Third Semester B.E./B.Tech. Degree Examination, Dec.2023/Jan.2024
Mathematics-III for EE Engineering

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. VTU Formula Hand Book is permitted.
3. M : Marks , L: Bloom's level , C: Course outcomes.
4. Mathematics handbook is permitted.

		Module – 1	M	L	C
Q.1	a.	Solve : $(D^4+8D^2+16)y=0$.	6	L1	CO1
	b.	Solve : $(D^2-3D+2)y=2\sinh x$	7	L2	CO1
	c.	Solve : $x^2y''-3xy'+5y=3\sin(\log x)$	7	L3	CO1
		OR			
Q.2	a.	Solve : $(D^4-4D^3-5D^2-36D-36)y=0$.	6	L1	CO1
	b.	Solve : $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=\sin 2x$.	7	L2	CO1
	c.	Solve : $(2x+1)^2\frac{d^2y}{dx^2}-2(2x+1)\frac{dy}{dx}-12y=3(2x+1)$.	7	L3	CO1
		Module – 2			
Q.3	a.	Find the curve at best fit of the form $y=ax^b$ to the following data : x: 1, 2, 3, 4, 5 y: 0.5, 2, 4.5, 8, 12.5	6	L2	CO2
	b.	Calculate the coefficient of correlation and obtain the lines of regression for the following data : x: 1, 2, 3, 4, 5, 6, 7, 8, 9 y: 9, 8, 10, 12, 11, 13, 14, 16, 15	7	L3	CO2
	c.	In a partially destroyed laboratory record of correlation data, following results only available : Variance of x is 9 and regression lines, $4x-5y+33=0$; $20x-9y=107$. Find (i) Mean value of x and y (ii) SD of y. (iii) Coefficient of correlation between x and y	7	L4	CO2
		OR			
Q.4	a.	Fit a curve of the form, $y=ax^2+bx+c$ to the following data : x : 1, 2, 3, 4, 5 y : 10, 12, 8, 10, 14	6	L2	CO2
	b.	If θ is the acute angle between the two regression lines relating the variables x and y, show that $\tan\theta=\left(\frac{1-r^2}{r}\right)\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$. Indicate the significance of the cases $r=0$ and $r=\pm 1$	7	L2	CO2

c. Ten competitor's in a music contest ranked by 3 judges A, B, C in the following order. Use the rank correlation coefficient to decide which pair judges have the nearest approach to common test of music. — 7 | L3 | C

A	1	6	5	10	3	2	4	9	7	8
B	3	5	8	4	7	10	2	1	6	9
C	6	4	9	8	1	2	3	10	5	7

Module – 3

Q.5 a. Find the Fourier series for the function $f(x) = x^2$ in the interval $-\pi \le x \le \pi$, hence deduce the $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. — 6 | L2 | C

b. Expand the function $f(x) = x(\pi - x)$ over the interval $(0, \pi)$ in half range cosine Fourier series hence deduce that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$ — 7 | L3 | C

c. The following table gives the variations of a periodic current A over a certain period T. — 7 | L3 | C

t(sec)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A(amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a current part of 0.75 amp in the current A and obtain the amplitude of the first harmonic.

OR

Q.6 a. Find the Fourier expansion of the function $f(x) = (\pi - x)^2$ over the interval $0 \le x \le 2\pi$. Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. — 7 | L2 | C

b. Expand the function $f(x) = \begin{cases} \frac{1}{4} - x & \text{for } 0 < x < \frac{1}{2} \\ 0 - \frac{3}{4} & \frac{1}{2} < x < 1 \end{cases}$ in the half range sine series. — 6 | L2 | C

c. Find the constant term and the first harmonic in the Fourier series for $f(x)$ given by the table. — 7 | L3 | C

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Module – 4

Q.7 a. Find the Fourier transform of the function, $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| \ge a \end{cases}$ — 6 | L2 | C

where a is a positive constant hence evaluate integrals,

$$\int_0^{\infty} \frac{\sin ax \cos ax}{x} dx$$

	b.	Find the Fourier cosine transform of $f(x)=e^{-ax}$, $a>0$, hence deduce that $\int_0^\infty \frac{\cos mx}{x^2+a^2}dx=\frac{\pi}{2a}e^{-am}$	7	L3	CO4
	c.	Find the inverse z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$.	7	L3	CO4

OR

Q.8	a.	Find the Fourier transform of $f(x)=e^{-\frac{x^2}{2}}$.	6	L2	CO4
	b.	Find the z-transform of $\sin n\theta$ and $\cos n\theta$ hence find $z\left\{\cos\left(\frac{n\pi}{2}\right)\right\}$ and $z\left\{\sin\left(\frac{n\pi}{2}\right)\right\}$.	7	L3	CO4
	c.	Solve the difference equation, $u_{n+2}-5u_{n+1}+6u_n=2$ given $u_0=3$, $u_1=7$, using z-transforms.	7	L3	CO4

Module – 5

Q.9	a.	Define (i) Type I and Type II errors. (ii) Confidence interval. (iii) Level of significance.	6	L1	CO5
	b.	The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that (i) Exactly 2 will be defective. (ii) At least 2 will be defective. (iii) None will be defective	7	L2	CO5
	c.	In normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and SD, given that A(0.5) = 0.19 and A(1.4) = 0.42, where A(Z) is the area under the standard normal curve from 0 to z.	7	L3	CO5

OR

Q.10 a. The pdf $P(x)$ of a variate X is given by the table : (6, L2, CO5)

x :	0	1	2	3	4	5	6
P(x) :	K	3K	5K	7K	9K	11K	13K

For what value of K, does this represent a valid probability distribution? Also find $P(x<4)$, $P(x\geq 5)$ and $P(3<x\leq 6)$.

b. Consider the sample consisting of nine numbers, 45, 47, 50, 52, 48, 47, 49, 53 and 51. The sample is drawn from a population whose mean is 47.5. Find whether the sample mean differs significantly from the population mean at 5% level of significance (Given $t_{0.05}$ (df = 8) = 2.31) (7, L3, CO5)

c. A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given by the table : (7, L3, CO5)

x	1	2	3	4	5	6
frequency	15	6	4	7	11	17

Test the hypothesis that the die is unbiased. Given $\chi^2_{0.05}(5)=11.07$ and $\chi^2_{0.01}(5)=15.09$.

