KLS Vishwanathrao Deshpande Institute of Technology

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(Approved by AICTE, New Delhi, Affiliated to VTU, Belagavi)

(Recognized Under Section 2(f) by UGC, New Delhi)

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	Dr. MEENAL M. KALIWAL
Course Name	:	AV Mathematics III for EC Engg.
Course Code	:	BMATEC301
Year of Question Paper	:	DEC. 2024 | JAN 2025
Date of Submission	:	16-06-2025

Faculty Member　　　HoD　　　Dean (Acad.)

Head of the Department
Dept. of Electronic & Communication Engg.
KLS V.D.I.T., HALIYAL (U.K.)

CBCS SCHEME

USN 2VD23EC098

BMATEC301/BEC301/BBM301

Third Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
AV Mathematics III for EC/ BM Engineering

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Statistical table and Mathematics formula handbook are allowed.
3. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Obtain the Fourier series of $f(x)=\frac{\pi-x}{2}$ in $0<x<2\pi$. Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+.....=\frac{\pi}{4}$.	6	L2	CO1
	b.	Find the Fourier series of $f(x)=\lvert x\rvert$ in $(-\ell\ \ \ell)$. Hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+......=\frac{\pi^2}{8}$.	7	L3	CO1
	c.	Expand $f(x)=2x-1$ as a cosine half range Fourier series in $0<x<1$.	7	L2	CO1
		OR			
Q.2	a.	Find the Fourier series of $f(x)=\begin{cases}1+\frac{2x}{\pi} & \text{in } -\pi<x<0\\ 1-\frac{2x}{\pi} & \text{in } 0<x<\pi\end{cases}$. Hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+......$	6	L2	CO1
	b.	Obtain the sine half range series of, $f(x)=\begin{cases}\frac{1}{4}-x & \text{in } 0<x<\frac{1}{2}\\ x-\frac{3}{4} & \text{in } \frac{1}{2}<x<1\end{cases}$	7	L2	CO1
	c.	Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data :	7	L1	CO1

x°:	0	45	90	135	180	225	270	315
y:	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$

		Module – 2	M	L	C
Q.3	a.	Find the Fourier transform of the function, $f(x)=\begin{cases}1 & \text{for } \lvert x\rvert\le a\\ 0 & \text{for } \lvert x\rvert>a\end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$.	6	L2	CO2
	b.	Find the Fourier sine and cosine transforms of $f(x)=e^{-\alpha x}$, $\alpha>0$.	7	L2	CO2
	c.	Find the Fourier sine transform of $\frac{e^{ax}}{x}$, $a>0$.	7	L3	CO2

		OR			
Q.4	a.	If $f(x) = \begin{cases} 1-x^2, & \lvert x \rvert < 1 \\ 0, & \lvert x \rvert \geq 1 \end{cases}$, find the Fourier transform of f(x) and hence find the value of, $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} dx$.	6	L2	CO2
	b.	Find the Fourier sine transform of $f(x) = e^{-\lvert x \rvert}$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx,\ m > 0$.	7	L3	CO2
	c.	Find the Discrete fast fourier of signal $= (0, 1, 4, 9)^T$	7	L3	CO2
		Module – 3			
Q.5	a.	Find the z-transform of, (i) $\cosh n\theta$ (ii) $\sinh n\theta$	6	L1	CO3
	b.	If $V(z) = \frac{2z^2+3z+12}{(z-1)^4}$, evaluate u_0, u_1 and u_2	7	L2	CO3
	c.	Find the inverse z-transform of, $\frac{z}{(z-1)(z-2)}$.	7	L2	CO3
		OR			
Q.6	a.	Solve by using z-transforms, $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = 0 = y_1$	6	L3	CO3
	b.	Find $z_T^{-1}\left[\frac{5z}{(3z-1)(2-z)}\right]$.	7	L2	CO3
	c.	Solve by using z-transforms $u_{n+2} - 5u_{n+1} + 6u_n = 2^n$ with $u_0 = 0 = u_1$.	7	L3	CO3
		Module – 4			
Q.7	a.	Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$.	6	L1	CO4
	b.	Solve $(D^2+1)y = x^2 + 4x - 6$.	7	L2	CO4
	c.	Using the method of variation of Parameters of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$	7	L3	CO4
		OR			
Q.8	a.	Solve $6\frac{d^2y}{dx^2} + 17\frac{dy}{dx} + 12y = e^{-x}$.	6	L2	CO4
	b.	Solve the Cauchy's differential equation, $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x)$.	7	L2	CO4
	c.	The charge q in a series circuit containing an Inductance L, Capacitance C, emf E satisfy the differential equation, $L\frac{d^2q}{dt^2} + \frac{q}{C} = E$. Express q in terms of t.	7	L3	CO4
		Module – 5			
Q.9	a.	Fit a second degree parabola $y = a + bx + cx^2$ into least square sense for the data and estimate y at x = 6. x: 1, 2, 3, 4, 5 y: 10, 12, 13, 16, 19	6	L1	CO5

<table>
<tr><td></td><td>b.</td><td>Find a correlation coefficient for the two variables x and y.
<table><tr><td>x:</td><td>92</td><td>89</td><td>87</td><td>86</td><td>83</td><td>77</td><td>71</td><td>63</td><td>53</td><td>50</td></tr><tr><td>y:</td><td>86</td><td>83</td><td>91</td><td>77</td><td>68</td><td>85</td><td>52</td><td>82</td><td>37</td><td>57</td></tr></table></td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>Ten students got the following percentage of marks in two subjects x and y. Compute the rank correlation coefficient.
<table><tr><td>x:</td><td>78</td><td>36</td><td>98</td><td>25</td><td>75</td><td>82</td><td>90</td><td>62</td><td>65</td><td>39</td></tr><tr><td>y:</td><td>84</td><td>51</td><td>91</td><td>60</td><td>68</td><td>62</td><td>86</td><td>58</td><td>53</td><td>47</td></tr></table></td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td colspan="6" align="center">OR</td></tr>
<tr><td>Q.10</td><td>a.</td><td>If θ is the angle between the lines of regression show that $\tan\theta = \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\left(\frac{1-r^2}{r}\right)$.</td><td>6</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b.</td><td>Obtain the lines of regression and hence find the coefficient of correlation for the data,
<table><tr><td>x:</td><td>1</td><td>3</td><td>4</td><td>2</td><td>5</td><td>8</td><td>9</td><td>10</td><td>13</td><td>15</td></tr><tr><td>y:</td><td>8</td><td>6</td><td>10</td><td>8</td><td>12</td><td>16</td><td>16</td><td>10</td><td>32</td><td>32</td></tr></table></td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>If $8x-10y+66=0$ and $40x-18y=214$ are the two regression lines. Find $\bar{x}$, $\bar{y}$ and r. Find σ_y if $\sigma_x=3$.</td><td>7</td><td>L2</td><td>CO5</td></tr>
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AY: 2025-26

