactions due to simple supports.

* Cantilever beam:





Here HA is the horizontal component of reaction RA in any direction due to fixed support

VA is the vertical component of reaction RA in any direction due to fixed support.

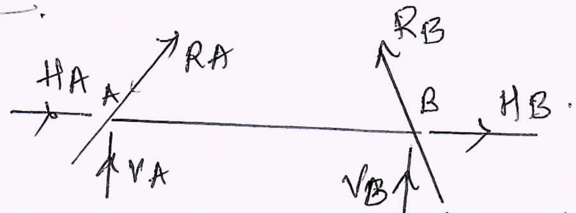
MA is resisting ~~mom~~ moment due to applied loads. —— 03M

ii) Indeterminate beams:

These are the beams whose equilibrium condition can not be analyzed using the equations of equilibrium as here the no of unknown forces are more than the no of equations of equilibrium ~~Exa~~ The equilibrium condition can be analyzed using the methods like conjugate beam method, Kani's method etc.

Example:

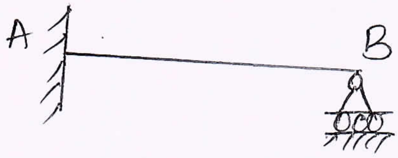
i) Both ends hinged beam.

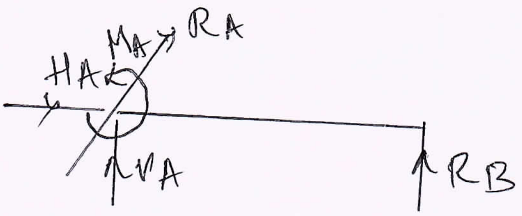


Here HA & VA be the components of reacting force RA in any direction due to hinged support at A.

HB & VB are the components of reacting force RB in any direction due to hinged support at B

ii) Propped cantilever beam.





Here HA & VA are components of reacting force RA due to fixed support at point A

MA is the resisting moment due to fixed support at point A

RB is the normal reaction due to roller support at point B.

— 03M

Total 06M.

Q.4 c) Find the support reactions for the beam shown in fig. Q4(c)

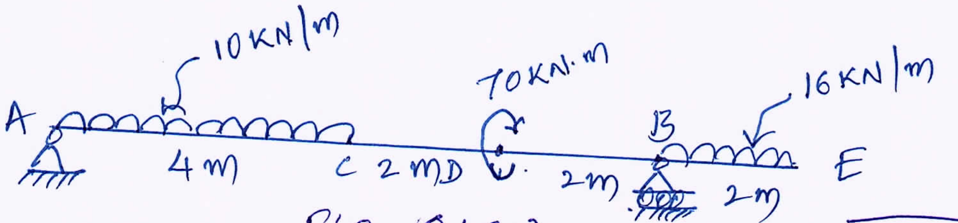
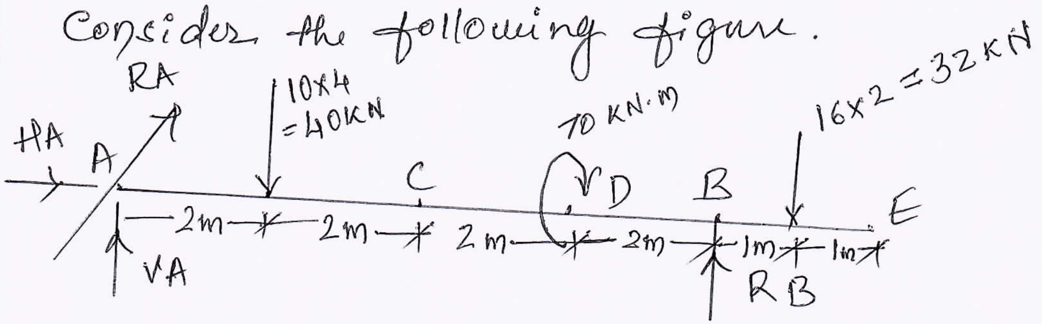


Fig. Q4(c)

(08M)

Sol^n Consider the following figure.



Let H_A & V_A be the horizontal & vertical components of reacting force R_A in any direction due to hinged support at point A.

Let R_B be the normal reaction due to roller support at point B. —— 02M

Applying equations of equilibrium.

$$\Sigma M_A = 0$$

$$\Rightarrow +40\times 2 + 70 - R_B \times 8 + 32\times 9 = 0$$ —— 03M

$$\Rightarrow \boxed{R_B = 54.75\,KN}$$ —— 01

$$\Sigma F_x = 0 \Rightarrow H_A = 0.$$

$$\Sigma F_y = 0 \Rightarrow +V_A - 40 - 54.75 - 32 = 0.$$

$$\Rightarrow \boxed{\begin{matrix} V_A = +126.75N \\ R_A = +126.75N \end{matrix}}$$ —— 02

Total 08M.

Module-3

Q.5 a) What are the assumptions made in the analysis of trusses. (04M)

Ans: Assumptions made in the analysis of trusses

i) The ends of the members are pin connected or hinged.

ii) The loads ~~at~~ act only at the joints.

iii) Self weights of the members are negligible.

iv) Members are having either uniform cross sections throughout or if they have varying cross section the centroid is located along the same longitudinal line.

---- 04M.