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Semester : III^{rd}

Subject with subject code: Mathematics III for EE stream

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MODULE - 01

1 a) we have,

$D^4 + 8D^2 + 16 = 0$

where,

$D = d/dt$

AE is,

$m^4 + 8m^2 + 16 = 0$

$(m^2+4)^2 = 0$

$\therefore$ roots are AE is $\pm 2i, \pm 2i$

The roots are repeated & imaginary roots.

The solution is,

$y = (c_1 + c_2 t)\cos 2t + (c_3 + c_4 t)\sin 2t$ is the general solution.

6 M

b) $(D^3 - 3D + 2)\,y = 2\sinh x$

AE is

$m^3 - 3m + 2 = 0$

Let $m = 1$ is a root and assume to find another two roots by inspection method.

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$\therefore m^2 + m - 2 = 0$

$-2m^2$

$+2m \quad -1m$

$m^2 + 2m - m - 2 = 0$

$m(m+2) - 1(m+2) = 0$

$(m-1)(m+2) = 0$

$m = 1, -2$ are the roots.

7 M

The solution is:

$$y_c = (c_1 + c_2 t)e^{t} + c_3 e^{-2t}.$$

$$y_P = \frac{2\sinh x}{D^3 - 3D + 2} = 2\left[\underset{D \Rightarrow 1}{\frac{e^{x}}{D^3 - 3D + 2}} - \underset{D \Rightarrow -1}{\frac{e^{-x}}{D^3 - 3D + 2}}\right]$$

$$y_P = 2\left[\underbrace{\frac{e^{x}}{(1)^3 - 3(1) + 2}}_{P_1} - \underbrace{\frac{e^{-x}}{(-1)^3 - 3(-1) + 2}}_{P_2}\right]$$

$$P_1 = 2\left[\frac{e^{x}}{(1)^3 - 3(1) + 2}\right] \Rightarrow D_r = 0 \quad \& \quad P_2 = 2\left[\frac{e^{-x}}{(-1)^3 - 3(-1) + 2}\right]$$

$$P_1 = 2 \cdot \frac{x \cdot e^{x}}{3D^2 - 3} \; [D \Rightarrow 1] \quad \& \quad P_2 = 2\left[\frac{e^{-x}}{-1 + 3 + 2}\right]$$

$$P_1 = 2 \cdot \frac{x \cdot e^{x}}{0} \; [D_r = 0] \quad \& \quad P_2 = 2\left[\frac{e^{-x}}{4}\right]$$

$$P_1 = 2 \, \frac{x^2 \cdot e^{x}}{6D} \; [D \Rightarrow 1] \quad \& \quad P_2 = \frac{e^{-x}}{2}.$$

$$P_1 = \cancel{2}^{1} \cdot \frac{x^2 \cdot e^{x}}{\cancel{6}\,3} = \frac{x^2 e^{x}}{3}.$$

Then,

$$y_P = P_1 - P_2$$

$$y_P = \frac{x^2 e^{x}}{3} - \frac{e^{-x}}{2}$$

Thus the general solution is,

$$y = y_c + y_P$$

$$= (c_1 + c_2 t)e^{t} + c_3 e^{-2t} + \frac{x^2 e^{x}}{3} - \frac{e^{-x}}{2}. \;//.$$

We have,

$$x^2 y'' - 3xy' + 5y = x^2 \sin(\log x) \quad \text{---} \; (1)$$

Put,

$$t = \log x \quad \text{or} \quad e^{t} = x.$$

Then we have,

$$xy' = Dy, \quad x^2 y'' = D(D-1)y \quad \text{where } D = d/dt$$

$[D(D-1)-3D+5]y = e^{2t}\sin t$

$(D^2-4D+5)y = e^{2t}\sin t$

AE is,

$m^2-4m+5=0$ and by solving

$$m = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$$

$$y_c = e^{2t}(C_1 \cos t + C_2 \sin t)$$

$$y_p = \frac{e^{2t}\sin t}{D^2-4D+5} \quad \text{Now } D \to D+2$$

$$y_p = e^{2t}\frac{\sin t}{(D+2)^2-4(D+2)+5} = e^{2t}\cdot\frac{\sin t}{D^2+1}$$

$$y_p = e^{2t}\cdot t\cdot\frac{\sin t}{2D} = \frac{-e^{2t}\cdot t\cos t}{2}$$

Complete solⁿ is,

$$y = y_c + y_p$$

$$y = x^2[C_1\cos(\log x) + C_2\sin(\log x)] - \frac{x^2\log x\cos(\log x)}{2}$$

7M

2a. $4D^4-8D^3-7D^2+11D+06=0$

A.E is

$4m^4-8m^3-7m^2+11m+06=0$

If $m=-1$: $(-1)^4-4(-1)^3-5(-1)^2-36(-1)-36=0$

$m=-1$ is root by inspection. Now by synthetic division

-1	4	-8	-7	$+11$	$+06$
	0	-4	12	-5	-6
	4	-12	5	06	0

Now

$4m^3-12m^2+5m+6=0$

If $m=2$: $32-48+10+6 = 48-48=0$

$$\begin{array}{c|cccc} 2 & 4 & -12 & 5 & 6 \\ & 0 & 8 & -8 & -6 \\ \hline & 4 & -4 & -3 & 0 \end{array}$$

Now,

$4m^2 - 4m - 3 = 0$

$4m^2 - 6m + 2m - 3 = 0$

$2m(2m-3) + 1(2m-3) = 0$

$(2m+1)(2m-3) = 0$

$m = -1/2,\ 3/2$

Hence the roots of the AE are $-1, 2, -1/2, 3/2$

Thus,

$y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-x/2} + c_4 e^{3x/2}$ is the general sol.

2b. $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 13y = \cos 2x$. 6M

we have,

$[D^2 - 4D + 13]y = \cos 2x$

AE is,

$m^2 - 4m + 13 = 0$

$m = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4.1.13}}{2.1} = 2 \pm 3i$

$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

$y_p = \dfrac{\cos 2x}{D^2 - 4D + 13}$

here,

$a = 2$ & hence replace D^2 by $-a^2 = -4$

$y_p = \dfrac{\cos 2x}{-4 - 4D + 13} = \dfrac{\cos 2x}{9 - 4D}$

Now multiply & divide by $(9 + 4D)$

$= \dfrac{(9+4D)\cos 2x}{(9+4D)(9-4D)} = \dfrac{9\cos 2x + 4D(\cos 2x)}{81 - 16D^2}$

$= \dfrac{9\cos 2x - 8\sin 2x}{145}$

Thus

$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \dfrac{9\cos 2x - 8\sin 2x}{145}$.

