KLS Vishwanathrao Deshpande Institute of Technology

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	Prof. Sudheendra Yalagi
Course Name	:	Digital Signal Processing
Course Code	:	BEC502
Year of Question Paper	:	DEC2024/JAN2025
Date of Submission	:	1/7/2025

Syalagi
Faculty Member

03.07.2025
HoD

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Dean (Acad.)

CBCS SCHEME

USN | | | | | | | | | | |

BEC502

Fifth Semester B.E/B.Tech. Degree Examination, Dec.2024/Jan.2025

Digital Signal Processing

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. M : Marks , L: Bloom's level , C: Course outcomes.

			M	L	C
		Module – 1			
1	a.	List and discuss different discrete time signals.	7	L2	CO1
	b.	Explain the steps of converting along to digital signal interms of frequencies.	7	L2	CO1
	c.	Discuss the advantages and limitations of Digital Signal Processing (DSP).	6	L2	CO1
		OR			
2	a.	With an example, explain how to verify any signal is periodic or Not.	6	L2	CO1
	b.	Derive the equation for output of a LTI system and list the steps of convolution.	8	L3	CO2
	c.	Write a program to generate : i) Circuit step sequence ii) Sinusoidal sequence.	6	L3	CO2
		Module – 2			
3	a.	Describe the properties of Z – transformation.	7	L3	CO2
	b.	Show that Discrete Fourier Transform (DFT) is a Liner Transformation.	7	L3	CO2
	c.	Compute the A-point DFT of $x(n) = \{1, 1, 0, 0\}$.	6	L3	CO2
		OR			
4	a.	Compute the N-point DFT of, $x(n) = e^{j\omega mn}$.	6	L3	CO2
	b.	State and prove symmetry property of DFT for real valued sequence.	6	L3	CO2
	c.	Compute circular convolution of sequences : $x_1(n) = \{2, 1, 2, 1\}$ and $x_2(n) = \{1, 2, 3, 4\}$.	8	L3	CO2
		Module – 3			
5	a.	State and prove circular item shift property of DFT.	6	L3	CO2
	b.	Compare DFT and FFT with examples.	6	L2	CO3
	c.	Compute Radix – 2 DIT FFT of the following – sequence, $x(n) = n + 1$, for $0 \le n \le 7$.	8	L3	CO3
		OR			
6	a.	State and prove Parseval's theorem for – DFT's.	6	L3	CO2
	b.	Explain overlap – save method used for the convolution of long input sequences.	6	L2	CO3
	c.	Develop an algorithm for Radix – 2 FFT without using built in function.	8	L3	CO3

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		Module – 4			
7	a.	Obtain the frequency response expression for the symmetric linear phase FIR filter.	8	L3	CO4
	b.	Compare different widows used to design FIR filters.	6	L2	CO4
	c.	Design an FIR filter using hamming window for N = 7. The desired frequency response is given by $H_d(\omega) = \begin{cases} e^{-j3\omega} & \lvert\omega\rvert \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} < \lvert\omega\rvert \pi \end{cases}$.	6	L3	CO4
		OR			
8	a.	Discuss the characteristics of practical frequency selective filters.	6	L3	CO4
	b.	Explain the steps of designing linear phase FIR high pass filter.	8	L2	CO4
	c.	Realize the system function of following FIR filter in cascade form. $H(z) = 1 - 2z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{2}z^{-4}$.	6	L3	CO4
		Module – 5			
9	a.	Explain the design procedure of analog Butter worth lowpass prototype – filter?	8	L3	CO5
	b.	Construct the system function in S – domain for N = A.	6	L3	CO5
	c.	Realize direct form – II for the IIR filter represented by $y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(x-2) = x(n) + \frac{1}{2}x(n-2)$.	6	L3	CO5
		OR			
10	a.	Design the digital IIR filter for following details. –3dB gain at 0.5π rads and the stop band automation of 15dB at 0.75π rads. Assume $T_s = 15$.	8	L3	CO5
	b.	Explain the significance of : i) Prewarping ii) Bilinear transformation.	6	L2	CO5
	c.	Obtain the direct form-I realization of following IIR filter : $H(z) = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}}$.	6	L3	CO5

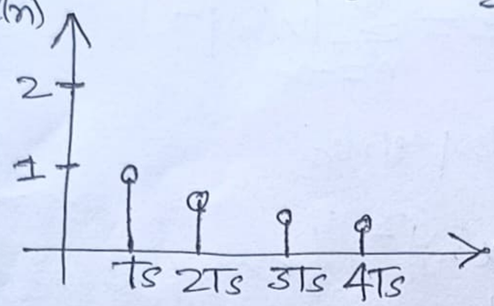
* * * * *

Module – 1

Q1 (a) List and discuss different discrete time signals

[Explaination – 7M]

Ans. – A discrete time signal is defined only at specific or regular time intervals. Ex – $x(n) = e^{-anTs} = e^{0}, e^{-aTs}, e^{-a2Ts}, e^{-a3Ts}$ etc



Thus discrete time signal has values only at 0, Ts, 2Ts, 3Ts etc, it is not defined over continuous time. Digital circuits such as Counters, Flip Flops, microprocessors etc are discrete time signals.

Different discrete time signals are listed below

1. Periodic and non-periodic signals
2. Even and Odd signals
3. Energy & power signals
4. Deterministic & Random signals

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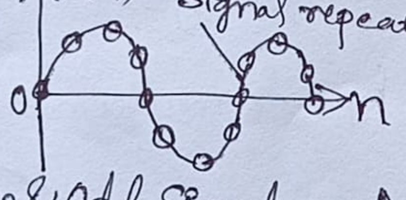
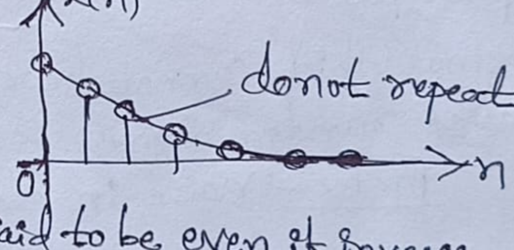
① **Periodic & non-periodic signals** – A signal is said to be periodic if it repeats at regular intervals. non-periodic signals donot repeat at regular intervals.

Ex





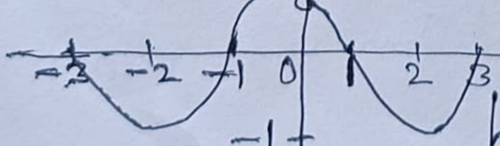
② **Even & Odd Signals** – A signal is said to be even if inversion of time axiss does not change the amplitude. ie for discrete time signal

if $\boxed{x(n) = x(-n)}$

A signal is said to be Odd signal if inversion of time axiss also inverts amplitude of the signal. ie for discrete time signal

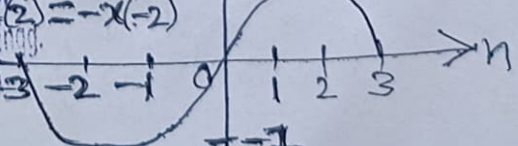
if $\boxed{x(n) = -x(-n)}$

Even Signal



here $x(2) = x(-2) = -1$

Odd Signal



here $x(2) = -x(-2)$

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(3) Energy & Power Signals

— A signal is said to be energy signal if its total energy is finite and non-zero. i.e. for energy signal $0 < E < \infty$

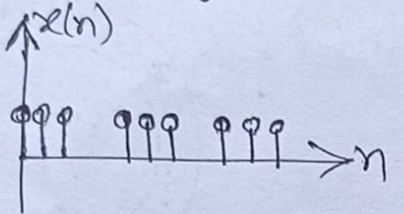
A signal is said to be power signal if its normalized power is non zero and finite i.e. for power signal, $0 < P < \infty$

For discrete time (DT) signals, $$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

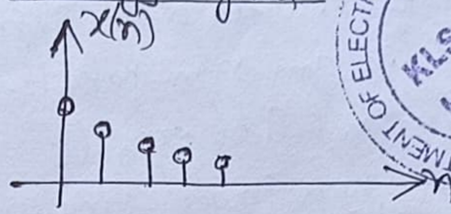
$$\& \; P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 \text{ if } x(n) \text{ is periodic}$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \text{ (for nonperiodic signals)}$$

Ex Power Signal



Energy Signal



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(4) Deterministic and Random Signals

A deterministic signal can be completely represented by mathematical equation at any time.