Q.5 b) State the laws of static friction. — (04M)

Ans: The laws of static friction are also called as laws of dry friction or coulomb's laws of dry friction proposed by Coulomb & Mozin.

i) Under the static conditions, the friction force opposes tendency for relative motion between the two surfaces in contact and acts tangential to the normal reaction.

ii) The limiting static friction force, which is the maximum value of friction force is directly proportional to the normal reaction between the two surfaces in contact.

iii) Limiting friction force is independent of the area of the two surfaces in contact.

iv) Limiting force of static friction depends on the nature and materials of the two surfaces in contact.

Q.5 c) A block weighing 4000N is resting on a horizontal surface supports another block of 2000N as shown in fig. Q5(c). Find the horizontal force F just to move the block to the left. Take coefficient of friction for all surfaces of contact to be 0.2

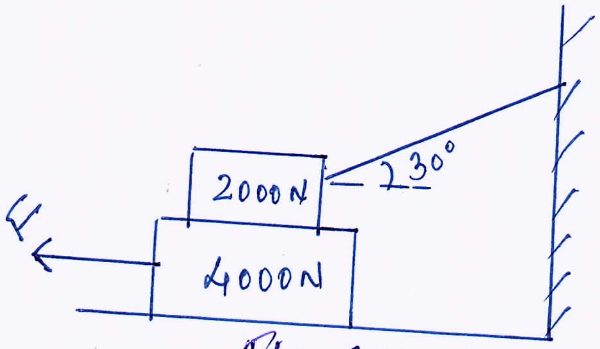
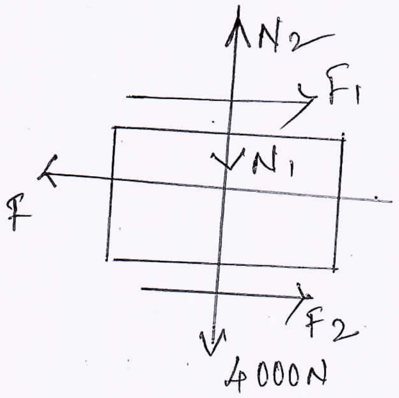
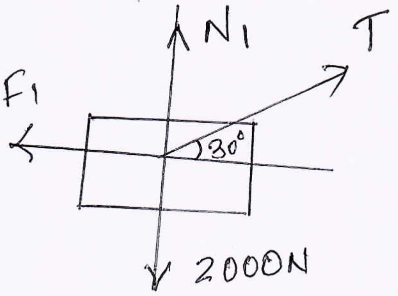


Fig Q5(c)

——— (12M)

Soln FBD for 2000N & 4000N blocks as shown





Applying eqⁿ of eqlᵐ for 2000N block.

$\Sigma F_y = 0 \Rightarrow N_1 + T\sin 30 - 2000 = 0$

$\Rightarrow N_1 = 2000 - 0.5T$ — ①

——— 04M

$\Sigma F_x = 0 \Rightarrow T\cos 30 - F_1 = 0$

$\Rightarrow T\cos 30 = F_1$

$F_1 = \mu N = 0.2 N_1$

$T\cos 30 = 0.2 N_1$

$T\cos 30 = 0.2(2000 - 0.5T)$ from eqⁿ ①

$0.866T = 400 - 0.1T$

$\Rightarrow T = \dfrac{400}{0.966}$

$\boxed{T = 414.07 N}$ ———— 04M

$\Rightarrow N_1 = 2000 - 0.5 \times 414.07$

$\Rightarrow N_1 = 1792.96 N$

From FBD of 4000N Block.

$\Sigma F_x = +F_1 + F_2 - F = 0.$

$F = F_1 + F_2$

$= \mu N_1 + \mu N_2$

Here $N_2 = N_1 + 4000 = 1792.96 + 4000$

$\Rightarrow N_2 = 5792.96 N$

$F = 0.2 \times 1792.96 + 0.2 \times 5792.96$

$\boxed{F = 1517.18 N}$ ———— 04M

Total 12M.

OR

Q.6 a) Explain different types of trusses. —(04M)

Ans: Depending upon the existance of members in the plane trusses are classified as.

i) Plane truss:

A truss in which all members lie in a single plane is called plane truss.

example: roof truss like king post-truss. ——— 02M

ii) Space truss:

A truss in which all the members do not lie in a single plane is called space truss

example: Tripod, transmission towers. ——— 02M.

Q.6 b) Explain: i) Angle of friction
ii) cone of friction. ——— (04M)

Ans: i) Angle of friction

The inclination of resultant of normal reaction & friction with respect to the normal reaction is called "angle of friction"

For limiting friction i.e friction at the verge of motion of the body.

$$\tan \phi_s = \frac{\mu_s N}{N} = \mu_s$$

where ϕ_s is called angle of limiting friction. ——— 02M

ii) Cone of friction:

Cone of friction is the inverted cone with semi central angle, equal to the limiting frictional angle made by the resultant of normal & friction force with the normal reaction

——— 02 M

Total 04M.