4) $(2x+1)\frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 3(2x+1)$

Put,

$t = \log(2x+1)$ or $e^t = 2x+1$

Then we have,

$(2x+1)y' = 2Dy$, $(2x+1)^2 y'' = 2^2 D(D-1)y$

Further, the given eqn becomes.

$[4D(D-1) - 4D - 12]y = 3e^t$

$[4D^2 - 4D - 4D - 12]y = 3e^t$

$4D^2 - 8D - 12 = 3e^t$

A.E is,

$4m^2 - 8m - 12 = 0$

$4m^2 - 12m + 4m - 12 = 0$

$4m(m-3) + 4(m-3) = 0$

$(m-3)(4m+4) = 0$

$\therefore m = 3, -1$ are the roots of the equation.

$y_c = c_1 e^{-x} + c_2 e^{3x}$

P.I is,

$y_p = \frac{3e^t}{4D^2 - 8D - 12}$ $D \Rightarrow 1$

$y_p = \frac{3e^t}{4(1)^2 - 8(1) - 12} \Rightarrow \frac{3e^t}{-16}$

Thus the general solution is,

$y = y_c + y_p$

$= c_1 e^{-x} + c_2 e^{3x} + \left\{ \frac{3e^t}{-16} \right\}$

$y = c_1 e^{-x} + c_2 e^{3x} + \frac{3(2x+1)}{-16}$ //

7M

MODULE-02:-

3a) Consider,

$y = ax^b$

$\therefore \log_e y = \log_e a + b\log_e x$ and let $Y = \log_e y$, $A = \log_e a$,

$X = \log_e x$.

The normal equation associated with $Y = A + bX$ are as follows.

$\Sigma Y = nA + b\Sigma X$

$\Sigma XY = A\Sigma X + b\Sigma X^2$ $(n=5)$

x	y	$X = \log_e x$	$Y = \log_e y$	XY	X^2
1	0.5	0	-0.6931	0	0
2	2	0.6931	0.6931	0.4804	0.4804
3	4.5	1.0986	1.5041	1.6524	1.2069
4	8	1.3863	2.0794	2.8827	1.9218
5	12.5	1.6094	2.5257	4.0649	2.5902
		$\Sigma X = 4.7874$	$\Sigma Y = 6.1092$	$\Sigma XY = 9.0804$	$\Sigma X^2 = 6.1993$

The normal equations become

$5A + 4.7874b = 6.1092$

$4.7874A + 6.1993b = 9.0804$

on solving

$A = -0.69315$, $b = 2$

$\log_e a = A \Rightarrow a = e^A = e^{-0.69015} = 0.5$

Thus,

$y = 0.5x^2$ is the required curve of fit.

6 M

3b) We first compute $\bar{x}$, $\bar{y}$ and denote $X = x - \bar{x}$, $Y = y - \bar{y}$ to form the table.

Here,

$\bar{x} = \frac{\Sigma x}{n} = \frac{45}{9} = 5$; $\bar{y} = \frac{\Sigma y}{n} = \frac{108}{9} = 12$.

$\therefore X = x - 5$ and $Y = y - 12$.

The relevant table is as follows.

x	y	X	Y	XY	X^2	Y^2
1	9	-4	-3	12	16	09
2	8	-3	-4	12	09	16
3	10	-2	-2	4	04	04
4	12	-1	0	0	01	00
5	11	0	-1	0	0	01
6	13	1	1	1	1	01
7	14	2	2	4	4	04
8	16	3	4	12	9	16
9	15	4	3	12	16	09
				$\Sigma XY = 57$	$\Sigma X^2 = 60$	$\Sigma Y^2 = 60$

We shall consider regression lines

$Y = \frac{\Sigma XY}{\Sigma X^2} \cdot X$ and $X = \frac{\Sigma XY}{\Sigma Y^2} \cdot Y$

$y - 12 = \frac{57}{60} \times (x - 5)$ & $x - 5 = \frac{57}{60}(y - 12)$

$y - 12 = 0.95(x - 5)$ & $x - 5 = 0.95(y - 12)$

$y - 12 = 0.95x - 4.75$ & $x - 5 = 0.95y - 11.4$

$y = 0.95x + 7.25$ & $x = 0.95y - 6.4$

These are the regression lines and we compute r as the geometric mean of the regression coefficients

$r = \sqrt{(\text{coeff of } x)(\text{coeff of } y)}$

$r = \sqrt{(0.95)(0.95)}$

$r = 0.9259$

7 M

... lines are,

$4x - 5y + 33 = 0$

$20x - 9y = 107$

Solve the system of linear equation

$4x - 5y = -33$ --- (1)

$20x - 9y = 107$ -- (2) multiply 1^{st} eqn by 5 to align the x coef

$20x - 25y = -165$ ---- (3)

substract eqn (2) from eqn (3) and simply.

$(20x - 25y) - (20x - 9y) = -165 - 107$

$-16y = -272$

$\boxed{y = 17}$

Then

$4x - 5(17) = -33$

$4x - 85 = -33$ implies

$4x = 52 \Rightarrow \boxed{x = 13}$

Hence the mean values are $x = 13$ and $y = 17$

ii) Finding SD of Y regression line of $\overline{Y}$ on $\overline{x}$ is

The formula for

$y - \overline{y} = \overline{xy}\,(x - \overline{x})$

The eqn $4x - 5y + 33 = 0$ is equivalent to

$y = (4)/5x + 33/5$

So the regression coefficient

$\overline{xy} = 4/5$

we know the variance $x = 9$.

S.D $y = 2.6$.

iii) coefficient of correlation

$r = 0.6$.

7M

The normal equations associated with $y = ax^2+bx+c$ are as follows.

$\Sigma y = a\Sigma x^2 + b\Sigma x + nc$.

$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$ $(n=5)$

$\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$

x	y	xy	x^2y	x^2	x^3	x^4
1	10	10	10	1	1	1
2	12	24	48	4	8	16
3	08	24	72	9	27	81
4	10	40	160	16	64	256
5	14	70	350	25	125	625
$\Sigma x = 15$	Σy 54	Σxy 168	Σx^2y 640	Σx^2 55	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$.

The normal equation become.

$54 = 55a + 15b + 5c$.

$168 = 225a + 55b + 15c$.

$640 = 979a + 225b + 55c$

on solving we get,

$a = 0.7143$, $b = -3.6857$, $c = 14$.