For Ex — Sinewave, Exponential Pulse, triangular wave, Square pulse etc.

A signal which cannot be represented by any mathematical equation is called random signal.

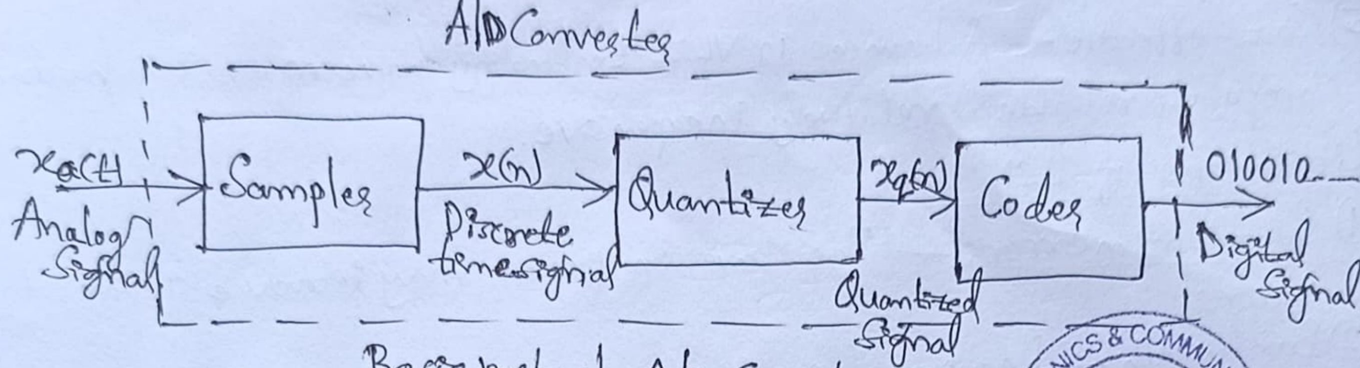
For Ex — noise in electronic components, Cables etc.

(5) Explain the steps of converting analog to digital signals in terms of frequencies.

[Explanation — 7M]

Ans — Most of the practicle signals such as video signals, audio signals, speech, biological signals, radar signals etc are analog in nature, to process analog signals by digital means, it is necessary to convert them into digital form. This process is called A/D (Analog to Digital) Conversion & corresponding devices are called ADC's.

The steps followed in Analog to Digital Signal Conversion are shown in below figure.



Basic parts of A/D Converter

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1. Sampling — This is conversion of CT (Continuous time) signal into a DT (Discrete time) signal, obtained by taking samples of the CT signal at discrete time instants if $x_a(t)$ is the input the sampler the o/p is $x_a(nT) \triangleq x(n)$, where T – Sampling Interval.

2. Quantization — This is the conversation of a DT continuous-valued signal into DT discrete valued (digital) signal. The value of each signal sample is represented by a value selected from a finite set of possible values.

The difference between the unquantized sample $x(n)$ & quantized o/p $x_q(n)$ is called quantization error.

3. Coding — In the coding each discrete value $x_q(n)$ is represented by a binary sequence.

ⓒ Discuss the advantages & limitations of Digital Signal Processing (DSP).

[Explaination – 6M]

Ans — Advantages of DSP:

① Noise Immunity — Digital signals are less susceptible to noise, distortion & interferance compared to analog signals, making them more reliable for transmission & storage.

② Flexibility & Programmability — DSP systems can be easily reprogra-mmable and customizable offering greater flexibility.

③ Ease of Storage & Transmission — Easily stored on various media like hard drives, flash memory & transmitted over long distance.

④ Security — Digital signals can be easily encrypted and compressed.

⑤ Cost-effective — Advances in VLSI technology have made DSP hardware Compact, reliable & relatively inexpensive.

Limitations of DSP

① Quantization errors — The conversion process may introduce quantization errors.

② Bandwidth — Requires large Bandwidth & requires specialized hardware or processing techniques.

③ Power Consumption — DSP with high processing power can consume significant amount of energy.

Q2 ⓐ With an example, explain how to verify any signal is periodic or not.

[Example Solving — 6M]

Ans — A signal is said to be periodic if it repeats at regular intervals. For a Discrete time signal to be periodic, only if its frequency is rational (Function of two integers)

Example — ① Determine whether the signal $x(n) = \cos(3\pi n)$ is periodic or not.

Solution — $x(n) = \cos(3\pi n)$

Comparing with $x(n) = \cos 2\pi f n$ $\quad \boxed{f = \frac{k}{N}}$

$$2\cancel{\pi} f \cancel{n} = 3\cancel{\pi}\cancel{n}$$

$$2f = 3 \quad \therefore f = \frac{3}{2} \text{ ratio of two integers (rational)}$$

Hence the signal is periodic with $N = 2$

② $x(n) = \sin(\pi + 0.2n)$

Solution — Comparing with $x(n) = \sin(2\pi f n + \theta)$ Where $\theta = \pi$ (phase shift)

$$\& \; 2\pi f \cancel{n} = 0.2\cancel{n}$$

$$f = \frac{0.2}{2\pi} = \frac{1}{10\pi} \text{ which is not rational}$$

Hence signal is non-periodic.

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(b) Derive the equation for output of a LTI System & list the steps of Convolution.

[Derivation - 4M, Steps of Convolution - 4M]

Ans — Response of LTI Systems to arbitrary inputs: The Convolution Sum.

Proof — Let $x(n)$ be the input signal, we denote the response $y(n,k)$ of the system to the input unit sample sequence at $n=k$ by symbol $h(n,k) = T[\delta(n-k)]$; $-\infty \leq k \leq \infty$

ie $y(n,k) = h(n,k) = T[\delta(n-k)]$ ——— (1)

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Where n — time index & k — shows the location of input impulse

If the impulse at the input is scaled by an amount $C_k = x(k)$ the output is also scaled that is,

$$C_k h(n,k) = x(k) h(n,k) \quad \text{——— (2)}$$

The input $x(n)$ is expressed as sum of weighted impulses

$$\therefore x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k) \quad \text{——— (3)}$$

Then the response of the system to $x(n)$ is the corresponding sum of weighted outputs

$$\text{ie } y(n) = T[x(n)] = T\left[\sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, T[\delta(n-k)]$$

$$\therefore \boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)} \quad \text{——— (4)}$$

Eqn (4) follows Time invariance property. ie $h(n-k) = T[\delta(n-k)]$

The process of computing convolution of $x(k)$ & $h(k)$ involves following four steps.