Department: Electronics & Communication Engg

Subject with Sub. Code: AV Mathematics-III for EC Engineering (BMATEC301) **Max. Marks: 100**

Semester / Branch / Division: III / ECE / A & B **Name of Faculty: Dr. Meenal Kaliwal**

Q.No.	Solution and Scheme	Marks
1a.	The Fourier series of $f(x)$ having period 2π is given by, $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$ $$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} \frac{\pi - x}{2}\,dx$$ $$= \frac{1}{2\pi}\left[\pi x - x^2/2\right]_{x=0}^{2\pi} = \frac{1}{2\pi}\left[2\pi^2 - 2\pi^2 - 0\right]$$ $$\Rightarrow \boxed{a_0 = 0}$$ $$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)\cos nx\,dx$$ Applying Bernoulli's rule of integration, $$a_n = \frac{1}{2\pi}\left[(\pi - x)\frac{\sin nx}{n} - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_{x=0}^{2\pi}$$ $$= \frac{1}{2\pi}\left[0 - \frac{\cos nx}{n^2}\right]_{x=0}^{2\pi} \quad \because \sin 2n\pi = 0 \ \& \ \sin 0 = 0$$ $$= -\frac{1}{2\pi n^2}\left[\cos 2n\pi - \cos 0\right] = 0$$ $$\therefore \boxed{a_n = 0}$$	2

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Q. No.	Solution	Marks
	$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\, dx$	
	$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi-x}{2}\right)\sin nx\, dx$	
	$= \frac{1}{2\pi}\left[(\pi-x)\left(-\frac{\cos nx}{n}\right)-(-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{x=0}^{2\pi}$	
	$=\frac{-1}{2\pi}\left[\frac{1}{n}\{(\pi-x)\cos n\pi\}\right]$ $\because \sin 2n\pi = 0$	2
	$= \frac{-1}{2n\pi}\{-\pi\cos 2n\pi - \pi\cos 0\}$	
	$b_n = -\frac{1}{2n\pi}[-\pi-\pi]$ $\cos 2n\pi = +1$	
	$= \frac{2\pi}{2n\pi} = \frac{1}{n}$	
	$\therefore f(x) = \sum_{n=1}^{\infty}\frac{1}{n}\sin nx \longrightarrow (1)$	
	To deduce the series : put $x = \pi/2$ in result (1)	
	$f(\pi/2) = \frac{\pi-\pi/2}{2} = \sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi/2)$	
	$\frac{\pi}{4} = \frac{\sin(\pi/2)}{1} + \frac{\sin\pi}{2} + \frac{\sin(3\pi/2)}{3} + \frac{\sin(4\pi)}{4} + \frac{\sin(5\pi/2)}{5} + \cdots$	2
	Thus, $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$	
		6

Q. No.	Solution	Marks
1b.	The period of $f(x) = l-(-l) = 2l$.	
	The Fourier series of period $2l$ is given by,	
	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$	
	To check $\lvert x\rvert$ is even or odd function.	
	$f(-x) = \lvert -x\rvert = \lvert x\rvert = f(x)$	
	Hence, $f(x)$ is even and $b_n = 0$.	
	$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{l}\int_0^l x\,dx = \frac{2}{l}\left[\frac{x^2}{2}\right]_{x=0}^{l}$	2
	$a_0 = \frac{1}{l}\left[l^2 - 0\right] = l$	
	$\Rightarrow \boxed{\frac{a_0}{2} = \frac{l}{2}}$	
	$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$	
	$= \frac{2}{l}\int_0^l x\cos\frac{n\pi x}{l}\,dx$	
	$= \frac{2}{l}\left\{x\frac{\sin n\pi x/l}{n\pi/l} - 1\times\frac{(-\cos n\pi x/l)}{(n\pi/l)^2}\right\}_{x=0}^{l}$	
	$= \frac{2}{l}\times\frac{l^2}{n^2\pi^2}\left[\cos n\pi x/l\right]_{x=0}^{l}$ $\left[\sin\frac{n\pi x}{l} = \sin\frac{n\pi \cancel{l}}{\cancel{l}} = \sin n\pi = 0\right.$	
	$a_n = \frac{2l}{n^2\pi^2}\left[\cos n\pi - \cos 0\right] = \frac{2l}{n^2\pi^2}\left[(-1)^n - 1\right]$	
	$\therefore\ a_n = -\frac{2l}{n^2\pi^2}\left\{1-(-1)^n\right\}$	2

Q. No.	Solution	Marks
	$\therefore f(x) = \frac{l}{2} + \sum_{n=1}^{\infty} -\frac{2l}{n^2\pi^2}\{1-(-1)^n\} \cos \frac{n\pi x}{l}$	1
	To deduce the series put $x=0$,	
	$f(x=0) = 0 = \frac{l}{2} + \sum_{n=1}^{\infty} -\frac{2l}{n^2\pi^2}\{1-(-1)^n\}$	
	$-\frac{l}{2} = -\frac{2l}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\{1-(-1)^n\}$	1
	$\therefore \frac{\pi^2}{4} = \sum_{n=1,3,5,---}^{\infty} \frac{2}{n^2}$	
	$\frac{\pi^2}{8} = \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+----$	
		6
1c.	Comparing $(0,1) \equiv (0,l) \Rightarrow l=1$	
	The cosine half range Fourier series is,	
	$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x$	1
	where $a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{1}\int_0^1 (2x-1)\,dx$	
	$= 2\left[2.\frac{x^2}{2} - x\right] = 2\left[x^2-x\right]_0^1 = 2(1-1)$	2
	$\therefore a_0 = 0 \Rightarrow \boxed{a_0/2 = 0}$	
	$a_n = \frac{2}{l}\int_0^l f(x)\cos n\pi x\,dx$	
	$= 2\int_0^1 (2x-1)\cos n\pi x\,dx$	
	Applying Bernoulli's rule of integration,	

