KLS Vishwanathrao Deshpande Institute of Technology

(Accredited by NAAC with "A" Grade)
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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	Dr. MEENAL M. KALIWAL
Course Name	:	Mathematics - I for EEE Stream
Course Code	:	BMATE101
Year of Question Paper	:	DEC. 2024 | JAN. 2025
Date of Submission	:	17-06-2025

Faculty Member

HoD 17.06.2025

Head of the Department

Dean (Acad) 17/6/25

First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

Mathematics - I for EEE Stream

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. VTU Formula Handbook is permitted.

			M	L	C
		Module – 1			
Q.1	a.	With usual notations, prove that $\tan\phi = r.\ {}^{d\theta}/_{dr}$.	6	L2	CO1
	b.	Show that the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ intersects orthogonally.	7	L2	CO1
	c.	Find the radius of curvature for the curve $x^3 + y^3 = 3axy$ at $\left({}^{3a}/_{2}, {}^{3a}/_{2}\right)$.	7	L3	CO1
		OR			
Q.2	a.	Find the angle of intersection between curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$.	7	L2	CO1
	b.	Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$.	8	L2	CO1
	c.	Using modern mathematical tool, write a program to plot the curve $r = 2\lvert\cos 2\theta\rvert$.	5	L3	CO1
		Module – 2			
Q.3	a.	Expand $\sqrt{1+\sin 2x}$ using Maclaurin's series expansion upto terms containing x^6.	6	L2	CO1
	b.	If $u = f\left({}^{x}/_{y}, {}^{y}/_{z}, {}^{z}/_{x}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$.	7	L2	CO1
	c.	Show that the function $f(x,y) = x^3 + y^3 - 3xy + 1$ is minimum at the point (1, 1).	7	L3	CO1
		OR			
Q.4	a.	If $u = {}^{xy}/_{z}$, $v = {}^{yz}/_{x}$ and $w = {}^{xz}/_{y}$ then show that $J\left(\dfrac{u,v,w}{x,y,z}\right) = 4$.	7	L2	CO1
	b.	Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.	8	L3	CO1
	c.	Using modern tool write a program to evaluate $\lim\limits_{x\to\infty}\left(1+{}^{1}/_{x}\right)^x$.	5	L3	CO5

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		Module – 3			
Q.5	**a.**	Solve $x\frac{dy}{dx} + y = x^3y^6$.	**6**	**L2**	**CO2**
	b.	Find the orthogonal trajectories of a family of curves $\frac{2a}{r} = 1 - \cos\theta$.	**7**	**L3**	**CO2**
	c.	Solve $xy(p^2) - (x^2 + y^2)p + xy = 0$.	**7**	**L2**	**CO2**
		OR			
Q.6	**a.**	Solve $(x^2 + y^2 + x)dx + xy\,dy = 0$.	**6**	**L2**	**CO2**
	b.	A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t.	**7**	**L3**	**CO2**
	c.	Solve $(px - y)(py + x) = a^2p$ by reducing into Clairaut's form using the substitution $X = x^2$ and $Y = y^2$.	**7**	**L2**	**CO2**
		Module – 4			
Q.7	**a.**	Evaluate $\int_{-1}^{+1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)\,dy\,dx\,dz$.	**6**	**L2**	**CO3**
	b.	Change the order of integration and evaluate $\int_{0}^{1}\int_{x}^{\sqrt{x}} xy\,dy\,dx$.	**7**	**L2**	**CO3**
	c.	Prove that $\beta(m, n) = \frac{\sqrt{m}.\sqrt{n}}{\sqrt{m+n}}$.	**7**	**L2**	**CO3**
		OR			
Q.8	**a.**	Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dxdy$ by changing into polar coordinates.	**6**	**L2**	**CO3**
	b.	Evaluate $\int_{0}^{\pi/2}\sqrt{\cot\theta}\,d\theta$ by expressing in terms of gamma functions.	**7**	**L2**	**CO3**
	c.	Using double integration find the area between the curves $y^2 = 4ax$ and $x^2 = 4ay$.	**7**	**L3**	**CO3**

		Module – 5			
Q.9	a.	Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$	6	L2	CO4
	b.	Investigate the values of λ and μ such that the system of equations $x+y+z=6$, $x+2y+3z=10$ and $x+2y+\lambda z=\mu$ may have i) Unique solution ii) Infinite solution iii) No solution	7	L3	CO4
	c.	Using Rayleigh's power method find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking $[1, 1, 1]^T$ as initial eigen vector.	7	L3	CO4
		OR			
Q.10	a.	Solve by using Gauss – Jordan method $x+y+z=9$, $x-2y+3z=8$ and $2x+y-z=3$.	7	L2	CO4
	b.	Solve by using Gauss – Siedel method $20x+y-2z=17$, $3x+20y-z=-18$ and $2x-3y+20z=25$.	8	L2	CO4
	c.	Using modern mathematical model, write a program to test the consistency of the equations $x+2y-z=1$, $2x-y+4z=2$ and $3x+3y+4z=1$.	5	L3	CO5

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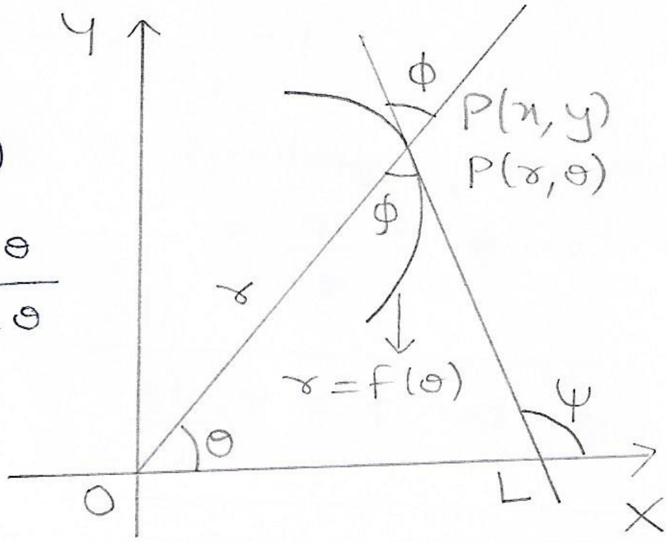
AY: 2025-26

