# KLS Vishwanathrao Deshpande Institute of Technology

(Accredited by NAAC with "A" Grade)
(Approved by AICTE, New Delhi, Affiliated to VTU, Belagavi)
(Recognized Under Section 2(f) by UGC, New Delhi)
Udyog Vidya Nagar, Haliyal - 581 329, Dist.: Uttara Kannada
Phone: 08284 - 220861, 220334, 221409, Fax: 08284 - 220813
www.klsvdit.edu.in | principal@klsvdit.edu.in | hodece@klsvdit.edu.in

## DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING


# University / Model Question Paper Scheme & Solution

| | | |
|---|---|---|
| Faculty Name | : | Prof. Plasin Francis Dias. |
| Course Name | : | Digital Communication |
| Course Code | : | BEC 503 |
| Year of Question Paper | : | Dec 2024 / Jan 2025 |
| Date of Submission | : | 20-06-2025 |

Faculty Member HoD Dean (Acad.)

20/06/2025

Head, Dept. of Electronic & Communication Engg.
KLS V.D.I.T., HALIYAL (U.K.)

20/6/25

Dean, Academics
KLS VDIT, HALIYAL

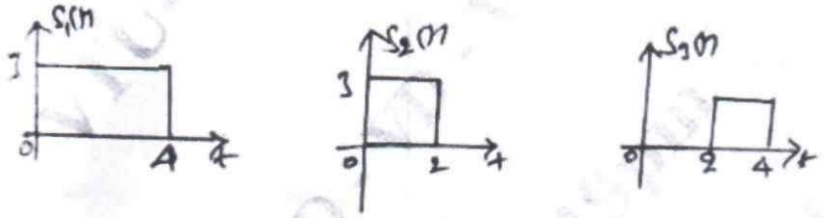
# CBCS SCHEME

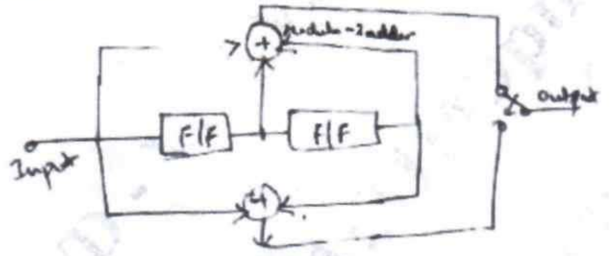
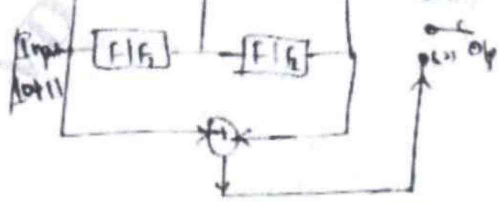
USN 2 V D 2 2 E C 1 0 9 **BEC503**

## Fifth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
## Digital Communication

Time: 3 hrs. Max. Marks: 100

***Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.***
***2. M : Marks , L: Bloom's level , C: Course outcomes.***

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a. | Explain Hilbert transform and its properties. | 6 | L2 | CO1 |
| | b. | Describe the canonical representation of bandpass signal. | 7 | L2 | CO1 |
| | c. | Describe the correlation receiver with neat diagram. | 7 | L2 | CO1 |
| | | **OR** | | | |
| Q.2 | a. | Apply gram Schmidt orthogonalizaiton procedure find the set of orthonormal basis function to represent the signals $S_1(t)$, $S_2(t)$ and $S_3(t)$ as shown in Fig.Q2(a). Also express each of these figures interms of set of basis function.   Fig.Q2(a) | 10 | L3 | CO1 |
| | b. | Derive the equation for converting continuous AWGN channel into a vector channel. | 10 | L2 | CO1 |
| | | **Module – 2** | | | |
| Q.3 | a. | Describe with a neat diagram, the generation and detection of BPSK signal. | 8 | L2 | CO2 |
| | b. | Define bandwidth efficiency. Tabulate the comment on the bandwidth efficiency of M-ary PSK signal. | 8 | L2 | CO2 |
| | c. | Encode the binary sequence using DPSK 11011011. Assume reference bit as 1. | 4 | L2 | CO2 |
| | | **OR** | | | |
| Q.4 | a. | Derive the expression for probability of error of QPSK signal. | 8 | L2 | CO2 |
| | b. | Discuss the non-coherent detection of BFSK signal. | 8 | L2 | CO2 |
| | c. | Calculate the average power required for a DPSK signal operation gat a data rate of 1000 bit/sec, over a band-pass channel having a bandwidth of 3000 Hz, $\frac{N_0}{2} = 10^{-10}\, w/H_z$ probability of error $P_e = 10^{-5}$. | 4 | L3 | CO2 |
| | | **Module – 3** | | | |
| Q.5 | a. | Define entropy and summaries its properties. | 6 | L2 | CO3 |
| | b. | A source has five symbols $S = \{S_1, S_2, S_3, S_4, S_5\}$ with probabilities $P = \{0.4, 0.2, 0.2, 0.1, 0.1\}$ respectively. compute the source code using Huffman binary coding. Also find the average length and entropy. | 8 | L3 | CO3 |
| | c. | Briefly discuss instantaneous code with an example. | 6 | L2 | CO3 |
| | | **OR** | | | |
| Q.6 | a. | Derive the expression for mutual information and summarize its properties. | 10 | L2 | CO3 |
| | b. | Derive the expression for the channel capacity of binary symmetric channel. | 10 | L3 | CO3 |