Q. No.	Solution	Marks
	$= 2\left[(2x-1)\dfrac{\sin n\pi x}{n\pi} - \dfrac{2(-\cos n\pi x)}{n^2\pi^2}\right]_{x=0}^{1}$	
	$= \dfrac{4}{n^2\pi^2}\left[\cos n\pi x\right]_{x=0}^{1}$ $\quad [\because \sin n\pi = 0 = \sin 0]$	3
	$a_n = \dfrac{4}{n^2\pi^2}\left[\cos n\pi - \cos 0\right] = \dfrac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$	
	$a_n = -\dfrac{4}{n^2\pi^2}\left\{1-(-1)^n\right\}$	
	$\therefore f(x) = \displaystyle\sum_{n=1}^{\infty} -\dfrac{4}{n^2\pi^2}\left\{1-(-1)^n\right\}\cos n\pi x$	1
		7
2a.	The Fourier series of $f(x)$ in $(-\pi,\pi)$ is	
	$f(x) = \dfrac{a_0}{2} + \displaystyle\sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$	
	$a_0 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\,dx = \dfrac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right\}$	
	$= \dfrac{1}{\pi}\left\{\displaystyle\int_{-\pi}^{0}\left(1+\dfrac{2x}{\pi}\right)dx + \int_{0}^{\pi}\left(1-\dfrac{2x}{\pi}\right)dx\right\}$	
	$= \dfrac{1}{\pi}\left\{\left(x+\dfrac{x^2}{\pi}\right)_{x=-\pi}^{0} + \left(x-\dfrac{x^2}{\pi}\right)_{x=0}^{\pi}\right\}$	1
	$= \dfrac{1}{\pi}\left\{0-\left(-\pi+\dfrac{\pi^2}{\pi}\right)+0-\left(\pi-\pi^2/\pi\right)\right\}$	
	$= \dfrac{1}{\pi}\left\{-(-\pi+\pi)+(\pi-\pi)\right\}$	
	$a_0 = 0 \Rightarrow \boxed{\dfrac{a_0}{2} = 0}$	

Q. No.	Solution	Marks
	$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right\}$	
	$= \frac{1}{\pi}\left\{\int_{-\pi}^{0}\left(1+\frac{2x}{\pi}\right)\sin nx\,dx + \int_{0}^{\pi}\left(1-\frac{2x}{\pi}\right)\sin nx\,dx\right\}$	
	$= \frac{1}{\pi}\left\{\left(1+\frac{2x}{\pi}\right)\left(-\frac{\cos nx}{n}\right) - \frac{2}{\pi}\left(\frac{-\sin nx}{n^2}\right)\Big\|_{x=-\pi}^{0} + \left(1-\frac{2x}{\pi}\right)\left(-\frac{\cos nx}{n}\right) - \left(-2/\pi\right)\left(-\frac{\sin nx}{n^2}\right)\Big\|_{x=0}^{\pi}\right\}$	
	$= \frac{1}{\pi}\left\{\left(1+\frac{2x}{\pi}\right)\left(-\frac{\cos nx}{n}\right)\Big\|_{x=-\pi}^{0} - \left(1-\frac{2x}{\pi}\right)\left(\frac{\cos nx}{n}\right)\Big\|_{x=0}^{\pi}\right.$	
	$= -\frac{1}{n\pi}\left\{\cos 0 - (1-2)\cos n\pi + (1-2)\cos n\pi - \cos 0\right\}$	2
	$b_n = -\frac{1}{n\pi}\left\{1 + \cos n\pi - \cos n\pi - 1\right\}$	
	$\boxed{\therefore b_n = 0}$	
	$f(x) = \frac{a_0}{2} + \sum \frac{4}{\pi^2 n^2}\left\{1-(-1)^n\right\}\cos nx$	
	But, $1-(-1)^n = \begin{cases} 1-(+1) = 0, & \text{if } n \text{ is even} \\ 1-(-1) = 2, & \text{if } n \text{ is odd} \end{cases}$	
	$f(x) = \frac{8}{\pi^2}\sum_{n=1,3,5,7,---}^{\infty} \frac{\cos nx}{n^2}$	
	$= \frac{8}{\pi^2}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ---\right]$	
	Now, put $x = 0$ in $f(x) \Rightarrow f(x) = 1$	

Q. No.	Solution	Marks
	$\therefore 1 = \frac{8}{\pi^2}\left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right)$	
	$\frac{\pi^2}{8} = \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$	1
		6
2b.	Sine half range series is	
	$f(x) = \sum_{n=1}^{\infty} b_n \sin n x \pi \longrightarrow (i)$	1
	$b_n = 2\int_0^1 f(x) \sin n\pi x \, dx$	
	$= 2\left\{\int_0^{1/2}\left(\frac{1}{4}-x\right)\sin n\pi x \, dx + \int_{1/2}^{1}\left(x-\frac{3}{4}\right)\sin n\pi x \, dx\right\}$	1
	Applying Bernoulli's rule of integration by parts,	
	$= 2\left\{\left[\left(\frac{1}{4}-x\right)\left(\frac{-\cos n\pi x}{n\pi}\right)-(-1)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right]_0^{1/2} + \left[\left(x-\frac{3}{4}\right)\left(-\frac{\cos n\pi x}{n\pi}\right)-\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right]_{1/2}^{1}\right\}$	1
	$= 2\left\{\frac{1}{4n\pi}\left(\cos\frac{n\pi}{2}+1\right)-\frac{1}{n^2\pi^2}\sin\frac{n\pi}{2} + \frac{1}{4n\pi}\left(-\cos n\pi-\cos\frac{n\pi}{2}\right)-\frac{1}{n^2\pi^2}\sin\frac{n\pi}{2}\right\}$	1
	$= 2\left\{\frac{1}{4n\pi}(1-\cos n\pi)-\frac{2}{n^2\pi^2}\sin n\pi/2\right\}$	
	$= \frac{1}{2n\pi}\left\{1-(-1)^n\right\} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$	1
	Putting this into (i), we get	

Q. No.	Solution	Marks
	$f(x) = \sum_{n=1}^{\infty} \left\{ \frac{1}{2n\pi}\left[1-(-1)^n\right] - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} \right\} \sin n\pi x$	1
		6
2c.	The interval of x is $0 \le x \le 2\pi$. We have to calculate a_0, a_1 & b_1.	

x°	y	$y\cos x$	$y \sin x$
0	2	2	0
45	1.5	1.0607	1.0607
90	1	0	1
135	0.5	-0.3535	0.3535
180	0	0	0
225	0.5	-0.3535	-0.3535
270	1	0	-1
315	1.5	1.0607	-1.0607
Total	8	3.4144	0

