CBCS SCHEME

USN | | | | | | | | | | |

BMATC101

First Semester B.E/B.Tech. Degree Examination, Dec.2024/Jan.2025
Mathematics–I for Civil Engineering Stream

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. VTU formula Handbook is permuted.

		Module – 1	M	L	C
1	a.	With usual notation prove that $\tan\phi = r\dfrac{d\theta}{dr}$.	6	L2	CO1
	b.	Find the angle between the curves $r = 4\sec^2(\theta/2)$ and $r = 9\operatorname{cosec}^2(\theta/2)$.	7	L2	CO1
	c.	Find the radius of curvature of the curve. $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.	7	L3	CO1
		OR			
2	a.	Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$.	6	L2	CO1
	b.	Derive the radius of curvature in Cartesian form.	7	L2	CO1
	c.	Using modern mathematical tool write a program to plot the curve $r = 2(1+\cos\theta)$.	7	L3	CO5
		Module – 2			
3	a.	Using Maclaurin's series expand $e^{\sin x}$ in powers of x upto the term containing x^5.	6	L2	CO2
	b.	If $u = f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$.	7	L3	CO2
	c.	Find the extreme values of the function : $f(x,y)=x^3+y^3-3x-12y+20$.	7	L3	CO2
		OR			
4	a.	Evaluate : i) $\lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$ ii) $\lim\limits_{x\to 0}(\sin x)^{\tan x}$.	6	L2	CO2
	b.	If : $u = x+3y^2-z^3$; $v = 4x^2yz$; $w = 2z^2-xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, –1, 0).	7	L3	CO2
	c.	Using modern mathematical tool write a program to evaluate $\lim\limits_{x\to 0}(\sin x)^{\tan x}$.	7	L3	CO5

Module – 3

5	a.	Solve : $x\frac{dy}{dx}+y\log y = xye^x$.	6	L2	CO3
	b.	Find the orthogonal trajectories of the family of curves $y^2 = 4ax$.	7	L3	CO3
	c.	Solve : $yp^2 + (x-y)p - x = 0$.	7	L3	CO3

OR

6	a.	Solve $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$.	6	L2	CO3
	b.	A bottle of mineral water at a room temperature of 72°F is kept in a refrigerator where the temperature is 44°F. After half am hour water cooled to 61°F. What is the temperature of the water in another half an hour?	7	L3	CO3
	c.	Find the general and singular solution of $xp^2 - py + kp + a = 0$.	7	L3	CO3

Module – 4

7	a.	Solve $(D^4 + 18D^2 + 81)y = 0$.	6	L2	CO3
	b.	Solve $D^3y + 8y = \sin(2x)$.	7	L2	CO3
	c.	Solve Legendre's linear differential equation : $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y = 4\cos[\log(1+x)]$.	7	L2	CO3

OR

8	a.	Solve $D^2y + 3Dy + 2y = 12x^2$.	6	L2	CO3
	b.	Solve by method of variation of parameters $\frac{d^2y}{dx^2}+y = \sec x \tan x$.	7	L2	CO3
	c.	Solve $x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+4y = x^4$.	7	L2	CO3

		Module – 5			
9	a.	Find the rank of the matrix $A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$.	6	L2	CO4
	b.	Using Gauss-elimination method solve : $x + y + z = 9$ $x - 2y + 3z = 8$ $2x + y - z = 3$.	7	L3	CO4
	c.	Using Rayleigh's power method find the dominant eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, taking initial vector as $[1, 1, 1]^T$.	7	L3	CO4
		OR			
10	a.	Apply Gauss – Jordan method to solve the equations : $x + y + z = 9$ $2x - 3y + 4z = 13$ $3x + 4y + 5z = 40$.	6	L2	CO4
	b.	Test for consistency and solve : $x + y + z = 6$ $x - y + 2z = 5$ $3x + y + z = 8$.	7	L2	CO4
	c.	Write a program to solve the system of equations using Gauss – Seidel method by taking initial approximations (0, 0, 0). Carry out 3 iterations. $x + y + 54z = 110$ $27x + 6y - z = 85$ $6x + 15y + 2z = 72$.	7	L3	CO5

* * * * *

AY: 2024-25

Department:Electrical &Electronics Engineering
Subject with Sub. Code:Mathematics I for Civil Stream(BMATC101)
Semester/Division/Branch:I/Civil
Name of Faculty: Prof. Akshata B Patil.

