Modified

CBCS SCHEME

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BMATM201

Second Semester B.E./B.Tech. Degree Examination, June/July 2024
Mathematics – II for Mechanical Engineering Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. Mathematics hand book is permitted.

		Module – 1	M	L	C
Q.1	a.	Evaluate $\int_0^a\int_0^b\int_0^c (x^2+y^2+z^2)\,dx\,dy\,dz$.	6	L2	CO1
	b.	Change the order of integration in $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$ and hence evaluate the same.	7	L2	CO1
	c.	Derive the relation between Beta and Gamma function.	7	L2	CO1
		OR			
Q.2	a.	Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$ by changing to polar coordinates.	7	L2	CO1
	b.	Using double integration, find the volume of the tetrahedron bounded by the planes x = 0, y = 0, z = 0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.	7	L2	CO1
	c.	Write a modern mathematical program to evaluate the integral $\int_0^3\int_0^{3-x}\int_0^{3-x-y} xyz\,dz\,dy\,dx$.	6	L3	CO5
		Module – 2			
Q.3	a.	If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then show that $\nabla r^n = nr^{n-2}\vec{r}$.	7	L2	CO2
	b.	Find the directional derivative of $\phi = x^2yz+4xz^2$ at (1, -2, -1) along $2\hat{i}-\hat{j}-2\hat{k}$.	7	L2	CO2
	c.	If $\vec{F}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$, find a, b, c such that curl $\vec{F}=\vec{0}$.	6	L2	CO2
		OR			
Q.4	a.	Find the work done in moving a particle in the force field $\vec{F}=3x^2\hat{i}+(2xz-y)\hat{j}+z\hat{k}$ along the straight line from (0, 0, 0) to (2, 1, 3).	7	L3	CO2

<table>
<tr><td></td><td>b.</td><td>Using Green's theorem, evaluate $\int_c (xy + y^2)dx + x^2 dy$, where C is the closed curve of the region bounded by y = x and $y = x^2$.</td><td>7</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>c.</td><td>Write a modern mathematical tool program to find the divergence of $\vec{F} = x^2yz\hat{i} + y^2zx\hat{j} + z^2xy\hat{k}$.</td><td>6</td><td>L3</td><td>CO2</td></tr>
<tr><td colspan="6">Module – 3</td></tr>
<tr><td>Q.5</td><td>a.</td><td>Form the partial differential equation from the relation
$z = f(y + 2x) + g(y - 3x)$.</td><td>6</td><td>L1</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Solve $\frac{\partial^2 z}{\partial x^2} = xy$ subject to the conditions $\frac{\partial z}{\partial x} = \log(1 + y)$ when x = 1 and z = 0 when x = 0.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Derive one dimensional heat equation.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.6</td><td>a.</td><td>Form the partial differential equation from the relation
$f(xy + z^2, x + y + z) = 0$</td><td>6</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>b.</td><td>Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when x = 0, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td></td><td>c.</td><td>Solve $(mz - ny)p + (nx - \ell z)q = \ell y - mx$.</td><td>7</td><td>L2</td><td>CO3</td></tr>
<tr><td colspan="6">Module – 4</td></tr>
<tr><td>Q.7</td><td>a.</td><td>Find the real root of the equation $\cos x - xe^x = 0$ in (0.5, 0.6) using the Regula – Falsi method correct to four decimal places, carryout there interations.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>The population of a town is given by the table
<table>
<tr><td>Year</td><td>1951</td><td>1961</td><td>1971</td><td>1981</td><td>1991</td></tr>
<tr><td>Population in thousands</td><td>19.96</td><td>39.65</td><td>58.81</td><td>77.21</td><td>94.61</td></tr>
</table>
Using Newton's forward interpolation formula, calculate the population in the year 1955.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Evaluate $\int_0^6 \frac{dx}{1 + x^2}$ by using Simpson's 1/3rd rule. [Take 6 equal parts].</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>Find a real root of the equation $x^3 + 5x - 11 = 0$ near to x = 1 using Newton – Raphson method. Carryout three iterations.</td><td>7</td><td>L3</td><td>CO4</td></tr>
</table>