| | | | | | |
|---|---|---|---|---|---|
| | | **Module – 4** | | | |
| **Q.7** | **a.** | Indicate the advantages and disadvantages of error control coding. Also differentiate between block code and convolution code. | 8 | L2 | CO4 |
| | **b.** | If 'C' is a valid code vector then show that $CH^T = 0$ where $H$ is parity check matrix of code. | 5 | L2 | CO4 |
| | **c.** | Design an encoder for the (7, 4) binary cyclic code generated by : $g(x) = 1 + x + x^3$ for the message vector [1001]. | 7 | L3 | CO4 |
| | | **OR** | | | |
| **Q.8** | **a.** | Describe the block diagram of generator and parity check matrix with equation. Also write the syndrome equation and list its properties. | 10 | L2 | CO4 |
| | **b.** | A (7, 4) Linear block code has :<br>$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$<br>i) All possible code vector<br>ii) Determine the Hamming weight of each code word<br>iii) If the received vector is [1100010]. Determine its syndrome correct the codeword. | 10 | L3 | CO4 |
| | | **Module – 5** | | | |
| **Q.9** | **a.** | For a given convolutional encoder shown in Fig.Q9(a), with D = 10011. Compute output sequence using transform domain approach. Also draw the code free diagram.<br><br><br>Fig.Q9(a) | 10 | L3 | CO5 |
| | **b.** | Describe the recursive systematic convolutional code encoder with an example. | 10 | L3 | CO5 |
| | | **OR** | | | |
| **Q.10** | **a.** | A convolution encoder has two flip-flop with two states, three modulo – 2 adders and an output multiplexer. The generator sequences of the encoder. $g^{(1)} = (1, 0, 1)$, $g^{(2)} = (1, 1, 0)$, $g^{(3)} = (1, 1, 1)$.<br>i) Generator matrix [G]<br>ii) Draw the encoder block diagram<br>iii) Calculate the codeword for the message input vector 11101. | 10 | L3 | CO5 |
| | **b.** | For a given convolution encoder shown in Fig.Q10(b). Build state table, state transaction table, sketch diagram and describe the Trellis diagram for the input message vector (10111).<br><br><br>Fig.Q10(b) | 10 | L3 | CO5 |

* * * * *

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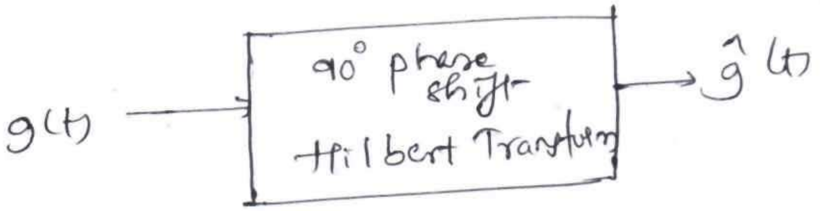
1 a. Hilbert transform and its properties (6M)

The Hilbert transform of g(t), which is denoted by $\hat{g}(t)$ is defined by,

$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau} \cdot d\tau$$

Hilbert transform of a signal g(t) is defined as the transform in which phase angle of all components of the signal is shifted by $\pm 90^\circ$




The functions g(t) and $\hat{g}(t)$ are said to constitute a Hilbert transform pair. The Hilbert transform is called a quadrature filter. Hilbert transform is a linear operation.

$$g(t) \xleftrightarrow{HT} \hat{g}(t)$$

$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau} \cdot d\tau$$

$$\Rightarrow \hat{g}(t) = g(t) * \frac{1}{\pi t} \quad //$$

Impulse response of HT

$$h(t) = \frac{1}{\pi t} \quad //$$

Frequency response of h(t) is,

$$h(t) \xleftrightarrow{FT} H(f)$$

(3M)

Ex 1: Time function $\delta(t)$ — Impulse response — frequency response //

its Hilbert transform is $1/\pi t$ //

Ex: 2 Time function $\cos 2\pi f_c t$ Hilbert transform $\sin 2\pi f_c t$ + (3M)

Properties of Hilbert Transform:

<u>Property 1:</u> A signal $g(t)$ and its Hilbert transform have same magnitude spectrum

$$|G(f)| = |\hat{G}(f)|$$

<u>Property 2:</u> If $\hat{g}(t)$ is the Hilbert transform of $g(t)$, then the Hilbert transform of $\hat{g}(t)$ is $-g(t)$

$$\arg[G(f)] = -\arg\{G(f)\}$$

<u>Property 3:</u> A signal $g(t)$ and its Hilbert transform $\hat{g}(t)$ are orthogonal over the entire time interval $(-\infty, \infty)$

Orthogonality of $g(t)$ and $\hat{g}(t)$ is described by,

$$\int_{-\infty}^{\infty} g(t) \cdot \hat{g}(t) \cdot dt = 0$$

→ Hilbert transform is applicable to any signals that is fourier transformable.

→ Hilbert transform differ from fourier transform that it operates exclusively in time domain. ie it does not change the domain of the signal.

→ $\hat{x}(t)$ is not an equivalent representation of $x(t)$.

→ Hilbert transformer is a $-90°$ phase shifter, without changing its amplitude.

→ $\hat{X}(f) = X(f)\,[-j\,\mathrm{sgn}(f)]$

$$= \begin{cases} -j\,X(f) & ; f > 0 \\ +j\,X(f) & ; f < 0 \end{cases}$$

1 b: Canonical representation of Band Pass signal. (7M)

$$S_+(t) = s(t) + j\hat{s}(t)$$

$$\therefore s(t) = Re\{S_+(t)\}$$

But $S_+(t) = \tilde{s}(t) \cdot e^{-j2\pi f_c t}$

$$\therefore s(t) = Re\{\tilde{s}(t) \cdot e^{j2\pi f_c t}\} \quad \text{—— (A)}$$

Since $\tilde{s}(t)$ is a complex valued quantity, we can express it in cartesian form as

$$\tilde{s}(t) = s_I(t) + j s_Q(t)$$

$$\therefore s(t) = Re\{[s_I(t) + j s_Q(t)]\, e^{j2\pi f_c t}\}$$

$$= Re\{[s_I(t) + j s_Q(t)][\cos(2\pi f_c t) + j\sin(2\pi f_c t)]\}$$

$$\therefore s(t) = \underbrace{s_I(t)\cos(2\pi f_c t)}_{\text{Inphase component}} - \underbrace{s_Q(t)\cdot\sin(2\pi f_c t)}_{\text{Quadrature components}} \quad \text{—— (1)}$$

This is canonical or Standard form.

$s_I(t)$ and $s_Q(t)$ are low pass signals, limited to $-w \le f \le w$. They can be extracted from $s(t)$.