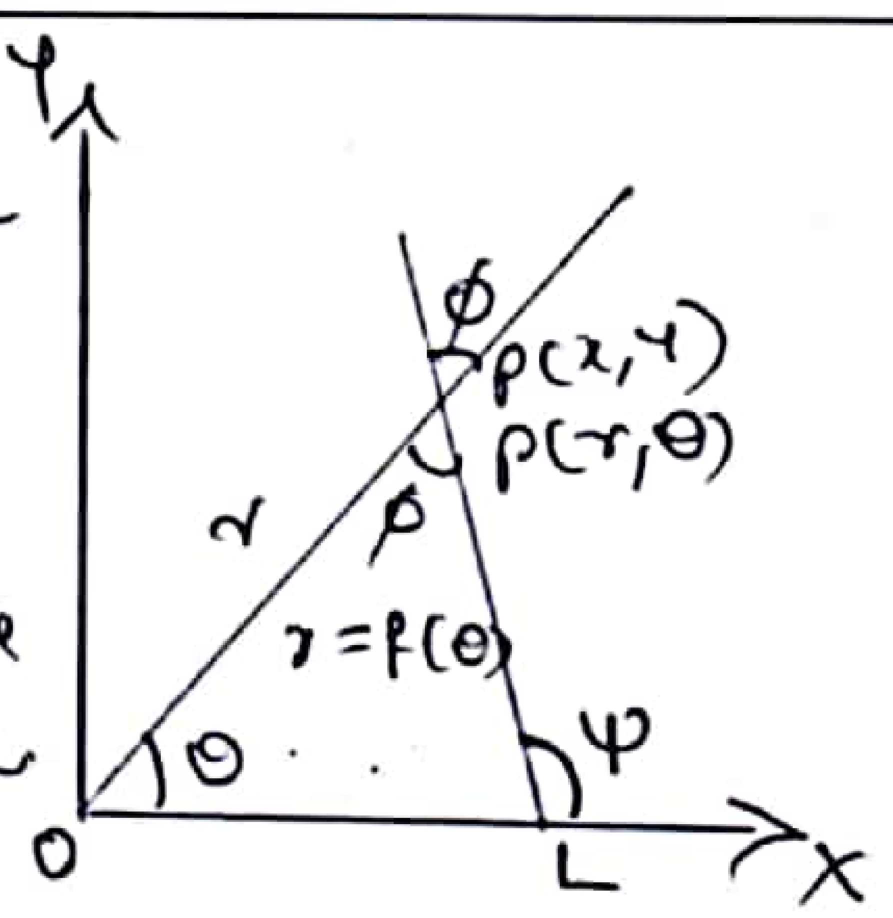
Q.No.	Solution and Scheme	Marks

MODULE - 01

1 a) Let $P(r,\theta)$ be any point on the curve $r = f(\theta)$.

$\therefore X\hat{O}P = \theta$ & $OP = r$

Let PL be the tangent to the curve at P subtending an angle ψ with the positive direction of the initial line & ϕ be the angle betn the radius vector OP and the tangent PL.



That is $O\hat{P}L = \phi$

From the figure,

$$\psi = \phi + \theta \quad ------ (1)$$

(2)

$$\tan\psi = \frac{\tan\phi + \tan\theta}{1 - \tan\phi\tan\theta}$$

Let (x,y) be the cartesian co-ordinates of P.

$x = r\cos\theta$, $y = r\sin\theta$ (2)

Then,

$\tan\psi = \frac{dy}{dx}$ = slope of the tangent PL.

$\tan\psi = \frac{dy/d\theta}{dx/d\theta}$ since x & y are functions of θ.

$$\tan\psi = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$$

Dividing both the Nr & Dr by $r'\cos\theta$ we have, (1)

$$\tan\psi = \frac{\frac{r\cos\theta}{r'\cos\theta} + \frac{r'\sin\theta}{r'\cos\theta}}{\frac{-r\sin\theta}{r'\cos\theta} + \frac{r'\cos\theta}{r'\cos\theta}}$$

$$\tan\psi = \frac{r/r' + \tan\theta}{1 - r/r' \cdot \tan\theta} \quad ------ (2)$$