3

$$a_0 = \frac{2}{N}\sum y = \frac{2}{8}(8) = 2$$

$$a_1 = \frac{2}{N}\sum y\cos x = \frac{2}{8}(3.4144) = 0.8536$$

$$b_1 = \frac{2}{N}\sum y \sin x = \frac{2}{8}(0) = 0$$

3

$\therefore$ Constant term $= a_0 = 2$

First cosine term $= a_1 = 0.8536$

First sine term $= b_1 = 0$

1

7

Q. No.	Solution	Marks
3a.	$F(u) = \int_{x=-\infty}^{\infty} f(x) e^{iux}\, dx$	
	$= \int_{-a}^{a} 1 \cdot e^{iux}\, dx$, since, $f(x) = \begin{cases} 1, & -a \le x \le a \\ 0, & \text{otherwise} \end{cases}$	
	$F(u) = \left[\frac{e^{iux}}{iu} \right]_{x=-a}^{a} = \frac{1}{iu} \left\{ e^{iua} - e^{-iua} \right\}$	
	$F(u) = \frac{1}{iu} \left\{ (\cos au + i \sin au) - (\cos au - i \sin au) \right\}$	
	$= \frac{1}{iu} (2i \sin au) = \frac{2 \sin au}{u}$	2
	$\therefore F(u) = \frac{2 \sin au}{u}$	
	Inverse Fourier tansform,	
	$\frac{1}{2\pi} \int_{-\infty}^{\infty} F(u) e^{-iux}\, dx = f(x)$	
	$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2 \sin au}{u} e^{-iux}\, du$	1
	$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin au}{u} \cdot e^{-iux}\, du$	
	put, $x = 0$, $f(x) = 1$, for $\lvert x \rvert \le a$.	
	Hence, $\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin au}{u}\, du = 1$, since $e^0 = 1$	
	$\frac{2}{\pi} \int_{0}^{\infty} \frac{\sin au}{u}\, du = 1$, since $\frac{\sin au}{u}$ is an even function of u	2
	$\therefore \int_{0}^{\infty} \frac{\sin au}{u}\, du = \frac{\pi}{2}$	

Q. No.	Solution	Marks
	putting $a=1$, $\int_0^\infty \frac{\sin u}{u}\, du = \pi/2$	
	$\boxed{\int_0^\infty \frac{\sin x}{x}\, dx = \pi/2}$	1
		6
3b.	Fourier cosine transform is, $F_c[e^{-\alpha x}] = \int_0^\infty e^{-\alpha x} \cos ux\, dx$	
	$F_c[e^{-\alpha x}] = \left[\frac{e^{-\alpha x}}{\alpha^2+u^2}(-\alpha \cos ux + u \sin ux)\right]_{x=0}^{\infty}$	1
	$= \frac{1}{\alpha^2+u^2}\left\{e^{-\infty} - e^0(-\alpha \cos 0 + 0)\right\}$	1
	$\boxed{F_c(u) = \frac{\alpha}{\alpha^2+u^2}}$	1
	Fourier sine transform is, $F_s(u) = \int_0^\infty f(x) \sin ux\, dx$	
	$= \int_0^\infty e^{-\alpha x} \sin ux\, dx$	1
	$= \left[\frac{e^{-\alpha x}}{\alpha^2+u^2}(-\alpha \sin ux - u \cos ux)\right]_0^\infty$	1
	$= \frac{1}{\alpha^2+u^2}\left[-\alpha(e^{-\infty} - 0) - u(e^{-\infty} - \cos 0)\right]$	
	$= \frac{1}{\alpha^2+u^2}\left[-\alpha(0) + u\right]$	
	$\boxed{\therefore F_s(u) = \frac{u}{\alpha^2+u^2}}$	1
		6

3c. Fourier Sine transform is,

$$F_s(u) = \int_0^{\infty} f(x) \sin ux \, dx$$

$$F_s(u) = \int_0^{\infty} \frac{e^{-ax}}{x} \sin ux \, dx \longrightarrow (i)$$ 1

Using differentiation under integral sign rule,

$$\frac{d}{du}\left[F_s(u)\right] = \int_0^{\infty} \frac{\partial}{\partial u}(\sin ux) \frac{e^{-ax}}{x} dx$$

$$= \int_0^{\infty} \frac{e^{-ax}}{x} \times x \cos ux \, dx$$ 1

$$= \int_0^{\infty} e^{-ax} \cos ux \, dx$$

$$= \left[\frac{e^{-ax}}{a^2+u^2}(-a \cos ux + u \sin ux)\right]_{x=0}^{\infty}$$ 1

$$= \frac{1}{a^2+u^2}\left[-a e^{-ax} \cos ux + u e^{-ax} \sin ux\right]_{x=0}^{\infty}$$

$$= \frac{1}{a^2+u^2}\left\{-a\left[\overset{(0)}{e^{-\infty}} - e^0 \cos 0\right] + u\left[e^{-\infty} - \sin 0\right]\right\}$$

$$= \frac{1}{a^2+u^2}\left[a + u(0)\right] = \frac{a}{a^2+u^2}$$ 2

$$\frac{d}{du}\left[F_s(u)\right] = \frac{a}{a^2+u^2}$$

$$F_s(u) = \tan^{-1}(u/a) + c,$$ on integrating wrt 'u' 1

To evaluate c, put $u = 0$

$$\therefore F_s(0) = \tan^{-1}(0) + c$$

Q. No.	Solution	Marks
	To evaluate $F_s(0)$ using eqn (i),	
	$F_s(u=0) = \int_0^\infty \frac{e^{-ax}}{x} \sin 0 \, dx$	
	$\Rightarrow F_s(0) = 0$	
	Thus, $F_s(u) = \tan^{-1}(u/a)$	1
		7
4a.	Here, $\lvert x\rvert \le 1 \Rightarrow -1 < x < 1$	
	$\lvert x\rvert > 1 \Rightarrow x \in (-\infty, \infty)$	
	$F(u) = \int_{-\infty}^{\infty} f(x) e^{iux} dx$	
	$= \int_{-1}^{1} (1-x^2) e^{iux} dx$	
	$= \left[(1-x^2)\frac{e^{iux}}{iu} - (-2x)\frac{e^{iux}}{i^2u^2} + (-2)\frac{e^{iux}}{i^3u^3} \right]_{x=-1}^{1}$	1
	$= \left[-\frac{2}{u^2}(xe^{iux}) + \frac{2}{iu}(e^{iux}) \right]_{x=-1}^{1}$	
	$= -\frac{2}{u^2}\{e^{iu} - (-1)e^{-iu}\} - \frac{2i}{u^3}(e^{iu} - e^{-iu})$	
	$= -\frac{2}{u^2}\{e^{iu} + e^{-iu}\} - \frac{2i}{u^3}(e^{iu} - e^{-iu})$	
	$= -\frac{2}{u^2}(2\cos u) - \frac{2i}{u^3}(2i \sin u)$	
	$= -\frac{4\cos u}{u^2} + \frac{4 \sin u}{u^3}$	
	$\therefore F(u) = \frac{4(\sin u - u\cos u)}{u^3}$	2
	Also, by Inverse Fourier transform, we have	