6 M

b) we know that if θ is acute, the angle between lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1m_2}$$

we have the line of regression

$$y - \bar{y} = r.\frac{\sigma y}{\sigma x}(x - \bar{x}) \quad \text{------- (1)}$$

& $$x - \bar{x} = r.\frac{\sigma x}{\sigma y}(y - \bar{y})$$

we write the second of the equation

$$y-\bar{y} = \frac{\sigma_y}{r \cdot \sigma_x}(x-\bar{x}) \quad \text{------} (2)$$

Slopes of eqⁿ ① & ② are respectively

$$m_1 = \frac{r \cdot \sigma_y}{\sigma_x} \quad \text{and} \quad m_2 = \frac{\sigma_y}{r \cdot \sigma_x}$$

Substituting these in the formula for $\tan\theta$

we have,

$$\tan\theta = \frac{\frac{\sigma_y}{r\sigma_x} - \frac{r\sigma_y}{\sigma_x}}{1 + \frac{r\sigma_y}{\sigma_x} \cdot \frac{\sigma_y}{r \cdot \sigma_x}}$$

$$= \frac{\frac{\sigma_y}{\sigma_x}\left(\frac{1}{r} - r\right)}{1 + \frac{\sigma_y^2}{\sigma_x^2}} = \frac{\frac{\sigma_y}{\sigma_x}\left(\frac{1-r^2}{r}\right)}{\frac{\sigma_x^2+\sigma_y^2}{\sigma_x^2}}$$

Thus,

$$\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$$

7 M

4c] we shall compute $\rho_{AB}, \rho_{BC}, \rho_{CA}$ with the help of the following table. where d is the difference in ranks

A	B	C	d^2_{AB}	d^2_{BC}	d^2_{CA}
1	3	6	4	9	25
6	5	4	1	1	4
5	8	9	9	1	16
10	4	8	36	16	4
3	7	1	16	36	4
2	10	2	64	64	0
4	2	3	4	01	1
9	1	10	64	81	1
7	6	5	1	01	4
8	9	7	1	4	1
			$\Sigma d^2_{AB} = 200$	$\Sigma d^2_{BC} = 214$	$\Sigma d^2_{CA} = 60$

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} \quad \text{and} \quad n = 10$$

$$\rho_{AB} = 1 - \frac{6 \times 200}{10(10^2-1)} = -0.21$$

$$\rho_{BC} = 1 - \frac{6 \times 214}{10(10^2-1)} = -0.297$$

$$\rho_{CA} = 1 - \frac{6 \times 60}{10(10^2-1)} = +0.636$$

It may be observed that ρ_{AB} and ρ_{BC} are negative which means their tastes (A & B ; B & C) are opposite.
But ρ_{CA} is positive and is nearer to 1.

Thus we conclude that the judges C and A have the nearest approach to common taste of music.

7M

MODULE - 03

5a. $f(x) = x^2$

The required Fourier series expansion is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{------} (1)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx$$

$$a_0 = \frac{1}{\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left[\frac{\pi^3}{3} - \frac{(-\pi)^3}{3}\right]$$

$$a_0 = \frac{2}{3}\pi^2$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx\,dx$$

$$a_n = \frac{1}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$a_n = \frac{2}{\pi n^2}\left[x \cos nx\right]_{-\pi}^{\pi}$$

$$a_n = \frac{2}{\pi n^2}\left[\pi \cos n\pi - (-\pi)\cos(-n\pi)\right]$$

$$= \frac{2}{\pi n^2}\left[2\pi\right]\cos n\pi = \frac{4}{n^2}(-1)^n$$

$$a_n = \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$$

$$b_n = 1/\pi \int_{-\pi}^{\pi} x^2 \sin nx\, dx$$

$$b_n = 1/\pi\left[x^2\left(-\frac{\cos nx}{n}\right) - 2x\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$b_n = 0$$

From eqn (1)

$$x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$x^2 = \frac{\frac{2}{3}\pi^2}{2} + \sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n \cos nx$$

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx.$$

GM

5b. Given

$f(x) = x(\pi - x)$ in $(0,\pi)$

the cosine half range series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx \quad ; \quad a_n = 2/\pi\int_0^{\pi} f(x)\cos nx\, dx$$

consider,

$$a_0 = 2/\pi \int_0^{\pi} x(\pi - x)\, dx$$

$$a_0 = 2/\pi\int_0^{\pi}(\pi x - x^2)\,dx = 2/\pi\left[\pi.\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{\pi}$$

$$a_0 = \frac{2}{\pi}\left[\pi.\frac{\pi^2}{2} - \frac{\pi^3}{3}\right] = \frac{2}{\pi}\left[\frac{\pi^3}{2} - \frac{\pi^3}{3}\right] = \frac{2}{\pi}\times\frac{\pi^3}{6}$$

$$\boxed{a_0 = \frac{\pi^2}{3}}$$

$$a_n = 2/\pi \int_0^\pi (\pi x - x^2)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[(\pi x - x^2)\cdot\left(\frac{\sin nx}{n}\right) - (\pi - 2x)\left(\frac{-\cos nx}{n^2}\right) + (-2)\left(\frac{-\sin nx}{n^3}\right)\right]_0^\pi$$

$$= \frac{2}{\pi}\left[\frac{1}{n^2}\left\{(\pi - 2x)\cos nx\right\}_0^\pi\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{n^2}\left\{-\pi\cos n\pi - \pi\cos 0\right\}\right]$$

$$= \frac{2}{\pi}\times\frac{-\pi}{n^2}\left[\cos n\pi + 1\right] = \frac{-2}{n^2}\left[(-1)^n + 1\right]$$

The required half range fourier Series expansion is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$f(x) = \frac{\frac{\pi^2}{3}}{2} + \sum_{n=1}^{\infty} \frac{-2}{n^2}\left[1 + (-1)^n\right]$$

$$f(x) = \frac{\pi^2}{6} - 2\sum_{n=1}^{\infty} \frac{1}{n^2}\left(1 + (-1)^n\right) //$$

7M

5c] Here the interval of x is $(0 \le x \le 2\pi)$ and the values of y at $x=0$ and $x=2\pi$ must be same by the periodic property $f(x+2\pi) = f(x)$. In the given problem the values of y at $x=0$ and 2π both are given and we must omit one of them. Let us omit the last value. The values of x in degree are 0, 60, 120, 180, 240, 300, and $N=6$

The relavant table is formulated below.