Department: Electronics & Communication Engg
Subject with Sub. Code: Mathematics-I for EEE Stream (BMATE101) **Max. Marks: 100**
Semester / Branch / Division: I / ECE / A & B **Name of Faculty: Dr. Meenal Kaliwal**

Q.No.	Solution and Scheme	Marks
1a	Let $P(r,\theta)$ be any point on the curve $r=f(\theta)$ $\therefore XOP=\theta$ and $OP=r$. Let, PL be the tangent to the curve at P subtending an angle ψ with the positive direction of the initial line (x-axis) and ϕ be the angle between the radius vector and the tangent PL. That is, $O\hat{P}L=\phi$.	1
	From the figure, we have $\psi=\theta+\phi$ $\tan\psi=\tan(\theta+\phi)$ $\tan\psi=\dfrac{\tan\phi+\tan\theta}{1-\tan\phi\tan\theta}$ $\to$ (1)  	1
	Let (x,y) be the cartesian coordinates of P so that we have, $x=r\cos\theta$, $y=r\sin\theta$ $\tan\psi=\dfrac{dy}{dx}$ = slope of the tangent PL $\tan\psi=\dfrac{dy/d\theta}{dx/d\theta}$, since x and y are functions of θ.	1

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Q. No.	Solution	Marks
	$\tan\psi = \dfrac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \dfrac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$ where $r' = dr/d\theta$	1
	Dividing both Numerator & Denominator by $r'\cos\theta$ we have,	
	$\tan\psi = \dfrac{\frac{r\cos\theta}{r'\cos\theta} + \frac{r'\sin\theta}{r'\cos\theta}}{\frac{-r\sin\theta}{r'\cos\theta} + \frac{r'\cos\theta}{r'\cos\theta}}$	1
	$\tan\psi = \dfrac{\frac{r}{r'} + \tan\theta}{1 - \frac{r}{r'}\tan\theta} \longrightarrow (2)$	
	Comparing equations (1) & (2),	
	$\tan\phi = \dfrac{r}{r'} = \dfrac{r}{\frac{dr}{d\theta}}$	1
	$\boxed{\therefore \tan\phi = r.\dfrac{d\theta}{dr}}$	
		6
b.	$r = a(1+\cos\theta)$	
	$\log r = \log a + \log(1+\cos\theta)$	
	Differentiating wrt 'θ',	
	$\dfrac{1}{r}\cdot\dfrac{dr}{d\theta} = 0 + \dfrac{-\sin\theta}{1+\cos\theta}$	
	$\cot\phi_1 = \dfrac{-2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$	

Q. No.	Solution	Marks

$$\cot \phi_1 = -\tan(\theta/2)$$
$$= \cot(\pi/2 + \theta/2)$$

$$\Rightarrow \phi_1 = \pi/2 + \theta/2$$ **3**

Consider, $r = b(1-\cos\theta)$

$$\log r = \log b + \log(1-\cos\theta)$$

Differentiating wrt 'θ',

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{(1-\cos\theta)} \times (\sin\theta)$$

$$\cot \phi_2 = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$$ **3**

$$\cot \phi_2 = \cot(\theta/2) \Rightarrow \phi_2 = \theta/2$$

$\therefore$ angle of intersection $= |\phi_1 - \phi_2|$

$$= |\pi/2 + \theta - \pi/2| = \pi/2$$ **1**

Thus, the given curves intersect each other orthogonally.

7

1c. $x^3 + y^3 = 3axy$

Differentiating wrt 'x',

$$3x^2 + 3y^2\frac{dy}{dx} = 3a\left(x\frac{dy}{dx} + y\right)$$

$$3(y^2 - ax)\frac{dy}{dx} = 3(ay - x^2)$$

$$\frac{dy}{dx} = y_1 = \frac{ay - x^2}{y^2 - ax}$$

At $(3a/2, 3a/2)$, $y_1 = \dfrac{3a^2/2 - 9a^2/4}{\frac{9a^2}{4} - \frac{3a^2}{2}} = -1$ **2**

Q. No.	Solution	Marks
	Next, $\dfrac{d^2y}{dx^2} = y_2 = \dfrac{(y^2-ax)(ay_1-2x)-(ay-x^2)(2yy_1-a)}{(y^2-ax)^2}$	1
	At $(3a/2, 3a/2) \Rightarrow y^2-ax = \dfrac{9a^2}{4} - \dfrac{3a^2}{2} = \dfrac{3a^2}{4}$	
	and $ay-x^2 = \dfrac{3a^2}{2} - \dfrac{9a^2}{4} = -3a^2/4$	
	Hence, at $(3a/2, 3a/2)$	
	$y_2 = \dfrac{(3a^2/4)(-a-3a)-(-3a^2/4)(-3a-a)}{(3a^2/4)^2}$	1
	$= \dfrac{-3a^3-3a^3}{9a^4/16} = \dfrac{16(-6a^3)}{9a^4} = \dfrac{-32}{3a}$	1
	$\therefore \rho = \dfrac{(1+y_1^2)^{3/2}}{y_2} = \dfrac{(1+1)^{3/2}}{-32/3a}$	1
	$= \dfrac{2\sqrt{2}\cdot 3a}{-32} = \dfrac{-3\sqrt{2}\,a}{16}$	
	$\boxed{\lvert\rho\rvert = \dfrac{3a}{8\sqrt{2}}}$	1
		7