Q.No.	Solution and Scheme	Marks
	comparing equation (1) & (2)	
	$\tan\phi = \frac{r}{r_1} = \frac{r}{\left(\frac{dr}{d\theta}\right)}$ or $\tan\phi = r\left(\frac{d\theta}{dr}\right)$	(1)
	Equivalently we can write in the form.	
	$\frac{1}{\tan\phi} = \frac{1}{r}\left(\frac{dr}{d\theta}\right)$ or $\cot\phi = \frac{1}{r}\left(\frac{dr}{d\theta}\right)$	6M.
1b)	$r = 4\sec^2(\theta/2)$: $r = 9\operatorname{cosec}^2(\theta/2)$	
	$\log r = \log 4 + 2\log\sec(\theta/2)$: $\log r = \log 9 + 2\log\operatorname{cosec}(\theta/2)$	(1)
	Differentiating these w.r.t 'θ'	
	$\frac{1}{r}\frac{dr}{d\theta} = \frac{2}{\sec(\theta/2)} \cdot \sec(\theta/2)\tan(\theta/2)\cdot 1/2$	
	$\therefore \frac{1}{r}\frac{dr}{d\theta} = \frac{-2\operatorname{cosec}(\theta/2)\cot(\theta/2)}{\operatorname{cosec}(\theta/2)}\cdot\frac{1}{2}$	(2)
	$\frac{1}{r}\frac{dr}{d\theta} = \tan(\theta/2)$ $\therefore \frac{1}{r}\frac{dr}{d\theta} = -\cot(\theta/2)$	(2)
	$\cot\phi_1 = \cot(\pi/2 - \theta/2)$ $\therefore \cot\phi_2 = \cot(-\theta/2)$	
	$\phi_1 = \pi/2 - \theta/2$ $\therefore \phi_2 = -\theta/2$	
	$\therefore \lvert\phi_1 - \phi_2\rvert = \lvert \pi/2 - \theta/2 + \theta/2\rvert = \pi/2$	(2)
	Thus the curves intersect each other orthogonally.	7M.
1c)	$\sqrt{x} + \sqrt{y} = \sqrt{a}$; $(x,y) = (a/4, a/4)$	
	Diff w.r.t 'x'	
	$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}y_1 = 0$ or $y_1 = -\frac{\sqrt{y}}{\sqrt{x}}$; At $\left(\frac{a}{4},\frac{a}{4}\right)$: $y_1 = -1$	(1)
	Next;	
	$y_2 = \frac{\sqrt{x}\left(\frac{-1}{2\sqrt{y}}y_1\right) + \sqrt{y}\left(\frac{1}{2\sqrt{x}}\right)}{x}$	(1)
	At $(a/4, a/4)$: $y_2 = \frac{\frac{1}{2}+\frac{1}{2}}{a/4} = \frac{4}{a}$	
	Substituting these in the formula for the centre of curvature.	(1)
	$\bar{x} = \frac{a}{4} + \frac{(1+1)}{(4/a)} = \frac{a}{4} + \frac{2a}{4} = \frac{3a}{4}$	
	$\bar{y} = \frac{a}{4} + \frac{(1+1)}{(4/a)} = \frac{a}{4} + \frac{2a}{4} = \frac{3a}{4}$.	

Q.No.	Solution and Scheme	Marks
	Centre of curvature : $(\bar{x}, \bar{y}) = (3a/4, 3a/4)$ Next we shall substitute the values of y_1 & y_2 in the formula for ρ. $\rho = \frac{(1+1)^{3/2}}{(4/a)} = \frac{2\sqrt{2}a}{4} = \frac{a}{\sqrt{2}}$	(2)
	circle of curvature $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$ That is, $(x-\frac{3a}{4})^2 + (y-\frac{3a}{4})^2 = (a/\sqrt{2})^2 = \frac{a^2}{2}$	(2)
	Thus the centre & circle of curvature is given by, $(\frac{3a}{4}, \frac{3a}{4})$ & $(x-\frac{3a}{4})^2 + (y-\frac{3a}{4})^2 = \frac{a^2}{2}$.	7M.
2a]	$r^m = a^m(\cos m\theta + \sin m\theta)$ Taking log on both sides $m\log r = m\log a + \log(\cos m\theta + \sin m\theta)$ Diff w.r.t. 'θ' $\frac{m}{r} dr/d\theta = \frac{-m\sin m\theta + m\cos m\theta}{\cos m\theta + \sin m\theta}$	(2)
	$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cos m\theta - \sin m\theta}{\cos m\theta + \sin m\theta}$ $= \frac{\cos m\theta(1-\tan m\theta)}{\cos m\theta(1+\tan m\theta)}$	(1)
	$\cot\phi = \cot(\pi/4 + m\theta) \Rightarrow \phi = \pi/4 + m\theta$ consider, $p = r\sin\phi$ $p = r\sin(\pi/4 + m\theta)$ $p = r[\sin(\pi/4)\cos m\theta + \cos(\pi/4)\sin m\theta]$	(2)
	$p = r/\sqrt{2}[\cos m\theta + \sin m\theta]$ Now, $r^m = a^m(\cos m\theta + \sin m\theta)$ ---- (1) $p = r/\sqrt{2}(\cos m\theta + \sin m\theta)$ ---- (2)	(1)
	using (2) in (1) $r^m = a^m \frac{p\sqrt{2}}{r} \Rightarrow r^{m+1} = \sqrt{2}a^m p$. is the pedal eqn.	6M.