x	y	$y\cos x$	$y\cos 2x$	$y\cos 3x$	$y\sin x$	$y\sin 2x$	$y\sin 3x$
0	1.98	1.98	1.98	1.98	0	0	0
60	1.3	0.65	−0.65	−1.3	1.1258	1.1258	0
120	1.05	−0.525	−0.525	1.05	0.9093	−0.9093	0
180	1.3	−1.3	1.3	−1.3	0	0	0
240	−0.88	0.44	0.44	−0.88	0.76208	−0.76208	0
300	−0.25	−0.125	0.125	0.25	0.2165	0.2165	0
Total	4.5	1.12	2.67	−0.2	3.01368	−0.32908	0

$a_0 = 2/N \Sigma y = \frac{1}{3}(4.5) - 1.5$

$a_1 = \frac{2}{N}\Sigma y\cos x = \frac{1}{3}(1.12) = 0.3733$

$a_2 = \frac{2}{N}\Sigma y\cos 2x = \frac{1}{3}(2.67) = 0.89$

$a_3 = 2/N \Sigma y\cos 3x = \frac{1}{3}(-0.2) = -0.0667$

$b_1 = 2/N \Sigma y\sin x = 1/3(3.01368) = 1.00456$

$b_2 = 2/N \Sigma y\sin 2x = 1/3(-0.32908) = -0.1097$

$b_3 = 2/N \Sigma y\sin 3x = \frac{1}{3}(0) = 0$

Fourier series upto third harmonic

$y = \frac{a_0}{2} + (a_1\cos x + b_1\sin x) + (a_2\cos 2x + b_2\sin 2x)$
$+ (a_3\cos 3x + b_3\sin 3x)$

Thus,

$y = 0.75 + (0.3733\cos x + 1.00456\sin x)$
$+ (0.89\cos 2x - 0.1097\sin 2x) + (-0.0667\cos 3x)$

7M

The fourier Series of period 2π is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Consider,

$$f(x) = (\pi - x)^2$$

$$f(2\pi - x) = (\pi - 2\pi - x)^2$$
$$= \cancel{\pi - \pi}\ (-\pi - x)^2$$
$$= [-(\pi - x)]^2$$
$$= (\pi - x)^2$$

$\therefore f(x)$ is even hence $\boxed{b_n = 0}$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} (\pi - x)^2\,dx = \frac{2}{\pi}\left[\frac{(\pi - x)^2}{2}\right]_0^{\pi} = \frac{1}{\pi}\left[(\pi - \pi)^2 - (\pi - 0)^2\right]$$

$$a_0 = \frac{1}{\pi}\left[0 - \pi^2\right] = -\pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx\,dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} (\pi - x)^2 \cos nx\,dx$$ Applying Bernoulis rule of Int

$$a_n = \frac{2}{\pi}\left[(\pi - x)^2 \cdot \frac{\sin nx}{n} - 2(\pi - x)\cdot(-1)\cdot -\frac{\cos nx}{n^2} + 2\cdot -\frac{\sin nx}{n^3}\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[\frac{-2}{n^2}\left((\pi - x)\cos nx\right)_0^{\pi}\right]$$

$$a_n = \frac{-4}{\cancel{\pi} n^2}\left[0 - \cancel{\pi}(-1)^n\right] = \frac{4}{n^2}(-1)^n$$

Thus the required Fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$(\pi - x)^2 = -\frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n \cos nx$$

7M

6b. $f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < 1/2 \\ x - 3/4 & \text{in } 1/2 < x < 1 \end{cases}$

$f(x)$ is defined in $(0,1)$. comparing with half range $(0, l)$ we have $l = 1$. The corresponding Sine half range Series is given by,

$f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x$ where, $b_n = \frac{2}{1}\int_0^1 f(x) \sin n\pi x \, dx$

$$b_n = 2\left\{ \int_0^{1/2} \left(\frac{1}{4} - x\right) \sin n\pi x \, dx + \int_{1/2}^{1} (x - 3/4) \sin n\pi x \, dx \right\}$$

Applying Bernoullis rule to each of the integrals

$$b_n = 2\left\{ \left[\left(\frac{1}{4} - x\right) \cdot -\frac{\cos n\pi x}{n\pi} - (-1) \cdot -\frac{\sin n\pi x}{n^2\pi^2}\right]_0^{1/2} \right.$$

$$\left. + \left[\left(x - \frac{3}{4}\right) \cdot -\frac{\cos n\pi x}{n\pi} - 1 \cdot \frac{-\sin n\pi x}{n^2\pi^2}\right]_{1/2}^{1} \right\}$$

$$= 2\left\{ \frac{-1}{n\pi}\left[\left(\frac{1}{4} - x\right) \cos n\pi x\right]_0^{1/2} - \frac{1}{n^2\pi^2}\left[\sin n\pi x\right]_0^{1/2} \right.$$

$$\left. - \frac{1}{n\pi}\left[\left(x - 3/4\right) \cos n\pi x\right]_{1/2}^{1} + \frac{1}{n^2\pi^2}\left[\sin n\pi x\right]_{1/2}^{1} \right\}$$

$$= 2\left\{ \frac{-1}{n\pi}\left(-\frac{1}{4}\cos\frac{n\pi}{2} - \frac{1}{4}\right) - \frac{1}{n^2\pi^2}\left(\sin\frac{n\pi}{2}\right) \right.$$

$$\left. - \frac{1}{n\pi}\left(\frac{1}{4}\cos n\pi + \frac{1}{4}\cos\frac{n\pi}{2}\right) + \frac{1}{n^2\pi^2}\left(0 - \sin\frac{n\pi}{2}\right) \right\}$$

$$= 2\left\{ \frac{1}{4n\pi}\left(\cos\frac{n\pi}{2} + 1 - \cos n\pi - \cos\frac{n\pi}{2}\right) - \frac{2}{n^2\pi^2}\sin\frac{n\pi}{2} \right\}$$

$$= \frac{1}{2n\pi}\left\{1 - (-1)^n\right\} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}.$$

Thus the Sine half range Series is given by,

$$f(x) = \sum_{n=1}^{\infty}\left[\frac{1}{2n\pi}\left\{1 - (-1)^n\right\} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}\right] \sin n\pi x$$

6 M

6Y.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

$f(x)=y$ in the interval $0 \le x \le 2\pi$ the relavent tabl is prepared as below

x^o	y	$\cos x$	$\sin x$	$y\cos x$	$y\sin x$.
0	1	1	0	1	0
60°	1.4	0.5	0.866	0.7	1.2124
120°	1.9	-0.5	0.866	-0.95	1.6454
180°	1.7	-1	0	-1.7	0
240°	1.5	-0.5	-0.866	-0.75	-1.299
300°	1.2	0.5	-0.866	0.6	-1.0392
Total	8.7			-1.1	0.5196

$a_0=\frac{2}{N}\Sigma y = \frac{2}{6}\times 8.7 = 2.9$.

