Modified

CBCS SCHEME

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BMATE201

Second Semester B.E./B.Tech. Degree Examination, June/July 2024
Mathematics – II for EEE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Find the directional derivatives of $\phi = x^2yz + 4xz^2$, at (1, 2, -1) along $2i - j - 2k$.	7	L3	CO1
	b.	Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.	7	L3	CO1
	c.	Show that the vector, $\vec{F} = \dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational.	6	L2	CO1
		OR			
Q.2	a.	Find the work done in moving a particle in the Force field $F = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line from (0, 0, 0) to (2, 1, 3).	7	L3	CO1
	b.	Using Green's theorem, evaluate $\oint (xy + y^2)dx + x^2dy$ over the region bounded by the curves $y = x$ and $y = x^2$.	7	L3	CO1
	c.	Using modern mathematical tools, write a code to find the divergence and curl of the vector $x^2yz\,\hat{i} + y^2zx\,\hat{j} + z^2xy\hat{k}$.	6	L2	CO5
		Module – 2			
Q.3	a.	Define a subspace. Show that the intersection of two subspaces of a vector space V is also a subspace of V.	7	L2	CO2
	b.	Define a basis for a vector space. Determine whether or not the vectors : (2, 2, 1), (1, 3, 7), (1, 2, 2) form a basis of R^3.	7	L2	CO2
	c.	Show that $T : R^2 \to R^3$ given by $T(x, y) = (x + y, x - y, y)$ is a linear transformation.	6	L2	CO2
		OR			
Q.4	a.	Define linearly independent set of vectors and linearly dependent set of vectors. Show that the vectors (1, 4, 9), (3, 1, 4), (9, 3, 12) are linearly dependent.	7	L2	CO2
	b.	Verify the Rank-Nullity theorem for $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.	7	L2	CO2
	c.	Using the modern mathematical tool, write the code to represent the reflection transformation $T : R^2 \to R^2$ and to find the image of vector (10, 0) when it is reflected about the y – axis.	6	L2	CO5
		Module – 3			
Q.5	a.	Find the Laplace Transform of, (i) $e^{-3t}(2\cos 5t - 3\sin 5t)$ (ii) $\dfrac{\cos at - \cos bt}{t}$	7	L2	CO3

	b.	Find the Laplace transform of the triangular wave function, $f(t)=\begin{cases} t, & \text{if } 0\le t\le a \\ 2a-t, & \text{if } a\le t\le 2a\end{cases}$	7	L2	CO3
	c.	Express $f(t)=\begin{cases} t^2, & 1<t\le 2 \\ 4t, & t>2\end{cases}$ interms of Heaviside unit step function and hence find $L(f(t))$.	6	L3	CO3
		OR			
Q.6	a.	Find $L^{-1}\left[\dfrac{2s^2-6s+5}{s^3-6s^2+11s-6}\right]$.	7	L2	CO3
	b.	Find $L^{-1}\left[\dfrac{1}{s(s^2+a^2)}\right]$ using convolution theorem.	7	L2	CO3
	c.	Solve the differential equation by using Laplace Transform method. $y''+6y'+9y=12t^2e^{-3t}$, $y(0)=y'(0)=0$	6	L3	CO3
		Module – 4			
Q.7	a.	By Newton-Raphson method, find the root of $x\sin x+\cos x=0$, near $x=\pi$. Carryout the iteration upto four decimal places of accuracy.	7	L2	CO4
	b.	Using Lagrange's interpolation formula, find y at x = 2, using the points (0, -12), (1, 0), (3, 6), (4, 12)	7	L2	CO4
	c.	Using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule, evaluate $\int_0^{0.6} e^{-x^2}$ by taking seven ordinates.	6	L3	CO4
		OR			
Q.8	a.	Find a real root of the equation $x^3-4x-9=0$ correct to three decimal places by the method of False position in (2, 3)	7	L2	CO4
	b.	Construct Newton's forward interpolation polynomial for the data : x: 0, 1, 2, 3 f(x): 1, 2, 1, 10	7	L2	CO4
	c.	Evaluate $\int_0^1 \frac{dx}{(1+x)^2}$ by using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule, by taking 6 equal intervals.	6	L3	CO4
		Module – 5			
Q.9	a.	Use Taylor series method to find y(0.2) from $\frac{dy}{dx}=2y+3e^x$, with $y(0)=0$.	7	L3	CO5
	b.	Using R-K method of order 4, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$ at $x=0.2$.	7	L3	CO5
	c.	Applying Milne's Predictor-Corrector method, find y(0.4), from $\frac{dy}{dx}=2e^x-y$, given that, $y(0)=2$, $y(0.1)=2.010$, $y(0.2)=2.040$, $y(0.3)=2.090$	6	L3	CO5
		OR			