Q. No.	Solution	Marks
	$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(u) e^{-iux}\, du \longrightarrow (1)$	
	putting $x = 1/2$ in (1) we have,	
	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{4(\sin u - u\cos u)}{u^3} e^{-iu/2}\, du = f(1/2)$	
	$= 1-\left(\frac{1}{2}\right)^2 = 3/4$	1
	$\frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin u - u\cos u}{u^3}\right)\left(\cos\frac{u}{2} - i\sin\frac{u}{2}\right) du = 3/4$	
	$\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{\sin u - u\cos u}{u^3}\cos\frac{u}{2}\, du = \frac{3\pi}{8}$	
	Equating real parts on both sides, we get	
	$\frac{2}{\pi}\int_{-\infty}^{\infty} \frac{\sin u - u\cos u}{u^3}\cos\frac{u}{2}\, du = 3/4$	
	$2\int_{0}^{\infty} \frac{\sin u - u\cos u}{u^3}\cos\left(\frac{u}{2}\right) du = \frac{3\pi}{8}$	
	[$\because$ integrand is even]	
	Replacing $\frac{u}{x}$ & dividing by 2, we get	
	$\int_{0}^{\infty} \frac{\sin x - x\cos x}{x^3}\cos\frac{x}{2}\, dx = \frac{3\pi}{16}$	2
		6

Q. No.	Solution	Marks

4b. Fourier sine transform is,

$$F_s(u) = \int_0^\infty f(x)\sin ux\,dx = \int_0^\infty e^{-|x|}\sin ux\,dx \qquad 1$$

$$= \int_0^\infty e^{-x}\sin ux\,dx \quad [\because |x| = x, \text{ since } x>0]$$

$$F_s(u) = \left[\frac{e^{-x}}{1+u^2}(-\sin ux - u\cos ux)\right]_{x=0}^{\infty} \qquad 1$$

$$= \frac{1}{1+u^2}\left\{-(e^{-\infty} - e^{0}\sin 0) - u(e^{-\infty} - \cos 0)\right\}$$

$$= \frac{1}{1+u^2}\{0 + u\} = u/1+u^2 \qquad 1$$

By Inverse Fourier sine transform,

$$\frac{2}{\pi}\int_0^\infty F_s(u)\sin ux\,du = f(x) \qquad 1$$

$$\frac{2}{\pi}\int_0^\infty \frac{u}{1+u^2}\sin ux\,du = f(x)$$

putting $x = m$ where $m>0$ we have

$$f(x) = e^{-|m|} = e^{-m}$$

$$\int_0^\infty \frac{u\sin mu}{1+u^2}\,du = \frac{\pi}{2}e^{-m} \qquad |m| = m,\ m>0 \qquad 1$$

By changing variable u to 'x',

$$\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx = \frac{\pi}{2}e^{-m} \qquad 1$$

6

Q. No.	Solution	Marks
4c.	$x[n] = \{0, 1, 49\}, N = 3$	
	$X[k] = \sum_{n=0}^{N-1} x[n] . e^{-j\frac{2\pi}{N}kn}$	1
	Let, $W_N = e^{-j\frac{2\pi}{N}}$, so $W_3 = e^{-j2\pi/3}$	
	$W_3^0 = 1$	1
	$W_3^1 = e^{-j2\pi/3} = -\frac{1}{2} - j\sqrt{3}/2$	
	$W_3^2 = e^{-j4\pi/3} = -\frac{1}{2} + j\sqrt{3}/2$	
	$X[0] = x[0].1 + x[1].1 + x[2].1 = 0+1+49 = 50$	1
	$X[1] = x[0].W_3^0 + x[1].W_3^1 + x[2].W_3^2$	
	$= 0 + 1\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + 49\left(-1/2 + j\sqrt{3}/2\right)$	1
	$= -\frac{50}{2} + j\frac{48\sqrt{3}}{2} = -25 + j24\sqrt{3}$	
	$X[2] = x[0].W_3^0 + x[1].W_3^2 + x[2].W_3^1$	
	$= 0 + 1.\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 49.\left(-\frac{1}{2} - j\sqrt{3}/2\right)$	
	$= -\frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{49}{2} - j\,49\frac{\sqrt{3}}{2}$	1
	$= -25 - j24\sqrt{3}$	
	$\therefore X[0] = 50,\ X[1] = -25 + j24\sqrt{3}$	1
	$= -25 + j41.57$	
	$X[2] = -25 - j24\sqrt{3} = -25 - j41.57$	1
		7

Q. No.	Solution	Marks
5a.	$\cosh n\theta = \frac{1}{2}\left[e^{n\theta}+e^{-n\theta}\right] = \frac{1}{2}\left[(e^{\theta})^n+(e^{-\theta})^n\right]$	
	$\cosh n\theta = \frac{1}{2}\{p^n+q^n\}$ where $p=e^{\theta}$, $q=e^{-\theta}$	1
	$Z_T[\cosh n\theta] = \frac{1}{2}\{Z_T(p^n)+Z_T(q^n)\}$	
	$= \frac{1}{2}\left\{\frac{z}{z-p}+\frac{z}{z-q}\right\}$	1
	using, $Z_T(k^n) = \frac{z}{z-k}$	
	$= \frac{z}{2}\left\{\frac{1}{z-e^{\theta}}+\frac{1}{z-e^{-\theta}}\right\}$	
	$= \frac{z}{2}\left\{\frac{z-e^{-\theta}+z-e^{\theta}}{(z-e^{\theta})(z-e^{-\theta})}\right\}$	
	$= \frac{z}{2}\left\{\frac{2z-(e^{\theta}+e^{-\theta})}{z^2-z(e^{\theta}+e^{-\theta})+1}\right\}$	
	$= \frac{z}{2}\left\{\frac{2z-2\cosh\theta}{z^2-2z\cosh\theta+1}\right\}$	2
	$Z_T[\cosh n\theta] = \frac{z(z-\cosh\theta)}{z^2-2z\cosh\theta+1}$	
	Next, $\sinh n\theta = \frac{e^{n\theta}-e^{-n\theta}}{2}$	
	$Z_T(\sinh n\theta) = \frac{z}{2}\left\{\frac{1}{z-e^{\theta}}-\frac{1}{z-e^{-\theta}}\right\}$	
	$= \frac{z}{2}\left\{\frac{z-e^{-\theta}-z+e^{\theta}}{(z-e^{\theta})(z-e^{-\theta})}\right\}$	
	$= \frac{z}{2}\left\{\frac{e^{\theta}-e^{-\theta}}{z^2-2z\cosh\theta+1}\right\}$	2
	$Z_T(\sinh n\theta) = \frac{z\sinh n\theta}{z^2-2z\cosh\theta+1}$	
		6