$a_1 = \frac{2}{N}\Sigma y\cos x = \frac{2}{6}(-1.1) = -0.367$

$b_1 = 2/N \, \Sigma y \sin x = 2/6(0.5196) = 0.1732$

Thus the constant term $a_0 = 2.9$.

$f(x) = a_0 + (a_1 \cos x + b_1 \sin x)$

$f(x). = 2.9 + (-0.367 \cos x + 0.1732 \sin x)$ //

7M

Module IV :-

7a. Complex Fourier transform of $f(x)$ is given by

$$F(u) = \int_{x=-\infty}^{\infty} f(x)\, e^{iux} \cdot dx.$$

$$F(u) = \int_{x=-a}^{a} 1 \cdot e^{iux} \cdot dx \quad \text{since } f(x) = \begin{cases} 1 & \text{for } -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

$$F(u) = \left[\frac{e^{iux}}{iu}\right]_{x=-a}^{a} = \frac{1}{iu}\left\{e^{iua} - e^{-iua}\right\}$$

$$F(u) = \frac{1}{iu}\left\{(\cos au + i\sin au) - (\cos au - i\sin au)\right\}$$

$$= \frac{1}{iu}(2i \sin au) = \frac{2\sin au}{u}$$

Thus,

$$F(u) = \frac{2\sin au}{u}$$

Let us evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$

we have obtained, $F(u) = \frac{2\sin au}{u}$

inverse fourier transform is $\frac{1}{2\pi}\int_{-\infty}^{\infty} F(u)\, e^{-iux}\, du = f(x)$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\sin au}{u} \cdot e^{-iux} \cdot du$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin au}{u} \cdot e^{-iux} \cdot du$$

Now, let put $x = 0$

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin au}{u}\, du = 1$$

$$\frac{2}{\pi}\int_{0}^{\infty} \frac{\sin au}{u}\, du = 1$$

$$\therefore \int_0^\infty \frac{\sin au}{u}\, du = \frac{\pi}{2}$$

putting $a = 1$, $\int_0^\infty \frac{\sin u}{u}\, du = \frac{\pi}{2}$

Thus by changing u to x, we have $\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$

6M

7b. we have,

$$F_c[e^{-ax}] = \frac{a}{a^2+u^2}$$

$$\int_0^\infty e^{-ax} \cos ux \, dx = \frac{a}{a^2+u^2} \quad \text{---- (1)}$$

Differentiating (1) wrt 'a' on both sides.

$$\int_0^\infty e^{-ax}(-x)\cos ux \, dx = \frac{(a^2+u^2)(1)-2a^2}{(a^2+u^2)^2} = \frac{u^2-a^2}{(a^2+u^2)^2}$$

or

$$\int_0^\infty (x e^{-ax})\cos ux \, dx = \frac{a^2-u^2}{(a^2+u^2)^2}$$

That is,

$$F_c[x e^{-ax}] = \frac{a^2-u^2}{a^2+u^2}$$

Further,

$$F_c[e^{-ax}] = a/a^2+u^2$$

$$e^{-ax} = 2/\pi \int_0^\infty \frac{a}{a^2+u^2} \cos ux \, du$$, by inverse Cosine transform

or $$\int_0^\infty \frac{\cos ux}{a^2+u^2} \, du = \frac{\pi}{2a} e^{-ax}.$$

7M

7c. Let,

$$\bar{u}(z) = \frac{2z^2+3z}{(z+2)(z-4)}$$

$$\frac{\bar{u}(z)}{z} = \frac{2z+3}{(z+2)(z-4)}$$

Let,

$$\frac{2z+3}{(z+2)(z-4)} = \frac{A}{z+2} + \frac{B}{z-4}.$$

or $2z+3 = A(z-4) + B(z+2)$

put,

$z = -2$; $-1 = A(-6)$ $\therefore$ $A = 1/6$

put,

$z = 4$; $11 = B(6)$ $\therefore$ $B = 11/6$.

Hence,

$$\frac{\bar{u}(z)}{z} = \frac{1}{6}\cdot\frac{1}{z+2} + \frac{11}{6}\cdot\frac{1}{z-4}$$

or $$\bar{u}(z) = \frac{1}{6}\cdot\frac{z}{z+2} + \frac{11}{6}\cdot\frac{z}{z-4}$$

$$Z_T^{-1}\left[\bar{u}(z)\right] = \frac{1}{6} Z_T^{-1}\left[\frac{z}{z+2}\right] + \frac{11}{6} Z_T^{-1}\left[\frac{z}{z-4}\right]$$

Thus,

$$Z_T^{-1}\left[\bar{u}(z)\right] = u_n = \tfrac{1}{6}\left\{(-2)^n + 11(4)^n\right\}.$$

7M

8a.

$$\phi(u) = \int_0^\infty e^{-x^2/2}\cdot \cos ux \, dx \quad \text{——— } (1)$$

$$\phi'(u) = \int_0^\infty e^{-x^2/2}\cdot \partial/\partial u(\cos ux)\, dx$$

$$\phi'(u) = \int_0^\infty e^{-x^2/2} (-x\sin ux)\, dx$$

$$\phi'(u) = \int_0^\infty \sin ux\left(-x\, e^{-x^2/2}\right) dx$$

Now, integrating by parts

$$\phi'(u) = \left[\sin ux\left(e^{-x^2/2}\right)\right]_{x=0}^{\infty} - \int_0^\infty e^{-x^2/2}(u\cos ux)\, dx$$

$$= 0 - u\int_0^\infty e^{-x^2/2}\cos ux\, dx$$

$$\phi'(u) = -u\,\phi(u) \quad \text{or} \quad \frac{\phi'(u)}{\phi(u)} = -u$$

$$\int \frac{\phi'(u)}{\phi(u)}\, du = -\int u\, du + c.$$

$$\log\phi(u) = \left(-u^2/2\right) + c \quad \text{or} \quad \phi(u) = e^{-u^2/2 + c}$$

hence,

$$\phi(u) = k\, e^{-u^2/2} \quad \text{where } k = e^{c}$$

To evaluate c, let us put $\boxed{u=0}$

$\phi(0) = k$ But, $\phi(0) = \int_0^{\infty} e^{-x^2/2}\, dx$ from eqⁿ (i)

put,

$x/\sqrt{2} = t \quad \therefore dx = \sqrt{2}\, dt$

$\phi(0) = \int_{t=0}^{\infty} e^{-t^2} \cdot \sqrt{2}\, dt$ But, $\int_{t=0}^{\infty} e^{-t^2}\, dt = \frac{\sqrt{\pi}}{2}$

hence,

$\phi(0) = \sqrt{\pi/2} = k$.

we now have,

$\phi(u) = \sqrt{\pi/2} \cdot e^{-u^2/2}$.