Q. No.	Solution	Marks
2a.	$r^n = a^n \cos n\theta$	
	$n \log r = n \log a + \log \cos n\theta$	
	Differentiating wrt 'θ',	
	$n \times \frac{1}{r} \times \frac{dr}{d\theta} = 0 - \frac{n \sin n\theta}{\cos n\theta}$	3
	$\frac{1}{r}\frac{dr}{d\theta} = -\tan n\theta$	
	ie. $\cot \phi_1 = \cot(\pi/2 + n\theta)$	
	$\Rightarrow \phi_1 = \pi/2 + n\theta$	
	Consider, $r^n = b^n \sin n\theta$	
	$n \log r = n \log b + \log \sin n\theta$	
	$\frac{n}{r}\frac{dr}{d\theta} = 0 + n \frac{\cos n\theta}{\sin n\theta}$	3
	$\frac{1}{r}\frac{dr}{d\theta} = \cot \phi_2 = \cot n\theta$	
	$\Rightarrow \phi_2 = n\theta$	
	$\therefore \lvert\phi_1 - \phi_2\rvert = \lvert\pi/2 + n\theta - n\theta\rvert = \pi/2$	1 / 7
b.	$r^m = a^m(\cos m\theta + \sin m\theta)$	
	$m \log r = m \log a + \log(\cos m\theta + \sin m\theta)$	
	Differentiating wrt 'θ',	
	$\frac{m}{r} \times \frac{dr}{d\theta} = \frac{+m\cos m\theta - m \sin m\theta}{\cos m\theta + \sin m\theta}$	1
	$\frac{1}{r} \times \frac{dr}{d\theta} = \frac{\cos m\theta - \sin m\theta}{\cos m\theta + \sin m\theta}$	
	$= \frac{\cos m\theta\,(1 - \tan m\theta)}{\cos m\theta\,(1 + \tan m\theta)}$	2

Q. No.	Solution	Marks
	$\cot\phi = \cot(\pi/4 + m\theta) \Rightarrow \phi = \pi/4 + m\theta$	1
	Consider, $p = r\sin\phi$	
	$p = r\sin(\pi/4 + m\theta)$	
	$= r\left[\sin(\pi/4)\cos m\theta + \cos(\pi/4)\sin m\theta\right]$	2
	$p = \dfrac{r}{\sqrt{2}}(\cos m\theta + \sin m\theta)$ $\left[\because \sin\pi/4 = 1/\sqrt{2},\ \cos\pi/4 = 1/\sqrt{2}\right.$	
	Now, we have $r^m = a^m(\cos m\theta + \sin m\theta) \rightarrow ①$	1
	$p = \dfrac{r}{\sqrt{2}}(\cos m\theta + \sin m\theta) \rightarrow ②$	
	Using ② in ①, we get	
	$r^m = a^m \cdot \dfrac{p\sqrt{2}}{r}$ or $r^{m+1} = \sqrt{2}\,a^m p$	
	Thus, $r^{m+1} = \sqrt{2}\,a^m p$ is the required pedal equation.	1
		8
2c.	$r = 2\lvert\cos 2\theta\rvert$	
	from pylab import *	1
	theta = linspace (0, 2 * pi, 1000)	1
	r = 2 * abs (cos (2 * theta))	1
	polar (Theta, r, 'r')	1
	show ( )	1
		5

Q. No.	Solution	Marks
3a.	Maclaurin's series expansion is, $y(x) = y(0) + x\,y_1(0) + \frac{x^2}{2!}y_2(0) + \frac{x^3}{3!}y_3(0) + \cdots$	
	Let, $y = \sqrt{1+\sin 2x}$ $= \sqrt{\cos^2 x + \sin^2 x + 2\sin x\cos x}$ $= \sqrt{(\cos x + \sin x)^2} = \cos x + \sin x$	2
	$y = \sin x + \cos x$ $\therefore y(0) = 1$	
	$y_1 = -\sin x + \cos x$ $\therefore y_1(0) = 1$	
	$y_2 = -\cos x - \sin x = -y$ $\therefore y_2(0) = -1$	
	$y_3 = -y_1$ $\therefore y_3(0) = -1$	3
	$y_4 = -y_2$ $\therefore y_4(0) = 1$	
	$y_5 = -y_3$ $\therefore y_5(0) = 1$	
	$y_6 = -y_4$ $\therefore y_6(0) = -1$	
	Substituting these values in $y(x)$, $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} - \frac{x^6}{720}$	1
		6
b.	Let, $u = f(p,q,r)$ where $p = \frac{x}{y},\ q = \frac{y}{z},\ r = \frac{z}{x}$	
	$\{u \to (p,q,r) \to (x,y,z)\} \Rightarrow u \Rightarrow x, y, z$	
	$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial u}{\partial q}\frac{\partial q}{\partial x} + \frac{\partial u}{\partial r}\frac{\partial r}{\partial x}$	1
	$= \frac{\partial u}{\partial p}\cdot\frac{1}{y} + \frac{\partial u}{\partial q}(0) + \frac{\partial u}{\partial r}\left(-\frac{z}{x^2}\right)$	1

Q. No.	Solution	Marks
	Hence, $x\dfrac{\partial u}{\partial x} = \dfrac{x}{y}\dfrac{\partial u}{\partial p} - \dfrac{z}{x}\dfrac{\partial u}{\partial r} \longrightarrow (1)$	1
	Similarly, by symmetry $y\dfrac{\partial u}{\partial y} = \dfrac{y}{z}\dfrac{\partial u}{\partial q} - \dfrac{x}{y}\dfrac{\partial u}{\partial p} \longrightarrow (2)$	1
	$z\dfrac{\partial u}{\partial z} = \dfrac{z}{x}\dfrac{\partial u}{\partial r} - \dfrac{y}{z}\dfrac{\partial u}{\partial q} \longrightarrow (3)$	1
	Thus, by adding (1), (2) & (3), $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$	2
		7
3c.	$f(x,y) = x^3+y^3-3xy+1$ $f_x = \dfrac{\partial f}{\partial x} = 3x^2-3y$ $f_y = \dfrac{\partial f}{\partial y} = 3y^2-3x$	2
	Let, $A = f_{xx} = \dfrac{\partial^2 f}{\partial x^2} = 6x$; $B = f_{yx} = \dfrac{\partial^2 f}{\partial y \partial x} = -3$ $C = f_{yy} = 6y$ At $(1,1) \Rightarrow f_x = 0$ & $f_y = 0$. Also, $A = 6$, $B = -3$, $C = 6$ $AC - B^2 = 27 > 0$	2
	Now at $(1,1)$, $f_x = 0$ and $f_y = 0$. $AC - B^2 > 0$, $A = 6 > 0$	1
	$\therefore$ $f(x,y)$ at $(1,1)$ satisfy the necessary & sufficient conditions for minima.	1
	Thus, $f(x,y)$ is minimum at $(1,1)$.	1
		7