Carrier frequency $f_c$ larger than low pass ~~band~~ bandwidth $w$, $s(t)$ is referred as pass band signal waveform mapping from $s_I(t)$ and $s_Q(t)$ combined into $s(t)$ is known as "Pass Band Modulation" 3M +

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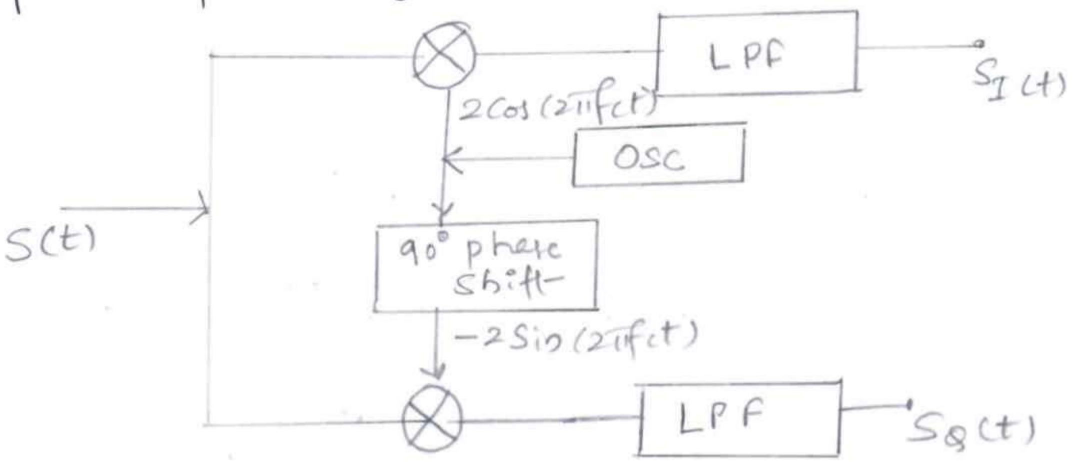
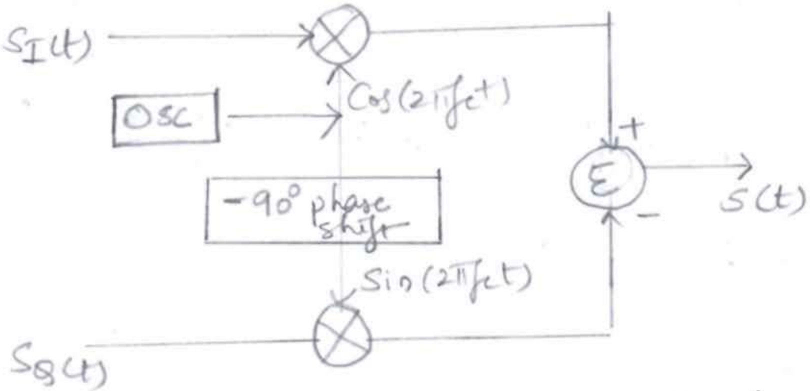
fig(a) Scheme for deriving in phase and quadrature component of band pass signal g(t) (4m)




fig(b) Scheme for reconstructing the band pass signal from its inphase and quadrature component.

1C. Correlation Receiver :- (7m)

for an AWGN, when the transmitted signals $s_1(t)$, $s_2(t)$ - - - $s_m(t)$ are equally likely is called a Correlation Receiver.

It consists of Two subsystems, which are ① Detector ② Maximum likelihood decoder.

①Detector:

Detector consists of M correlators supplied with a set of orthonormal basis functions $\phi_1(t)$, $\phi_2(t)$ - - - - - $\phi_N(t)$ that are generated locally. This bank of correlators operates on the received signal $x(t)$, $0 \le t \le T$, to produce the observation vector $\vec{x}$.

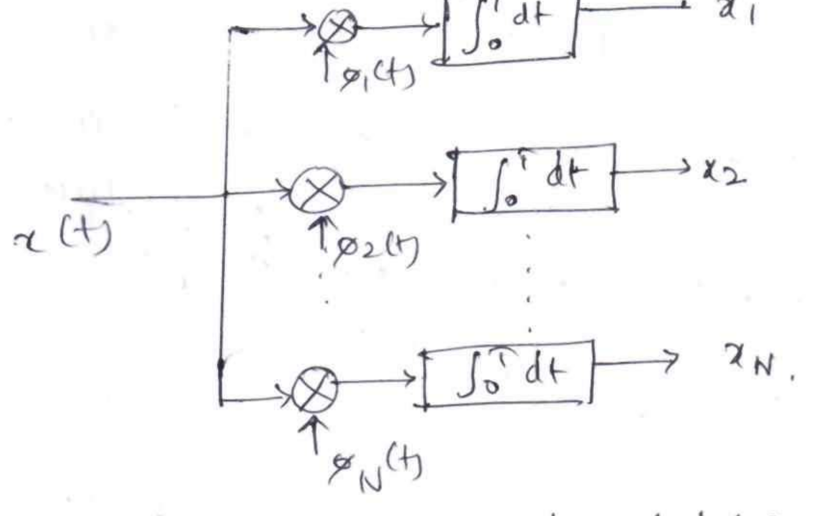



fig 1. Detector or demodulator

Observation Vector
$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$ //

3M+

Maximum likelihood Decoder:

• Maximum likelihood decoder operates on the observation vector x to produce an estimate $\hat{m}$ of the transmitted symbol $m_i$, $i = 1, 2 \dots m$ in such a way that the average probability of symbol Error is minimized.

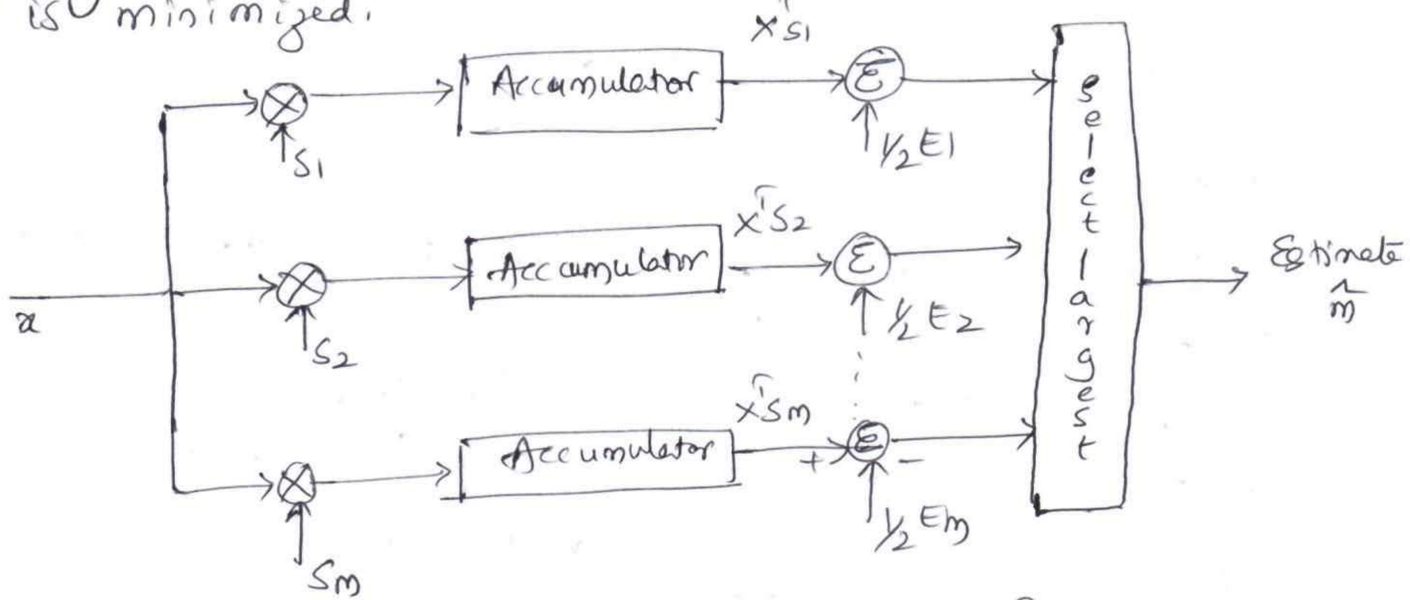



fig 2. Signal Transmission Decoder.