Q.6 c) Analyse the frame & tabulate the members forces for the frame shown in fig. Q6(c)

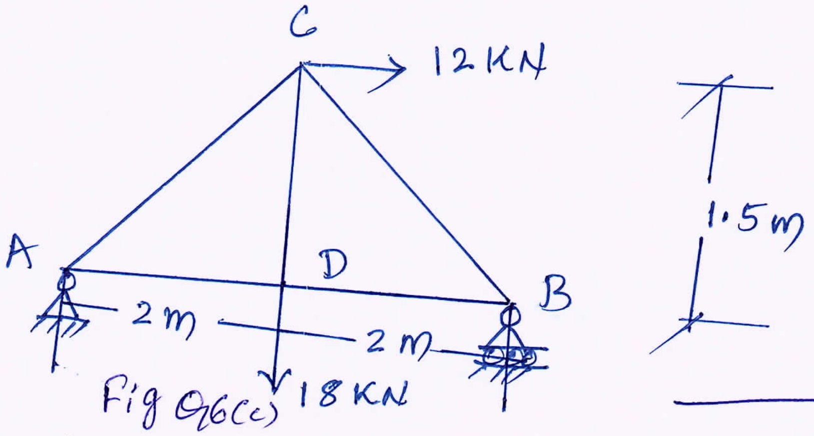
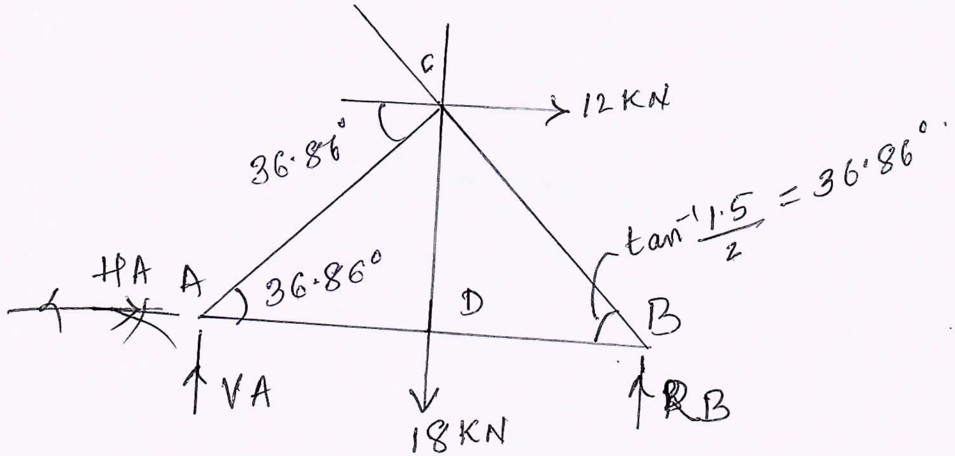


Fig Q6(c)

(12M)

Sol^n Consider the following figure.



Considering the equillibrium of entire truss.

$\Sigma M_A = 0 \Rightarrow +12 \times 1.5 + 18 \times 2 - R_B \times 4 = 0$

$\Rightarrow \boxed{R_B = 13.5 \text{ KN}}$

$\Sigma F_x = 0 \Rightarrow H_A + 12 = 0$

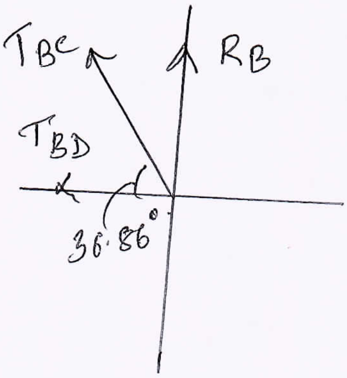
$\Rightarrow \boxed{H_A = -12 \text{ KN}}$

$\Sigma F_y = 0 \Rightarrow -18 + V_A + 13.5 = 0$

$\Rightarrow \boxed{V_A = 4.5 \text{ KN}}$

— 04M

FBD for point B.



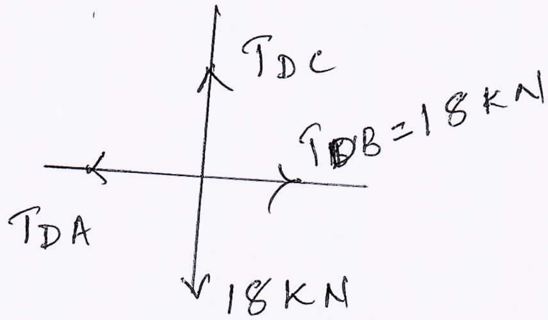
$\Sigma F_y = 0 \Rightarrow +13.5 + T_{BC} \sin 36.86° = 0$

$\Rightarrow \boxed{T_{BC} = -22.5 \text{ KN}}$

$\Sigma F_x = 0 \Rightarrow -T_{BD} - (-22.5) \cos 36.86° = 0.$

$\Rightarrow \boxed{T_{BD} = +18.00 \text{ KN}}$ —— 02

FBD for point D:



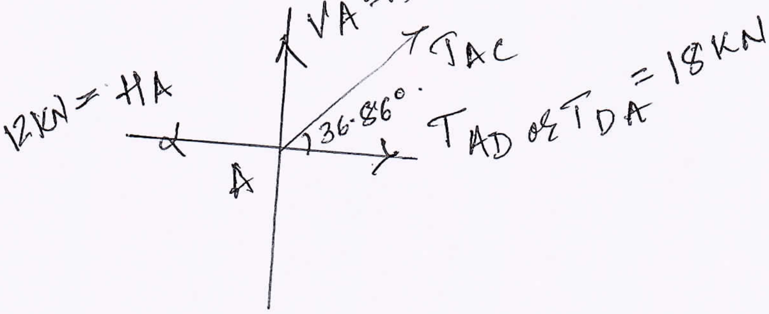
$\Sigma F_x = 0 \Rightarrow +18 - T_{DA} = 0$

$\Rightarrow \boxed{T_{DA} = +18 \text{ KN}}$

$\Sigma F_y = 0 \Rightarrow -18 + T_{DC} = 0$

$\Rightarrow \boxed{T_{DC} = 18 \text{ KN}}$ —— 02

FBD for point A:



$\Sigma F_x = 0$.