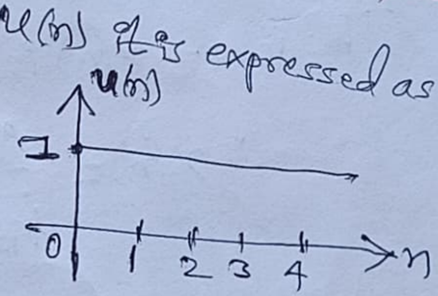
1. Folding — fold $h(k)$ about $k=0$ to obtain $h(-k)$.
2. Shifting — Shift $h(-k)$ by n_0 to the right (left) if n_0 is +ve (-ve) to obtain $h(n_0-k)$.
3. Multiplication — Multiply $x(k)$ by $h(n_0-k)$ to obtain product sequence $v_{n_0}(k) = x(k) \cdot h(n_0-k)$.
4. Summation — Sum all the values of the product sequence $v_{n_0}(k)$ to obtain the value of the output at time $n=n_0$.

ⓒ Write a program to generate (i) Unit step sequence (ii) Sinusoidal Sequence

[03 marks each, total-6M]

<u>Ans</u> — (i) Unit Step Sequence is denoted by $u(n)$ it is expressed as

$$u(n) = \begin{cases} 0 & \text{if } n<0 \\ 1 & \text{if } n \geq 0 \end{cases}$$



<u>Program</u> —
```
clc;
n=-5:1:5
x=(n>=0);
Subplot(3,2,1)
Stem(n,x,'r-','MarkerSize',10)
Xlabel('n')
ylabel('u(n)')
title('Unit step Sequence')
```

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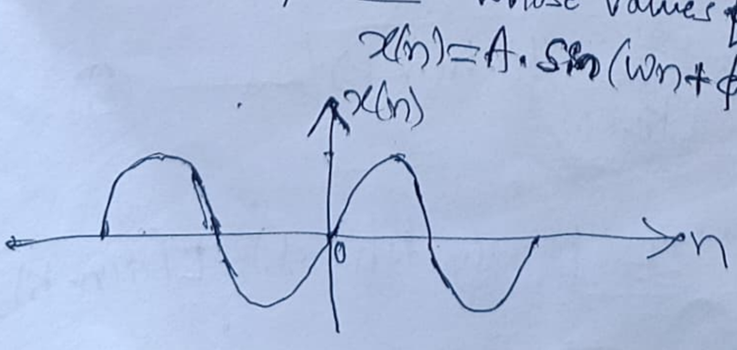
(ii) <u>Sinusoidal Sequence</u> — Whose values follows a sine or cosine function

$$x(n) = A \cdot \sin(wn + \phi)$$



Program —
```
Clc;
n=0:1:40
y=Sin(0.1*pi*n);
Subplot(3,2,4)
Stem(n,y,'g','MarkerSize',10)
xlabel('n')
ylabel('x(n)')
title('Sinusoidal Sequence')
```

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Module - 2

Q3 (a) Describe the properties of Z-transformation.

[Explaination - 7M]

Ans - The Z transform has following properties

① Linearity Property

if $x(n) \xleftrightarrow{ZT} X(Z)$ & $y(n) \xleftrightarrow{ZT} Y(Z)$

then according to linearity property,

$ax(n)+by(n) \xleftrightarrow{ZT} aX(Z)+bY(Z)$

② Time shifting property

if $x(n) \xleftrightarrow{ZT} X(Z)$

then according to time shifting property,

$x(n-m) \xleftrightarrow{ZT} Z^{-m}X(Z)$

③ Multiplication by exponential sequence property states that

$a^n x(n) \xleftrightarrow{ZT} X(Z/a)$

④ Time Reversal Property

if $x(n) \xleftrightarrow{ZT} X(Z)$ then according to time reversal property,

$x(-n) \xleftrightarrow{ZT} X(1/Z)$

⑤ Differentiation in Z Domain or Multiplication by n property states that,

if $x(n) \xleftrightarrow{ZT} X(Z)$ then, $n^k x(n) \xleftrightarrow{ZT} (-1)^k Z^k \frac{d^k X(Z)}{dZ^k}$

⑥ Convolution Property

if $x(n) \xleftrightarrow{ZT} X(Z)$ & $y(n) \xleftrightarrow{ZT} Y(Z)$

then according to convolution property,

$$x(n) * y(n) \xleftrightarrow{ZT} X(Z) \cdot Y(Z)$$

⑦ Correlation property

if $x(n) \xleftrightarrow{ZT} X(Z)$

& $y(n) \xleftrightarrow{ZT} Y(Z)$ then according to Correlation property

$$x(n) \otimes y(n) \xleftrightarrow{ZT} X(Z) \cdot Y(Z^{-1})$$

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ⓑ Show that Discrete Fourier Transform (DFT) is a Linear Transformation.

[Explaination – 7M]

Ans – The DFT as a linear transformation

The formula's for DFT & IDFT are given as below,

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \; ; \; k = 0,1,2 \ldots\ldots N-1 \quad \text{—— ①}$$

$$\& \; x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} \; ; \; n = 0,1,2 \ldots\ldots N-1 \quad \text{—— ②}$$

Where, $W_N = e^{-j2\pi/N}$ —— ③

Computation of each point of the DFT can be accomplished by N complex multiplications & (N−1) complex additions, hence an N point DFT can be computed in a total of N^2 complex multiplications & N(N−1) complex additions. In matrix from this can be represented as,

$$x_N = \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} \quad \& \quad X_N = \begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix}$$

$$W_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

The N point DFT can be expressed in matrix form as,

$$X_N = W_N x_N$$

If we assume that inverse of W_N exists then

$$x_N = W_N^{-1} X_N$$

this is expression for IDFT, in matrix form can be given as,

$$x_N = \frac{1}{N} W_N^{*} X_N$$

Where W_N^{*} is complex conjugate of W_N, hence,

$$W_N^{-1} = \frac{1}{N} W_N^{*}$$

Which becomes, $\boxed{W_N W_N^{*} = N I_N}$

Hence DFT is completely linear transformation.

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ⓒ Compute 4 point DFT of $x(n) = \{1,1,0,0\}$

[Example — 6M]

Ans — given $x(n) = \{1,1,0,0\}$

By using matrix method since $N=4$, we have

$$X(k) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

∴ 4 point DFT $X(k) = \{2, 1-j, 0, 1+j\}$

Q4 ⓐ Compute the N point DFT of, $x(n) = e^{j\omega m n}$

[Example – 6M]

Ans – given $x(n) = e^{j\omega m n}$

We know That, $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$

$$= \sum_{n=0}^{N-1} e^{j\omega m n}\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} \left[e^{j\omega m} \cdot e^{-j\frac{2\pi}{N}k}\right]^{n}$$

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We have $\sum_{N_1}^{N_2} a^n = \frac{a^{N_1} - a^{N_2+1}}{1-a}$; $a \neq 1$

here $a = e^{j\omega m} \cdot e^{-j\frac{2\pi}{N}k}$ & $N_1 = 0$ & $N_2 = N-1$

$$\therefore X(k) = \frac{1 - \left[e^{j\omega m} \cdot e^{-j\frac{2\pi}{N}k}\right]^{N}}{1 - \left[e^{j\omega m} \cdot e^{-j\frac{2\pi}{N}k}\right]}$$

$e^{-j2\pi k} = \cancel{\cos(2\pi k)}^{1} - j\cancel{\sin(2\pi k)}^{0} = 1 - 0 = 1$

$$\therefore \boxed{X(k) = \frac{1 - e^{j\omega m}}{1 - \left[e^{j\omega m} \cdot e^{-j\frac{2\pi}{N}k}\right]}}$$

ⓑ State and prove Symmetric property of DFT for real valued Sequence.