			Marks	Level	CO
	b.	Using Newton's divided difference formula, evaluate f(4) from the following table x: 0, 2, 3, 6 f(x): -4, 2, 14, 158	7	L3	CO4
	c.	Evaluate $\int_0^1 \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $(3/8)^{th}$ rule.	6	L3	CO4
		Module – 5			
Q.9	a.	Using Taylor's series method, find y(0.1) considering upto fourth degree term if y(x) satisfies the equation $\frac{dy}{dx} = x - y^2, y(0) = 1$.	6	L2	CO4
	b.	Using Runge-Kutta method of fourth order, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 taking h = 0.2.	7	L3	CO4
	c.	Given that $\frac{dy}{dx} = x - y^2$ and the data y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762. Compute y at x = 0.8 applying Milne's method.	7	L3	CO4
		OR			
Q.10	a.	Using modified Euler's method, compute y(1.1) given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and y = 1 at x = 1 by taking h = 0.1.	7	L3	CO5
	b.	Apply Runge-Kutta fourth order method, to find an approximate value of y when x = 0.2, given that $\frac{dy}{dx} = x + y$ and y = 1 when x = 0.	7	L3	CO5
	c.	Using modern mathematical tools with a program to find y when x = 1.4, given $\frac{dy}{dx} = x^2 + (y/2)$, y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514 using predictor corrector method.	6	L3	CO5

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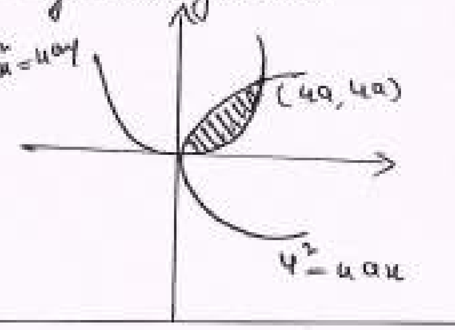
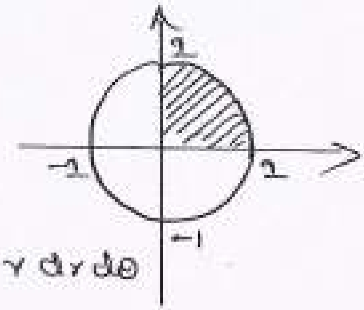
AY: 202 -202

Department: Mathematics
Semester / Branch: 1Sem CIVIL **Name of Faculty: Prof.Akshata Patil**
Subject with Sub. Code: Mathematics-I for C Eng Streams(

Q.No.	Solution and Scheme	Marks
	Module 1:	
1a]	$I=\int_{z=0}^{a}\int_{y=0}^{b}\int_{x=0}^{c}(x^2+y^2+z^2)\,dx\,dy\,dz$	2
	$=\int_{z=0}^{a}\int_{y=0}^{b}\left[\frac{c^3}{3}+cy^2+cz^2\right]dy\,dx$	2
	$=\int_{z=0}^{a}\left[\frac{bc^3}{3}+\frac{b^3c}{3}+bcz^2\right]dz$	2
	$=\frac{abc^3}{3}+\frac{ab^3c}{3}+\frac{a^3bc}{3}$	
	$=\frac{abc}{3}\left[a^2+b^2+c^2\right]$	6M
1b	$I=\int_{x=0}^{4a}\int_{y=x^2/4a}^{2\sqrt{ax}}dy\,dx$	
	$\frac{x^2}{4a}=2\sqrt{ax}$ or $x^4=64a^3x$	2
	ie $x(x^3-64a^3)=0 \Rightarrow x=0$ and $x=4a$.	1
	From, $y=x^2/4a$ we get $y=0$ and $y=4a$	
	Thus the points of intersection of the parabolas $y=x^2/4a$ and $y=2\sqrt{ax}$ are $(0,0)$ and $(4a,4a)$	2
	on changing the order of Integration	1
	$I=\int_{y=0}^{4a}\int_{x=y^2}^{2\sqrt{ay}}dx\,dy$ 	1
		7M