$\Rightarrow -12 + 18 + T_{AC} \cos 36.86° = 0$.

$\Rightarrow T_{AC} = -\frac{6}{\cos 36.86°}$.

$\boxed{T_{AC} = -7.50 kN}$ —— 02

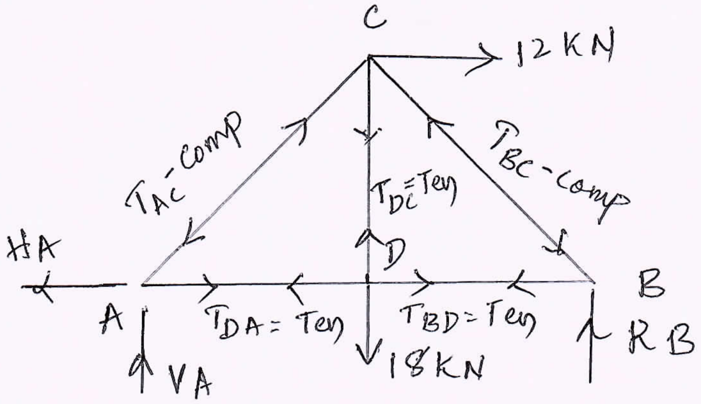
Arthmatic check:

$\Sigma F_y = 0$.

$\Rightarrow +4.5 + (-7.5) \sin 36.86° = 0$.

$+4.5 - 4.5 = 0$

$0 = 0$.



—— 02M

Total 12M.

Module – 4.

Q.7 a) State and Prove parallel axis theorem. (08M)

Ans: **Parallel axis theorem:**

Statement: MI about any axis in the plane of area is equal to the sum of MI about a parallel centroidal axis and the product of area and the square of the distance between the two parallel axes.

Mathematically

$$I_{AB} = I_{GG} + A y_c^2$$

Where

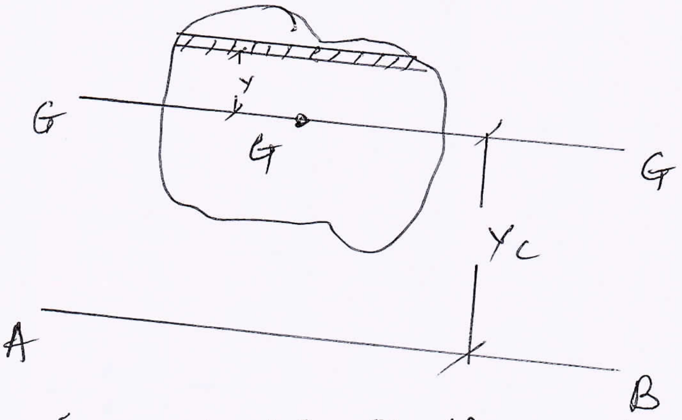
I_{GG} = MI about centroidal axis

I_{AB} = MI about any axis parallel to centroidal axis.

A = Area of plane figure.

y_c = the distance between AB & the centroidal axis.

Proof: Consider plane area A as shown in fig.



The meaning of AB is the axis parallel to centroidal axis GG. The meaning of y_c as above.