[Statement & Proof – 6M]

Ans – Symmetric property for real valued Sequence

if $x(n) \xleftrightarrow{DFT} X(k)$ & $x(n)$ is real, then according to Symmetric property

$$X(N-k) = X^{*}(k) = X(-k)$$

Proof – WKT, $X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$ —— ①

Now $X(N-k) = \sum_{n=0}^{N-1} x(n)\, W_N^{(N-k)n}$

$$X(N-k) = \sum_{n=0}^{N-1} x(n)\, W_N^{Nn} \cdot W_N^{-kn}$$

WKT $W_N^{Nn} = \cos\left(\frac{2\pi}{N} Nn\right) - j\sin\left(\frac{2\pi}{N} Nn\right)$

$= 1 - 0 = 1$ Always

$\therefore X(N-k) = \sum_{n=0}^{N-1} x(n)\, W_N^{-kn} = X^*(k)$

$\therefore \boxed{X(N-k) = X(-k)}$

ⓒ Compute the circular convolution of sequences $x_1(n) = \{2,1,2,1\}$ & $x_2(n) = \{1,2,3,4\}$

Ans: Given $x_1(n) = \{2,1,2,1\}$ & $x_2(n) = \{1,2,3,4\}$ [Example-8]

We have $y(n) = x_1(n) \circledast_4 x_2(n)$

First find DFT of $x_1(n) \xleftrightarrow{DFT} X_1(k)$

then find DFT of $x_2(n) \xleftrightarrow{DFT} X_2(k)$

$Y(k) = X_1(k) \cdot X_2(k)$

Now, $y(n) = IDFT\{Y(k)\}$

By using matrix method

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$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+1+2+1 \\ 2-\cancel{j}-2+\cancel{j} \\ 2-1+2-1 \\ 2+\cancel{j}-2-\cancel{j} \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$\therefore X_1(k) = \{6,0,2,0\}$

Now,

$$\begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$\therefore X_2(k) = \{10, -2+2j, -2, -2-2j\}$

Now $Y(k) = X_1(k) \cdot X_2(k) = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix}$

Now $y(n) = \text{IDFT}\{Y(k)\}$ here $N=4$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix}$$

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$$= \frac{1}{4} \begin{bmatrix} 56 \\ 64 \\ 56 \\ 64 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

$\therefore y(n) = x_1(n) \circledast_4 x_2(n) = \{14, 16, 14, 16\}$

Module-3

Q5 a) State and prove Circular shift property of DFT.

[Statement & proof — 6M]

Ans — if $x(n) \xleftrightarrow{DFT} X(k)$ then

$$x(n-n_0)_N \xleftrightarrow{DFT} X(k)\, e^{-j\frac{2\pi}{N}kn_0}$$

Proof — WKT IDFT is given by $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn}$

put $n = n - n_0$

ie $x(n-n_0) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}k(n-n_0)}$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn} \cdot e^{-j\frac{2\pi}{N}kn_0}$$

ie $x(n-n_0) = x(n) \cdot e^{-j\frac{2\pi}{N}kn_0}$ Taking DFT on both sides

$\text{DFT}\{x(n-n_0)\} = \text{DFT}\{x(n)\}\, e^{-j\frac{2\pi}{N}kn_0}$

$\therefore \text{DFT}\{x(n-n_0)\} = X(k)\, e^{-j\frac{2\pi}{N}kn_0}$ //

(b) Compare DFT and FFT with examples

[Comparision—6M]

Ans— Discrete Fourier Transform — DFT is a mathematical transformation that converts a sequence of data points in the time domain into its frequency domain representation.

The Fast Fourier Transform (FFT) is an efficient algorithm used to implement the DFT.

The FFT significantly reduces the computational cost associated with DFT making it practicle for real word applications.

The FFT algorithm reduces the number of computations significantly. Due to its speed FFT is used in real time signal processing, large datasets & various applications where computational efficiency is critical.

As the value of 'N' increases DFT needs very high number of computations. DFT is very slow & requires more complex additions & multiplications as 'N' increases, it will be cleared from below example

N	DFT Complex Multiplication (N^2)	DFT Complex Addition $N(N-1)$	FFT Complex Multiplication $(N/2)\log_2 N$	FFT Complex Addn $N\log_2 N$
2	4	2	1	2
4	16	12	4	8
8	64	56	12	24
16	256	240	32	64
32	1024	992	80	160

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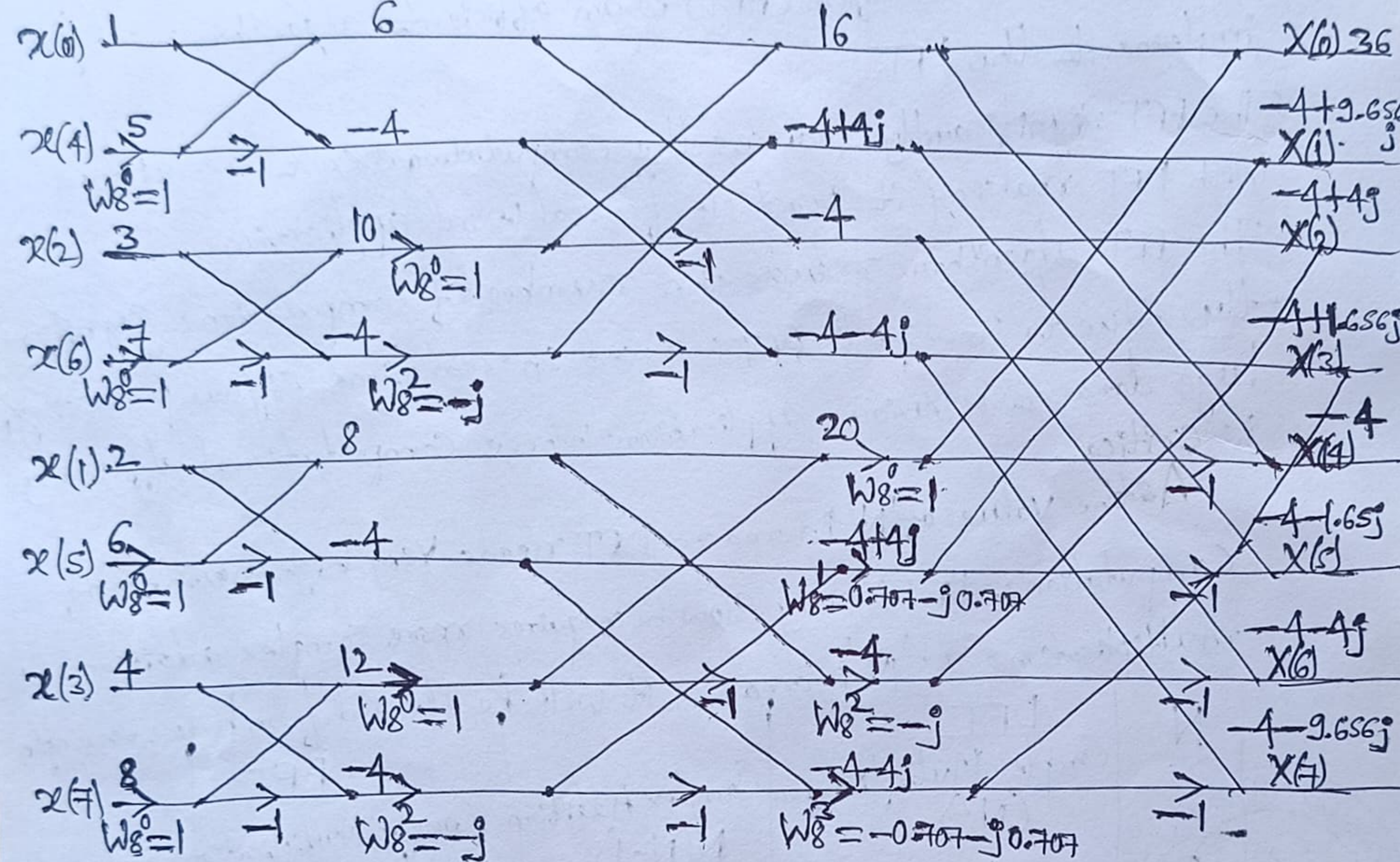
From the above table it is clear that as 'N' value increases the DFT computations becomes more complex & slow as compared with FFT Algorithms.