Q.10	**a.**	Solve by using modified Euler's method, $y' = 1 + \frac{y}{x}$, $y(1) = 2$ at $x = 1.2$ and $x = 1.4$.	7	**L3**	**CO5**
	b.	Using the Runge-Kutta method of fourth order find $y(1.1)$, given $\frac{dy}{dx} = xy^{\frac{1}{3}}$, taking $h = 0.1$, $y(1) = 1$.	7	**L3**	**CO5**
	c.	Using modern mathematical tools, write a code to find $y(1.4)$, given $\frac{dy}{dx} = x^2 + \frac{y}{2}$, $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$, by Milne's Predictor and Corrector method.	6	**L3**	**CO5**

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AY: 2024-25

Department:Electrical &Electronics Engineering
Subject with Sub. Code:Mathematics II for EE Stream(BMATE101)
Semester/Division/Branch:II/EE
Name of Faculty: Prof. Akshata B Patil.

Q.No.	Solution and Scheme	Marks
	Module 1:-	
Q.1 a]	$D.D = \nabla\phi\cdot\hat{n} = \nabla\phi\cdot\frac{\vec{a}}{\lvert\vec{a}\rvert}$	1
	$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$	2
	$= (2xyz + 4z^2)\hat{i} + x^2z\hat{j} + (x^2y + 8xz)\hat{k}$	1
	$(\nabla\phi)_{(1,2,-1)} = 0\hat{i} - \hat{j} - 6\hat{k}$	
	$\frac{\vec{a}}{\lvert\vec{a}\rvert} = \frac{2i - j - 2k}{\sqrt{4+1+4}} = \frac{2i-j-2k}{\sqrt{9}} = \frac{2i-j-2k}{3}$	2
	$D.D = \nabla\phi\cdot\hat{n} = \frac{1}{3}(1+12) = \frac{13}{3}$	1
		7M
1b.	$\vec{F} = \nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$	1
	$\vec{F} = (3x^2 - 3yz)\hat{i} + (3y^2 - 3zx)\hat{j} + (3z^2 - 3xy)\hat{k}$	2
	$\operatorname{div}\vec{F} = \frac{\partial}{\partial x}(3x^2 - 3yz) + \frac{\partial}{\partial y}(3y^2 - 3zx) + \frac{\partial}{\partial z}(3z^2 - 3xy)$	2
	$= 6(x+y+z)$	
	$\operatorname{curl}\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3(x^2-yz) & 3(y^2-zx) & 3(z^2-xy) \end{vmatrix}$	2
	$= 0\hat{i} + 0\hat{j} + 0\hat{k}$	
		7M

Q.No.	Solution and Scheme	Marks
1c	$\vec{F} = \dfrac{x\hat{i} + y\hat{j}}{x^2+y^2}$	
	Solenoidal if $\nabla \cdot \vec{F} = 0$	
	$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right)$	
	$= \dfrac{(x^2+y^2)(1) - x(2x)}{(x^2+y^2)^2} + \dfrac{(x^2+y^2)(1) - y(2y)}{(x^2+y^2)^2}$	3
	$= \dfrac{-x^2+y^2+x^2-y^2}{(x^2+y^2)^2} = 0$	
	Irrotational if $\nabla \times \vec{F} = 0$	
	$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} & 0 \end{vmatrix}$	3
	$= \cancel{\frac{\partial}{\partial z}\left[0 - \frac{y}{x^2+y^2}\right] - \frac{\partial}{\partial y}\left[\frac{x}{x^2+y^2}\right]}$	
	$= \frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}\left(\frac{y}{x^2+y^2}\right) - \left[\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}\left(\frac{x}{x^2+y^2}\right)\right]$	
	$+ \left[\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right) - \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)\right]$	
	$= 0 - 0 - 0 + 0$	
	$\nabla \times \vec{F} = 0$	6M