Q.No.	Solution and Scheme	Marks
	$I = \int_{y=0}^{4a} \left[2\sqrt{ay} - \frac{y^2}{4a} \right] dy$	
	$I = \frac{4}{3}\sqrt{a}\,(4a)^{3/2} - \frac{1}{4a}\frac{(4a)^3}{3}$	
	$I = \frac{32}{3}a^2 - \frac{16}{3}a^2 = \frac{16}{3}a^2$	
7 c]	$\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta \longrightarrow (1)$	
	$\Gamma n = 2\int_0^{\infty} e^{-x^2} \cdot x^{2n-1} \cdot dx \longrightarrow (2)$	
	$\Gamma m = 2\int_0^{\infty} e^{-y^2} \cdot y^{2m-1} \cdot dy \longrightarrow (3)$	2
	$\Gamma(m+n) = 2\int_0^{\infty} e^{-r^2} \cdot r^{2(m+n)-1} \cdot dr \longrightarrow (4)$	
	$\Gamma m \cdot \Gamma n = 4\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} \cdot x^{2n-1} \cdot y^{2m-1} dx\,dy \longrightarrow (5)$	2
	Changing the polar form.	
	Put $x = r\cos\theta$; $y = r\sin\theta$; $dx\,dy = r\,dr\,d\theta$	
	r varies from 0 to ∞ and θ varies from 0 to $\pi/2$	
	$\therefore \Gamma m\,\Gamma n = 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2} (r\cos\theta)^{2n-1} (r\sin\theta)^{2m-1} \cdot r\,dr\,d\theta$	2
	$= 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2} \cdot r^{2(m+n)-1} \cdot \sin^{2m-1}\theta \cos^{2n-1}\theta \, dr\,d\theta$	
	$= \left[2\int_{\theta=0}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta \right] \left[2\int_{r=0}^{\infty} e^{-r^2} \cdot r^{2(m+n)-1} \cdot dr \right]$	2
	$= \beta(m,n)\,\Gamma(m+n) \Longrightarrow \therefore \beta(m,n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m+n)}$	
		7M

Q.No.	Solution and Scheme	Marks
2a.	Let, $I=\int_{y=0}^{1}\int_{x=0}^{\sqrt{1-y^2}}(x^2+y^2)\,dx\,dy$ 	2
	Changing to polar form $x=r\cos\theta$, $y=r\sin\theta$, $dxdy=r\,dr\,d\theta$ $x^2+y^2=r^2$; r varis from 0 to 1 θ varis from 0 to $\pi/2$	2
	$I=\int_{\theta=0}^{\pi/2}\int_{r=0}^{1}r^3\,dr\,d\theta=\int_{\theta=0}^{\pi/2}\left[\frac{r^4}{4}\right]_{r=0}^{1}d\theta$	2
	$I=\frac{1}{4}\int_{\theta=0}^{\pi/2}d\theta=\frac{1}{4}\times\pi/2=\pi/8.$	1
		7M
2b.	Volume $V=\iint z\,dx\,dy$ $=\int_{x=0}^{a}\int_{y=0}^{b(1-x/a)}c\left[1-x/a-y/b\right]dy\,dx$	2
	$=c\int_{x=0}^{a}\left[y-\frac{x}{a}y-\frac{1}{b}\frac{y^2}{2}\right]_0^{b(1-x/a)}dx$ $\therefore c\int_{x=0}^{a}\left[b(1-x/a)-\frac{x}{a}b(1-x/a)-\frac{1}{2b}b^2(1-x/a)^2\right]dx$	2
	$=c\int_{x=0}^{a}\left[\frac{b}{2}-\frac{b}{a}x+\frac{b}{2a^2}x^2\right]dx$ $=c\left[\frac{b}{2}x-\frac{b}{2a}x^2+\frac{b}{2a^2}\cdot\frac{x^3}{3}\right]_0^a$	2
	$=c\left[\frac{b}{2}a-\frac{b}{a}\cdot\frac{a^2}{2}+\frac{b}{6a^2}\cdot a^3\right]$ $=\frac{abc}{6}$ Cubic units.	1
		7M

Q.No.	Solution and Scheme	Marks
2c.	Mathematical Program to evaluate $\int_0^3 \int_0^{3-x} \int_0^{3-x-y} xyz \, dz\, dy\, dx$	2
	`from sympy import*` `x, y, z = symbols('x y z')`	2
	`w1 = integrate((x*y*z), (z,0,3-x-y), (y,0,3-x), (x,0,3))`	2
	`print(w1)`	6M

Module 2:

Q.No.	Solution and Scheme	Marks
3a	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ $r = \lvert\vec{r}\rvert = \sqrt{x^2+y^2+z^2}$ $\quad \therefore r^2 = x^2+y^2+z^2$	2
	Differentiating partially wrt x, y, z $\frac{\partial r}{\partial x} = x/r$, $\frac{\partial r}{\partial y} = \frac{y}{r}$, $\frac{\partial r}{\partial z} = \frac{z}{r}$	2
	$\nabla r^n = \frac{\partial}{\partial x}(r^n)\hat{i} + \frac{\partial}{\partial y}(r^n)\hat{j} + \frac{\partial}{\partial z}(r^n)\hat{k}$ $= nr^{n-1}\left(\frac{x}{r}\right)\hat{i} + nr^{n-1}\left(\frac{y}{r}\right)\hat{j} + nr^{n-1}\left(\frac{z}{r}\right)\hat{k}$	2
	$= \frac{nr^{n-1}}{r}\left[x\hat{i} + y\hat{j} + z\hat{k}\right]$	1
	$\nabla r^n = nr^{n-2}\,\vec{r}$	7M
3b.	$\phi = x^2yz + 4xz^2$ $\nabla\phi = (2xyz + 4z^2)\hat{i} + x^2z\hat{j} + (x^2y + 8xz)\hat{k}$	2
	$(\nabla\phi)_{(1,-2,-1)} = 8\hat{i} - \hat{j} - 10\hat{k}$ $\vec{d} = 2\hat{i} - \hat{j} - 2\hat{k}$	2
	$\therefore \hat{d} = \frac{\vec{d}}{\lvert\vec{d}\rvert} = \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k})$	2
	$\therefore$ Directional derivative $= \nabla\phi \cdot \hat{d}$ $= (8\hat{i} - \hat{j} - 10\hat{k}) \cdot \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k})$	1
	$= \frac{1}{3}(16 + 1 + 20) = \frac{37}{3}$ //	7M

Q.No	Solution and Scheme	Marks

3c.

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (x+y+az) & (bx+2y-z) & (x+cy+2z) \end{vmatrix} = \vec{0}$$ — 3

ie $i(c+1) - j(1-a) + k(b-1) = \vec{0}$ — 2

$c+1=0$, $1-a=0$, $b-1=0$

ie $a=1$, $b=1$, $c=-1$ are the required values. — 2

6M

4a. Let,

$O=(0,0,0)$ & $A=(2,1,3)$ equation of straight line $\frac{x}{2} = \frac{y}{1} = \frac{z}{3} = t$ — 2

$x = 2t \Rightarrow dx = 2dt$

$y = t \Rightarrow dy = dt$

$z = 3t \Rightarrow dz = 3dt$; $0 \le t \le 1$ — 2

$\vec{F}\cdot d\vec{r} = 3x^2\,dx + (2xz-y)\,dy + z\,dz$

$= (24t^2 + 12t^2 - t + 9t)\,dt$ — 2

$= (36t^2 + 8t)\,dt$

$\therefore \int_C \vec{F}\cdot d\vec{r} = \int_{t=0}^{1} [36t^2 + 8t]\,dt = 36\left[\frac{t^3}{3}\right]_{t=0}^{1} + 8\left[\frac{t^2}{2}\right]_{t=0}^{1}$ — 1

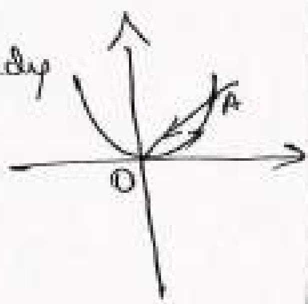
$= 12 + 4 = 16$

7M

4b Let, $(0,0)$ & $(1,1)$ are the points of intersection

Green's theorem in a plane is

$$\oint_C [M\,dx + N\,dy] = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] dx\,dy$$ — 2



$M = xy + y^2 \Rightarrow \frac{\partial M}{\partial y} = x + 2y$ — 2

$N = x^2 \Rightarrow \frac{\partial N}{\partial x} = 2x$

$\therefore \frac{\partial M}{\partial y}$ $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = x - 2y$

Solution and Scheme

Using green's theorem

$$\int_C (xy+y^2)dx + x^2 dy = \int_{x=0}^{1}\int_{y=x^2}^{x} (x-2y)\,dy\,dx$$ 2

$$= \int_{x=0}^{1} \left[xy - y^2\right] dx$$ 1

$$= \int_{x=0}^{1} (x^2 - x^2 - x^3 + x^4)\,dx = \int_{x=0}^{1}(x^4 - x^3)\,dx$$

$$= \left[\frac{x^5}{5} - \frac{x^4}{4}\right]_{x=0}^{1} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20}$$