Consider an elemental parallel strip dA at a distance y from the centroidal axis

$$\text{then} \quad I_{AB} = \Sigma(y+y_c)^2 dA$$

$$= \Sigma(y^2+y_c^2+2yy_c)dA$$

$$= \Sigma y^2 dA + \Sigma y_c^2 dA + \Sigma 2y y_c dA$$

$$\text{Now} \quad \Sigma y^2 dA = MI \text{ about } GG = I_{GG}$$

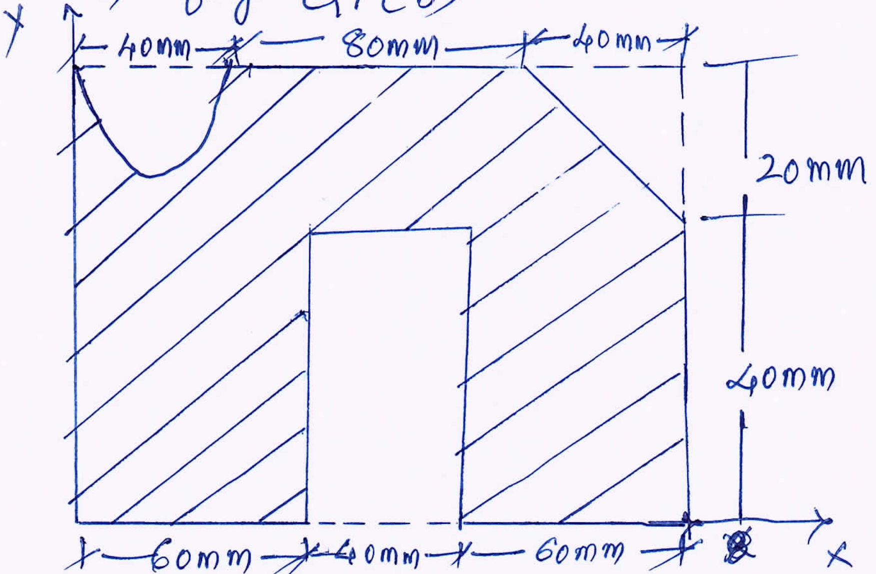
$$\Sigma 2y_c y dA = 0$$

$$\Rightarrow \quad I_{AB} = I_{GG} + \Sigma y_c^2 dA$$

$$\boxed{I_{AB} = I_{GG} + A y_c^2}$$

Thus proved.

Q.7 b) Locate the centroid of shaded area shown in fig. Q7(b)

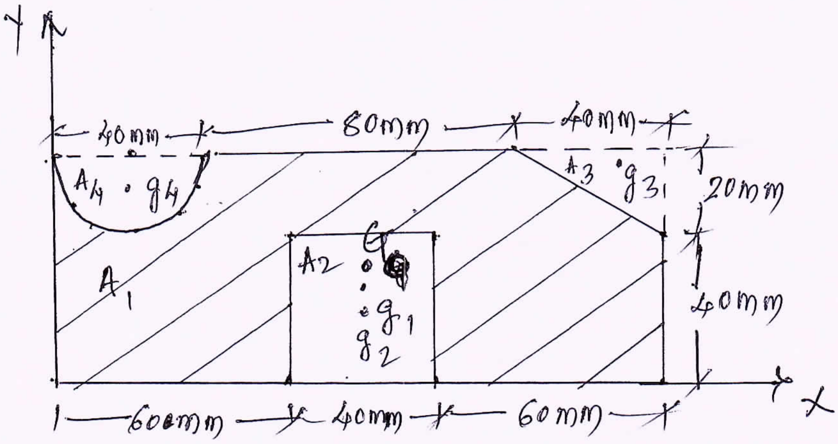


—— (12M)

Fig Q7(b)

Solⁿ

Consider the following figure.



Let A_1 (rectangle of size 160mm × 60m), A_2 (~~rectangle of~~ square of size 40mm × 40mm), A_3 (triangle), A_4 (semicircle) be the segmental areas & g_1, g_2, g_3 & g_4 be their positions of centroids as shown in fig.

$$A_1 = 160 \times 60 = 9600 mm^2$$

$$A_2 = 40 \times 40 = 1600 mm^2$$

$$A_3 = \frac{1}{2} \times 20 \times 40 = 400 mm^2$$

$$A_4 = \cancel{\phi}\ \frac{\pi \times 20^2}{2} = 628.32 mm^2$$

$$\Sigma A = A_1 - A_2 - A_3 - A_4$$

$\Sigma A = 9600 - 1600 + 400 - 628.32$

$= 6971.68\ mm^2$ ———————— 04

Let $\bar{X}$ & $\bar{Y}$ be the coordinates of Centroid G of given composite area.

$$\bar{X} = \frac{A_1\bar{x}_1 - A_2\bar{x}_2 - A_3\bar{x}_3 - A_4\bar{x}_4}{\Sigma A}$$

where $\bar{x}_1, \bar{x}_2, \bar{x}_3$ & $\bar{x}_4$ be the x coordinates of g_1, g_2, g_3 & g_4 respectively.

$$\bar{X} = \frac{9600 \times 80 - 1600 \times 80 - 400 \times (160 - \frac{1}{3} \times 40) - 628.32 \times 20}{6971.68}$$

$$= \frac{640000 - 58666.68 - 12566.4}{6971.68}$$

$$\boxed{\bar{X} = 81.58\ mm}$$ ———————— 04

$$\bar{Y} = \frac{A_1\bar{y}_1 - A_2\bar{y}_2 - A_3\bar{y}_3 - A_4\bar{y}_4}{\Sigma A}$$

where $\bar{y}_1, \bar{y}_2, \bar{y}_3$ & $\bar{y}_4$ be the y coordinates of g_1, g_2, g_3 & g_4 respectively.

$$\bar{Y} = \frac{9600 \times 30 - 1600 \times 20 - 400 \times (60 - \frac{1}{3} \cdot 20) ~~60 - 4 \times 20~~ - (60 - \frac{4 \times 20}{3\pi}) \times 628.32}{6971.68}$$

$$= \frac{256000 - 21332 - 32366.02}{6971.68}$$

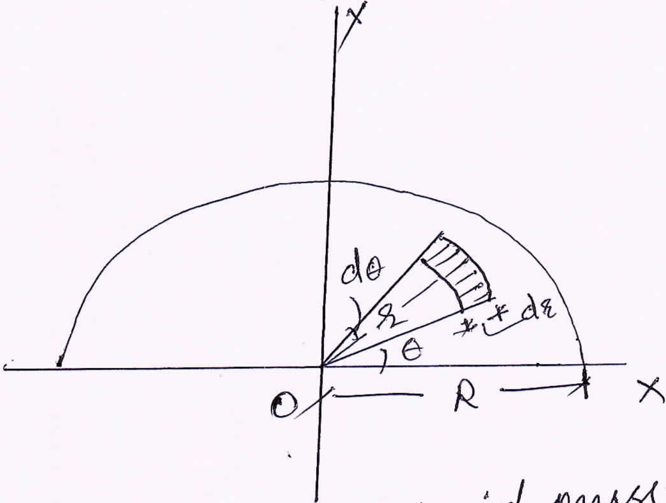
$$\boxed{\bar{Y} = 29.01\ mm}$$ ———————— 04M

Total 12M.