Q.No.	Solution and Scheme	Marks
2a.	Let, $F = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ at $(0,0,0)$ to $(2,1,3)$	2
	$\int_C \vec{F}.\,dR = \int_C 3x^2 dx + (2xz - y)\,dy + z\,dz$ The equation of the st. line from $(0,0,0)$ to $(2,1,3)$ are $\frac{x}{2} = \frac{y}{1} = \frac{z}{3} = t$	2
	$\therefore x = 2t$, $y = t$, $z = 3t$ and $t=0$ to $t=1$ Work done, $\int_C F.\,dR = \int_0^1 (36t^2 + 8t)\,dt = 16.$	3
		7M
2b.	By the Green's theorem $\oint (xy + y^2)\,dx + x^2\,dy$ bounded by the curves $y = x$ and $y = x^2$ here, $M = xy + y^2$ and $N = x^2$,	3
	Then, $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 2x - x - 2y = x - 2y$ $\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy\,dx$	2
	$= \int_0^1 \int_{x^2}^{x} (x - 2y)\,dy\,dx$ $= \int_0^1 (xy - y^2)\,dx$	2
	$\int_C M\,dx + N\,dy = \int_0^1 (x^4 - x^3)\,dx = -\frac{1}{20}$ //	
		7M

Q.No.	Solution and Scheme	Marks

2 C.

Divergence

```
from Sympy.vector export*
from Sympy import Symbol
N = CoordSys3D('N')
x,y,z = Symbols ('xyz')
A = N.x**2*N.y*N.z*N.i+
    N.y**2*N.z*N.x*N.j+
    N.z**2*N.x*N.y*N.k
```

2

```
delop = Del()
div A = delop.dot(A)
display (div A)
print(f"\n Divergence of
      {A} is \n")
display (divergence A)
```

1

Curl

```
from Sympy.vector export*
from Sympy import Symbol
N = CoordSys3D('N')
x,y,z = Symbols ('xyz')
A = N.x**2*N.y*N.z*N.i
    N.y**2*N.z*N.x*N.j
    N.z**2*N.x*N.y*N.k.
```

2

```
delop = Del( )
curl A = delop.Cross (A)
display (curl A)
print(f "\n curl of {A} is \n")
display (curl (A))
```

1

6M

Q.No.	Solution and Scheme	Marks
	Module 2:	
3a.	Definition:	
	Let U & W be subspaces of vector space	2
	V. $0 \in U$ & $0 \in W$, hence $0 \in U \cap W$	
	suppose u & v belong to intersection	1
	$U \cap W$ then $u, v \in U$ & $u, v \in W$, Further U & W	
	are suppose subspace for any scalars	2
	$a, b \in k$,	
	$au + bv \in U$ & $au + bv \in W$	
	Thus,	2
	$au + bv \in U \cap W$ $\therefore$ $U \cap W$ is also a subspace.	
		7M
3b.	Definition:	
	3 vectors in R^3 forms a basis iff they are linearly independent.	
	$x(2,2,1) + y(1,3,7) + z(1,2,2) = (0,0,0)$	2
	$2x+y+z=0$; $2x+3y+2z=0$; $x+7y+2z=0$	
	Let, $AX = 0$ hence,	
	$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 7 & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \sim \begin{bmatrix} 1 & 7 & 2 \\ 2 & 3 & 2 \\ 2 & 1 & 1 \end{bmatrix}$	2
	$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 2R_1$	
	$\begin{bmatrix} 1 & 7 & 2 \\ 0 & -11 & -2 \\ 0 & -13 & -3 \end{bmatrix}$	2
	$R_2 \to -R_2$	
	$R_3 \to -R_3$	
	$\begin{bmatrix} 1 & 7 & 2 \\ 0 & 11 & 2 \\ 0 & 13 & 3 \end{bmatrix}$	

Q.No.	Solution and Scheme	Marks

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 7 & 2 \\ 0 & 11 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 7 & 2 \\ 0 & 2 & 1 \\ 0 & 11 & 2 \end{bmatrix}$$

$R_3 \to 11R_2 - 2R_3$

$$\begin{bmatrix} 1 & 7 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 7 \end{bmatrix}$$