(c) Compute Radix-2 DIT-FFT of the following sequence $x(n) = n+1$ for $0 \le n \le 7$

[Example — 8M]

Ans — Given $x(n) = n+1$; $0 \le n \le 7$, Let us construct the signal $x(n)$ put $n = 0$ to 7 then $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8\}$ Algorithm to be used is DIT-FFT.

Solution —



$\therefore X(k) = \{36, -4+9.656j, -4+4j, -4+1.656j, -4, -4-1.656j, -4-4j, -4-9.656j\}$

Q6 (a) State and prove Parseval's theorem for DFT's

[Statement & Proof — 6M]

Ans — Parseval's Theorem

if $x_1(n) \xleftrightarrow{DFT} X_1(k)$ & $x_2(n) \xleftrightarrow{DFT} X_2(k)$

then, $$\sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k) \cdot X_2^*(k)$$

Proof — $$x_2(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_2(k)\, e^{j\frac{2\pi}{N}kn}$$

$$\& \; x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_2^*(k)\, e^{-j\frac{2\pi}{N}kn}$$

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Now $\sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \sum_{n=0}^{N-1} x_1(n) \cdot \frac{1}{N} \sum_{k=0}^{N-1} X_2^*(k)\, e^{-j\frac{2\pi}{N}kn}$

Rearranging above equation to,

$$= \frac{1}{N} \sum_{k=0}^{N-1} X_2^*(k) \cdot \sum_{n=0}^{N-1} x_1(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X_2^*(k) \cdot X_1(k)$$

$\therefore \sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k) \cdot X_2^*(k)$ // hence the proof

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(b) Explain overlap—Save method used for the convolution of the long input Sequences.

[Explaination—6M]

Ans— Overlap—Save Method

→ $h(n)$ of length 'M' & $x(n)$ Sequenced into blocks of 'L'

Step① — Select Value of $N = 2^M$

Step② The length of $h(n)$ is made 'N' by padding L−1 Zeros

$\cancel{N = L+M-1}$ $[N = M+L-1]$

$$h(n) = \{h(0), h(1) \ldots h(M-1), 0, 0, \ldots 0\}$$

Step③ The sequence $x(n)$ is divided into Sub sequences of long 'N' as

$$x_1(n) = \{0, 0, \ldots 0, x(0), x(1), \ldots x(L-1)\}$$

$$x_2(n) = \{x(L-M+1), \ldots x(L-1), x(L), x(L+1)\}$$

$$x_3(n) = \{x(2L-M+1), \ldots x(2L-1), x(2L), x(2L+1)\}$$

Step④ Calculate $Y_1(k) = X_1(k) \cdot H(k)$

$\boxed{y_1(n) = x_1(n) \textcircled{N} h(n)}$ $y_1(n) = \text{IDFT}\{Y_1(k)\}$

Step⑤ Repeat Step④ to obtain $y_2(n)$, $y_3(n)$ & so on.

Step⑥ first M−1 Samples of $y_1(n)$, $y_2(n)$, $y_3(n)$ are discarded and remaining Samples are fitted one after other to get final sequence.

ⓒ Develop an algorithm for Radix-2 FFT without using built in function.

[Algorithm - 8M]

Ans – The Decimation In Time (DIT) algorithm is one of the most widely used methods for efficiently computing the Fast Fourier Transform (FFT) of a sequence. The Radix-2 FFT Algorithm operates on an input sequence whose length N is a power of 2 (ie $N = 2^M$)

The key to the DIT Radix-2 FFT is the decimation step where the input is divided into two smaller sequences, this is done by splitting the sequence into Odd & Even indexed terms.

Step ① Divide & Conquer – Given a sequence $x(n)$ of length $N = 2^m$

Split into two sequences

Even indexed : $x[0], x(2), x(4), x(6)$

Odd indexed : $x(1), x(3), x(5), x(7)$

Step ② FFT of Sub sequences – Once the sequence has divided into smaller subsequences of length 1, The DFT is computed

Step ③ Combine Results using Butterfly operation

Butterfly combines two smaller DFT's to compute corresponding DFT of larger sequence

$$X(k) = X_{even}(k) + W_N^k \cdot X_{odd}(k)$$

$$X[k+N/2] = X_{even}(k) - W_N^k X_{odd}(k)$$

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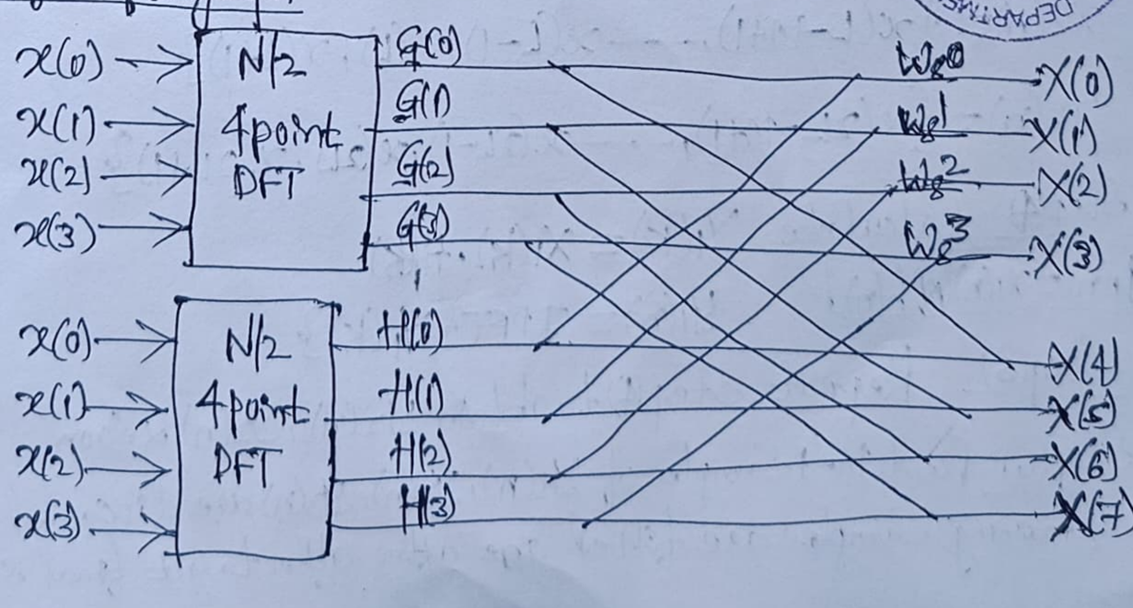
Signal flowgraph



Module – 4

Q7 (a) Obtain the frequency response expression for the symmetric linear phase FIR filter.