Q. No.	Solution	Marks
4a.	$u = xy/z, \quad v = yz/x, \quad w = xz/y$	
	$\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$	
	$= \begin{vmatrix} y/z & x/z & -xy/z^2 \\ -yz/x^2 & z/x & y/x \\ z/y & -xz/y^2 & x/y \end{vmatrix}$	2
	$= \frac{y}{z}\left\{\frac{z}{x}\times\frac{x}{y} + \frac{y}{x}\times\frac{xz}{y^2}\right\} - \frac{x}{z}\left\{\left(-\frac{yz}{x^2}\right)\left(\frac{x}{y}\right) - \frac{y}{x}\times\frac{z}{y}\right\} - \frac{xy}{z^2}\left\{\left(-\frac{yz}{x^2}\right)\left(-\frac{xz}{y^2}\right) - \frac{z}{x}\times\frac{z}{y}\right\}$	2
	$= \frac{y}{z}\left\{\frac{z}{y}+\frac{z}{y}\right\} - \frac{x}{z}\left\{-\frac{z}{x}-\frac{z}{x}\right\} - \frac{xy}{z^2}\left\{\frac{z^2}{xy} - \frac{z^2}{xy}\right\}$	1
	$= \frac{y}{z}\times\frac{z}{y}\{1+1\} + \frac{x}{z}\times\frac{z}{x}\{1+1\} - 0$	1
	$= 1+1+1+1 = 4$	1
	$\therefore J = 4$	
		7

Q. No.	Solution	Marks

4b.

$f(x,y) = x^3 + y^3 - 3x - 12y + 20$

$f_x = \frac{\partial f}{\partial x} = 3x^2 - 3$; $f_y = \frac{\partial f}{\partial y} = 3y^2 - 12$

Let, $f_x = 0$ & $f_y = 0$

$3x^2 - 3 = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$3y^2 - 12 = 0 \Rightarrow y^2 - 4 = 0 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$ — **2**

$\therefore$ $(1,2), (1,-2), (-1,2), (-1,-2)$ are the stationary points.

Let $A = f_{xx}$, $B = f_{xy}$, $C = f_{yy}$

	$(1,2)$	$(1,-2)$	$(-1,2)$	$(-1,-2)$
$A = 6x$	$6 > 0$	6	-6	$-6 < 0$
$B = 0$	0	0	0	0
$C = 6y$	12	-12	12	-12
$AC - B^2$	$72 > 0$	$-72 < 0$	$-72 < 0$	$72 > 0$
Conclusion	Min point	Saddle pt.	Saddle pt.	Max. pt

4

Maximum value of $f(x,y)$ is $f(-1,-2) = 38$

Minimum value of $f(x,y)$ is $f(1,2) = 2$ — **2**

8

4c.

```
from sympy import *          1
from math import inf         1
x = Symbol('x')              1
l = Limit((1+1/x)**x, x, inf).doit()   1
display (l)                  1
```

5

Q. No.	Solution	Marks

5a.

$$x\frac{dy}{dx} + y = x^3 y^6$$

Dividing by 'x',

$$\frac{dy}{dx} + \frac{y}{x} = x^2 y^6 \quad \to (1)$$ which is a Bernoulli's eqn $\frac{dy}{dx} + Py = Qy^n$

Here, $P = \frac{1}{x}$, $Q = x^2$

Let, $v = y^{1-n} = y^{-5} \Rightarrow \frac{dv}{dx} = -5y^{-6}\frac{dy}{dx}$ — 1

Multiply eqn (1) by y^{-6},

$$y^{-6}\frac{dy}{dx} + \frac{1}{x}y^{-5} = x^2$$

Now, use $v = y^{-5}$ and $\frac{dv}{dx} = -5y^{-6}\frac{dy}{dx}$

So : $$-\frac{1}{5}\frac{dv}{dx} + \frac{1}{x}v = x^2$$

$$-\frac{dv}{dx} + \frac{5}{x}v = 5x^2$$

$$\Rightarrow \frac{dv}{dx} - \frac{5}{x}v = -5x^2 \quad \to (2)$$ which is a linear eqn in v. — 1

Integrating factor : $I.F. = e^{\int P\,dx} = e^{\int -\frac{5}{x}dx}$

$$= e^{-5\log x} = x^{-5}$$ — 1

General solution is,

$$v \times I.F. = \int (Q \times I.F.)\,dx + c$$

$$x^{-5}v = \int(-5x^2 \times x^{-5})\,dx + c$$

$$= -\int 5x^{-3}\,dx + c = \frac{5}{2}x^{-2} + c$$ — 2

$$v = x^5\left(\frac{5}{2}x^{-2} + c\right) = \frac{5}{2}x^3 + Cx^5 \longrightarrow (3)$$

Q. No.	Solution	Marks
	Substitute $v = y^{-5}$ in eqn (3)	
	$y^{-5} = \frac{5}{2}x^3 + Cx^5$	
	$\therefore y = \left(\frac{1}{\frac{5}{2}x^3 + Cx^5}\right)^{1/5}$	1
		6
5b.	We have $\frac{2a}{r} = 1 - \cos\theta$	
	$\Rightarrow \log 2a - \log r = \log(1 - \cos\theta)$	
	Differentiating w.r.t. 'θ' we have,	1
	$-\frac{1}{r} \cdot \frac{dr}{d\theta} = \frac{\sin\theta}{1 - \cos\theta}$	
	Replacing $\frac{dr}{d\theta}$ by $-r^2 \frac{d\theta}{dr}$ and simplifying	1
	$-\frac{1}{r}\left(-r^2 \frac{d\theta}{dr}\right) = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$	1
	$r\frac{d\theta}{dr} = \cot(\theta/2)$	
	$\tan(\theta/2)\, d\theta = dr/r$	1
	$\int \frac{dr}{r} - \int \tan(\theta/2)\, d\theta = c$	1
	$\log r - \frac{\log \sec(\theta/2)}{(1/2)} = c$	
	$\log[r/\sec^2(\theta/2)] = \log b$	1
	$r/\sec^2(\theta/2) = b$	1
	or $r\cos^2(\theta/2) = b$	
	$r \cdot \frac{1}{2}(1 + \cos\theta) = b$ or $r(1 + \cos\theta) = 2b$	
	Thus, $2b/r = 1 + \cos\theta$	7