GM

8b $Z_T[k^n] = \frac{z}{z-k}$

let,

$k = e^{i\theta}$

$Z_T[(e^{i\theta})^n] = \frac{z}{z-e^{i\theta}}$

$Z_T[e^{in\theta}] = \frac{z}{z-e^{i\theta}} \times \frac{z-e^{-i\theta}}{z-e^{-i\theta}}$

$Z_T[\cos n\theta + i \sin n\theta] = \frac{z^2 - z e^{-i\theta}}{z^2 - z e^{-i\theta} - z e^{i\theta} + e^{0}}$

$Z_T[\cos n\theta] + i Z_T[\sin n\theta] = \frac{z^2 - z[\cos\theta - i\sin\theta]}{z^2 - z(e^{-i\theta} + e^{i\theta}) + 1}$

$Z_T[\cos n\theta] + i Z_T[\sin n\theta] = \frac{z^2 - z\cos\theta + iz\sin\theta}{z^2 - z(2\cos\theta) + 1}$

$Z_T[\cos n\theta] + i Z_T[\sin n\theta] = \frac{z^2 - 2\cos\theta}{z^2 - 2z\cos\theta + 1} + i\frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$

equating real & imaginary part.

$Z_T[\cos n\theta] = \frac{z^2 - 2\cos\theta}{z^2 - 2z\cos\theta + 1}$

$Z_T[\sin n\theta] = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$

Now find $\cos(n\pi/2)$ & $\sin(n\pi/2)$

$$Z_T\left[\underset{t=\pi/2}{\cos n\pi/2}\right] = \frac{z(z-\cos\pi/2)}{z^2-2z\cos\pi/2+1}$$

$$= \frac{z(z-0)}{z^2-0+1} = z^2/z^2+1$$

$$Z_T[\cos n\pi/2] = z^2/z^2+1$$

$$Z_T\left[\underset{t=\pi/2}{\sin n\pi/2}\right] = \frac{z\sin\pi/2}{z^2-2z\cos\pi/2+1} = \frac{z}{z^2-0+1}$$

$$Z_T[\sin n\pi/2] = z/z^2+1$$

7M

8C] Taking z-transform on both sides

$$Z_T(u_{n+2}) - 5Z_T(u_{n+1}) + 6Z_T(u_n) = Z_T(0)$$

$$z^2[\bar{u}(z) - u_0 - u_1 z^{-1}] - 5z[\bar{u}(z) - u_0] + 6\bar{u}(z) = 0$$

$$[z^2-5z+6]\bar{u}(z) - u_0(z^2-5z) - u_1 z = 0$$

$$[z^2-5z+6]\bar{u}(z) = u_0(z^2-5z) + u_1 z$$

or $$\bar{u}(z) = u_0 \cdot \frac{z^2-5z}{z^2-5z+6} + u_1 \cdot \frac{z}{z^2-5z+6}$$

$$Z_T^{-1}[\bar{u}(z)] = u_0 Z_T^{-1}\left[\frac{z^2-5z}{(z-2)(z-3)}\right] + u_1 Z_T^{-1}\left[\frac{z}{(z-2)(z-3)}\right] \quad \text{--- (1)}$$

$$P(z) = \frac{z^2-5z}{(z-2)(z-3)}$$

$$\frac{P(z)}{z} = \frac{z-5}{(z-2)(z-3)} = \frac{A}{z-2} + \frac{B}{z-3}$$

or

$$z-5 = A(z-3) + B(z-2)$$

Put,

$z=3$: $-2 = B(1)$ $\therefore B = -2$

$z=2$: $-3 = A(-1)$ $\therefore A = 3$

7M

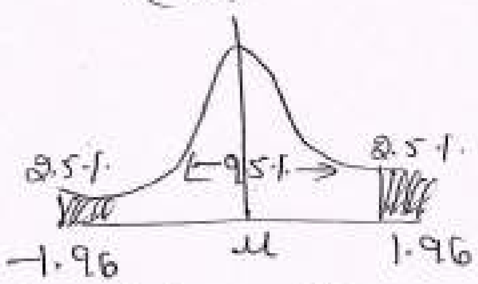
-: Module 5 :-

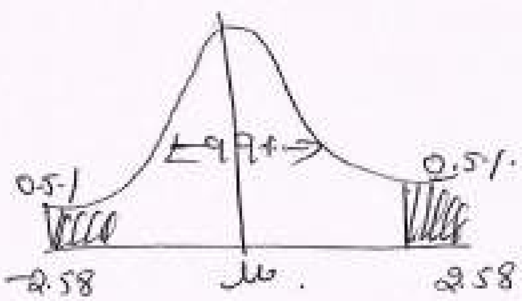
9a i] **Type I Error** :- If a hypothesis is rejected while it should have been accepted is known as Type I Error.

Type II Error :- If a hypothesis is accepted while it should have been rejected is known as Type II Error.

ii) **Confidence interval**:

Suppose that we have normal population with mean μ and S.D σ. If $\bar{x}$ is the sample mean of a random sample size n. the quantity Z defined by $z = \frac{\bar{x}-\mu}{(\sigma/\sqrt{n})}$ is called standard normal variate.





iii) **Level of significance**:

The probability level below which we reject the hypothesis is known as the level of significance.