OR

Q.8 a) Determine the centroid of a semi circle of radius R by the method of integration. (08M)

Ans: Consider the semi circle of radius R as shown below in fig.



Due to symmetry centroid must lie on y-axis. Let its distance from diametral axis be $\bar{y}$. To find $\bar{y}$ Consider an element at a distance r from the Centre O of semicircle, radial width being dr and bound by radii at θ and θ+dθ.

The elemental area can be treated as a rectangle of sides rdθ and dr. Hence

Area of element = rdθ × dr.

Its moment about diametral axis x is given by

$$r d\theta \cdot dr \cdot r \sin\theta = r^2 \sin\theta \, dr \, d\theta \text{ —— 04M}$$

∴ Total moment of area about diametral axis

$$= \int_0^\pi \int_0^R r^2 \sin^2\theta \, dr \cdot d\theta = \int_0^\pi \left[\frac{r^3}{3}\right]_0^R \sin\theta \, d\theta$$

$$= \frac{R^3}{3}\left[-\cos\theta\right]_0^\pi$$

$$= \frac{R^3}{3}[1+1]$$

$$= \frac{2R^3}{3}$$

Area of Semicircle $= \frac{\pi R^2}{2}$

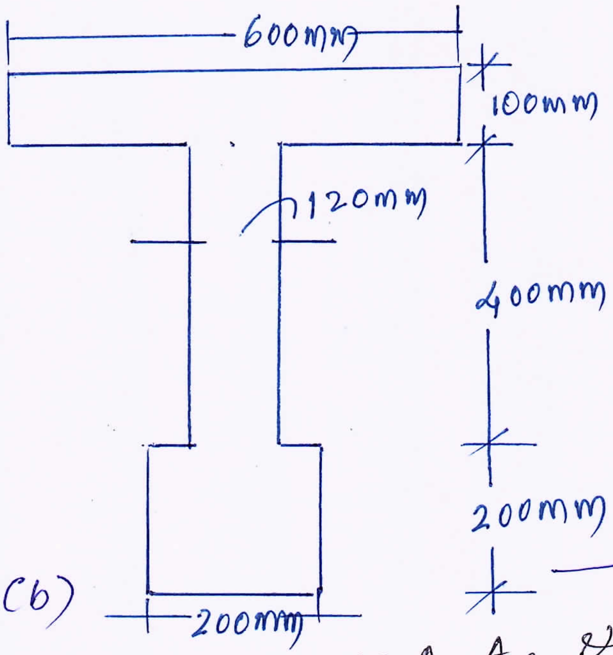
$$\bar{y} = \frac{\text{Moment of area}}{\text{Total area}}$$

$$= \frac{\frac{2R^3}{3}}{\frac{\pi R^2}{2}}$$

$$\boxed{\bar{y} = \frac{4R}{3\pi}}$$ ——— 04M

Total 08M.

Q.8 b) Determine the moment of inertia of a pre-stressed concrete beam section shown in fig. Q8(b) about horizontal & vertical axis passing through centroid.

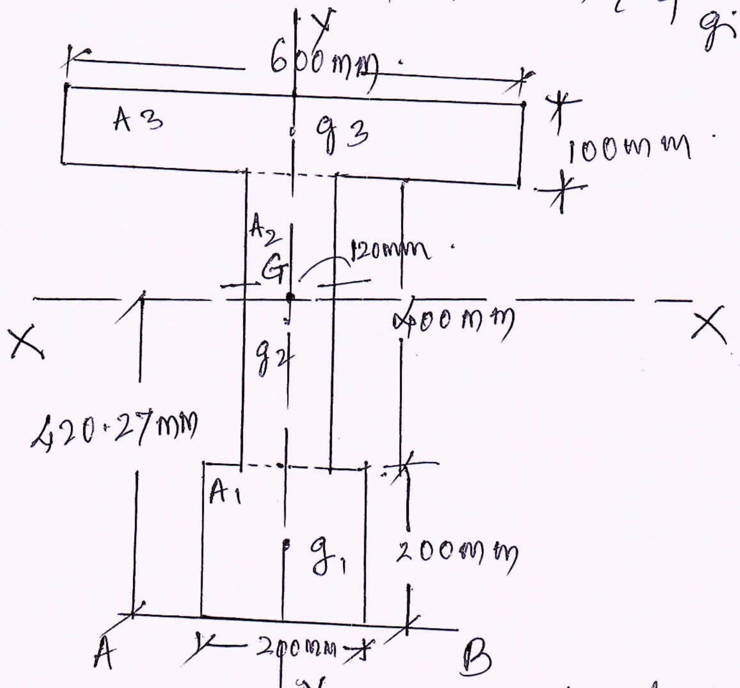


(12M)

Fig Q8(b)

Soln Consider the following fig. Let A_1, A_2, & A_3 be the segmental areas & g_1, g_2 & g_3 be their centroids as shown in fig.