Q.No.	Solution and Scheme	Marks
2b.	Consider a certesian curve $y = f(x)$	
	$y_1' = \frac{dy}{dx} = \tan\psi$	
	$y_2'' = \frac{d^2y}{dx^2} = \sec^2\psi \cdot \frac{d\psi}{dx} = (1+\tan^2\psi)\frac{d\psi}{ds}\cdot\frac{ds}{dx}$	(2)
	$y'' = [1+(y')^2]\cdot\frac{d\psi}{ds}\cdot\frac{ds}{dx}$	(1)
	Since, WKT	
	$\frac{d\psi}{ds} = 1/\rho$, $\frac{ds}{dx} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{1/2} = \left\{1+(y')^2\right\}^{1/2}$	(2)
	$y'' = \frac{\{1+(y')^2\}^{3/2}}{\rho}$	(2)
	or $\rho = \frac{\{1+(y')^2\}^{3/2}}{y''}$	7M.
2c)	$r = 2(1+\cos\theta)$	
	# plot Cardioid r=2(1+costheta)	5
	from pylab import*	
	theta = linspace(0, 2*np.pi, 1000)	
	r1= 2 + 2* cos(theta)	(2)
	polar(theta,r1,'r')	7M.
	show( )	
	MODULE -02	
3a)	we have, maclaurin's expansion	
	$y(x) = y(0) + xy_1(0) + \frac{x^2}{2!}y_2(0) + \frac{x^3}{3!}y_3(0) + \frac{x^4}{4!}y_4(0) + \frac{x^5}{5!}y_5(0)$ --- (i)	
	Let,	
	$y = e^{\sin x}$; $y(0) = e^0 = 1$	
	$y_1 = e^{\sin x}\cdot\cos x$; $y_1(0) = y(0)\cdot\cos 0 = 1$	(4)
	$y_1 = y\cos x$	
	$y_2 = -y\sin x + \cos x \cdot y_1$; $y_2(0) = 0+1 = 1$	
	$y_3 = -(y\cos x + y_1\sin x) + (\cos x\, y_2 - y_1\sin x)$	
	$\therefore y_3(0) = -1-0+1 = 0$	

Q.No.	Solution and Scheme	Marks
	$y_4 = -y_2 - 2(y_1\cos x + \sin x\, y_2) + (\cos x\, y_3 - \sin x\, y_2)$	
	$= -y_2 - 2y_1\cos x - 3\sin x\, y_2 + \cos x . y_3$	(1)
	$y_4(0) = -1-2-0+0 = -3$	
	$\therefore y_4(0) = -3$.	
	$\therefore y_5(0) = -8$.	
	From eq (1)	
	$y(x) = e^{\sin x} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} - - - - - -$	6M.
3b)	Here we need to convert the given function u into a Composite function.	
	Let, $u = f(p, q, r)$ where $p = \frac{x}{y}$, $q = \frac{y}{z}$, $r = \frac{z}{x}$	(1)
	ie $\{u \to (p, q, r) \to (x, y, z)\} \Rightarrow u \to x, y, z$	
	$\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} . \frac{\partial p}{\partial x} + \frac{\partial u}{\partial q} . \frac{\partial q}{\partial x} + \frac{\partial u}{\partial r} . \frac{\partial r}{\partial x}$	(2)
	$\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} . \frac{1}{y} + \frac{\partial u}{\partial q} . 0 + \frac{\partial u}{\partial r}\left(-\frac{z}{x^2}\right)$	
	Hence,	
	$x\frac{\partial u}{\partial x} = \frac{x}{y}\frac{\partial u}{\partial p} - \frac{z}{x}\frac{\partial u}{\partial r}$ - - - - - - (1)	(2)
	lly by symmetry	
	$y\frac{\partial u}{\partial y} = \frac{y}{z}\frac{\partial u}{\partial q} - \frac{x}{y}\frac{\partial u}{\partial p}$ - - - - (2)	
	$z\frac{\partial u}{\partial z} = \frac{z}{x}\frac{\partial u}{\partial r} - \frac{y}{z}\frac{\partial u}{\partial q}$ - - - - - - (3)	(2)
	Thus by adding (1), (2) & (3)	
	we get,	7M.
	$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0}$	

Q.No.	Solution and Scheme	Marks
3 c]	$f(x,y) = x^3+y^3-3x-12y+20$	
	$f_x = 3x^2-3$, $f_y = 3y^2-12$	
	We shall find points (x,y) such that $f_x=0$ & $f_y=0$	
	ie $3x^2-3=0$ and $3y^2-12=0$ or $x^2-1=0$ & $y^2-4=0$	(2)
	ie $x=\pm 1$, $y=\pm 2$	
	$\therefore (1,2)(1,-2)(-1,2)(-1,-2)$ are the stationary points.	
	Let, $A = f_{xx}$, $B = f_{xy}$; $C = f_{yy}$.	

	(1,2)	(1,-2)	(-1,2)	(-1,-2)
$A=6x$	$6>0$	6	-6	$-6<0$
$B=0$	0	0	0	0
$C=6y$	12	-12	12	-12
$AC-B^2$	$72>0$	$-72<0$	$-72<0$	$72>0$
Conclusion	min. pt	Saddle pt	Saddle pt	Max. pt

(2)

maximum value of $f(x,y)$ is,

$f(-1,-2) = -1-8+3+24+20 = 38$ (2)

minimum value of $f(x,y)$ is $f(1,2) = 1+8-3-24+20 = 2$

Thus,

maximum value is 38 and minimum value is 2. (1)

7M

---

4 a] $k = \lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$ $[1^\infty$ form$]$ (2)