The observation vector x is multiplied by m signal vectors $S_1, S_2 \dots S_m$ and the resulting products are successively summed in accumulators, to get the corresponding set of inner products

$$\{ x^T S_k, \quad k = 1, 2, 3 \dots m \}$$

Finally the largest in the resulting set of number is selected and corresponding decision is made on the transmitted signal.

The optimum receiver is commonly referred to as a correlation receiver.

(4m)

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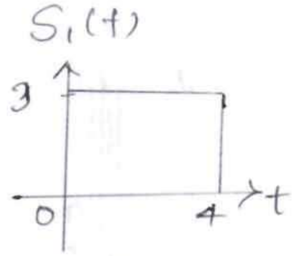
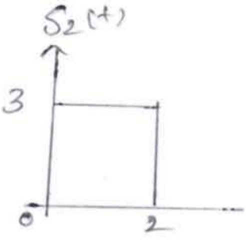
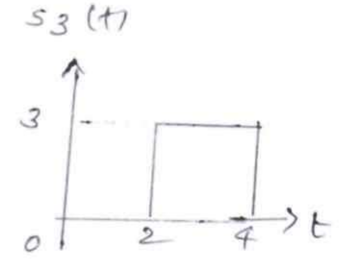
Q 2a Gram Schmidt orthogonalization Procedure




Step 1 : first Basis function.

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$$

(i) N = Number of basis function = ?

(ii) Express $S_1, S_2, S_3$ in terms of basis function.

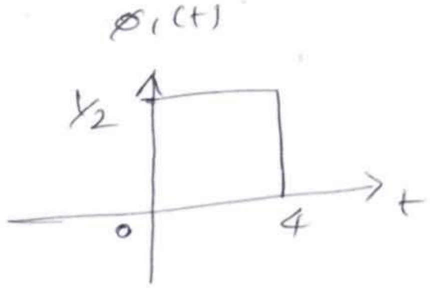
$$E_1 = \int_0^T S_1^2(t) \cdot dt$$

$$= \int_0^4 3^2 \cdot dt$$

$$= 9[t]_0^4$$

$$= 9[4-0]$$

$E_1 = 36$ Joules




$$\therefore \phi_1(t) = \frac{3}{\sqrt{36}} = \frac{3}{\sqrt{9 \times 4}} = \frac{\cancel{3}}{\cancel{3} \times 2}$$

$$\therefore \phi_1(t) = \frac{1}{2}$$

$$\therefore S_1(t) = \phi_1(t) \cdot \sqrt{E_1}$$

$$\therefore S_1(t) = \sqrt{36}\, \phi_1(t) = \underline{6\, \phi_1(t)}$$

OR $\quad \phi_1(t) = \frac{S_1(t)}{\sqrt{36}} = \frac{1}{6} S_1(t)$

$$\phi_1(t) = \begin{cases} \frac{1}{2} & ; \ 0 \le t \le 4 \\ 0 & ; \ \text{o.w.} \end{cases}$$

Number of orthonormal functions (N) = Number of Independent signals
= 2

ie $S_2(t)$, $S_3(t)$ are independent signals

and $S_1(t)$ is dependent signals.
because $S_1(t) = S_2(t) + S_3(t)$

i.e. linear combination of two signals.

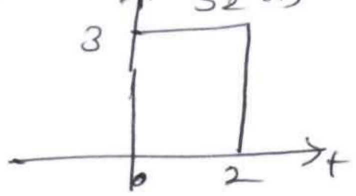
Step 2: To find $2^{nd}$ orthonormal function

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\, \phi_j(t)$$

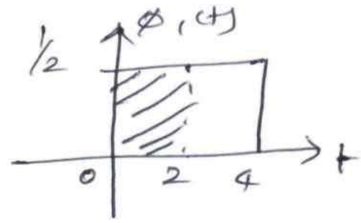
$\hookrightarrow$ $2^{nd}$ intermediate signal

$i=2$

$$g_2(t) = s_2(t) - \sum_{j=1}^{1} s_{2j}\, \phi_j(t)$$




$$g_2(t) = s_2(t) - s_{21}\, \phi_1(t)$$

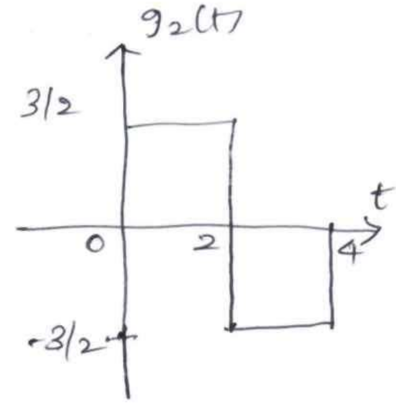



$$s_{21} = \int_0^T s_2(t) \cdot \phi_1(t) \cdot dt$$

$$= \int_0^2 (3)(1/2) \cdot dt = 3/2 \left[t\right]_0^2$$

$$= \frac{3}{2}\left[2-0\right] = 3$$

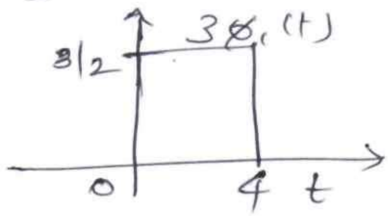
$$\therefore s_{21} = 3$$




$$g_2(t) = s_2(t) - 3\phi_1(t)$$

$$= \begin{cases} 3 - 3/2 = +3/2 & ; \quad 0 \le t \le 2 \\ (0 - 3/2) & ; \quad 2 \le t \le 4 \end{cases}$$