Determine Centroid G with respect base AB of given section as shown



Let $\bar{x}$ & $\bar{y}$ be the coordinates of G

Due to symmetry $\bar{x} = 0$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{\Sigma A}$$

~~At~~ where $\bar{y}_1, \bar{y}_2$ & $\bar{y}_3$ be the y coordinates of Centroid G.

$$\bar{y} = \frac{200\times200\times100 + 120\times400\times400 + 600\times100\times650}{200\times200 + 120\times400 + 600\times100}$$

$$= \frac{4\times10^6 + 19.2\times10^6 + 39\times10^6}{4\times10^4 + 4.8\times10^4 + 6\times10^4}$$

$$= \frac{62.2\times10^6}{14.8\times10^4}$$

$$\boxed{\bar{y} = 420.27 mm.}$$ ———— 04M

Applying parallel axis theorem.

$$I_{xx} = I_{GG} + Ah^2$$

$$I_{xx} = 200 \times \frac{200^3}{12} + 200 \times 200 (420.27 - 200)^2$$

$$+ 120 \times \frac{400^3}{12} + 120 \times 400 (420.27 - 400)^2$$

$$+ 600 \times \frac{100^3}{12} + 600 \times 100 (650 - 420.27)^2$$

$$= \overset{1333.33}{\cancel{6.6667}} \times 10^5 + 19407.54916 \times 10^5$$

$$+ 6400 \times 10^5 + 197.2189 \times 10^5$$

$$+ 500 \times 10^5 + \overset{31666.55 \times 10^5}{\cancel{530.3237 \times 10^5}}$$

$$\boxed{I_{xx} = \overset{5.949310}{\cancel{2.836842}} \times 10^9 \, mm^4} \quad \text{———} \quad 06$$

$$I_{yy} = \frac{200 \times 200^3}{12} + \frac{400 \times 120^3}{12} + \frac{100 \times 600^3}{12}$$

$$I_{yy} = 1333.3333 \times 10^5 + 576 \times 10^5 + 18000 \times 10^5$$

$$\boxed{I_{yy} = 1.990933 \times 10^9 \, mm^4} \quad \text{———} \quad 02M$$

Total 12M.

Module - 5

Q.9 a) Derive the equations of motion. (06M)

Ans: Consider a body which is having the details of motion as follows.

Let
u = initial velocity
v = Final velocity
s = Distance covered
a = accelaration
t = time.

* Equation for final velocity v

Change of velocity = Final velocity - Initial velocity

Change of velocity = $v - u$

Devide LHS & RHS by time t.

$$\frac{\text{Change of velocity}}{t} = \frac{v-u}{t}$$

$$\Rightarrow a = \frac{v-u}{t}$$

$$\Rightarrow \boxed{v = u+at}$$ —— 03M

* Equation for distance covered (s)

Average velocity $= \frac{v+u}{2}$ — ①

But by def^n Avg. velocity $= \frac{\text{dist covered}}{\text{time}} = \frac{s}{t}$

$$\Rightarrow S = t \cdot \text{Avg velocity}$$

$$= t\left(\frac{u+v}{2}\right)$$

But $v = u+at$

$$\Rightarrow S = t\left(\frac{u+u+at}{2}\right)$$

$$S = \frac{2ut + at^2}{2}$$

$$\boxed{S = ut + \tfrac{1}{2}at^2}$$ —— 03M.

Total 06M.

Q.9 b) What is superelevation? why is it necessary. — (04M)

Ans. (Out of syllabus)

OR.

* Super elevation: The raising of the out edge of the ~~cur~~ curved track or road

is called "Superelevation". It is also called as "Cant" or "banking". ——— 02M

It is necessary to make load equal on the two wheels for the average speed (V). Also to avoid the centrifugal force. normal to the surface of track or road.

Mathematically.

$$\text{Superelevation} = h = \frac{BV^2}{gR}$$

where B = width of the highway.

V = Average speed.

g = Accelaration du to gravity

R = resultant force of weight (W) and centrifugal force. ——— 02M

Total 04M.

Q.9 c). A ball is dropped from the top of a tower 30m high. At the same instant another ball is thrown upward from the ground with an initial velocity of 15m/sec. when & where do they cross. ——— (10M)

Soln

For the motion of first ball.

Given $u = 0$, $s = 30 - h$ (considering two balls cross each other at a height of h from the ground after t seconds)

$a = 9.8\ m/sec^2$

Using the equation

$$S = ut + \frac{1}{2}at^2$$

$$30 - h = 0 \times t + \frac{1}{2} \times 9.81 \times t^2$$

$$\Rightarrow 30 - h = \frac{9.81}{2} t^2 \; ---- \; ①$$ ——— 04M

For the motion of second ball.

$u = 15m/sec$, $s = h$ and $a = -9.81m/sec^2$ ——— 4M

$$h = 15t - \frac{1}{2} 9.81t^2 \; — \; ②$$

① + ②

$$30 - h + h = \cancel{\frac{9.81}{2} t^2} + 15t - \cancel{\frac{1}{2} 9.81^2 t^2}$$

$$\Rightarrow 15t = 30$$

$$\Rightarrow \boxed{t = 2 \text{ Sec}}$$ --- Ans.