$\log k = \lim_{x\to 0}\frac{1}{x^2}\log\left(\frac{\tan x}{x}\right)$ $[0/0$ form$]$

$\log k = \lim_{x\to 0}\frac{1}{3}\sec^2 x\left(\frac{\tan x}{x}\right) = \frac{1}{3}\times 1\times 1 = 1/3$

$\log k = 1/3$ (1)

$\boxed{k = e^{1/3}}$

Q.No.	Solution and Scheme	Marks
ii)	$K = \lim_{x \to 0} (\sin x)^{\tan x}$ ---- $[0^0 \text{ form}]$	
	$\log K = \lim_{x \to 0} \tan x \log \sin x$	(2)
	$\log K = \lim_{x \to 0} \dfrac{\log \sin x}{\cot x}$ $[\infty/\infty \text{ form}]$	(1)
	$\log K = -\frac{1}{2} \lim_{x \to 0} \tan x = 0$	
	$\therefore \boxed{K = 1}$	6M.
4b]	$u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$	
	$\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$	(1)
	$= \begin{vmatrix} 1 & 6y & -3z^2 \\ 8xyz & 4x^2z & 4x^2y \\ -y & -x & 4z \end{vmatrix}$	(2)
	It will be easier if the elements of the determinant are evaluated at $(1,-1,0)$	
	$\therefore \dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0) = \begin{vmatrix} 1 & -6 & 0 \\ 0 & 0 & -4 \\ 1 & -1 & 0 \end{vmatrix}$	(2)
	On expanding $1(0-4) + 6(0+4) + 0 = 20$	(2)
	Thus, $J_{(1,-1,0)} = 20.$	7M

Q.No.	Solution and Scheme	Marks
4c]	from sympy import Limit, symbol, sin, tan	(2)
	X = Symbol ('x')	
	L = Limit (sin(x) ** tan(x), x, 0).doit( )	(2)
	display (L)	(1)
	Out put: The limit of $(\sin(x))^{\wedge(\tan(x))}$ as x approaches 0 is : 1.	(2)
		7M
	MODULE-03.	
5a]	$x\frac{dy}{dx} + y\log y = xye^x$	(2)
	Divide by xy	
	$\therefore \frac{1}{y}\frac{dy}{dx} + \frac{\log y}{x} = e^x$ ------ (1)	(2)
	putting $\log y = z$	
	$\frac{dz}{dx} + \frac{z}{x} = e^x$	(2)
	$I.F = e^{\int 1/x\,dx} = x$	
	The solution is,	
	$x\log y = e^x(x-1) + c.$	6M
5b]	$y^2 = 4ax$	
	Consider,	(1)
	$y^2/x = 4a$ ----- (1)	
	Diff (1) w.r.t 'x'	
	$\frac{x.2y\frac{dy}{dx} - y^2.1}{x^2} = 0$ or $2xy\frac{dy}{dx} - y^2 = 0$	(2)
	$2x\frac{dy}{dx} - y = 0$ is the DE of the given family.	
	Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$	(2)
	we have,	
	$2x\left(-\frac{dx}{dy}\right) - y = 0$	(2)
	or $2x\,dx + y\,dy = 0$	
	$\int 2x\,dx + \int y\,dy = C$	7M
	ie $x^2 + y^2/2 = C$ or $2x^2 + y^2 = 2c = k$	
	$2x^2 + y^2 = k$ is the required O.T.	

Q.No.	Solution and Scheme	Marks
5c]	$yp^2+(x-y)p-x=0$ The given eq^n is, $yp^2+(x-y)p-x=0$ $\therefore p=\dfrac{-(x-y)\pm\sqrt{(x-y)^2+4xy}}{2y}$ $p=\dfrac{(y-x)\pm(x+y)}{2y}$	(2)
	$p=\dfrac{y-x+x+y}{2y}$ or $\dfrac{y-x-x-y}{2y}$ $p=1$ or $p=-x/y$ we have,	(1)
	$\dfrac{dy}{dx}=1 \Rightarrow y=x+c$ or $(y-x-c)=0$ Also, $\dfrac{dy}{dx}=\dfrac{-x}{y}$ or $y\,dy+x\,dx=0$	(2)
	Integrating $\int y\,dy+\int x\,dx=k.$ $\dfrac{y^2}{2}+\dfrac{x^2}{2}=k$ or $y^2+x^2=2k.$ or $(x^2+y^2-c)=0$ Thus the general solution is,	(2)
	$(y-x-c)(x^2+y^2-c)=0.$	7M.
6a]	Let, $M=4xy+3y^2-x$ and $N=x(x+2y)=x^2+2xy$ $\dfrac{\partial M}{\partial y}=4x+6y$ and $\dfrac{\partial N}{\partial x}=2x+2y$ consider,	(1)
	$\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}=2x+4y=2(x+2y)$ ----- Close to N. Now,	(1)
	$\dfrac{1}{N}\left(\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}\right)=\dfrac{2(x+2y)}{x(x+2y)}=\dfrac{2}{x}=f(x)$ Hence, $e^{\int f(x)dx}$ is an integrating factor.	
	$e^{\int f(x)dx}=e^{\int 2/x\,dx}=e^{2\log x}=e^{\log(x^2)}=x^2$	(1)