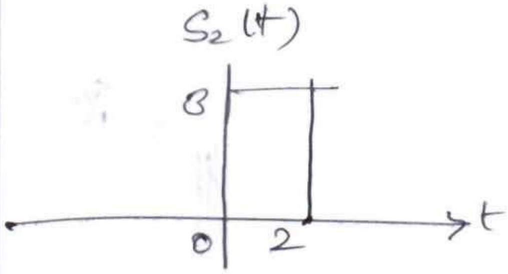
$$E_{g_2}(t) = \int_0^T g_2^2(t) \cdot dt = \int_0^2 (3/2)^2 \cdot dt + \int_2^4 (-3/2)^2 \cdot dt$$

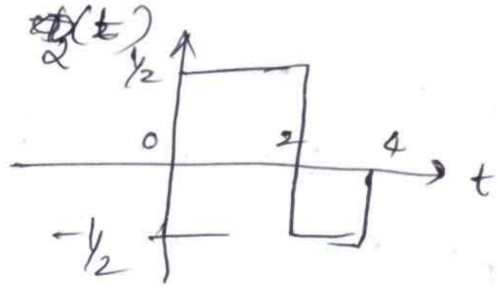



$$= \frac{9}{4} \cdot \left[t\right]_0^2 + \frac{9}{4}\left[t\right]_2^4$$

$$= \frac{9}{4}\left[2-0\right] + \frac{9}{4}\left[4-2\right]$$

$$E_{g_2} = \frac{9}{4}\left[2+2\right] = \frac{9}{4} \times 4 = 9 \text{ Joules}$$




$$\therefore S_1(t) = 6\phi_1(t)$$

$$S_2(t) = 3[\phi_1(t) + \phi_2(t)]$$

$$S_2(t) = 3\phi_1(t) + 3\phi_2(t)$$

$$S_3(t) = S_1(t) - S_2(t)$$

$$= 6\phi_1(t) - 3\phi_1(t) - 3\phi_2(t)$$

$$\therefore S_3(t) = 3\phi_1(t) - 3\phi_2(t)$$

Signal constallation Diagram

$S_1(t) = 6\phi_1(t)$

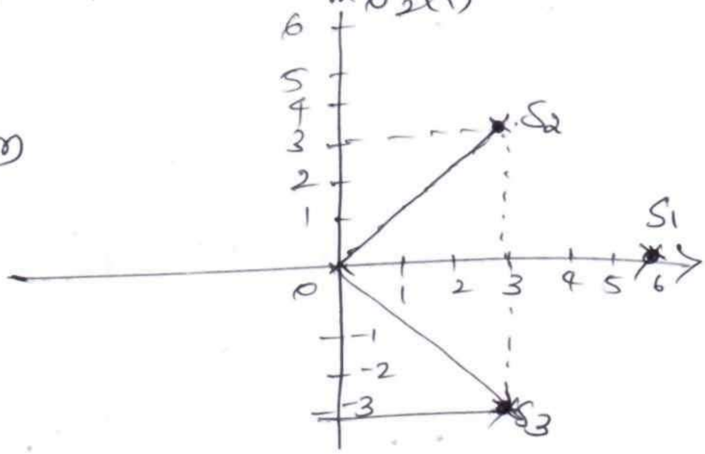
$S_2(t) = 3\phi_1(t) + 3\phi_2(t)$

$S_3(t) = 3\phi_1(t) - 3\phi_2(t)$

→ There are two basis function ($\phi_1, \phi_2$)

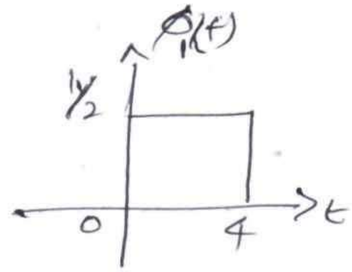
∴ It is two dimensional.

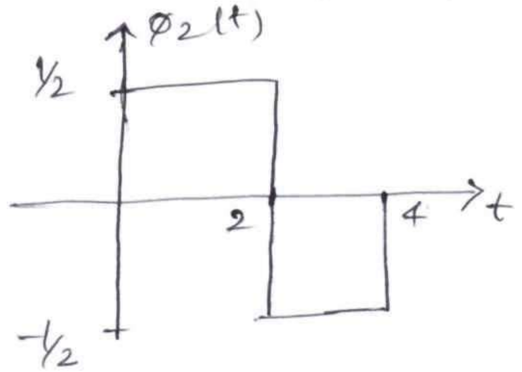
2D signal constallation Diagram



$2^{nd}$ Basis function

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E g_2(t)}} = \frac{g_2(t)}{\sqrt{9}} = \frac{1}{3} g_2(t)$$





$$\phi_2(t) = \begin{cases} \frac{1}{2} ; & 0 \le t \le 2 \\ -\frac{1}{2} & 2 \le t \le 4 \end{cases}$$

$$\int_0^4 \phi_1(t) \cdot \phi_2(t) \cdot dt = 0$$

$$= \int_0^2 (\tfrac{1}{2})(\tfrac{1}{2}) \cdot dt + \int_2^4 (\tfrac{1}{2})(-\tfrac{1}{2}) \cdot dt$$

$$= \frac{1}{4} [t]_0^2 - \frac{1}{4} [t]_2^4$$

$$= \frac{1}{4}[2-0] - \frac{1}{4}[4-2]$$

$$= \frac{1}{4}[2-2] = 0$$

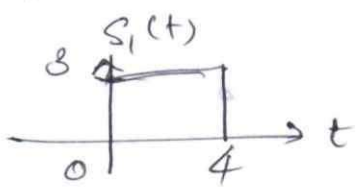
Therefore $\phi_1(t)$ and $\phi_2(t)$ are orthogonal to each other

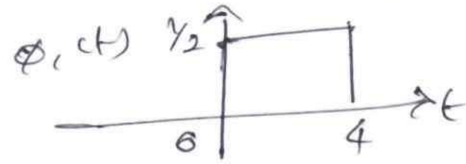
$3^{rd}$ Basis function

$$\phi_3(t) = 0$$

Because only two Basis function are existing, since number of independent message signals are two.

Express $s_1(t)$, $s_2(t)$ and $s_3(t)$ interms of Basis function.