Rank $[A] = 3$ = No of unknowns — 1

hence,

$\therefore$ unique solution and $x = 0$, $y = 0$, $z = 0$

Hence three vectors are linearly independent — 7M

3c Let, — 1

$u = (x_1, y_1)$, $v = (x_2, y_2) \in R^2$

$\therefore T(u+v) = T(x_1+x_2, y_1+y_2)$

$= (x_1+x_2+y_1+y_2, x_1+x_2-y_1-y_2, y_1+y_2)$ — 2

$= (x_1+y_1, x_1-y_1, y_1) + (x_2+y_2, x_2-y_2, y_2)$

$= T(u) + T(v)$

Also for $a \in R$ & $u \in R^2$

$T(au) = T(ax_1, ay_1)$

$= (ax_1 + ay_1, ax_1 - ay_1, ay_1)$ — 2

$= a(x_1+y_1, x_1-y_1, y_1)$

$= aT(u)$

$\therefore T$ is a linear transformation — 1

6M

Q.No.	Solution and Scheme	Marks
4a	Definition: $x(1,4,9)+y(3,1,4)+z(9,3,12) = (0,0,0)$ $x+3y+9z=0$; $4x+y+3z=0$; $9x+4y+12z=0$	1
	Consider, $AX=0$ $A=\begin{bmatrix}1&3&9\\4&1&3\\9&4&1\end{bmatrix}$	2
	$R_2 \to R_2-4R_1$; $R_3 \to R_3-9R_1$ $A=\begin{bmatrix}1&3&9\\0&-11&-33\\0&-23&-69\end{bmatrix}$	2
	$R_2 \to -R_2/11$; $R_3 \to -R_3/11$ $A=\begin{bmatrix}1&3&9\\0&1&3\\0&1&3\end{bmatrix}$ $R_3 \to R_3-R_2$ $A=\begin{bmatrix}1&3&9\\0&1&3\\0&0&0\end{bmatrix}$	2
	Rank $[A]=2<3$ (No of unknowns) So the vectors are linearly dependent.	7M
4b	Let, $v=(x,y,z)\in R^3$ Null T $=\{v\in V \mid T(v)=0\}$ $T(x,y,z)=0$ $(x+2y-z,\ y+z,\ x+y-2z)=(0,0,0)$	2
	$x+2y-z=0$; $y+z=0$; $x+y-2z=0$ $x=-3z$; $y=-z$	

Q.No.	Solution and Scheme	Marks

$\therefore \{(x, y, z)\} = \{(3z, -z, z)\} = z(3, -1, 1)$

Thus $\{(3, -1, 1)\}$ is a basis of nullity of T

& Nullity $T = 1$

As $\{(1,0,0), (0,1,0), (0,0,1)\}$ generates R^3 2

$\Rightarrow \{T(1,0,0), T(0,1,0), T(0,0,1)\}$ generates range of T.

$\Rightarrow \{(1,0,1), (2,1,1), (-1,1,-2)\}$ generates of T.

To find the basis of range

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 + R_2$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$ 2

$R_3 \Rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $\{(1,0,1), (0,1,-1)\}$ form a basis of Range of T. $\dim[\text{Range}(T)] = 2$

$\therefore$ Rank (T) + Nullity $(T) = 2 + 1 = 3 = \dim(R^3)$ 1

Hence Rank Nullity theorem verified.

7M

Q.No.	Solution and Scheme	Marks
4c	import numpy as np import matplotlib.pyplot as plt V = np.array([[1,0,0]]) origin = np.array([[0,0,0],[0,0,0]]) A = np.matrix([[-1,0],[0,1]]) V1 = np.matrix(V) V2 = A * np.transpose(V1) V2 = np.array(V2) plt.quiver(*origin, V[:,0], V[:,1], color['b'], scale= 50) plt.quiver(*origin, V2[0,:], V2[1,:], color=['r'] scale = 50) plt.show	2 2 2 6M
	Module 3:	
5a	i] $e^{-3t}(2\cos 5t - 3\sin 5t)$ Taking Laplace on both sides $L[e^{-at} f(t)] = [L(2\cos 5t - 3\sin 5t)]_{s \Rightarrow s+3}$ $= \frac{2(s+3)}{(s+3)^2+25} - 3 \frac{\times 5}{(s+3)^2+25}$ $= \frac{2s-9}{s^2+6s+34}$ Thus, $L[e^{-at} f(t)] = \frac{2s-9}{s^2+6s+34}$	2 2

Q.No.	Solution and Scheme	Marks
	ii) $\frac{\cos at - \cos bt}{t}$	
	Let,	
	$L\left[\frac{\cos at - \cos bt}{t}\right] = \int_s^\infty L(\cos at - \cos bt)\, ds$	2
	$= \int_s^\infty \left[\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right] ds$	
	$= \frac{1}{2} \log \frac{s^2+b^2}{s^2+a^2}$	1
		7M
5b	$f(t) = \begin{cases} t \; . \text{ if } 0 \le t \le a \\ 2a-t \text{ if } a \le t \le 2a \end{cases}$	
	Let,	
	$L[f(t)] = \frac{1}{1-e^{-2as}} \int_0^{2a} e^{-st} f(t)\, dt$	1
	$= \frac{1}{1-e^{-2as}} \left[\int_0^a e^{-st} . t\, dt + \int_a^{2a} e^{-st}(2a-t)\, dt\right]$	2
	$= \frac{1-2e^{-as}+e^{-2as}}{s^2(1-e^{-2as})} = \frac{(1-e^{-as})^2}{s^2(1+e^{-as})(1-e^{-as})}$	2
	$= \frac{1-e^{-as}}{s^2(1+e^{-as})} = \frac{1}{s^2} \tanh \frac{as}{2}$	2
		7M