Q.No.	Solution and Scheme	Marks

Multiplying the given equation by x^2

$M = 4x^3y + 3x^2y^2 - x^3$ and $N = x^4 + 2x^3y$ (1)

$\frac{\partial M}{\partial y} = 4x^3 + 6x^2y$ and $\frac{\partial N}{\partial x} = 4x^3 + 6x^2y$

solution of the exact equation is

$\int M\,dx + \int N(y)\,dy = C$

$\int (4x^3y + 3x^2y^2 - x^3)\,dx + \int 0\,dy = C$ (2)

Thus,

$x^4y + x^3y^2 - \frac{x^4}{4} = C$. is the required solution. 6M.

6b] According to the Newton's law of Cooling, the expression for the temperature function at any time t is, (1)

$T = t_2 + (t_1 - t_2)e^{-kt}$

we have by data,

$t_1 = 72$, $t_2 = 44$ and $T = 61$ when $t = 30$ mts. (1)

$\therefore T = 44 + 28e^{-kt}$

i) By applying the initial condition

$61 = 44 + 28e^{-30k}$ or $e^{-30k} = \frac{17}{28}$ or $e^{30k} = \frac{28}{17}$ (2)

$e^{30k} = \frac{28}{17} \Rightarrow 30k = \log_e(28/17)$ $\therefore k \simeq 0.0166$

we have to find T when $t = 30 + 30 = 60$ (mts)

$(T)_{t=60} = 44 + 28e^{(-0.0166)60} \simeq 54.3$. (2)

Thus the temperature of the mineral water after another half an hour [1 hr = 60 mts] is 54.3° F. (1)

7M.

6c] $xp^2 - py + kp + a = 0$

$xp^2 + kp + a = py$

$y = \frac{p(xp+k)+a}{p}$ or $y = xp + k + \frac{a}{p}$ (1)

$y = px + \left[k + \frac{a}{p}\right]$ ------ (1)

Q.No.	Solution and Scheme	Marks
	Here ① is in the Clairauts form $y = px + f(p)$ whose general solution is	
	$y = cx + f(c)$	(2)
	Thus the general solution is $y = cx + \left[k + \frac{a}{c}\right]$	
	Now, diff w.r.t 'c'	
	$0 = x - \frac{a}{c^2}$ or $\frac{a}{c^2} = x$	(1)
	or $c^2 = a/x \Rightarrow c = \sqrt{a/x}$	
	General solution becomes,	
	$y = \left(\sqrt{a/x}\right)x + k + a\left(\sqrt{x/a}\right)$	(2)
	or $y - k = 2\sqrt{ax} \Rightarrow (y-k)^2 = 4ax$	(1)
	Thus the singular solution is,	7M.
	$(y-k)^2 = 4ax$.	
	MODULE-04	
7 a]	$(D^4 + 18D^2 + 81)y = 0$	(2)
	$[(D^2)^2 + 18D^2 + 81]y = 0$	
	A.E is	(2)
	$(m^2 + 9)^2 = 0$	
	Roots $\pm i3, \pm i3$	(2)
	$y = (c_1 + c_2x)\cos 3x + (c_3 + c_4x)\sin 3x$	6M.
7b]	$D^3y + 8y = \sin(2x)$	
	A.E is	
	$D^3 + 8 = 0$	(1)
	Roots $m = -2, 1 \pm \sqrt{3}i$	(2)
	C.F $= c_1e^{-2x} + e^{x}(c_2\cos\sqrt{3}x + c_3\sin\sqrt{3}x)$	
	P.I $= \frac{\sin 2x}{D^2.D + 8} = \frac{1}{-4D + 8}\sin 2x$	(2)
	$= \frac{1}{16}[\cos 2x + \sin 2x]$	
	$y = C.F + P.I$	(2)
	$y = c_1e^{-2x} + e^{x}[c_2\cos\sqrt{3}x + c_3\sin\sqrt{3}x] + \frac{1}{16}[\cos 2x + \sin 2x]$	7M.