2b. Equation for converting continuous AWGN channel into a vector channel. 10M

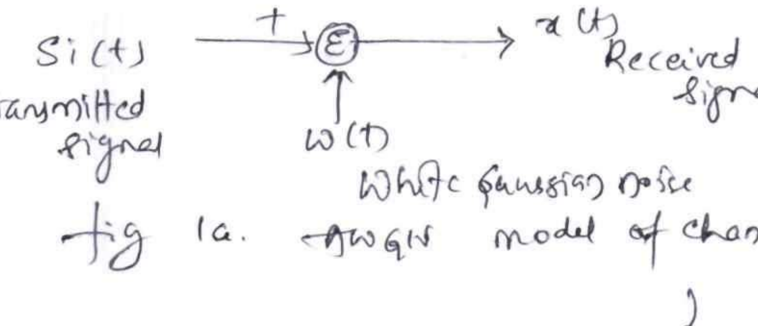



fig 1a. AWGN model of channel.

$$x(t) = S_i(t) + w(t) \quad \left\{ \begin{array}{l} 0 \le t \le T \\ i = 1, 2, \cdots M \end{array} \right. \quad \text{—— ①}$$

$w(t)$ is sample function of the white Gaussian noise process $W(t)$ of zero mean and power spectral density $N_0/2$

The output of correlator $j$, is the sample value of random variable $X_j$, whose sample value is defined by

$$x_j = \int_0^T x(t) \cdot \phi_j(t) \cdot dt \quad \text{—— ②}$$

$$= S_{ij} + w_j \quad , \quad j = 1, 2, \cdots N$$

first component $S_{ij}$ is the deterministic component of $x_j$ due to transmitted signal $S_i(t)$ by $S_{ij}$

$$S_{ij} = \int_0^T S_i(t) \cdot \phi_j(t) \cdot dt$$

$w_j$ is sample value of random variable $W_j$ due to channel noise $w(t)$

$$w_j = \int_0^T w(t) \cdot \phi_j(t) \cdot dt$$

New stochastic process $x'(t)$ whose sample function $x'(t)$ is related to received signal $x(t)$

$$x'(t) = x(t) - \sum_{j=1}^{N} x_j \phi_j(t) \quad \text{—— ③}$$

Substitute ①, ② in ③

Using Eq$^n$ $S_i(t) = \sum_{j=1}^{N} S_{ij} \phi_j(t) \quad \left\{ \begin{array}{l} 0 \le t \le T \\ i = 1, 2 \cdots M \end{array} \right.$

$$x'(t) = S_i(t) + w(t) - \sum_{j=1}^{N} (S_{ij} + w_j) \phi_j(t)$$

$$= w(t) - \sum_{j=1}^{N} w_j \phi_j(t)$$

$$= w'(t)$$

$x''(t)$ depends on $w(t)$

$$\therefore x(t) = \sum_{j=1}^{N} x_j \phi_j(t) + x'(t)$$

$$= \sum_{j=1}^{N} x_j \phi_j(t) + w'(t)$$

3 (a) Generation and Detection of BPSK.

In binary PSK system, pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0 defined by,

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad \text{—— (1)} \quad 0 \le t \le T_b$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi), \quad 0 \le t \le T_b \quad \text{—— (2)}$$

where $T_b$ is bit duration.

$E_b$ is the transmitted signal energy / bit.

Generation of BPSK: The Generator (transmitter) consists of two components.

1. Polar NRZ level Encoder, which represents symbols '1' and '0' (zero) of the incoming binary sequence by amplitude levels $+\sqrt{E_b}$ and $-\sqrt{E_b}$
2. Product Modulator which multiplies the output of the polar NRZ encoder by the basis function $\phi_1(t)$ acts as the "carrier" of the binary PSK signal.

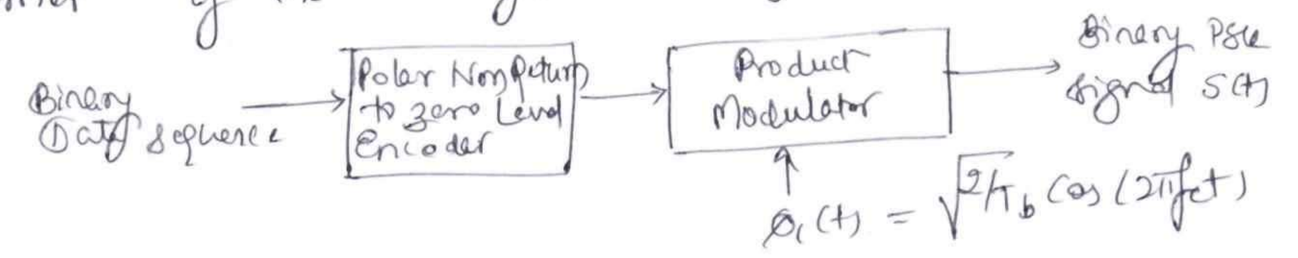



fig 3a1. Block diagram for binary PSK Transmitter.

Binary PSK Receiver

The two basic components in the binary PSK Receiver,

(i) Correlator, which correlates the received signal $x(t)$ with the basis function $\phi_1(t)$ on a bit by bit basis.

(ii) Decision Device, which compares the correlator output against a zero threshold.

Assume binary symbol '1' and '0' are equiprobable.

If the threshold is exceeded, a decision is made in favour of symbol 1, if not decision is made in favour of symbol '0'.

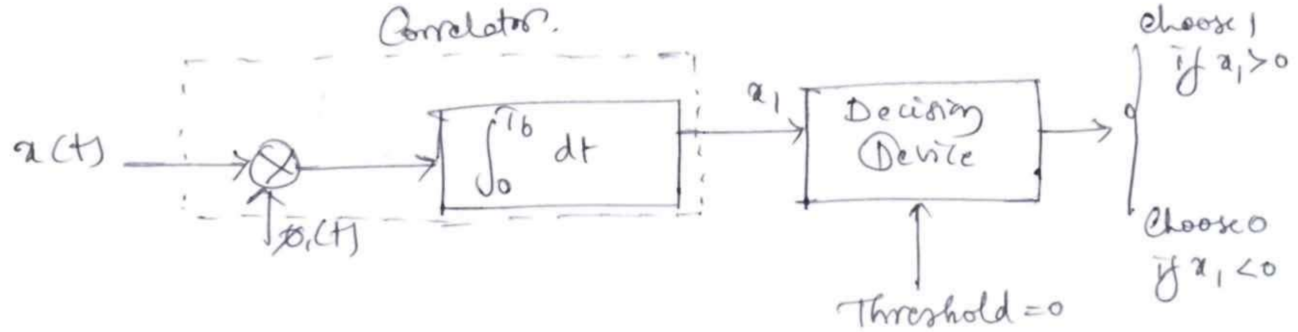



fig 3a2. Coherent binary PSK Receiver

3b. Bandwidth Efficiency of M-ary PSK signal.