Q.No.	Solution and Scheme	Marks
5c.	$f(t) = \begin{cases} t^2, & 1 < t \le 2 \\ 4t, & t > 2 \end{cases}$	1
	$f(t) = t^2[u(t-1) - u(t-2) + 4t\,u(t-2)]$	
	$f(t) = t^2 u(t-1) + (4t - t^2) u(t-2)$	2
	$L[f(t)] = L[t^2 u(t-1) + L[(4t - t^2) u(t-2)]$	
	$= e^{-s} L(t+1)^2 + e^{-2s} L[4(t+2) - (t+2)^2]$	2
	$= e^{-s}\left[\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right] + e^{-2s}\left[\frac{2!}{s^3} + \frac{4}{s}\right]$	1
		6M
6a.	$L^{-1}\left[\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right]$	
	$\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-3}$	1
	$2s^2 - 6s + 5 = A(s-2)(s-3) + B(s-1)(s-3) + (s-2)(s-1)$	
	Put, $s = 2$, $2(2)^2 - 6(2) + 5 = B(+1)(-1)$	2
	$8 - 12 + 5 = -B$	
	$\therefore \boxed{B = -1}$	
	Put, $s = 3$, $2(3)^2 - 6(3) + 5 = C(1)(2)$	1
	$18 - 18 + 5 = 2C$	
	$\therefore \boxed{C = 5/2}$	
	Put, $s = 1$, $2(1)^2 - 6 + 5 = A(-1)(-2) + B(0) + C(0)$	
	$1 = 2A$	1
	$\therefore \boxed{A = 1/2}$	
	$\therefore L^{-1}\left[\frac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}\right] = L^{-1}\left[\frac{1/2}{s-1}\right] + L^{-1}\left[\frac{-1}{s-2}\right] + L^{-1}\left[\frac{5/2}{s-3}\right]$	1
	$= \frac{1}{2} e^{t} - e^{2t} + \frac{5}{2} e^{3t}$	1
		7M

Q.No.	Solution and Scheme	Marks
6b.	$L^{-1}\left[\dfrac{1}{s(s^2+a^2)}\right]$	
	$F(s) = 1/s \quad ; \quad G(s) = \dfrac{1}{s^2+a^2}$	2
	$f(t) = 1 \quad ; \quad g(t) = \dfrac{\sin at}{a}$	1
	$L^{-1}[F(s)\cdot G(s)] = f(t) * g(t) = \int_{u=0}^{t} f(t-u)\, g(u)\, du$	1
	$= \int_{u=0}^{t} 1 \cdot \dfrac{\sin au}{a}\, du$	2
	$= \dfrac{1}{a}\left[\dfrac{-\cos au}{a}\right]_{u=0}^{t}$	1
	$= \dfrac{-1}{a^2}(1-\cos at)$	7M
6c.	$y'' + 6y' + 9y = 12t^2 e^{-3t}, \quad y(0) = y'(0) = 0$	
	Taking Laplace on both sides	1
	$L[y''(t) + 6y'(t) + 9y(t)] = L[12t^2 e^{-3t}]$	
	$s^2 L[y(t)] - s\,y(0) - y'(0) + 6[s\,L(y(t)) - y(0)] + 9L[y(t)] = 12\, L[t^2 \cdot e^{-3t}]$	2
	$(s^2+6s+9) L[y(t)] = 12\, L[t^2 \cdot e^{-3t}]$	
	$(s+3)^2 L[y(t)] = 12 \cdot \dfrac{2}{(s+3)^3}$	2
	$L[y(t)] = \dfrac{24}{(s+3)^5}$	1
	$y(t) = L^{-1}\left[\dfrac{24}{(s+3)^5}\right] = e^{-3t} \cdot t^4$	6M