Q.No.	Solution and Scheme	Marks
7c]	$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos[\log(1+x)]$	(1)
	Put, $1+x=e^z \quad \therefore z=\log(1+x)$	(1)
	$(D^2+1)y=4\cos z$	
	A.E is $m^2+1=0$	(1)
	$\therefore$ roots are $m=\pm i$	(2)
	$C.F=C_1\cos 2x+C_2\sin 2x$	
	$P.I=\frac{4\cos z}{D^2+1}=2z\sin z$	(2)
	$y=C.F+P.I$	
	$=C_1\cos[\log(1+x)]+C_2\sin[\log(1+x)]+2\log(1+x)\sin[\log(1+x)]$	7M.
8 a]	$[D^2+3D+2]y=12x^2$	
	AE is $m^2+3m+2=0$ or $(m+1)(m+2)=0$	(1)
	$\Rightarrow m=-1,-2$	(1)
	$\therefore y_c=C_1e^{-x}+C_2e^{-2x}$	
	$y_p=\frac{12x^2}{D^2+3D+2}$	
	we need to divide for obtaining the PI	(3)
	$2+3D+D^2 \,\big)\, 12x^2 \,\big(\, 6x^2-18x+21$ $12x^2+36x+12$ $-36x-12$ $-36x-54$ 42 42 00	
	Hence, $y_p=6x^2-18x+21$	(1)
	Complete solution : $y=y_c+y_p$	
	Thus, $y=C_1e^{-x}+C_2e^{-2x}+6x^2-18x+21$	6M.

Q.No.	Solution and Scheme	Marks

8b we have,

$[D^2+1]\,y = \sec x \tan x$.

AE is $m^2+1=0 \Rightarrow \pm i$ (1)

$y_c = c_1\cos x + c_2\sin x$

$y = A(x)\cos x + B(x)\sin x$.

$y_1 = \cos x$ $\quad$ $y_2 = \sin x$

$y_1' = -\sin x$ $\quad$ $y_2' = \cos x$ (2)

$W = y_1y_2' - y_2y_1' = 1$. ; $\phi(x) = \sec x \tan x$

$A' = -\sin x \sec x \tan x$; $B' = \cos x \sec x \tan x$

$A' = -\tan^2 x = 1-\sec^2 x$; $B' = \tan x$ (2)

$A = \int (1-\sec^2 x)\,dx + k_1$; $B = \int \tan x\,dx + k_2$

$A = x - \tan x + k_1$; $B = \log(\sec x) + k_2$

Substituting these in $y = A\cos x + B\sin x$

$y = \{x - \tan x + k_1\}\cos x + \{\log(\sec x) + k_2\}\sin x$ (2)

$y = k_1\cos x + k_2\sin x + x\cos x - \sin x + \sin x\log(\sec x)$

Thus, 7 M.

$y = k_1\cos x + k_2\sin x + x\cos x + \sin x\log(\sec x)$.

8c] we have,

$x^2y'' + 5xy' + 4y = x^4$ ------ (1)

Put, (1)

$x = e^z$ $\quad$ $\therefore z = \log x$

we have, (1)

$xy' = Dy$; $x^2y'' = D(D-1)y$

Hence eq (1) becomes

$[D(D-1) + 5D + 4]\,y = e^{4z}$ (2)

$[D^2 - D + 5D + 4]\,y = e^{4z}$

$[D^2 + 4D + 4]\,y = e^{4z}$

Q.No.	Solution and Scheme	Marks
	A-E is	
	$m^2+4m+4=0$	
	i.e. $m=-2,-2$	(1)
	$y_c=(c_1+c_2 z)e^{-2z}$	
	$y_p = \dfrac{e^{4z}}{D^2+4D+4} = \dfrac{1}{36}e^{4z}$	(2)
	$y = y_c+y_p = [c_1+c_2\log x]\,x^{-2}+\frac{1}{36}x^4$	7M
	MODULE - 05	
9a]	$A=\begin{bmatrix}0&2&3&4\\2&3&5&4\\4&8&13&12\end{bmatrix}$	
	$R_1 \leftrightarrow R_2$	(1)
	$A \sim \begin{bmatrix}2&3&5&4\\0&2&3&4\\4&8&13&12\end{bmatrix}$	
	$R_3 \to -2R_1+R_3$	(1)
	$A \sim \begin{bmatrix}2&3&5&4\\0&2&3&4\\0&2&3&4\end{bmatrix}$	(1)
	$R_3 \to -R_2+R_3$	
	$A \sim \begin{bmatrix}2&3&5&4\\0&2&3&4\\0&0&0&0\end{bmatrix}$	(1)
	$\frac{1}{2}\cdot R_1$, $\frac{1}{2}\cdot R_2$	
	$A \sim \begin{bmatrix}1&3/2&5/2&2\\0&1&3/2&2\\0&0&0&0\end{bmatrix}$	(1)
	The matrix A in the row echelon form is having two non zero rows.	(1)
	Thus, $\boxed{\rho(A)=2}$	6M