Band width of a M-ary PSK is given by

$$BW = \frac{2R_b}{\log_2 M} \text{ Hz.}$$

where $R_b = \frac{1}{T_b}$ bit rate

$M$ = number of symbols.

Band width efficiency of M-ary PSK signal is the ratio of bit rate by,

$$\rho = \frac{R_b}{BW} = \frac{\log_2 M}{2}$$

| M | 2 | 4 | 8 | 16 | 32 | 64 |
|---|---|---|---|---|---|---|
| ρ(bit/(s/Hz)) | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |

i) As M increases B.W efficiency (ρ) increases

(ii) If M increases resulting function Bandwidth decreases and increase in data rate occurs.

(iii) As the number of state that is M in M-ary PSK is increased, the Bandwidth efficiency (ρ) is improved, at the expense of "Error performance."

To ensure no degradation in error performance, we have to increase ($E_b/N_0$) to compensate for the increase in M

3(C) Encoding binary sequence using DPSK. 11011011
Reference bit : 1.

| $\{b_k\}$ | $\{d_k\}$ | $\{d_{k-1}\}$ | Phase |
| --- | --- | --- | --- |
| | 1 Reference bit | | 0 |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | $\pi$ |
| 1 | 0 | 0 | $\pi$ |
| 0 | 1 | 0 | $\pi$ |
| 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 |

4a. Expression for probability of Error of QPSK Signal.

QPSK system operating on AWGN channel, the received signal $x(t)$ is given by

$$x(t) = S_i(t) + w(t) \quad \left.\right\} \quad \begin{array}{l} 0 \le t \le T \\ i = 1, 2, 3, 4 \end{array}$$

$w(t) \rightarrow$ Sample function of white Gaussian noise process of zero mean and power spectral density $N_0/2$.

From QPSK Transmitter, the two correlator outputs $x_1$ and $x_2$ are defined as,

$$x_1 = \int_0^T x(t) \cdot \phi_1(t) \cdot dt$$

$$= \sqrt{E} \cos\left[(2i-1)\pi/4\right] + w_1 \quad \text{—— (1)}$$

$$= \pm\sqrt{E/2} + w_1$$

$$x_2 = \int_0^T x(t)\, \phi_2(t) \cdot dt$$

$$= \sqrt{E} \sin\left[(2i-1)\pi/4\right] + w_2 \quad \text{—— (2)}$$

$$= \pm\sqrt{E/2} + w_2$$

$x_1$ and $x_2$ are sample values of independent Gaussian random variables with mean values equal to $\pm\sqrt{E/2}$ and $\mp\sqrt{E/2}$ respectively and with common variance equal to $N_0/2$.

Decision Rule: $S_1(t)$ was transmitted, if the received signal point associated with the observation vector $x$ falls inside region $Z_1$; $S_2(t)$ was transmitted if the received signal falls inside region $Z_2$ and so on for other two regions $Z_3$ and $Z_4$.

Erroneous Decision: Signal $S_4(t)$ is transmitted, but noise $w(t)$ is such that the received signal point falls outside region $Z_4$.

Average Probability of Symbol Error:

QPSK receiver equivalent to two binary PSK receiver working in parallel and using two carriers that are inphase quadrature.

The inphase channel $x_1$ and quadrature channel output $x_2$ (two elements of observation vector $x$) viewed as individual outputs of two binary PSK Receivers.

From (1) and (2), characterization of two binary PSK Receivers are signal energy per bit equal to $E/2$. Noise spectral density equal to $N_0/2$.

Average probability of symbol error of BPSK,

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

∴ Average probability of bit error in the inphase and quadrature paths of coherent QPSK Receivers is,

$$P' = Q\left(\sqrt{E/N_0}\right)$$

The decision device in phase path accounts for one of the two bits constituting a symbol (dibit) of QPSK signal. decision device in quadrature path takes care of the other dibit.

Average probability of correct detection, results from combining two channels (paths)

$$P_C = (1-P')^2$$

$$= \left[1 - Q\left(\sqrt{\frac{E}{N_0}}\right)\right]^2$$

$$= 1 - 2Q\left(\sqrt{E/N_0}\right) + Q^2\left(\sqrt{E/N_0}\right)$$

∴ Average probability of symbol error for QPSK

$$\therefore P_e = 1 - P_C$$

$$= 2Q\left(\sqrt{E/N_0}\right) - Q^2\left(\sqrt{E/N_0}\right)$$

In region where $(E/N_0) >> 1$ ignore quadratic term Average probability symbol error for QPSK receiver is

$$P_e \cong 2Q\left(\sqrt{\frac{E}{N_0}}\right)$$

4b. Binary Frequency Shift Keying Using Noncoherent Detection,

Transmitted signal in binary fsk, is represented as

$$S_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases}$$

where $T_b \rightarrow$ bit duration

$f_i \rightarrow$ Carrier frequency $\rightarrow f_1, f_2$ Equals two possible values.

Signals representing two frequencies are orthogonal.

$f_i = n_i / T_b$

$n_i \leftarrow$ Integer

frequency $f_1$ represents $\longrightarrow$ symbol 1

frequency $f_2$ represents $\longrightarrow$ symbol 0.

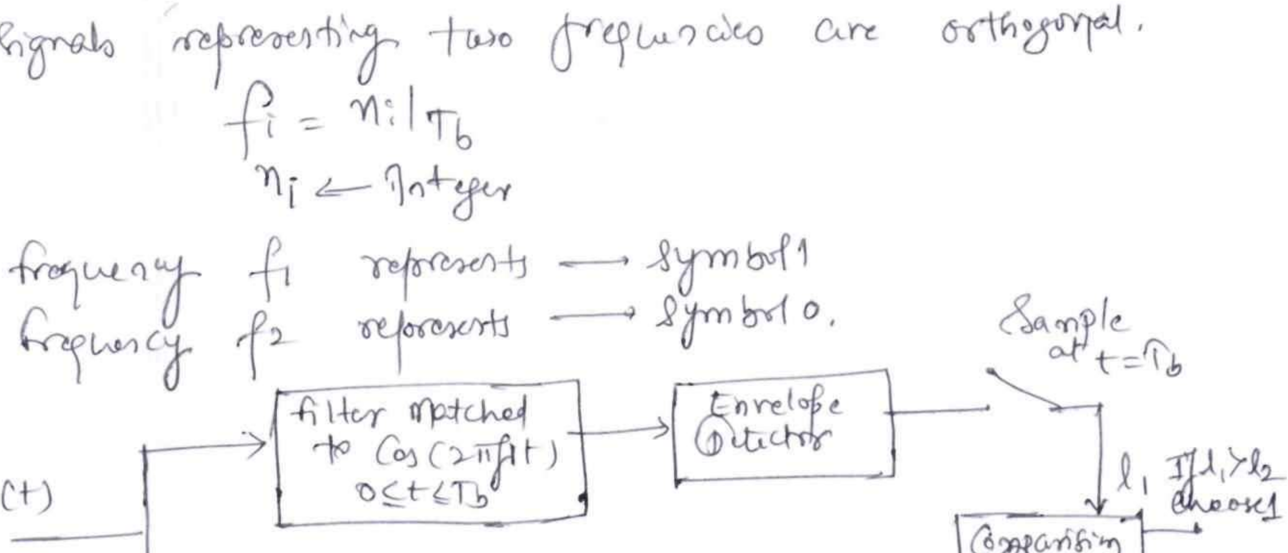



fig 4b. Noncoherent Receiver for detection of binary fsk

Receiver consists of pair of matched filters followed by envelope detectors.