Q.No.	Solution and Scheme	Marks
	Module - 4 :	
7a	Let,	
	$f(x) = x\sin x + \cos x$	
	$\therefore f'(x) = x\cos x + \sin x - \sin x = x\cos x$	
	Also,	
	$x_0 = \pi$	
	$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = \pi - \dfrac{f(\pi)}{f'(\pi)} = \pi - \dfrac{(\pi\sin\pi + \cos\pi)}{\pi\cos\pi}$	2
	$\therefore x_1 = \pi - 1/\pi = 2.8233$	
	Now,	
	$x_2 = 2.8233 - \left[\dfrac{2.8233\sin(2.8233) + \cos(2.8233)}{2.8233\cos(2.8233)}\right]$	2
	$\therefore x_2 = 2.7986$	
	Now,	
	$x_3 = 2.7986 - \left[\dfrac{2.7986\sin(2.7986) + \cos(2.7986)}{2.7986\cos(2.7986)}\right]$	2
	$\therefore x_3 = 2.7984$.	
	And,	
	$x_4 = 2.7984 - \left[\dfrac{2.7984\sin(2.7984) + \cos(2.7984)}{2.7984\cos(2.7984)}\right]$	1
	$x_4 = 2.7984$	
	Thus the required real root is 2.7984.	7M

Q.No.	Solution and Scheme	Marks
7b	Let,	

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 4$

$y_0 = -12 \quad y_1 = 0 \quad y_2 = 6 \quad y_3 = 12$

we have Lagrange's interpolation formula.

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

2.

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

Now,

$$y = f(x) = \frac{(x-1)(x-3)(x-4)}{(-1)(-3)(-4)}(-12) + 0 + \frac{x(x-1)(x-4)6}{(3)(2)(-1)}$$

2

$$+ \frac{x(x-1)(x-3)12}{(4)(3)(1)}$$

$$f(x) = (x-1)(x-3)(x-4) - x(x-1)(x-4) + x(x-1)(x-3)$$

$$= (x-1)[(x^2-7x+12) - (x^2-4x) + (x^2-3x)]$$

2

$$= (x-1)[x^2-6x+12] = x^3-7x^2+18x-12$$

Thus the required polynomial is

$f(x) = x^3 - 7x^2 + 18x - 12$

1

Now,

$f(2) = 2^3 - 7(2)^2 + 18(2) - 12 = 4$

Thus,

$f(2) = 4$

7M

Q.No.	Solution and Scheme	Marks
7c	Length of each subinterval $(h) = \frac{0.6-0}{6} = 0.1$ and $n = 6$.	
	The values of x and $y = e^{-x^2}$ correct to four decimal places are tabulated.	1

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$y = e^{-x^2}$	1	0.99	0.9608	0.9139	0.8521	0.7788	0.6977
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

2

Simpson's $1/3^{rd}$ rule for $n=6$ is given by

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$ 1

$$\therefore \int_0^{0.6} e^{-x^2}.dx = \frac{0.1}{3}\left[(1+0.6977+4(0.99+0.9139+0.7788) + 2(0.9608+0.8521)\right] = 0.5351$$ 2

Thus,

$$\int_0^{0.6} e^{-x^2}.dx = 0.5351$$ ~~0~~

~~6~~ 6M

Q.No.	Solution and Scheme	Marks
8a	$f(x) = x^3 - 4x - 9 = 0$	
	The roots lies in the interval $(2,3)$	1
	$f(2) = -9 < 0$, $f(3) = 6 > 0$	

$\underline{\underline{I^{st}}}$ $\underline{\underline{Iteration}}$:

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$ 2

$$= \frac{2(6) - 3(-9)}{6 - (-9)} = 2.6$$

$f(x_1) = -1.824$

$\therefore$ Roots are lies in $(2.6, 3)$

Q.No.	Solution and Scheme	Marks
	II nd Iteration:	
	$x_2 = \frac{(2.6)(6) - 3(-1.824)}{6 - (-1.824)}$	
	$x_2 = 2.6933$	2
	$f(2.6933) = -0.236 < 0$	
	$\therefore$ Roots lies in $(2.6933, 3)$	
	III rd Iteration:	
	$x_3 = \frac{(2.6933)(6) - 3(-0.236)}{6 - (-0.236)}$	2
	$x_3 = 2.7049$	
	$f(2.7049) = -0.029 < 0$	
	$\therefore$ Roots lies in $(2.7049, 3)$	
	IV th Iteration:	
	$x_4 = \frac{(2.7049)(6) - 3(-0.029)}{6 - (-0.029)}$	
	$x_4 = 2.7063$	7M

8b.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
		1		
			-2	
1	2			12
		-1		
2	1		10	
		9		
3	10			

2.

$$k = \frac{x_n - x_0}{h} = \frac{x - 0}{1} = x$$

Q.No.	Solution and Scheme	Marks

Newton's forward interpolation formula.