Q. No.	Solution	Marks
5c.	$xy(p^2) - (x^2+y^2)p + xy = 0$	
	which is a quadratic equation in p.	
	$\therefore p = \dfrac{(x^2+y^2) \pm \sqrt{(x^2+y^2)^2 - 4x^2y^2}}{2xy}$	1
	$= \dfrac{(x^2+y^2) \pm (x^2-y^2)}{2xy}$	
	$p = \dfrac{x^2+y^2+x^2-y^2}{2xy}$ or $p = \dfrac{x^2+y^2-x^2+y^2}{2xy}$	2
	i.e. $p = \dfrac{x}{y}$ or $p = y/x$	
	$\dfrac{dy}{dx} = \dfrac{x}{y}$ or $y\,dy - x\,dx = 0$	
	$\Rightarrow \int y\,dy - \int x\,dx = k$	
	$\dfrac{y^2}{2} - \dfrac{x^2}{2} = k$ or $y^2 - x^2 = 2k$	1
	$(y^2 - x^2 - c) = 0$	
	Also, $\dfrac{dy}{dx} = \dfrac{y}{x}$ or $\dfrac{dy}{y} - \dfrac{dx}{x} = 0$	1
	$\Rightarrow \int \dfrac{dy}{y} - \int \dfrac{dx}{x} = k$	
	$\log y - \log x = k$	
	or $\log(y/x) = \log c$	
	$y = cx \Rightarrow y - cx = 0$	1
	Thus, the general solution is given by	
	$(y^2 - x^2 - c)(y - cx) = 0$	1
		7

Q. No.	Solution	Marks
6a.	Let, $M = x^2 + y^2 + x$ and $N = xy$	
	$\frac{\partial M}{\partial y} = 2y$ and $\frac{\partial N}{\partial x} = y$	1
	$\therefore \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - y = y$ --- near to N.	
	Now, $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{y}{xy} = \frac{1}{x} = f(x)$	1
	Hence, $I.F. = e^{\int f(x)\,dx} = e^{\int 1/x\,dx} = e^{\log x}$	1
	$= x$	
	Multiplying the given equation by x, we have	
	$M = x^2 + xy^2 + x^2$ and $N = x^2 y$	
	$\frac{\partial M}{\partial y} = 2xy$ and $\frac{\partial N}{\partial x} = 2xy$	1
	The solution is,	
	$\int M\,dx + \int N\,dy = c$	
	$\int (x^3 + xy^2 + x^2)\,dx + \int 0\,dy = c$	1
	Thus, $\frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{x^3}{3} = c$, is the required solution.	1
		6
6b.	$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L} \longrightarrow (1)$	
	This is of the form $\frac{dy}{dx} + Py = Q$, whose solution	
	is given by $y e^{\int P dx} = \int Q e^{\int P dx} dx + c$	
	Applying for the form of DE as in (1) we have,	
	$i e^{\int \frac{R}{L} dt} = \int \frac{E}{L} e^{\int R/L\, dt} \int dt + c$	2

Q. No.	Solution	Marks
	That is, $ie^{Rt/L} = \frac{E}{L}\int e^{Rt/L}\,dt + c$	
	$ie^{Rt/L} = \frac{E}{L}\frac{e^{Rt/L}}{(R/L)} + c$	1
	$ie^{Rt/L} = \frac{E}{R}e^{Rt/L} + c$	1
	$i = \frac{E}{R} + ce^{-(Rt/L)} \longrightarrow (2)$	1
	using the initial condition $i=0, t=0$ $0 = E/R + c \Rightarrow c = -E/R$	1
	Thus, $i = \frac{E}{R}[1 - e^{-Rt/L}]$	1
		7
6c.	$X = x^2 \Rightarrow \frac{dX}{dx} = 2x$	1
	$Y = y^2 \Rightarrow \frac{dY}{dy} = 2y$	
	Now, $p = \frac{dy}{dx} = \frac{dy}{dY}\cdot\frac{dY}{dX}\cdot\frac{dX}{dx}$ and let $P = \frac{dY}{dX}$	1
	i.e. $p = \frac{1}{2y}\cdot P\cdot 2x$ or $p = \frac{x}{y}P$	1
	$p = \frac{x}{y}P$ $p = \frac{\sqrt{X}}{\sqrt{Y}}P$	
	Consider, $(px-y)(py+x) = a^2p$	
	$\left[\frac{\sqrt{X}}{\sqrt{Y}}P\sqrt{X} - \sqrt{Y}\right]\left[\frac{\sqrt{X}}{\sqrt{Y}}P\sqrt{Y} + \sqrt{X}\right] = \frac{2\sqrt{X}}{\sqrt{Y}}P$	1