[Derivation – 8M]

Ans – Frequency response of a linear phase filter

$h(n) \xleftrightarrow{DTFT} H(\omega)$

For FIR filter, $H(\omega) = H_r(\omega) e^{j\theta(\omega)}$

Consider if N is Even

The Z transform is given by $H(Z) = \sum_{n=0}^{N-1} h(n) Z^{-n}$ —— ①

Eqs ① broken into two summations as,

$$H(Z) = \sum_{n=0}^{\frac{N}{2}-1} h(n) Z^{-n} + \sum_{n=N/2}^{N-1} h(n) Z^{-n} \quad \text{—— ②}$$

Let ~~m = N-1-n~~ in 2nd summation,

$$H(Z) = \sum_{n=0}^{\frac{N}{2}-1} h(n) Z^{-n} + \sum_{m=N/2-1}^{0} h(N-1-m) Z^{-(N-1-m)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n) Z^{-n} + \sum_{m=0}^{\frac{N}{2}-1} h(N-1-m) Z^{-(N-1-m)} \quad \text{—— ③}$$

m can be replaced by n

$$\therefore H(Z) = \sum_{n=0}^{N/2-1} h(n) Z^{-n} + \sum_{n=0}^{\frac{N}{2}-1} h(N-1-n) \cdot Z^{-(N-1-n)} \quad \text{—— ④}$$

Since $h(N-1-n) = h(n)$ for $n = 0, 1, 2, \ldots, N-1$

$$\therefore H(Z) = \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[Z^{-n} + Z^{-(N-1-n)} \right]$$

put $Z = e^{j\omega}$ to find frequency response of the digital filter

$$H(e^{j\omega}) = H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{-j\omega n} + e^{-j\omega(N-1-n)} \right]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} 2h(n) e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\frac{e^{-j\omega\left[n-\left(\frac{N-1}{2}\right)\right]} + e^{j\omega\left[n-\left(\frac{N-1}{2}\right)\right]}}{2} \right]$$

$$\therefore H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} 2h(n) \cos\left(\omega\left(n - \left(\frac{N-1}{2}\right)\right)\right)$$

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Comparing this with

$$H(w) = H_r(w)\, e^{j\theta(w)}$$

$$H_r(w) = \sum_{n=0}^{\frac{N}{2}-1} 2h(n) \cos\left(w\left(n - \left(\frac{N-1}{2}\right)\right)\right) \text{ ; N is Even} \quad —(5)$$

$$\& \ \theta(w) = \begin{cases} -w\left(\frac{N-1}{2}\right) & \text{; if } H_r(w) > 0 \\ -w\left(\frac{N-1}{2}\right) + \pi & \text{; if } H_r(w) < 0 \end{cases} \quad —(6)$$

(6) Compare different windows used to design FIR filters

Ans — Commonly used window types for FIR filter design are.

[Comparison — 6M]

① Rectangular window — It is defined as,

$$W_r(n) = \begin{cases} 1 & ; 0 \le n \le N-1 \\ 0 & ; \text{Elsewhere} \end{cases}$$

② Bartlett window (or Triangular window)

$$W_B(n) = \begin{cases} 1 - \dfrac{2\left|n - \left(\frac{N-1}{2}\right)\right|}{N-1} & ; 0 \le n \le N-1 \\ 0 & ; \text{otherwise} \end{cases}$$

③ Hanning window

$$W_{Han}(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) & ; 0 \le n \le N-1 \\ 0 & ; \text{otherwise} \end{cases}$$

④ Hamming window

$$W_{Ham}(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) & ; 0 \le n \le N-1 \\ 0 & ; \text{otherwise} \end{cases}$$

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© Design an FIR filter using Hamming window for N=7. The desired frequency response is given by

$$H_d(w) = \begin{cases} e^{jw3} & |w| \le \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} < |w| < \pi \end{cases}$$

[Example—6M]

Ans— By definition of Inverse DTFT,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w)\, e^{jwn}\, dw = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{-j3w} \cdot e^{jwn}\, dw$$

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$$= \frac{\sin\left[\frac{3\pi}{4}(n-3)\right]}{\pi(n-3)} \; ; n \neq 3$$

Now hd(n) for n=3 is, $h_d(3) = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} e^{0}\, dw = \frac{3\pi/4}{\pi} = \frac{3}{4}$

Impulse response of Lowpass filter is, $h(n) = h_d(n)\, W_{Ham}(n)$

Given N=7, $W_{Ham}(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right) & ; 0 \le n \le 6 \\ 0 & ; \text{otherwise} \end{cases}$

Since N is odd, the frequency response of the FIR filter is,

$$H(e^{jw}) = H(w) = e^{-jw\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h(n)\cos\left[w\left(n-\left(\frac{N-1}{2}\right)\right)\right]\right]$$

put N=7 then,

$$H(w) = e^{-j3w}\left[h(3) + 2h(0)\cos 3w + 2h(1)\cos 2w + 2h(2)\cos w\right]$$

$h(n) = h_d(n) \times W_{Ham}(n)$

$$\therefore H(w) = e^{-j3w}\left[0.75 + 2\times 0.006\cos 3w + 2\times -0.049\cos 2w + 2\times 0.173\cos w\right]$$

$$H(w) = e^{-j3w}\left[0.75 + 0.012\cos 3w - 0.098\cos 2w + 0.346\cos w\right]$$

Q8 (a) Discuss the Characteristics of practical frequency selective filters.

[Explaination—6M]

Ans— As compared with ideal filters, the practical frequency selective filters dont have sharp transition between passbands & stopbands, The characteristics are.

1. Passband Ripple— In the pass band, the filters gain or magnitude is not perfectly constant, it may fluctuate this is called passband Ripple.

2. Stopband Attenuation— In the stopband the filter doesnot completly blocks signals. It attenuates ~~them~~, meaning it reduces their amplitude

3. Transition Bandwidth— It is the frequency range between the passband & stopband where the filter response is changing from the passing to attenuating the signal.

4. Roll off rate— Roll off rate describes how quickly the filter's response decreases (rolls off) in the transition band measured interms of decibels per decade.

5. Practicle filters must be Cousal ie their output depends only on past & present inputs, not on the future.

(b) Explain the steps of designing linear phase FIR highpass filter.