Sub in ① or ②

$$30 - h = \frac{9.81}{2}(2)^2$$

$$\Rightarrow h = 10.38 \text{ m}.$$

Check: $h = 15 \times 2 - \frac{1}{2} 9.81 \times 2^2$ ——— 02M

$$\Rightarrow h = 10.38 \text{ m}.$$

Total 10M.

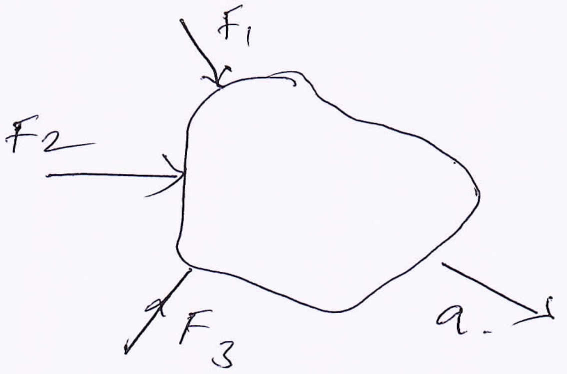
OR.

Q.10 a) State & explain D'Alembert's principle. —(04M)

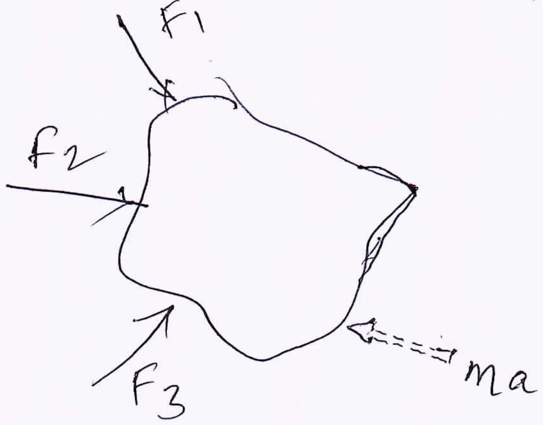
Ans: D'Alembert's Principle:

It states that "The system of forces acting on a Moving body is in dynamic equilibrium with the inertia force of the body".

Consider a fig. A as shown below.

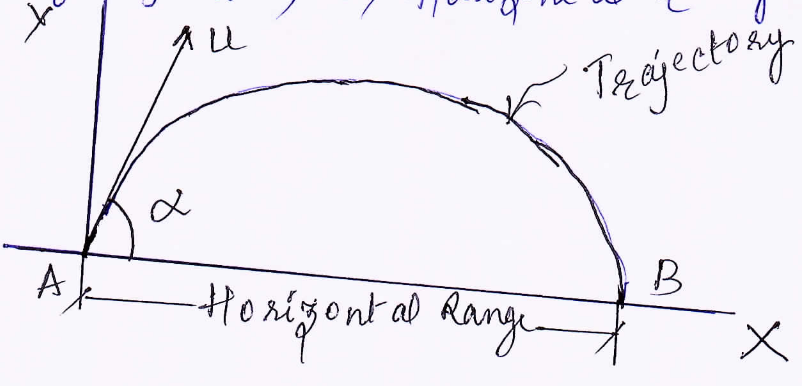


The body is subjected to a system of forces causing the body to move with an accelaration a in the direction of the resultant. Then apply a force equal to ma in the reversed direction of accelaration shown in fig B below.



Now according to D' Alembert's principle, the equations of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ may be used for the system of forces shown in fig B.

10 b) Define the following with neat sketch.
i) Angle of Projection ii) Horizontal Range iii) Time of flight — (04M)



i) **Angle of projection**: The angle between the direction of projection and horizontal direction is called as angle of projection. in fig α is angle of projection.

ii) **Horizontal range**: The horizontal distance through which the projectile travels in the flight is called horizontal range or simply range or range of the particle. It is shown as in figure.

iii) **Time of flight**: The time interval during which the projectile is in motion is called the time of flight.

Q.10 c) A cricket ball is thrown by a player from a height of 2m above the ground at an angle of 30° to the horizontal with a velocity of 20m/sec is caught by another fieldsman at a height of 1m from the ground. Find the distance between the two players. —— (12M)

Soln Consider the following fig.

Given,

Initial velocity $= u = 20\,m/sec.$

Angle of projection $= \alpha = 30°.$

$y_0 = -(2.0 - 1) = -1.0m$ ———— 04M

Time of flight is given by the expression

$y_0 = u \sin\alpha \times t - \frac{1}{2} g t^2$

$-1 = 20 \sin 30 \times t - \left(\frac{1}{2} 9.81 t^2\right).$

$-1 = 10t - 4.905t^2$

~~$t^2 - 2.038t - 0.2035$ 0~~ : $t^2 - 2.038t - 0.2038 = 0$

— 04M

Solving above quadratic eqn.

~~t = 4.445 sec.~~

$t = 2.133$ sec or $t = -0.0955$

time will not be −ve

$\therefore t = 2.133$ secs.

$\therefore$ The distance between the two players.

$= \text{Range} = u \cos\alpha \times t$

$= 20 \cos 30 \times 2.133$

$= 36.94\,M.$ ———— 04M

Total 12M.

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