Q. No.	Solution	Marks

5b. We have, $u_0 = \lim_{z\to\infty} \bar{u}(z)$

$$= \lim_{z\to\infty} \frac{2z^2+3z+12}{(z-1)^4}$$

$$= \lim_{z\to\infty} \frac{z^2\left[2+3/z+12/z^2\right]}{z^4(1-1/z)^4}$$ 2

$$= 0 \times \frac{2}{1} = 0$$

$\therefore u_0 = 0$

$$u_1 = \lim_{z\to\infty} z\left[\bar{u}(z) - u_0\right]$$

$$= \lim_{z\to\infty} z\left\{\frac{2z^2+3z+12}{(z-1)^4} - 0\right\}$$

$$= \lim_{z\to\infty} z \times z^2 \frac{\left[2+3/z+12/z^2\right]}{z^4(1-1/z)^4}$$ 2

$u_1 = 0$

$$u_2 = \lim_{z\to\infty} z^2\left[\bar{u}(z) - u_0 - u_1/z\right]$$

$$= \lim_{z\to\infty} z^2\left[\frac{2z^2+3z+12}{(z-1)^4} - 0 - 0\right]$$

$$= \lim_{z\to\infty} \cancel{z^2} \times \cancel{z^2}\frac{\left[2+3/z+12/z^2\right]}{\cancel{z^4}(1-1/z)^4}$$

$$= \frac{2+0+0}{(1-0)^4} = 2$$

$\therefore u_2 = 2$ 2

6

Q. No.	Solution	Marks
5c.	Let, $\bar{u}(z) = \dfrac{z}{(z-1)(z-2)}$	1
	$\dfrac{\bar{u}(z)}{z} = \dfrac{1}{(z-1)(z-2)}$	1
	$\dfrac{1}{(z-1)(z-2)} = \dfrac{A}{z-1} + \dfrac{B}{z-2}$	1
	$1 = A(z-2) + B(z-1)$ put, $z=1 \Rightarrow A = -1$ $z=2 \Rightarrow B = 1$	
	$\therefore \bar{u}(z) = -\dfrac{z}{z-1} + \dfrac{z}{z+2}$	2
	$\therefore Z^{-1}[\bar{u}(z)] = Z^{-1}\left[\dfrac{z}{z+2}\right] - Z^{-1}\left[\dfrac{z}{z-1}\right]$	1
	$u_n = 2^n - 1$, using $Z_T^{-1}\left[\dfrac{z}{z-k}\right] = k^n$	1
		7
6a.	$Z_T[y_{n+2}] + 2Z_T[y_{n+1}] + Z_T[y_n] = Z_T(n)$	
	$z^2[\bar{y}(z) - y_0 - y_1 z^{-1}] + 2z[\bar{y}(z) - y_0] + \bar{y}(z) = \dfrac{z}{(z-1)^2}$	1
	$(z^2 + 2z + 1)\bar{y}(z) = \dfrac{z}{(z-1)^2}$, by using given values $y_0 = 0 = y_1$	
	$\bar{y}(z) = \dfrac{z}{(z-1)^2(z+1)^2}$	1
	Using partial fractions, $\dfrac{z}{(z-1)^2(z+1)^2} = A.\dfrac{z}{z-1} + B.\dfrac{z}{(z-1)^2} + C.\dfrac{z}{z+1} + D.\dfrac{z}{(z+1)^2} \rightarrow (1)$	
	put, $z=1 \Rightarrow B = 1/4$ & $z=-1$; $D = 1/4$	1

Q. No.	Solution	Marks

6c.

$$u_{n+2} - 5u_{n+1} + 6u_n = 2^n, \quad u_0 = 0 = u_1$$

$$Z_T[u_{n+2}] - 5Z_T[u_{n+1}] + 6Z_T[u_n] = Z_T[2^n]$$

$$z^2[\bar{u}(z) - u_0 - u_1/z] - 5[\bar{u}(z) - u_0]z$$ 1

$$+ 6\bar{u}(z) = \frac{z}{z-2}$$

$$(z^2 - 5z + 6)\bar{u}(z) = z^2u_0 + zu_1 - 5zu_0 + \frac{z}{z-2}$$ 1

$$\bar{u}(z) = \frac{z}{(z-2)} \times \frac{1}{(z-2)(z-3)}$$

$$\bar{u}(z) = \frac{z}{(z-3)(z-2)^2}$$

1

$$\frac{z}{(z-3)(z-2)^2} = \frac{A}{z-3} + \frac{B}{z-2} + \frac{C}{(z-2)^2}$$

$$z = A(z-2)^2 + B(z-2)(z-3) + C(z-3)$$

put, $z = 2 \Rightarrow C = -2$

$z = 3 \Rightarrow A = 3$

put, $z = 0 \Rightarrow B = -3$

2

$$\frac{z}{(z-3)(z-2)^2} = \frac{3}{z-3} - \frac{3}{z-2} - \frac{2}{(z-2)^2}$$

$$Z^{-1}\left[\frac{1}{(z-3)(z-2)^2}\right] = 3Z^{-1}\left[\frac{z}{z-3}\right] - 3Z^{-1}\left[\frac{z}{z-2}\right] - 2Z^{-1}\left[\frac{z}{(z-2)^2}\right]$$

1

$$\bar{u}(z) = 3 \times 3^n - 3 \times 2^n - 2 \times 2^{n-1}$$

$$= 3^{n+1} - 3.2^n - 2^n$$

$$\bar{u}(z) = \underline{\underline{3^{n+1} - 2^{n+2}}}$$ 1

Q. No.	Solution	Marks

7a.