Q.No.	Solution and Scheme	Marks
9b)	$x+y+z=9$ $x-2y+3z=8$ $2x+y-z=3$ The Augmented matrix of the system is, $[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 1 & -2 & 3 & : & 8 \\ 2 & 1 & -1 & : & 3 \end{bmatrix}$	(2)
	$R_2 \to -R_1+R_2$, $R_3 \to -2R_1+R_2$ $[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & -1 & -3 & : & -15 \end{bmatrix}$	(1)
	$R_3 \to R_2 + (-3)R_3$ $[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 11 & : & 44 \end{bmatrix}$	(1)
	Hence we have, $x+y+z=9$ $-3y+2z=-1$ $11z=44 \quad \therefore z=4$	(2) (1)
	By Back substitution, $-3y+8=-1 \quad \therefore y=3$, Also $x=2$. Thus, $x=2,\ y=3,\ z=4$ is the required solution.	7M.
9c)	$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ taking initial vector $[1,1,1]^T$	
	$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \lambda^{(1)}X^{(1)}$	(1)
	$Ax^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 7.34 \\ -2.67 \\ 4.01 \end{bmatrix} = 7.34\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \lambda^{(2)}x^{(2)}$	(1)
	$Ax^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 7.82 \\ -3.63 \\ 4.01 \end{bmatrix} = 7.82\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \lambda^{(3)}x^{(3)}$	(1)

Q.No.	Solution and Scheme	Marks
	$$Ax^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \begin{bmatrix} 7.94 \\ -3.89 \\ 3.99 \end{bmatrix} = 7.94 \begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \lambda^{(4)} x^{(4)}$$	(1)
	$$Ax^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 7.98 \\ -3.97 \\ 3.99 \end{bmatrix} = 7.98 \begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(5)} x^{(5)}$$	(1)
	$$Ax^{(5)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 4 \end{bmatrix} = 8 \begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(6)} x^{(6)}$$	(1)
	Thus the dominant eigen value is 8 and the corresponding eigen vector is $[1, -0.5, 0.5]'$ or $[2, -1, 1]'$ equivalently.	(1)
		7 M.
10a)	$x + y + z = 9$ $2x - 3y + 4z = 13$ $3x + 4y + 5z = 40$	
	$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{array}\right]$$	(1)
	$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 3R_1$	
	$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{array}\right]$$	(1)
	$R_3 \to R_3 + \frac{1}{5}R_2$	
	$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 0 & 12/5 & 12 \end{array}\right]$$	(1)
	$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 5 & -2 & 5 \\ 0 & 0 & 12 & 60 \end{array}\right]$$	(1)
	$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 5 & 0 & 15 \\ 0 & 0 & 1 & 5 \end{array}\right]$$	

Scanned with OKEN Scanner

Q.No.	Solution and Scheme	Marks

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & 3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$ (1)

Thus, (1)

~~$x=4$~~, $x = 1$, $y = 3$, $z = 5$

6M.

10b) $$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & -1 & 2 & : & 5 \\ 3 & 1 & 1 & : & 8 \end{bmatrix}$$ is the Augmented matrix.

We now perform elementary row operation.

$R_2 \rightarrow -R_1 + R_2$, $R_3 \rightarrow -3R_1 + R_3$ (2)

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -2 & 1 & : & -1 \\ 0 & -2 & -2 & : & -10 \end{bmatrix}$$ (1)

$R_3 \rightarrow -R_2 + R_3$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & -2 & 1 & : & -1 \\ 0 & 0 & -3 & : & -9 \end{bmatrix}$$

$\therefore \rho[A] = 3$ and $\rho[A:B] = 3$ That is $r = 3$

and $n = 3$.

Since, $\rho[A] = \rho[A:B] = 3$ $(r = n = 3)$ The given system of equations is consistent and will have unique solution. (2)

$x + y + z = 6$ ------ (i)

$-2y + z = -1$ ------ (ii)

$-3z = -9$ ------ (iii)

From eq (iii) $z = 3$, substituting this value in eq (ii) we get $y = 2$. Finally substituting these values in eq (i) we get $x = 1$ (2)

Thus, $x = 1$, $y = 2$, $z = 3$. is the unique solution.

7M.

Q.No.	Solution and Scheme	Marks

10 c)

```
f1 = lambda x, y, z: (85-6*y+z)/27
f2 = lamba x, y, z : (72-6*x-2*z)/15
f3 = lamba x, y, z : (110-x-y)/54
```
(1)

```
x = 0
y = 0
z = 0
count = 1
e = float(input('Enter tolerance error:'))
# Gauss Seidal Iteration
```
(2)

```
Print('\n count\tx\ty\tz\n')
condition = True
while condition:
        x1 = f1(x0, y0, z0)
        y1 = f2(x1, y0, z0)
        z1 = f3(x1, y1, z0)
print('%d\t%0.4f\t%0.4f\n'%(count, x1, y1, z1))
```
(2)

```
e1 = abs(x0-x1);
e2 = abs(y0-y1);
e3 = abs(z0-z1);
```
(2)

```
count += 1
x0 = x1
y0 = y1
z0 = z1
condition = e1>e and e2>e and e3>e
Print('\n solution: x=%0.3f, y=%0.3f and z=%0.3f\n'%(x,y,z
```
7M.

Prof. Akshata B. Patil

HEAD
Dept. of Electrical & Electronics Engg.
KLS's V. D. Institute of Technology
HALIYAL-581 329

Dean, Academics
KLS VDIT, HALIYAL