Q. No.	Solution	Marks

$$\frac{(PX-Y)}{\sqrt{Y}}(P+1)\sqrt{X} = \frac{a^2\sqrt{X}}{\sqrt{Y}}P$$ 1

$$\frac{(PX-Y)}{\cancel{\sqrt{Y}}}(P+1)\cancel{\sqrt{X}} = a^2\frac{\cancel{\sqrt{X}}}{\cancel{\sqrt{Y}}}P$$

or $PX - Y = \dfrac{a^2P}{P+1}$ 1

$Y = PX - \dfrac{a^2P}{P+1}$, this is in the Clairaut's form

General solution is,

$$Y = cX - \frac{a^2c}{c+1}$$ 1

7

7a

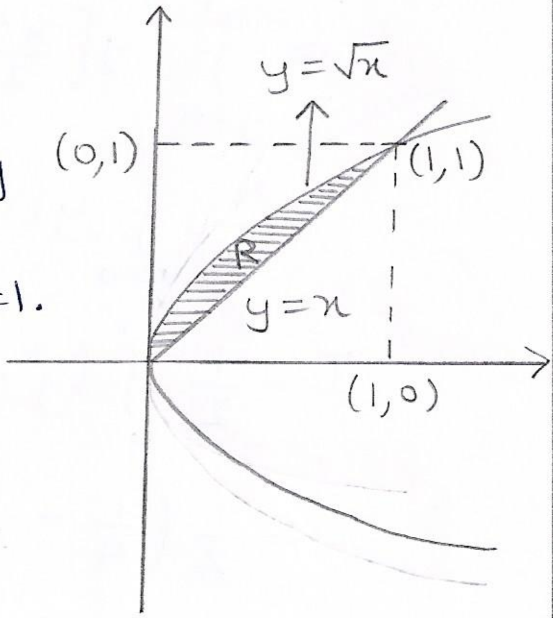
$$I = \int_{z=-1}^{1}\int_{x=0}^{z}\int_{y=x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$$

$$= \int_{z=-1}^{1}\int_{x=0}^{z}\left[xy + \frac{y^2}{2} + zy\right]_{y=x-z}^{x+z} dx\,dz$$ 1

$$= \int_{z=-1}^{1}\int_{x=0}^{z}\left\{x(\overline{x+z}-\overline{x-z}) + \frac{1}{2}\left[(x+z)^2 - (x-z)^2\right] + \left[\overline{x+z}-\overline{x-z}\right]z\right\}dx\,dz$$ 1

$$= \int_{z=-1}^{1}\int_{x=0}^{z}(2xz + 2xz + 2z^2)\,dx\,dz$$ 1

$$= \int_{z=-1}^{1}\int_{x=0}^{z}(4xz + 2z^2)\,dx\,dz$$

Q. No.	Solution	Marks
	$= \int_{z=-1}^{1} \left[z(2x^2) + 2z^2(x) \right]_{x=0}^{z}$	1
	$= \int_{z=-1}^{1} (2z^3 + 2z^3)\, dz = \int_{z=-1}^{1} 4z^3\, dz$	1
	$= \left[z^4 \right]_{z=-1}^{1} = 0$	
	$\therefore\ \underline{\underline{I = 0}}$	1
		6
7b.	$I = \int_{x=0}^{1} \int_{y=x}^{\sqrt{x}} xy\, dy\, dx$	
	* Identify the region of integration R bounded by the curves $y = x$, $y = \sqrt{x}$ between the lines $x = 0$, $x = 1$.	
		
	* Find the points of intersection of $y = x$ and $y = \sqrt{x}$ by equating their RHS.	
	i.e. $x = \sqrt{x} \Rightarrow x^2 = x$ or $x(x-1) = 0$	
	$x = 0, 1$. This gives us $y = 0$, $y = 1$ & hence the points of intersection are $(0,0)$ and $(1,1)$.	2
	* $y = x$ is a straight line passing through the origin making an angle 45° with the x-axis and $y = \sqrt{x}$ or $y^2 = x$ is a parabola symmetrical about the x-axis.	1

Q. No.	Solution	Marks

On changing the order of integration, we must have constant limits for y and variable limits for x.

From, the figure we observe that y-varies from 0 to 1 and x varies from y^2 to y. — 1

Hence, we have on changing the order of integration,

$$I = \int_{y=0}^{1} \int_{x=y^2}^{y} xy \, dx\, dy$$ — 1

$$= \int_{y=0}^{1} y\left[\frac{x^2}{2}\right]_{x=y^2}^{y} dy$$

$$= \frac{1}{2}\int_{y=0}^{1} y(y^2-y^4)\, dy$$

$$I = \frac{1}{2}\int_{0}^{1} (y^3-y^5)\, dy = \frac{1}{2}\left[\frac{y^4}{4}-\frac{y^6}{6}\right]_{y=0}^{1}$$ — 1

$$= \frac{1}{2}\left(\frac{1}{4}-\frac{1}{6}\right) = \underline{\underline{\frac{1}{24}}}$$ — 1

7

7c. We have by the definition of Beta and Gamma functions,

$$\beta(m,n) = 2\int_{\theta=0}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta \quad \rightarrow (1)$$ — 1

$$\Gamma n = 2\int_{0}^{\infty} e^{-x^2} x^{2n-1}\, dx \longrightarrow (2)$$ — 1

$$\Gamma m = 2\int_{0}^{\infty} e^{-y^2} y^{2m-1}\, dy \longrightarrow (3)$$

Q. No.	Solution	Marks

$$\Gamma(m+n) = 2\int_0^\infty e^{-r^2} r^{2(m+n)-1}\, dr \longrightarrow (4)$$

Now, $$\Gamma(m)\cdot\Gamma(n) = 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} x^{2n-1} y^{2m-1}\, dx\,dy \longrightarrow (5)$$ 1

Let us evaluate RHS by changing into polars.

putting $x = r\cos\theta$, $y = r\sin\theta$, we have $x^2+y^2 = r^2$. 1

Also, $dx\,dy = r\,dr\,d\theta$. r varies from 0 to ∞, θ varies from 0 to $\pi/2$.