$D^3y + 6D^2y + 11Dy + 6y = 0$

$(D^3 + 6D^2 + 11D + 6)y = 0$ — 1

$f(D)y = 0 \Rightarrow f(D) = D^3 + 6D^2 + 11D + 6$ — 1

$D = -1$, is a root by inspection method.

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & 0 & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$ — 1

— 2

$D^2 + 5D + 6 = 0$

Solving, $D = -2, -3$

The general solution is,

$y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{-3x}$ — 1

6

b.

$(D^2+1)y = x^2 + 4x - 6$

$D^2 + 1 = 0 \Rightarrow D^2 = -1 \Rightarrow D = \pm i$

$y_c = c_1 \cos x + c_2 \sin x$ — 1

$y_p = \dfrac{x^2 + 4x - 6}{D^2 + 1}$ — 1

$= (1 + D^2)^{-1}(x^2 + 4x - 6)$ — 1

$= [1 - D^2 + (D^2)^2 - \cdots\cdots](x^2 + 4x - 6)$

$= x^2 + 4x - 6 - D^2(x^2 + 4x - 6)$

$= x^2 + 4x - 6 - 2$ — 1

$y_p = x^2 + 4x - 8$

$\therefore$ The complete solution is,

$y = c_1 \cos x + c_2 \sin x + x^2 + 4x - 8$ — 2

6

Q. No.	Solution	Marks

7c.

$$D^2y - 6Dy + 9y = e^{3x}$$

$$f(D) = D^2 - 6D + 9 = 0$$

$$D^2 - 3D - 3D + 9 = 0$$

$$\Rightarrow \quad D(D-3) - 3(D-3) = 0$$

$$\Rightarrow \quad (D-3)^2 = 0 \Rightarrow D = 3, 3$$ 1

$$\therefore y_c = (c_1 + c_2 x)e^{3x}$$

Particular integral $= (U + Vx)e^{3x}$, then 1

$$y_1 = e^{3x}, \quad y_2 = xe^{3x}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & x3e^{3x} + e^{3x} \end{vmatrix}$$ 1

$$= 3xe^{6x} + e^{6x} - 3xe^{6x} = e^{6x}$$

$$U = -\int \frac{y_2 X}{W}\,dx = -\int \frac{xe^{3x}(e^{3x})}{e^{6x}}\,dx = -\int x\,dx$$ 1

$$= -\frac{x^2}{2}$$

$$V = \int \frac{y_1 X}{W}\,dx = \int \frac{e^{3x} \times e^{3x}}{e^{6x}}\,dx = \int 1.dx = x$$ 1

$$\therefore P.I. = y_p = x - \frac{x^2}{2}$$ 1

Hence, the complete solution is

$$y = y_c + y_p$$

$$y = (c_1 + c_2 x)e^{3x} + x - \frac{x^2}{2}$$ 1

7

Q. No.	Solution	Marks
8a.	The given equation is, $(6D^2+17D+12)y = e^{-x}$ $(3D+4)(2D+3)=0$ $3D+4=0$, $D=-4/3$ $2D+3=0$, $D=-3/2$	2
	$y_c = c_1 e^{-4/3x} + c_2 e^{-3x/2}$ $y_p = \frac{e^{-x}}{6D^2+17D+12} = \frac{e^{-x}}{6(-1)^2+17(-1)+12}$	2
	$y_p = e^{-x}$ Complete solution is $y = c_1 e^{-\frac{4x}{3}} + c_2 e^{-\frac{3}{2}x} + e^{-x}$	2
		6
8b.	put $t=\log x$ or $x=e^t$ $xy' = Dy$, $x^2y'' = D(D-1)y$, where $D=\frac{d}{dt}$	1
	$[D(D-1)+D+8]y = 65\cos t$ $[D^2-D+D+8] = 65\cos t$ $D^2+8 = 65\cos t$	1
	Auxilliary equation is $D^2+8=0$ $D^2=-8 \Rightarrow D = \pm\sqrt{8}\,i = \pm 2\sqrt{2}\,i$ $\therefore y_c = c_1\cos 2\sqrt{2}t + c_2 \sin 2\sqrt{2}t$	2
	Also, $y_p = \frac{65\cos t}{D^2+8}$ put $D^2=-1 \Rightarrow y_p = \frac{65\cos t}{7}$	1
	Complete solution : $y = y_c + y_p$ $y = c_1\cos 2\sqrt{2}t + c_2\sin 2\sqrt{2}t + \frac{65\cos t}{7}$ $= c_1\cos 2\sqrt{2}(\log x) + c_2\sin 2\sqrt{2}(\log x) + \frac{65\cos(\log x)}{7}$	1

Q. No.	Solution	Marks
8c.	$L\dfrac{d^2q}{dt^2} + \dfrac{q}{C} = E$, is a second order linear differential equation representing an L-C circuit with an applied constant EMF E.	
	$\dfrac{d^2q}{dt^2} + \dfrac{q}{LC} = \dfrac{E}{L}$, let $\omega^2 = 1/LC$	1
	$\dfrac{d^2q}{dt^2} + q\omega^2 = E/L$	
	Solving, $\dfrac{d^2q}{dt^2} + \omega^2 q = 0$, we get	
	$q(t) = A\cos\omega t + B\sin\omega t$ $\left[\begin{array}{l} D^2+\omega^2 = 0 \\ D^2 = -\omega^2 \\ D = \pm\omega i \end{array}\right.$	2
	Now, solving non-homogenous equation,	
	$\dfrac{d^2q}{dt^2} + \omega^2 q = E/L$	
	Let, $q = Q \Rightarrow 0 + \omega^2 Q = E/L$ [Q is a constant]	1
	hence, $\dfrac{d^2Q}{dt^2} = 0$	
	$\Rightarrow Q = E/L\omega^2$	
	$Q = \dfrac{E}{L\times\frac{1}{LC}} = EC$ $[\because \omega^2 = 1/LC]$	
	Thus, the particular solution is $q = EC$	1
	Thus, complete solution is $q = q_c + q_p$	
	$q(t) = q_c(t) + q_p(t)$	1
	$q(t) = A\cos\omega t + B\sin\omega t + EC$	
	$q(t) = A\cos\left(\dfrac{t}{\sqrt{LC}}\right) + B\sin\left(\dfrac{t}{\sqrt{LC}}\right) + EC$	1
		7

Q. No.	Solution	Marks

9a. The normal equations associated with $y = ax^2 + bx + c$ are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + c$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$ 1

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 \quad (n=5)$$

x	y	xy	x^2y	x^2	x^3	x^4
1	10	10	10	1	1	1
2	12	24	48	4	8	16
3	13	39	117	9	27	81
4	16	64	256	16	64	256
5	19	95	475	25	125	625
$\Sigma x = 15$	$\Sigma y = 70$	$\Sigma xy = 232$	$\Sigma x^2y = 906$	$\Sigma x^2 = 55$	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$

3

$$55a + 15b + 5c = 70$$

$$225a + 55b + 15c = 232$$

$$979a + 225b + 55c = 906$$

on solving, $a = 0.2857$, $b = 0.4857$, $c = 9.4$

$$y = 0.29x^2 + 0.49x + 9.4$$ 2

At, $x = 6$, $y = 22.78$

6

9b. Let the values of two groups correspond to variates x, y respectively. For convienience so that we can denote $z = x - y$ and prepare the following table. Here, $n = 10$.