7M

4c. Program to find divergence of $\vec{F} = x^2yz\,\hat{i} + y^2zx\,\hat{j} + z^2xy\,\hat{k}$

```
from Sympy.vector import *
from Sympy import Symbols
N = CoordSys3D('N')
x, y, z = Symbols('x y z')
A = N.x ** 2 * N.y * N.z * N.i + N.y ** 2 * N.z * N.x
    * N.j + N.z ** 2 * N.x * N.y * N.k
delop = Del( )
divA = delop.dot(A)
display(div A)
Print (f"\n Divergence of {A} is \n")
display (divergence (A))
```

2 2 2

6M

Module-3:-

5a. $\frac{\partial z}{\partial x} = p = 2f'(y+2x) - 3g'(y-3x) \longrightarrow (1)$

$\frac{\partial z}{\partial y} = q = f'(y+2x) + g'(y-3x) \longrightarrow (2)$ 2

$\frac{\partial^2 z}{\partial x^2} = r = 4f''(y+2x) + 9g''(y-3x) \longrightarrow (3)$

$\frac{\partial^2 z}{\partial x \partial y} = s = 2f''(y+2x) - 3g''(y-3x) \longrightarrow (4)$ 2

$\frac{\partial^2 z}{\partial y^2} = t = f''(y+2x) + g''(y-3x) \longrightarrow (5)$ 2

Adding eqn (3) & (4)

$r + s = 6\left[f''(y+2x) + g''(y-3x)\right]$

$r + s = 6t$ from eqn (5)

6M

Solution and scheme

5b $\frac{\partial}{\partial x}\left[\frac{\partial z}{\partial x}\right] = xy$

Integrating wrt 'x' treating y as constant 2

$\frac{\partial z}{\partial x} = \frac{x^2}{2}y + f(y) \longrightarrow (1)$ Int again wrt 'x'

$z = \frac{x^3}{6}y + x f(y) + g(y) \longrightarrow (2)$

Given $\frac{\partial z}{\partial x} = \log(1+y)$ when $x=1$; substituting 2

in eqn (1)

$\log(1+y) = \frac{1}{2}y + f(y) = f(y) = \log(1+y) - \frac{1}{2}y$

Also when $x=0,\ z=0$ Sub in eqn (2) 2

$g(y) = 0$

$\therefore$ The solution is $z = \frac{x^3}{6}y + x\left[\log(1+y) - \frac{1}{2}y\right]$ 1

7m

5c.

Consider a thin beam of heat conducting material OA placed along the x axis with one end of the beam coinciding the origin. 2

Let $OP = x$ & $OQ = x + \delta x$. The amount of heat contained in PQ is $H = \rho s u\, \delta x \rightarrow (1)$

where ρ is the density & s is the specific heat of the material which are taken as constant.

$\therefore$ Time rate of increase of heat in PQ is 2

$\frac{\partial H}{\partial t} = \rho s \frac{\partial u}{\partial t}\delta x \longrightarrow (2)$

By Fourier law of heat conduction

$q = -k\frac{\partial u}{\partial x} \longrightarrow (3)$ where k is the thermal conductivity of the material.

Let $q_1 \rightarrow$ rate of flow of heat into PQ. 2

$q_2 \rightarrow$ rate of flow of heat out of PQ.

Then net rate of heat flow into PQ is

$q_1 - q_2 = -k\left\{\left(\frac{\partial u}{\partial x}\right)_Q - \left(\frac{\partial u}{\partial x}\right)_P\right\} \longrightarrow (4)$

Solution and scheme

According to the law of balance energy

$$\frac{\partial H}{\partial t} = q_1 - q_2 \longrightarrow (5)$$

$$\therefore \rho s \frac{\partial u}{\partial t}\delta x = k\left\{\left(\frac{\partial u}{\partial x}\right)_{Q} - \left(\frac{\partial u}{\partial x}\right)_{P}\right\}$$

$$\frac{\partial u}{\partial t} = c^2 \cdot \frac{1}{\delta x}\left\{\left(\frac{\partial u}{\partial x}\right)_{(x+\delta x, t)} - \left(\frac{\partial u}{\partial x}\right)_{(x,t)}\right\}$$ 1

where

$$c^2 = k/\rho s$$

Taking the limit as $\delta x \to 0$

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad \text{or} \quad u_t = c^2 u_{xx}.$$