Now, equation (5) can be rewritten in the form

$$\Gamma(m)\cdot\Gamma(n) = 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2}(r\cos\theta)^{2n-1}(r\sin\theta)^{2m-1}\, r\,dr\,d\theta$$ 1

$$= 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2} r^{2m+2n-1}\sin^{2m-1}\theta\cos^{2n-1}\theta\, dr\,d\theta$$ 1

$$= \left[2\int_{r=0}^{\infty} e^{-r^2} r^{2(m+n)-1}\, dr\right]\left[2\int_{\theta=0}^{\pi/2}\sin^{2m-1}\theta\cos^{2n-1}\theta\, d\theta\right]$$

$\therefore\ \Gamma(m)\cdot\Gamma(n) = \Gamma(m+n)\cdot\beta(m,n)$ by using (1) & (4) 1

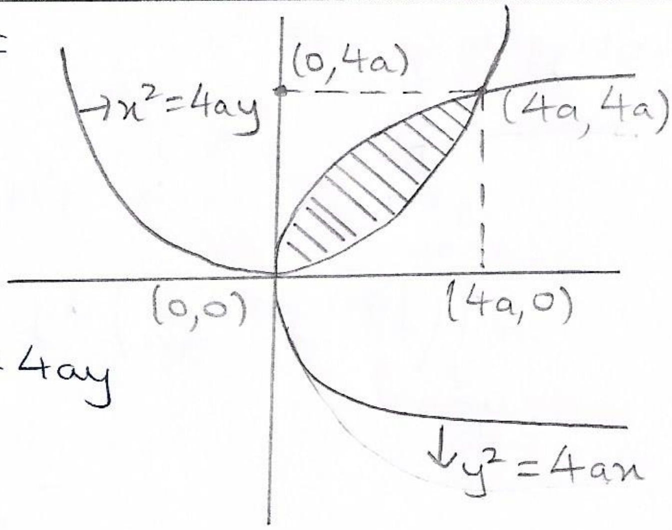
Thus, $$\beta(m,n) = \frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)}$$

7

Q. No.	Solution	Marks
8a.	In polars, $x = r\cos\theta$, $y = r\sin\theta$ $\therefore x^2+y^2 = r^2$ and $dx\,dy = r\,dr\,d\theta$	1
	Since, x, y varies from 0 to ∞, r also varies from 0 to ∞. In the first quadrant θ varies from 0 to $\pi/2$.	
	Hence, $I = \int_{\theta=0}^{\pi/2}\int_{r=0}^{\infty} e^{-r^2}\, r\,dr\,d\theta$	1
	put, $r^2 = t$. $\therefore r\,dr = dt/2$ t also varies from 0 to ∞.	
	$I = \int_{\theta=0}^{\pi/2}\int_{t=0}^{\infty} e^{-t}\, dt/2\, d\theta$	1
	$= \frac{1}{2}\int_{\theta=0}^{\pi/2}\left[e^{-t}\right]_{t=0}^{\infty} d\theta = -\frac{1}{2}\int_{\theta=0}^{\pi/2}(-1)\,d\theta$	2
	$= \frac{1}{2}\left[\theta\right]_{0}^{\pi/2} = \pi/4$	
	$\boxed{\therefore I = \pi/4}$	1
		6
8b.	Let, $I = \int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta = \int_0^{\pi/2}\frac{\sqrt{\cos\theta}}{\sqrt{\sin\theta}}\,d\theta$	1
	$= \int_0^{\pi/2}\sin^{-1/2}\theta\,\cos^{1/2}\theta\,d\theta$	1
	$I = \frac{1}{2}\beta\left(\frac{-1/2+1}{2}, \frac{1/2+1}{2}\right)$	2
	$I = \frac{1}{2}\beta(1/4, 3/4) = \frac{1}{2}\frac{\Gamma(1/4)\cdot\Gamma(3/4)}{\Gamma(1)} = \frac{1}{2}\pi\sqrt{2}$	3
	$\boxed{I = \pi/\sqrt{2}}$	
		7

Q. No.	Solution	Marks

8c. Step 1: Find points of intersection



From, $y^2 = 4ax$

$$x = y^2/4a$$

substitute into $x^2 = 4ay$

$$\left(\frac{y^2}{4a}\right)^2 = 4ay$$

$$\Rightarrow \frac{y^4}{16a^2} = 4ay$$

$$y^4 = 64a^3y \Rightarrow y^4 - 64a^3y = 0$$

$$y(y^3 - 64a^3) = 0 \Rightarrow y = 0$$

$$\& \; y^3 - 64a^3 = 0 \text{ or } y = 4a$$

2

At, $y = 0$; $x = 0$

At, $y = 4a$: $x = \frac{(4a)^2}{4a} = 4a$

So, the region is bounded between the points (0,0) and (4a,4a). Hence, the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ are (0,0) and (4a,4a).

On changing the order of integration, we have y varying from 0 to 4a (vertical strip) and x varying from $y^2/4a$ to $\sqrt{4ay}$ (horizontal strip) i.e. left curve $x = y^2/4a$ and right curve $x = \sqrt{4ay}$.

1

Area is $$A = \int_{y=0}^{4a} \int_{x=y^2/4a}^{\sqrt{4ay}} dx\,dy$$

1

Q. No.	Solution	Marks
	Integrating wrt 'x',	
	$A = \int_{y=0}^{4a} [x]_{x=y^2/4a}^{\sqrt{4ay}} \, dy$	
	$= \int_{y=0}^{4a} \left(\sqrt{4ay} - \frac{y^2}{4a}\right) dy = 2\sqrt{a}\int_{y=0}^{4a} \sqrt{y}\, dy - \frac{1}{4a}\int_0^{4a} y^2\, dy$	
	$\int_0^{4a} \sqrt{y}\, dy = \left[\frac{2}{3} y^{3/2}\right]_0^{4a} = \frac{2}{3}(4a)^{3/2} = \frac{2}{3} \times 8 a^{3/2}$	
	$= \frac{16}{3} a^{3/2}$	
	$\int_0^{4a} y^2\, dy = \left[\frac{y^3}{3}\right]_0^{4a} = \frac{(4a)^3}{3} = \frac{64a^3}{3}$	
	$A = 2\sqrt{a} \times \frac{16}{3} a^{3/2} - \frac{1}{4a} \times \frac{64a^3}{3}$	2
	$= \frac{32}{3}a^2 - \frac{64a^2}{12} = \frac{16a^2}{3}$	
	Thus, $A = \frac{16a^2}{3}$	1
		7

Q. No.	Solution	Marks
9a.	$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$	
	$R_1 \leftrightarrow R_2$ $\quad A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$	1
	$R_2 \to R_2 - 2R_1, \; R_3 \to R_3 - R_1$ $\quad A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$	1
	$R_2 \leftrightarrow R_4$ $\quad A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & -5 & -9 & 1 \end{bmatrix}$	1
	$R_3 \to R_3 + 2R_2 \;, \; R_4 \to R_4 + 5R_2$ $\quad A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & -4 \end{bmatrix}$	1
	$R_3 \leftrightarrow R_4$ and $R_3 \to -\frac{1}{4} R_3$ $\quad A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	1
	$\therefore \; \rho[A] = 3$	1
		6

Q. No.	Solution	Marks

9b.