Ans— Design steps

[Explaination—8M]

Step 1 We begin with the desired frequency response Specification $H_d(\omega)$ and determine the corresponding unit sample response $h_d(n)$ by finding the inverse ~~DTFT~~ of $H_d(\omega)$ using,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega \quad \text{——— (1)}$$

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Step② – Find the impulse response h(n) and the filter co-efficients hd(n),

$$h(n) = hd(\omega) \cdot W(n)$$

Where W(n) is the window and can be

- Rectangular window
- Bartlett window
- Hamming window
- Hanning window
- Blackman window

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Step③ Depending on whether N/M is odd or even, find frequency response

if N is odd then

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h(n)\cos\left[\omega\left(n-\left(\frac{N-1}{2}\right)\right)\right]\right]$$

if N is even then,

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} 2h(n)\cos\left(n-\left(\frac{N-1}{2}\right)\right)$$

ⓒ Realize the system function of following FIR filter in cascade form

$$H(z) = 1 - 2z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{2}z^{-4}$$

[Realization–6M]

Ans – Given $H(z) = 1 - 2z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{2}z^{-4}$

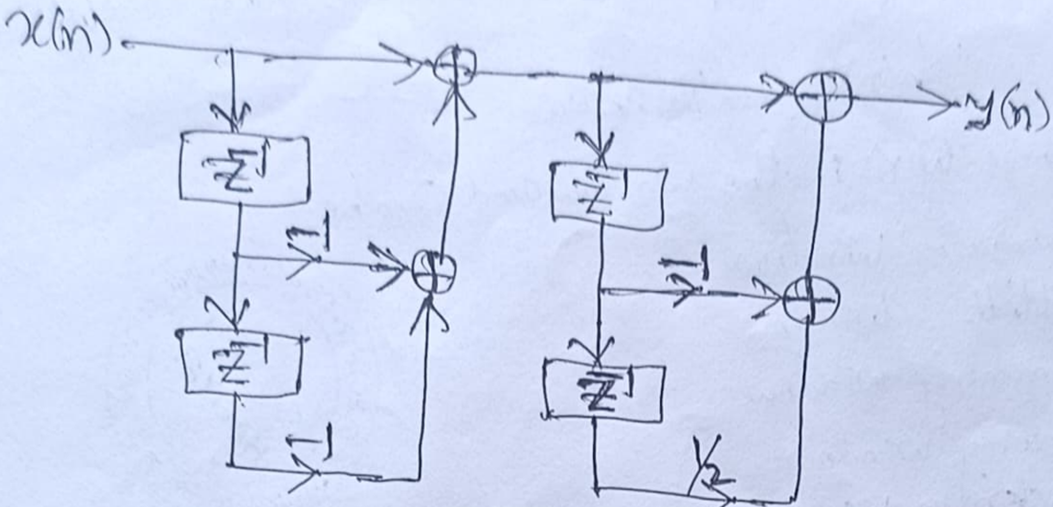
$$= \frac{z^4}{z^4}\left[1 - 2z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{2}z^{-4}\right]$$

$$= \frac{1}{z^4}\left[z^4 - 2z^3 + \frac{1}{2}z^2 + \frac{1}{2}z - \frac{1}{2}\right]$$

Factorized as,

$$H(z) = \underbrace{(1 - z^{-1} - z^{-2})}_{H_1(z)}\underbrace{\left(1 - z^{-1} + \frac{1}{2}z^{-2}\right)}_{H_2(z)}$$

The cascade form structure can be obtained as,



Module-5

Q9 a) Explain the design procedure of Analog Butterworth Lowpass Prototype filter.

[Explanation - 8M]

Ans - In the design of the filter the requirements are normally given in terms of a set of critical frequencies (eg - Ω_p, Ω_s and gains k_p & k_s). The set of conditions for Lowpass filter are,

$$0 \geq 20\log|H(j\Omega)| \geq k_p \quad \text{; for all } \Omega \leq \Omega_p \quad \text{--- (A)}$$

$$20\log|H(j\Omega)| \leq k_s \quad \text{; for all } \Omega \geq \Omega_s \quad \text{--- (B)}$$

The magnitude frequency response of Butterworth LPF is,

$$|H_n(j\Omega)|^2 = \frac{1}{\left|1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}\right|} \quad \text{--- (1)}$$

From equation (A),

$$20\log|H(j\Omega)| \geq k_p \quad \text{; for } \Omega \leq \Omega_p$$

$$\log|H(j\Omega)|^2 \geq \frac{k_p}{10}$$

$$\log\left|\frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}}\right| \geq \frac{k_p}{10}$$

$$\cancel{\log 1}^{\,0} - \log\left[1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}\right] = \frac{k_p}{10}$$

$$\text{At } \Omega = \Omega_p \text{ ; } \left(\frac{\Omega_p}{\Omega_c}\right) = 10^{-\frac{k_p}{10}} - 1 \quad \text{--- (2)}$$

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lly from eqn (B)
$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2n} = 10^{\frac{-k_s}{10}} - 1 \quad \text{--- (3)}$$

Divide eqn (3) by (2)
$$\left(\frac{\Omega_p}{\Omega_s}\right)^{2n} = \frac{10^{\frac{-k_p}{10}} - 1}{10^{\frac{-k_s}{10}} - 1} \quad \text{--- (4)}$$

Taking log on both sides,
$$2n \log\left(\frac{\Omega_p}{\Omega_s}\right) = \log\left[\frac{10^{\frac{-k_p}{10}} - 1}{10^{\frac{-k_s}{10}} - 1}\right]$$

$$\therefore \quad n = \frac{\log\left[\left(10^{\frac{-k_p}{10}} - 1\right) \Big/ \left(10^{\frac{-k_s}{10}} - 1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)} \quad \text{--- (5)}$$

Steps involved in design of Butterworth filter from the given specification $H(j\Omega)$.

(1) Determine the order of the filter n.
(2) Select the cutoff frequency Ω_c.
(3) Find the transfer function of normalized Butterworth LPF $H_n(s) = \frac{1}{B_n(s)}$
(4) From analog lowpass to lowpass frequency transformation, find the desired transfer function which is given by,
$$H_a(s) = H_n(s)\Big|_{s \rightarrow \frac{s}{\Omega_c}}$$

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(b) Construct the system function in S-domain for N=4

[Construction of system function - 6M]

Ans — Given N=4, for n even: $S_k = 1 \angle \frac{\pi}{2n} + \frac{k\pi}{n}$; $k = 0, 1, 2 \ldots 2n-1$

We have the poles S_0 to S_7 given as,

$S_0 = 1 \angle \frac{\pi}{8} = 1 \angle 22.5^\circ$

$S_1 = 1 \angle \frac{\pi}{8} + \frac{\pi}{4} = 1 \angle 67.5^\circ$

$S_2 = 1 \angle \frac{\pi}{8} + \frac{\pi}{2} = 1 \angle 112.5^\circ$

$S_3 = 1 \angle \frac{\pi}{8} + \frac{3\pi}{4} = 1 \angle 157.5^\circ$

$S_4 = 1 \angle \frac{\pi}{8} + \pi = 1 \angle 202.5^\circ$

$S_5 = 1 \angle \frac{\pi}{8} + \frac{5\pi}{4} = 1 \angle 247.5^\circ$

$S_6 = 1 \angle \frac{\pi}{8} + \frac{3\pi}{2} = 1 \angle 292.5^\circ$

$S_7 = 1 \angle \frac{\pi}{8} + \frac{7\pi}{4} = 1 \angle 337.5^\circ$

Now $H_4(s) = \dfrac{1}{(s-s_2)(s-s_3)(s-s_4)(s-s_5)}$

$$= \frac{1}{(s-s_2)(s-s_2^*)(s-s_3)(s-s_3^*)} \quad \because s_2 = s_5^*,\ s_3 = s_4^*$$

$$= \frac{1}{[(s+0.3827)^2+(0.9239)^2][(s+0.9239)^2+(0.3827)^2]}$$

$$\therefore H_4(s) = \frac{1}{\cancel{(s^2+0.3827)^2+}\,[(s^2+0.7654s+1)(s^2+1.8478s+1)]}$$