6M

1b) The probability of Defective pen $p = \frac{1}{10} = 0.1$ that is probability of success $p = 0.1$

so that, probability of Non-defective pen

$q = 1 - p = 1 - 0.1 = 0.9$; $n = 12$

Here, 11

$p(x) = nC_x \, p^x \, q^{n-x}$

$p(x) = 12C_x (0.1)^x (0.9)^{12-x}$

i) P[Exactly 2 are defective]

$p(x=2) = 12C_2 (0.1)^2 (0.9)^{12-2}$

$= 12C_2 (0.1)^2 (0.9)^{10} = 0.2301$

$$p(z) = 3.\frac{z}{z-2} - 2.\frac{z}{z-3}$$

$$Z_T^{-1}[p(z)] = 3Z_T^{-1}\left[\frac{z}{z-2}\right] - 2Z_T^{-1}\left[\frac{z}{z-3}\right]$$

$$Z_T^{-1}\left[\frac{z^2-5z}{z^2-5z+6}\right] = 3.2^n - 2.3^n \quad \text{-----} \quad (2)$$

Next,

$$q(z) = \frac{z}{(z-2)(z-3)}$$

$$\frac{q(z)}{z} = \frac{1}{(z-2)(z-3)} = \frac{C}{z-2} + \frac{D}{z-3}$$

$$1 = C(z-3) + D(z-2)$$

put,

$z = 2 \quad : 1 = C(-1) \qquad \therefore C = -1$

$z = 3 \quad : 1 = D(1) \qquad \therefore D = 1$

$$q(z) = \frac{-z}{z-2} + \frac{z}{z-3}$$

$$Z_T^{-1}[q(z)] = -Z_T^{-1}\left[\frac{z}{z-2}\right] + Z_T^{-1}\left[\frac{z}{z-3}\right]$$

$$Z_T^{-1}\left[\frac{z}{z^2-5z+6}\right] = -2^n + 3^n \quad \text{------} \quad (3)$$

Using (2) & (3) in eqn (1)

$$Z_T^{-1}[\bar{u}(z)] = u_0\{3.2^n - 2.3^n\} + u_1\{-2^n + 3^n\}$$

$$u_n = (3u_0 - u_1)2^n + (-2u_0 + u_1)3^n$$

Let,

$c_1 = 3u_0 - u_1$ and $c_2 = -2u_0 + u_1$

Thus,

$u_n = c_1.2^n + c_2.3^n$. where c_1 & c_2 are arbitrary constants is the general solution of the given DE.

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ii) P[at least 2 are defective]

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - [12C_1 (0.1)^1 (0.9)^{12-1} + 12C_0 (0.1)^0 (0.9)^{12-0}]$

$= 1 - [0.3765 + 0.2824]$

$= 1 - [0.6589]$

$P(x \geq 2) = 0.3410$

iii) None of them are defective.

$P(x=0) = 12C_0 (0.1)^0 (0.9)^{12-0}$

$P(x=0) = 0.2824.$

9c] Let μ and σ be the mean & S.D of the normal dist

By data,

$P(x < 45) = 0.31$ & $P(x > 64) = 0.08$

we have standard Normal variate.

$z = \frac{x-\mu}{\sigma}$ where $x = 45$

$z = \frac{45-\mu}{\sigma} = z_1$ & $x = 64$, $z = \frac{64-\mu}{\sigma} = z_2$

we have

$P(z < z_1) = 0.31$ and $P(z > z_2) = 0.08$

$0.5 + \phi(z_1) = 0.31$ & $0.5 - \phi(z_2) = 0.08$

$\phi(z_1) = -0.19$ & $\phi(z_2) = 0.42$

Refering to the normal probability table.

$0.1915 (\approx 0.19) = \phi(0.5)$ & $0.4192 (\approx 0.42) = \phi(1.4)$

$\phi(z_1) = -\phi(0.5)$ and $\phi(z_2) = \phi(1.4)$

$z_1 = -0.5$ and $z_2 = 1.4$

$\frac{45-\mu}{\sigma} = -0.5$ and $\frac{64-\mu}{\sigma} = 1.4$

$\mu - 0.5\sigma = 45$ and $\mu + 1.4\sigma = 64$

By solving we get,

$\mu = 50$ & $\sigma = 10$

Thus,

mean $= 50$ and S.D $= 10$

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the probability distribution is valid if $P(x) \geq 0$ and $\sum P(x) = 1$

Hence we must have $k \geq 0$ and

$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$49k = 1 \Rightarrow \boxed{k = 1/49}$

$P(x<4) = P(0) + P(1) + P(2) + P(3)$

$= 1/49 + 3 \times 1/49 + 5 \times 1/49 + 7 \times 1/49$

$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49}$

$P(x<4) = \frac{16}{49}$

$P(x \geq 5) = P(5) + P(6)$

$= 11k + 13k = 24k = \frac{24}{49}.$

$P(3 < x \leq 6) = P(4) + P(5) + P(6)$

$= 9k + 11k + 13k$

$= 33k.$

$P(3 < x \leq 6) = \frac{33}{49}$

2] Let,

6 M

$t = \frac{\bar{x} - \mu}{\sigma} \cdot \sqrt{n}$

$\mu = 47.5$, $n = 9$

$\bar{x} = \frac{\sum x}{n} = \frac{45+47+50+52+48+47+49+53+51}{9}$

$= 49.11$

$$V = \sum \frac{(x-\bar{x})^2}{(n-1)} = \frac{(45-49.11)^2 + (47-49.11)^2 + (50-49.11)^2 + (52-49.11)^2 + (48-49.11)^2 + (47-49.11)^2 + (49-49.11)^2 + (53-49.11)^2 + (51-49.11)^2}{8}.$$

$V = 6.8608$

$\sigma = \sqrt{V} = \sqrt{6.8608}$

$\sigma = 2.6193$

$t = \dfrac{49.11 - 47.05}{2.6193} \times \sqrt{9}$

$t = 1.845$

2.306 Accepted at 5% level of significance

7M

10 b) we have,

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

Assuming die is unbiased & the expected frequency for every face is

$P(1) = 1/6 = P(2) = P(3) = P(4) = P(5) + P(6)$

accepted frequency $= \dfrac{15+6+4+7+11+17}{6} = \dfrac{60}{6} = 10$

x	O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
1	15	10	5	25	2.5
2	6	10	−4	16	1.6
3	−4	10	−6	36	3.6
4	7	10	−3	9	0.9
5	11	10	1	1	0.1
6	17	10	7	49	4.9

$$\chi^2 = \sum_{i=1} \frac{(O_i - E_i)^2}{E_i}$$

$E_i = 10$

$$\chi^2 = \sum_{i=1}^{6} \frac{(O_i - E_i)^2}{E_i} = 13.6$$

Now,

$\chi^2_{13.6} > 11.07 \; (\chi^2_{0.05})$ Rejected at 5% of level sign

$\chi^2 < 15.09 \; (\chi^2_{0.01})$ Accepted at 1% level of significan

7M.

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