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

2

$R_3 \to R_3 - R_2$

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

(a) We must have $\rho[A] = \rho[A:B] = 3$, $\rho[A]$ will be 3 if $(\lambda-3) \neq 0$ since the other two entries in the last row of A are zero. 1

If, $(\lambda-3) \neq 0$ or $\lambda \neq 3$ irrespective of the value of μ, $\rho[A:B]$ will also be 3.
Hence, the system will have unique solution if $\lambda \neq 3$.

(b) Infinite solutions: Here we have $n=3$ & we need $\rho[A] = \rho[A:B] = r < 3$. We must have $r=2$ since first row and second row are non zero.

$\therefore \rho[A] = \rho[A:B] = 2$ only when the last row of $[A:B]$ is completely zero. This is possible if $\lambda - 3 = 0$, $\mu - 10 = 0$. Hence, the system will have infinite solution if $\lambda = 3$ and $\mu = 10$. 2

(c) No solution: We must have $\rho[A] \neq \rho[A:B]$

Q. No.	Solution	Marks

By case (a) $\rho[A] = 3$ if $\lambda \neq 3$ and hence if $\lambda = 3$ we obtain $\rho[A] = 2$.

If, we impose $(\mu - 10) \neq 0$ then $\rho[A:B]$ will be 3, — **2**

Hence, the system has no solution if $\lambda = 3$ and $\mu \neq 10$. — **7**

9c.

$$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$ — **1**

$$AX^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 7.34 \\ -2.67 \\ 4.01 \end{bmatrix}$$ — **1**

$$= 7.34 \begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 7.82 \\ -3.63 \\ 4.01 \end{bmatrix}$$ — **1**

$$= 7.82 \begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \begin{bmatrix} 7.94 \\ -3.89 \\ 3.99 \end{bmatrix}$$ — **1**

$$= 7.49 \begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \lambda^{(4)} X^{(4)}$$

Q. No.	Solution	Marks
	$Ax^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 7.98 \\ -3.97 \\ 3.99 \end{bmatrix}$	1
	$= 7.98\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(5)} X^{(5)}$.	
	$Ax^{(5)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 4 \end{bmatrix}$	1
	$= 8\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(6)} X^{(6)}$	
	Thus, the dominant eigen value is 8 and the corresponding eigenvector is $[1, -0.5, 0.5]$ or $[2, -1, 1]'$ equivalently.	1
		7
10a	$[A : B] = \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$	1
	$R_2 \to R_2 - R_1, \; R_3 \to R_3 - 2R_1$	
	$[A : B] \sim \left[\begin{array}{ccc\|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$	2
	$R_1 \to 3R_1 + R_2, \; R_3 \to R_2 - 3R_3$	
	$[A : B] \sim \left[\begin{array}{ccc\|c} 3 & 0 & 5 & 26 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 11 & 44 \end{array}\right]$	2
	$\sim \left[\begin{array}{ccc\|c} 3 & 0 & 5 & 26 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right] \; R_3 \to R_3/11$	

Q. No.	Solution	Marks
	$R_1 \to R_1 - 5R_3, \quad R_2 \to R_2 - 2R_3$	
	$\sim \left[\begin{array}{ccc\|c} 3 & 0 & 0 & 6 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$	1
	Hence, we have $3x = 6, \ -3y = -9, \ z = 4$	1
	Thus, $x = 2, \ \underline{y = 3}, \ z = 4.$	7
10b.	$x = \frac{1}{20}[17 - y + 2z]$	
	$y = \frac{1}{20}[-18 - 3x + z]$	1
	$z = \frac{1}{20}[25 - 2x + 3y]$	
	Let, $x = 0, y = 0, z = 0.$	1
	$\therefore x^{(1)} = \frac{17}{20} = 0.85, \ y^{(1)} = -1.0275, \ z^{(1)} = 1.0109$	
	Second iteration : $x^{(2)} = 1.0025, \ y^{(2)} = -0.9998$	2
	$z^{(2)} = [25 - 2(1.0025) + 3(-0.9998)]$	
	$= 0.9998$	
	Third iteration:	2
	$x^{(3)} = \frac{1}{20}[17 - (-0.9998) + 2(0.9998)] = 0.99997 \simeq 1$	
	$y^{(3)} = \frac{1}{20}[-18 - 3(0.99997) + 0.9998]$	
	$= -1.0000055 \simeq -1$	
	$z^{(3)} = \frac{1}{20}[25 - 2(0.99997) + 3(-1.0000055)]$	2
	$= 1.0000022 \simeq 1$	
	Thus, $x = 1, \ \underline{y = -1}, \ z = 1$	1
		8

Q. No.	Solution	Marks
10c.	`A = np.matrix([[1,2,-1],[2,1,4],[3,3,4]])`	1
	`B = np.matrix([[1],[2],[3]])`	
	`AB = np.concatenate((A,B), axis=1)`	1
	`rA = np.linalg.matrix_rank(A)`	
	`rAB = np.linalg.matrix_rank(BA)`	1
	`n = A.shape[1]`	
	`if (rA == rAB):`	
	`    if (rA == n):`	1
	`        print("The system has unique solution")`	
	`        print(np.linalg.solve(A,B))`	
	`    else:`	
	`        print("The system has infinitely many solutions")`	1
	`else:`	
	`    print("The system of equations is inconsistent")`	5

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