Q. No.	Solution	Marks

9c.

x	y	R_x	R_y	$d = R_x - R_y$	d^2
78	84	4	3	1	1
36	51	9	9	0	0
98	91	1	1	0	0
25	60	10	6	4	16
75	68	5	4	1	1
82	62	3	5	-2	4
90	86	2	2	0	0
62	58	7	7	0	0
65	53	6	8	-2	4
39	47	8	10	-2	4
					$\Sigma d^2 = 30$

3

$n = 10$ (number of students)

$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 30}{10(10^2-1)}$$

$$= 1 - \frac{180}{10 \times 99} = 1 - \frac{180}{990} = 1 - 0.1818$$

3

$\therefore \rho = 0.818$

Conclusion: This indicates a strong positive correlation between the student's performance in the two subjects. 1

7

Q. No.	Solution	Marks

10a. Regression of y on x:

$$y-\bar{y} = b_{yx}(x-\bar{x})$$

where $b_{yx} = r.\dfrac{\sigma_y}{\sigma_x}$ — 1

Regression of x on y:

$$x-\bar{x} = b_{xy}(y-\bar{y}) \text{ where } b_{xy} = r.\frac{\sigma_x}{\sigma_y}$$ — 1

Let the slopes of the two regression lines be

$$m_1 = b_{yx} \quad \& \quad m_2 = 1/b_{xy} = \frac{1}{r.\frac{\sigma_x}{\sigma_y}}$$

$$\therefore m_2 = \frac{\sigma_y}{r\,\sigma_x}$$ — 1

If, θ is the angle between two lines with slopes m_1 and m_2 then

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$ — 1

$$\tan\theta = \left|\frac{r.\frac{\sigma_y}{\sigma_x} - \frac{\sigma_y}{r.\sigma_x}}{1+\left(r.\frac{\sigma_y}{\sigma_x}\right)\left(\frac{\sigma_y}{r\sigma_x}\right)}\right| = \left|\frac{\frac{\sigma_y}{\sigma_x}\left(r-\frac{1}{r}\right)}{1+\frac{\sigma_y^2}{\sigma_x^2}}\right|$$ — 1

$$\tan\theta = \left|\frac{\sigma_y\,\sigma_x}{\sigma_x^2+\sigma_y^2}\left(r-\frac{1}{r}\right)\right|$$

$$= \left|\frac{\sigma_x\,\sigma_y}{\sigma_x^2+\sigma_y^2}\times\left(-\frac{(1-r^2)}{r}\right)\right|$$

$$\tan\theta = \frac{\sigma_x\,\sigma_y}{\sigma_x^2+\sigma_y^2}\left(\frac{1-r^2}{r}\right)$$ [taking the absolute value] — 1

Q. No.	Solution	Marks
b.	$\bar{x} = \frac{\Sigma x}{n} = \frac{70}{10} = 7$, $\bar{y} = \frac{\Sigma y}{n} = \frac{150}{10} = 15$ $X = x - \bar{x}$ and $Y = y - \bar{y}$	1

x	y	X	Y	X^2	Y^2	XY
1	8	-6	-7	36	49	42
3	6	-4	-9	16	81	36
4	10	-3	-5	9	25	15
2	8	-5	-7	25	49	35
5	12	-2	-3	4	9	6
8	16	1	1	1	1	1
9	16	2	1	4	1	2
10	10	3	-5	9	25	-15
13	32	6	17	36	289	102
15	32	8	17	64	289	136
$\Sigma x = 70$	$\Sigma y = 150$			$\Sigma X^2 = 204$	$\Sigma Y^2 = 818$	$\Sigma XY = 360$

2

$$Y = \frac{\Sigma XY}{\Sigma X^2} \cdot X \qquad X = \frac{\Sigma XY}{Y^2} \cdot Y$$

$$y - 15 = \frac{360}{204}(x-7) \qquad x - 7 = \frac{360}{818}(y-15)$$

$$y - 15 = 1.76(x-7) \qquad x - 7 = 0.44(y-15)$$

$$y = 1.76x + 2.68 \qquad x = 0.44y + 0.4$$

2

are the lines of regression.

$r = \sqrt{(\text{coeff. of } x)(\text{coeff. of } y)} = \sqrt{(1.76)(0.44)}$ 1

The sign of r is positive since both the regression coefficients are positive.

Thus, $r = 0.88$

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Q. No.	Solution	Marks
10c.	We know that the regression lines passing through $\bar{x}$ and $\bar{y}$.	
	$8\bar{x} - 10\bar{y} = -66$	2
	$40\bar{x} - 18\bar{y} = 214$	
	Solving, we get $\bar{x} = 13$, $\bar{y} = 17$	
	Rewrite the equation of the regression lines to find the regression coefficients	
	$10y = 8x + 66$ or $y = 0.8x + 6.6 \longrightarrow ①$	1
	$40x = 18y + 214$ or $x = 0.45y + 5.35 \longrightarrow ②$	
	Solving ① & ②, $r\dfrac{\sigma_y}{\sigma_x} = 0.8$; $r\dfrac{\sigma_x}{\sigma_y} = 0.45$	
	$\therefore$ Correlation coefficients $r = \sqrt{0.8 \times 0.45} = \pm 0.6$	1
	Thus, $\underline{\underline{r = 0.6}}$ since both regression coefficients are positive.	1
	Also, $\sigma_x = 3$, by data & we have $r\dfrac{\sigma_y}{\sigma_x} = 0.8$	1
	which gives $0.6\,\sigma_y = 2.4$	
	Thus, $\underline{\underline{\sigma_y = 4}}$	1
		7

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