$$f(x) = f(0) + \frac{x}{1}\Delta f(0) + \frac{x(x-1)}{2}\Delta^2 f(0) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(0)$$ 2

$$f(x) = 1 + x + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{6}(12)$$ 2

$$f(x) = 2x^3 - 7x^2 + 6x + 1$$ 1

7M

8c. Take $n = 1$

Length of each strip $(h) = \frac{1-0}{6} = 1/6$. 1

x	0	1/6	2/6	3/6	4/6	5/6	1
$y = \frac{1}{(1+x)^2}$	1	$\frac{36}{49}$	$\frac{9}{16}$	$\frac{4}{9}$	$\frac{9}{25}$	$\frac{36}{121}$	$\frac{1}{4}$
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

2

$$I = \frac{3h}{8}\left[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)\right]$$ 2

$$= \frac{3}{8} \times 1/6 \left[(1 + 1/4) + 3\left(\frac{36}{49} + \frac{9}{16} + \frac{9}{25} + \frac{36}{121}\right) + 2\left(\frac{4}{9}\right)\right]$$ 1

$$I = 0.50018$$

6M

Module 5:

9a] $y' = 2y + 3e^x$ and $y(0) = 0$. That is $x_0 = 0$, $y_0 = 0$ 1

Taylor series expansion is

$$y(x) = y(x_0) + (x - x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \cdots \quad (1)$$ 1

Consider,

$y' = 2y + 3e^x$; $y'(0) = 2y(0) + 3e^0 = 3$

$y'' = 2y' + 3e^x$; $y''(0) = 9$ 3

$y''' = 2y'' + 3e^x$; $y'''(0) = 21$

$y^{iv} = 2y''' + 3e^x$; $y^{iv}(0) = 45$

Q.No.	Solution and Scheme	Marks
	$y(x) = y(0) + x y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \frac{x^4}{4!} y^{iv}(0)$	
	$= 0 + 3x + \frac{9}{2}x^2 + \frac{21}{6}x^2 + \frac{45}{24}x^4$	2
	$y(0.2) = 0.8110.$	7M
9b	we have, $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$; $x_0=0$, $y_0=1$, $h=0.2$	
	$f(x,y) = \frac{y^2-x^2}{y^2+x^2}$ we shall compute k_1, k_2, k_3, k_4	1
	$k_1 = hf(x_0, y_0) = 0.2 f(0,1) = (0.2)1 = 0.2$	
	$k_2 = hf\left[x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right] = 0.2 f(0.1, 1.1) = 0.1967$	3
	$k_3 = hf\left[x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right] = 0.2 f(0.1, 1.0984) = 0.1967$	
	$k_4 = hf[x_0+h, y_0+k_3] = 0.2 f(0.2, 1.1967) = 0.1891$	
	we have,	2
	$y(x_0+h) = y_0 + 1/6[k_1+2k_2+2k_3+k_4]$	
	$y(0.2) = 1 + 1/6[0.2 + 2(0.1967) + 2(0.1967) + 0.1891]$	
	$y(0.2) = 1.19598 \simeq 1.196.$	1 7M
9c	(table below)	2
	we have milne's predictor formula.	
	$y_4^{(p)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$	2
	$y_4^{(p)} = 2 + \frac{4(0.1)}{3}[2(0.2003) - 0.4028 + 2(0.6097)] = 2.1623$	
	Now,	
	$y_4' = 2e^{0.4} - 2.1623 = 0.8213$	

x	y	$y' = 2e^x - y$
$x_0 = 0$	$y_0 = 2$	$y_0' = 2e^0 - 2 = 0$
$x_1 = 0.1$	$y_1 = 2.010$	$y_1' = 2e^{0.1} - 2.01 = 0.2003$
$x_2 = 0.2$	$y_2 = 2.040$	$y_2' = 2e^{0.2} - 2.04 = 0.4028$
$x_3 = 0.3$	$y_3 = 2.090$	$y_3' = 2e^{0.3} - 2.09 = 0.6097$
$x_4 = 0.4$	$y_4 = 2$	