7M

6a. Let,

$$f(u,v) = 0 \longrightarrow (1) \quad \text{where } u = xy + z^2,\ v = x+y+z$$

$$\frac{\partial u}{\partial x} = y + 2zp \qquad \frac{\partial v}{\partial x} = 1+p$$

$$\frac{\partial u}{\partial y} = x + 2zq \qquad \frac{\partial v}{\partial y} = 1+q$$ 2

Differentiating eqn (1) partially wrt x & y

$$\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} = 0 \Rightarrow \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} = -\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} \longrightarrow (2)$$

$$\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} = -\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} \longrightarrow (3)$$ 2

Dividing eqn (2) by eqn (3)

$$\frac{\partial u/\partial x}{\partial u/\partial y} = \frac{\partial v/\partial x}{\partial v/\partial y} \Rightarrow \frac{y+2zp}{x+2zq} = \frac{1+p}{1+q}$$

$$(y+2zp)(1+q) = (1+p)(x+2zq)$$ 2

$$y + yq + 2zp + 2zpq = x + 2zq + xp + 2zpq$$

$(2z-x)p + (y-2z)q = x-y$ is the required PDE

6M

Solution and Scheme.

6b	Suppose z is a function of one variable x only. Then the ODE is $(D^2+1)z=0$ where $D = d/dx$ A.E $m^2+1=0 \Rightarrow m^2=-1 \Rightarrow m = \pm i$ $\therefore$ The solution of ODE is $z = c_1\cos x + c_2 \sin x$ $\therefore$ The solution of PDE is $z = f(y)\cos x + g(y)\sin x \rightarrow (1)$ Differentiating eqn (1) partially w.r.t x. $\frac{\partial z}{\partial x} = -f(y)\sin x + g(y)\cos x \longrightarrow (2)$ Given $z = e^y$ and $\frac{\partial z}{\partial x} = 1$ when $x=0$ substitute in eqn (1) & (2) we get. $f(y) = e^y$, $g(y) = 1$ Sub in eqn (1) $\therefore$ The Solution is $z = e^y\cos x + \sin x$	2 2 2 1 7M
6c.	PDE is of the form $Pp+Qq=R$ The auxillary equation is, $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx} \longrightarrow (1)$ Using multiplier l, m, n each ratio in eqn (1) becomes $l\,dx + m\,dy + n\,dz = 0$ Integrating $lx+my+nz = c_1$ using multiplier x, y, z each ratio in eqn (1) equal T $x\,dx + y\,dy + z\,dz = 0$ Integrating $x^2+y^2+z^2 = 2c_2$ $\therefore$ The general solution of PDE is $\phi(lx+my+nz,\ x^2+y^2+z^2) = 0$	2 2 2 1 7M
7a	**Module 4:-** Let. $f(x) = \cos x - x e^x$ Given lies in $(0.5, 0.6)$ $\underline{I^{st}\ Iteration}$: $a=0.5$, $f(a) = 0.0532$, $b = 0.6$ $f(b) = -0.2679$ $x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)}$ $x_1 = +0.5166$; $f(0.5166) = 0.00352 > 0$	 2 2

Solution and Scheme.

∴ The root lies in (0.5166, 0.6)

<u>II</u>

$\underline{\text{II}^{nd}}$ Iteration: $a = 0.5166$, $f(a) = 0.00352$, $b = 0.6$

$f(b) = -0.2679$ | 2

∴ $x_2 = +0.5177$; $f(0.5177) = 0.00017$

The root lies in $(0.5177, 0.6)$

III^{rd} Iteration:-

$a = 0.5177$, $f(a) = 0.00017$, $b = 0.6$, $f(b) = 0.2679$ | 1

∴ $x_3 = 0.5177$

∴ The required approximate root correct to 4 decimal places is 0.5177. | 7M

7b The forward difference table is a follow.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1951	19.96	19.69			
			-0.53		
1961	39.65	19.16		-0.23	
1971	58.81	18.4	-0.76		-0.01
1981	77.21	17.4	-2	-0.24	
1991	94.61				

3

Using Newton's forward interpolation formula

$$r = \frac{x - x_0}{h} = \frac{1955 - 1951}{10} = 0.4$$ 2

$$\therefore y(1955) = 19.96 + (0.4)(19.69) + \frac{(0.4)(-0.6)}{2}(-0.53)$$

$$+ \frac{(0.4)(-0.6)(-1.6)}{6}(-0.23) + \frac{(0.4)(-0.6)(-1.6)(-2.24)}{24}(-0.01)$$ 2

$y = 27.84$ // | 7M

solution and scheme.