(c) Realize direct form-2 for the IIR filter represented by,

$$y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-2)$$

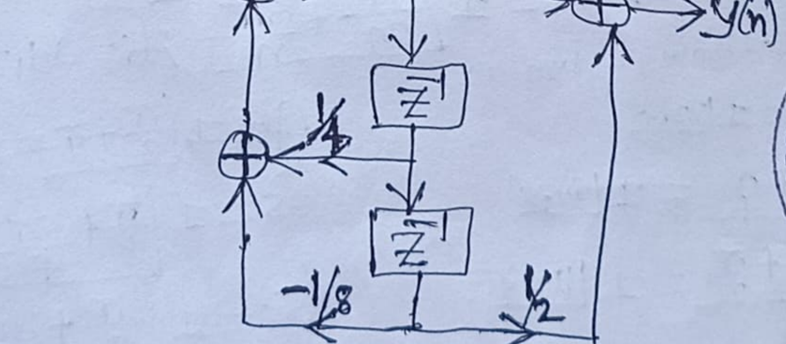
Ans- Given- $y(n) - \frac{1}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-2)$ [Realization-6M]

$$\therefore y(n) = x(n) + \frac{1}{2}x(n-2) + \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2)$$

taking Z transform on both sides,

$$Y(z) - \frac{1}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z) + \frac{1}{2}z^{-2}X(z)$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{2}z^{-2}}{1-\frac{1}{4}z^{-1}+\frac{1}{8}z^{-2}}$$



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Q10 (a) Design the digital IIR filter for following details −3dB gain at 0.5π rads and the stopband attenuation of 15dB at 0.75π rads. Assume T = 15.

[Example−8M]

Ans— Given— $\Omega_p = 0.5\pi$ rad/sec, $\Omega_s = 0.75\pi$ rad/sec

$k_p = -3dB$ & $k_s = 15dB$, $T = 15sec$

Solution—① To find order of the filter,

$$n = \frac{\log\left[\left(10^{\frac{-k_p}{10}} - 1\right) \Big/ \left(10^{\frac{-k_s}{10}} - 1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$n = \frac{\log\left[\left(10^{0.30} - 1\right) \Big/ \left(10^{-0.015} - 1\right)\right]}{2\log\left[\frac{0.5}{0.75}\right]}$$

$$= 0.932$$

Selecting next smallest integer $n = 1$

② The corresponding digital frequencies are,

$W_p = \Omega_p T = 0.5\pi \times 15 = 7.5\pi$ rad/sec

$W_s = \Omega_s T = 0.75\pi \times 15 = 11.25\pi$ rad/sec

③ Prewrap the frequencies using $T = 1sec$,

$$\Omega_p' = \frac{2}{T}\tan\left(\frac{W_p}{2}\right) = 2\tan\left(\frac{7.5\pi}{2}\right) = 1.453$$

$$\Omega_s' = \frac{2}{T}\tan\left(\frac{W_s}{2}\right) = 2\tan\left(\frac{11.25\pi}{2}\right) = 0.447$$

④ The transfer function for ($\Omega_c = 1$ rad/sec) is given by,

$$H(s) = \frac{1}{s+1}$$

$$\therefore H_a(s) = H_1(s)\Big|_{s \to \frac{\Omega_p'}{s} = \frac{1.453}{s}}$$

$$\therefore \boxed{H_a(s) = \frac{s}{s+1.453}}$$

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(b) Explain the Significance of (i) Prewrapping, (ii) Bilinear transformation.

[Explanation—6M]

Ans— (i) Significance of Prewrapping

In Converting Analog filter to Digital filter we use the formula

$$S \rightarrow \frac{2}{T}\left[\frac{1-Z^{-1}}{1+Z^{-1}}\right]$$

Analog in S domain Converted digital in Z-Domain

The frequency which is transformed to digital frequency the relationship between them is non linear & this is Called 'Wrapping'. This process is called Frequency wrapping hence Frequency Prewrapping is needed to make frequency linear. The prewrapping is done using

$$\Omega = \frac{2}{T}\left[\frac{2\sin\omega}{1+1+2\cos\omega}\right]$$

$$= \frac{2}{T}\left[\frac{\sin\omega}{1+\cos\omega}\right]$$

$$\boxed{\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)}$$

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(ii) Bilinear transformation — It is method of converting Analog frequency specifications to digital frequency specifications & Vice versa if the following specifications are given

Passband edge frequency $= \omega_p$

Stop band edge frequency $= \omega_s$

Now prewrapped frequencies are, $\Omega_p' = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right)$ —— (1)

& $\Omega_s' = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right)$ —— (2)

Bilinear transformation is the efficient method of converting Analog filter to Digital filter that is Prewrapping is done before applying Bilinear transformation to eliminate nonlinearities

(c) Obtain the direct form-I realization of following IIR filter

$$H(z) = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}}$$

[Example-6M]

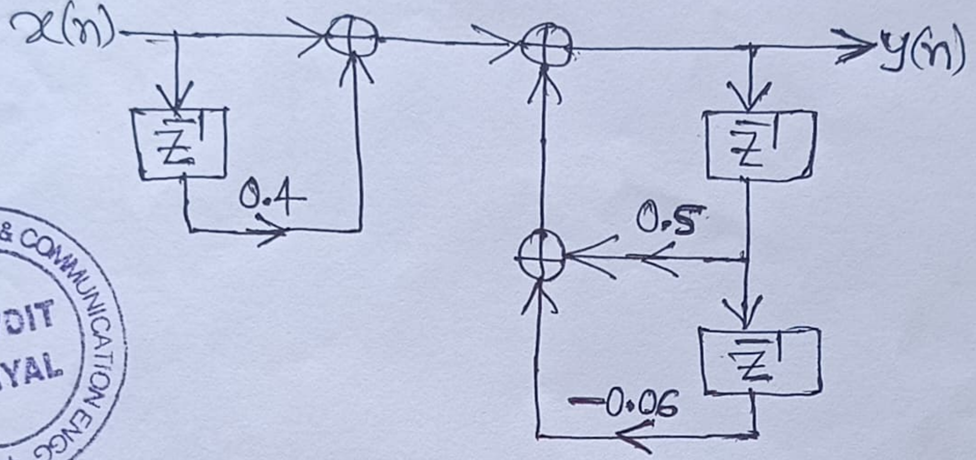
Ans— Given $H(z) = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}} = \frac{Y(z)}{X(z)}$

Taking Inverse Z-transform,

$$y(n) - 0.5y(n-1) + 0.06y(n-2) = x(n) + 0.4x(n-1)$$

$$\therefore y(n) = x(n) + 0.4x(n-1) + 0.5y(n-1) - 0.06y(n-2)$$

Direct form-I structure is shown below,



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