Q.No.	Solution and Scheme	Marks
	we have milne's corrector formula.	
	$y_4^{(C)} = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4')$	
	$\therefore y_4^{(C)} = 2.04 + \frac{0.1}{3}[0.4028 + 4(0.6097) + 0.8213]$	2
	$= 2.1621$	
	Now,	
	$y_4' = 2e^{0.4} - 2.1621 = 0.8215$	
	Applying again	
	$y_4^{(C)} = 2.04 + \frac{0.1}{3}[0.4028 + 4(0.6097) + 0.8215]$	1
	$= 2.1621$	
	Thus,	
	$y(0.4) = 2.1621$	6M
10(a)	I Stage:	
	$x_0 = 1$, $y_0 = 2$, $h = 0.2$	
	$f(x,y) = 1 + y/x$	
	Eulers formula	
	$y_1^{(0)} = y_0 + hf(x_0, y_0)$	1
	$= 2 + 0.2f(01, 2)$	
	$= 2 + 0.2(3) = 2.6$	
	modified Euler's formula.	
	$y_1^{(1)} = y_0 + h/2[f(x_0, y_0) + f(x_1, y_1^{(0)})]$	2
	$= 2 + (0.1)[3 + (1 + y_1^{(0)}/x_1)]$	
	$= 2 + 0.1[4 + 2.6/1.2]$	
	$y_1^{(1)} = 2.6167$	
	$y_1^{(2)} = y_0 + h/2[f(x_0, y_0) + f(x_1, y_1^{(1)})]$	2
	$= 2 + (0.1)[4 + 2.6167/1.2]$	
	$= 2.6181$	
	$y_1^{(3)} = y_0 + h/2[f(x_0, y_0) + f(x_1, y_1^{(2)})]$	2
	$= 2 + (0.1)[4 + 2.6181/1.2]$	
	$= 2.6182$	
	$y(1.2) = 2.6182$.	

Q.No.	Solution and Scheme	Marks
	<u>Stage II:</u>	
	$x_0 = 1.2$, $y_0 = 2.6182$	
	$y_1^{(0)} = y_0 + hf(x_0, y_0)$	
	$= 2.6182 + 0.2(3.1818)$	
	$= 3.2546$	
	$y_1^{(1)} = y_0 + h/2[f(x_0,y_0) + f(x_1, y_1^{(0)})]$	
	$= 2.6182 + 0.1[4.1818 + 3.2546/1.4]$	
	$= 3.2689$	
	$y_1^{(2)} = y_0 + h/2[f(x_0,y_0) + f(x_1, y_1^{(1)})]$	
	$= 2.6182 + 0.1[4.1818 + 3.2689/1.4]$	
	$= 3.2699$	
	$y_1^{(3)} = y_0 + h/2[f(x_0,y_0) + f(x_1, y_1^{(2)})]$	
	$= 2.6182 + 0.1[4.1818 + 3.2699/1.4]$	
	$= 3.2699$	
	Thus,	
	$y(1.4) = 3.2699 \simeq 3.27$	7M
10b]	$f(x,y) = xy^{1/3}$, $x_0 = 1$, $y_0 = 1$, $h = 0.1$	
	$k_1 = hf(x_0, y_0) = 0.1 f(1,1) = 0.1$	2
	$k_2 = hf\left(x_0 + h/2,\ y_0 + \frac{k_1}{2}\right)$	
	$= 0.1 f\left[1 + \frac{0.1}{2},\ 1 + \frac{0.1}{2}\right] = 0.1 f[1.05, 1.05]$	2
	$k_2 = 0.1067$	
	$k_3 = hf[x_0 + h/2,\ y_0 + k_2/2]$	
	$= 0.1 f[1.05, 1.05335] = 0.1068$	2
	$k_4 = hf[x_0 + h,\ y_0 + k_3]$	
	$= 0.1 f[1.1, 1.1068] = 0.1138$	
	$y(x_0 + h) = y_0 + 1/6[k_1 + 2(k_2 + k_3) + k_4]$	1
	$= 1 + 1/6[0.1 + 2(0.1067 + 0.1068) + 0.1138]$	
	$y(1.1) = 1.1068$	7M

Q.No.	Solution and Scheme	Marks
10 c)	from sympy import	
	def Milne (g, x0, h, y0, y1, y2, y3):	2.
	x, y = Symbols ('x, y')	
	f = lambdify([x,y], g)	
	x_1 = x0+h	
	$x_2 = x_1 + h$	
	$x_3 = x_2 + h$	
	$x_4 = x_3 + h$	
	y10 = f (x0, y0)	2
	y11 = f (x1, y1)	
	y12 = f (x2, y2)	
	y13 = f (x3, y3)	
	y4p = y0+(4* h/3)* (2* y11 - y12 + 2* y13)	
	Print ('Predicted value of y4', y4p)	
	y14 = f (x4, y4p)	
	for i in range (1, 4):	
	y4 = y2 + (h/3) * (y14 + 4 * y13 + y12)	2
	Print ('Corrected value of y4 etation %.d	
	y14 = f (x4, y4)	
	Milne('x**2+y/2', 1, 0.1, 2, 2.2156, 2.4649, 2.7514)	6M.

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Q.No.	Solution and Scheme	Marks