7c. $n = 6 \quad \therefore h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
$y = 1/1+x^2$	1	0.5	0.2	0.1	0.05884	0.0385	0.027

2

Simpson's $1/3^{rd}$ rule when $n=6$

$$I = \int_a^b y\, dx = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$ 2

$$\therefore I = \int_0^6 \frac{dx}{1+x^2} = \frac{1}{3}\left[(1+0.027)+4(0.5+0.1+0.0385)\right.$$

$$\left.+2(0.2+0.05884)\right]$$ 2

$I = 1.3662$

6M

8a $f(x) = x^3+5x-11$

$f'(x) = 3x^2+5 \quad ; \quad x_0 = 1$

I^{st} Iteration:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x)} = 1 - \frac{f(1)}{f'(1)} = 1.625$$ 2

II^{nd} Iteration:

$$x_2 = 1.625 - \frac{f(1.625)}{f'(1.625)} = 1.5154$$ 2

III^{rd} Iteration: 2

$$x_3 = 1.5154 - \frac{f(1.5154)}{f'(1.5154)} = 1.5106$$ 1

$\therefore$ Thus the required root is 1.5106 //

7M

Solution and Scheme

8b] The divided difference table is as follows.

x	$f(x)$	1^{st} D.D	2^{nd} D.D	3^{rd} D.D
0	-4			
		3		
2	2		3	
		12		1
3	14		9	
		48		
6	158			

3

Using Newton's divided difference formula

$f(4) = -4 + (4-0)(3) + (4-0)(4-2)(3) + (4-0)(4-2)$ 2

$(4-3)(1)$

$= -4 + 12 + 24 + 8 = 40$ 2

$f(4) = 40$

7M

8c. $n = 6$ $\quad \therefore h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

x	0	$1/6$	$2/6$	$3/6$	$4/6$	$5/6$	1
$y = 1/1+x$	1	$6/7$	$3/4$	$2/3$	$3/5$	$6/11$	$1/2$

2

Simpson's $3/8^{th}$ rule for $n=6$ is

$I = \int_a^b y\, dx = \frac{3}{8} h \left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) \right] + 2y_3]$ 2

$\therefore I = \int_0^1 \frac{1}{1+x}\, dx = \frac{1}{16}\left[(1+1/2) + 3(6/7 + 3/4 + 3/5 + 6/11) + 2 \times 2/3 \right]$ 2

$I = 0.6932$

6M

Solution and Scheme

Module 5:-

9a) Taylor's series expansion is,

$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \cdots$ 2

By data,

$x_0 = 0,\ y_0 = 1,\ y' = x - y^2$

$\therefore y(x) = y(0) + x y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \cdots \quad (1)$

Consider,

$y' = x - y^2 \implies y'(0) = 0 - 1^2 = -1$ 2

$y'' = 1 - 2yy' \implies y''(0) = 1 - 2(1)(-1) = 3$

$y''' = 0 - 2[yy'' + (y')^2] \implies y'''(0) = -2[(1)(3) + (-1)^2] = -8$

$y^{iv} = -2[yy''' + 3y'y''] \implies y^{iv}(0) = -2[(1)(-8) + 3(-1)(3)]$ 2

$y^{iv}(0) = 34$

Substituting in eq (1) by taking $x = 0.1$

$y(0.1) = 1 + (0.1)(-1) + \frac{(0.1)^2}{2}(3) + \frac{(0.1)^3}{6}(-8) + \frac{(0.1)^4}{24} \times 34$ 6M

$y(0.1) = 0.9138$ //

9b) $f(x,y) = \frac{y-x}{y+x}$, $x_0 = 0$, $y_0 = 1$, $h = 0.2$

$\therefore k_1 = hf(x_0, y_0) = (0.2)f(0,1) = 0.2\left[\frac{1-0}{1+0}\right] = 0.2$ 2

$k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right] = (0.2)f[0.1, 1.1] = 0.1667$

$k_3 = hf\left[x_0 + h/2, y_0 + \frac{k_2}{2}\right] = (0.2)f[0.1, 1.0835] = 0.1662$ 2

$k_4 = hf[x_0 + h, y_0 + k_3] = (0.2)f[0.2, 1.1662] = 0.1414$

$y(x_0 + h) = y_0 + 1/6[k_1 + 2k_2 + 2k_3 + k_4]$ 2

$y(0.2) = 1 + 1/6[0.2 + 2(0.1667) + 2(0.1662) + 0.1414]$ 1

$\therefore y(0.2) = 1.1679$ //

7M

Solution and scheme

9c)

x	y	
$0 = x_0$	$0 = y_0$	$y_0' = 0$
$0.2 = x_1$	$0.02 = y_1$	$y_1' = 0.1996$
$0.4 = x_2$	$0.0795 = y_2$	$y_2' = 0.3937$
$0.6 = x_3$	$0.1762 = y_3$	$y_3' = 0.5689$
$0.8 = x_4$	$y_4 = ?$	

2

$y_4^{(p)} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$

$= 0 + \frac{4(0.2)}{3}\left[2(0.1996) - 0.3937 + 2(0.5689)\right]$ 2

$= 0.3049$

$y_4' = x_4 - y_4^2 = 0.707$

$y_4^c = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$ 2

$= 0.0795 + \frac{0.2}{3}\left[0.3937 + 4(0.5689) + 0.707\right]$

$= 0.3046$

$y_4' = x_4 - y_4^2 = 0.7072$ 1

substituting this value in the corrector formula

$y_4^{(c)} = 0.3046$ 7M

$\therefore y_4 = y(0.8) = 0.3046$ //

10a) By data

$\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x}$ or $\frac{dy}{dx} = \frac{1-xy}{x^2}$ 2

$f(x,y) = \frac{1-xy}{x^2}$; $x_0 = 1$, $y_0 = 1$, $h = 0.1$

$f(x_0, y_0) = 0$, $x_1 = x_0 + h = 1.1$

$y(x_1) = y_1 = y(1.1) = ?$

From Euler's formula 2

$y_1^{(0)} = y_0 + h f(x_0, y_0)$

$y_1^{(0)} = 1$

Solution and scheme

By modified Euler's formula.

$y_1^{(1)} = y_0 + h/2 \left[f(x_0, y_0) + f(x_1, y_1^{(0)}) \right]$

$= 1 + 0.05 \left[\frac{1-(1.1)(1)}{(1.1)^2} \right] = 0.9959$ 2

$y_1^{(2)} = 1 + 0.05 \left[\frac{1-(1.1)(0.9959)}{(1.1)^2} \right] = 0.99605$

$y_1^{(3)} = 1 + 0.05 \left[\frac{1-(1.1)(0.99605)}{(1.1)^2} \right] = 0.99605$ 1

$\therefore y(1.1) = 0.99605$

7M

10b) $x_0 = 0,\ y_0 = 1,\ h = 0.2,\ f(x_0, y_0) = 1$

$k_1 = hf(x_0, y_0) = 0.2 \times 1 = 0.2000$

$k_2 = hf\left[x_0 + h/2,\ y_0 + \frac{k_1}{2}\right] = 0.2 \times f[0.1, 1.1] = 0.2400$ 2

$k_3 = hf\left[x_0 + h/2,\ y_0 + \frac{k_2}{2}\right] = 0.2 \times f[0.1, 1.12] = 0.2440$

$k_4 = hf[x_0 + h,\ y_0 + k_3] = 0.2 \times f[0.2, 1.244] = 0.2888$ 2

$y(x_0 + h) = y_0 + 1/6\,[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1 + 1/6\,[0.2 + 2(0.2400) + 2(0.2440) + 0.2888]$ 2

$y(x_0 + h) = 1.2428$ // 1

7M

10c) mathematical program to find $y(1.4)$

$x_0 = 1$

$y_0 = 2$

$y_1 = 2.2156$

$y_2 = 2.4649$ 2

$y_3 = 2.7516$

$h = 0.1$

$x_1 = x_0 + h$

$x_2 = x_1 + h$

$x_3 = x_2 + h$

Solution and scheme.

```
x4 = x3 + h
dif f(x,y):
    return x**2 + (y/2)
y10 = f(x0, y0)
y11 = f(x1, y1)
y12 = f(x2, y2)
y13 = f(x3, y3)
y4p = y0 + (4*h/3) * (2*y11 - y12 + 2*y13)
Print ('predicted value of y4 is % 3.3f' %
                                          y4p)
y14 = f (x4, y4p);
for i in range(1, 4):
    y4 = y2 + (h/3) * (y14 + 4*y13 + y12);
Print ('corrected value of y4 after \t
iteration % d is \t % 3.5 f \t, % (i, y4))
y14 = f (x4, y4);
```

2

2

GM

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