filter in upper path matched to $\cos 2\pi f_1 t$, filter in lower path matched to $\cos(2\pi f_2 t)$. Signaling Interval $0 \le t \le T_b$. Envelope detector output sampled at $t = T_b$ and values are compared. Envelope samples upper and lower path as $l_1$ and $l_2$.

Decision of Receiver : Symbol 1 if $l_1 > l_2$

Symbol 0 if $l_1 < l_2$

$l_1 = l_2$ random guess symbol '1' or '0'

BER for noncoherent binary fsk: $P_e = \frac{1}{2} \exp\left(-E_b/2N_0\right)$

4C. $P_{avg} = ?$ DPSK signal

Data Rate $= 1000$ bits/sec $\therefore T_b = \frac{1}{1000}$ sec $= 0.001$ sec

BW $= 3000$ Hz

$N_0/2 = 10^{-10}$ W/Hz $\Rightarrow N_0 = 2 \times 10^{-10}$ W/Hz

$P_e = 10^{-5}$

$$P_e = \frac{1}{2} e^{-E_b/N_0}$$

$$P_e = \frac{1}{2} e^{-PT_b/N_0}$$

$$10^{-5} = \frac{1}{2} e^{-\left(P \times \frac{1}{1000}\right) / 2 \times 10^{-10}}$$

$$\therefore 2 \times 10^{-5} = e^{-P/2 \times 10^{-7}}$$

$$\therefore P = 2.164 \times 10^{-6} \;//$$

5 a. Entropy and its properties (6M)

Average information content of symbols in long Independent sequence is called Entropy.

The average information per message will be

$$\text{Average information} = \frac{\text{Total Information}}{\text{Number of Messages}} = \frac{I\,total}{L}$$

Average Information is represented by Entropy. It is denoted by H. Thus,

$$\text{Entropy } (H) = \frac{I\,total}{L}$$

Total Information

$$I(total) = P_1 L \log 2\left(\frac{1}{P_1}\right) + P_2 L \log_2\left(\frac{1}{P_2}\right) + \cdots + P_m L \log_2\left(\frac{1}{P_m}\right)$$

$$\therefore \text{Entropy } (H) = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + \cdots P_m \log_2\left(\frac{1}{P_m}\right)$$

$$\therefore \text{Entropy}: H = \sum_{k=1}^{m} P_k \log_2 \frac{1}{P_k}$$ 3m

Properties of Entropy: + 3m

1. Entropy is zero if the event is sure or it is impossible ie $H = 0$.
2. When $P_k = 1/m$ for all the $m$ symbols, then the symbols are equally likely, for such source entropy is given as,

$$H = \log_2 m.$$

3. Upper bound on Entropy is given as

$$H_{max} = \log_2 m.$$

5b. A source has five symbols $S = \{S_1, S_2, S_3, S_4, S_5\}$ (8M)

Probabilities $P = \{0.4, 0.2, 0.2, 0.1, 0.1\}$

Computation of source code using Huffman binary coding

Average length = ?

Entropy = ?

* Arrange probabilities in decreasing order

| Message | Probabilities of Message | | | |
|---|---|---|---|---|
| $m_0$ | 0.4 1 | 0.4 1 | 0.4 | 0.6 0 |
| $m_1$ | 0.2 10 | 0.2 10 | 0.4 00 | 0.4 1 |
| $m_2$ | 0.2 000 | 0.2 000 | 0.2 10 | |
| $m_3$ | 0.1 | 0.2 100 | | |
| $m_4$ | 0.1 | | | |

0100

1100

Lowest probability hence put in the last column

4m

$$\text{Length} \quad L_i = \sum_{i=1}^{n} P_i (l)$$

+ 2m

$$= 0.4(1) + 0.2(2) + 0.2(3) + 0.1(4) + 0.1(6)$$

$$\therefore L = \underline{\underline{2.2}}$$

$$\text{Entropy} \quad H = \sum_{i=1}^{n} P_i \log \frac{1}{P_i}$$

+ 2m

$$= 0.4 \log_2 \frac{1}{0.4} + 0.2 \log_2 \frac{1}{0.2} + 0.2 \log \frac{1}{0.2} + 0.1 \log_2 \frac{1}{0.1} + 0.1 \log_2 \frac{1}{0.1}$$

$$= \underline{\underline{2.121}}$$

5 C Instantaneous code with an example (6m)

Instantaneous code is known as a prefix code. It is a uniquely decodable code, where no code word is a prefix of any other code word. This property allows immediate decoding of symbols, without need of waiting for the end of code word sequence. 3m

Example 1 :

+ 3m

symbol A is represented by code word 0
Symbol B is represented by code word 10
Symbol C is represented by code word 11

The received sequence "01101" can be decoded as "ACB"

Ex: 2 { 1, 01, 000, 001 } is instantaneous code.

Example 3: { 3, 11, 22 }

The entire set of possible encoded values must not contain any values that start with any other value in the set.

Codes where each codeword can be decoded as soon as it is received, is known as instantaneous code.

6a. Mutual Information and Properties: (10M)

The mutual information is defined as the amount of information transferred when $x_i$ is transmitted and $y_i$ is received. It is represented by $I(x_i, y_i)$.

$$I(x_i, y_j) = \log \frac{P(x_i/y_j)}{P(x_i)} \text{ bits.} \quad \text{——— (A)}$$

$I(x_i, y_i)$ is the mutual information

$P(x_i | y_i)$ is the conditional probability that $x_i$ was transmitted and $y_i$ is received.

$P(x_i)$ is the probability of symbol $x_i$ for transmission

The average mutual information is represented by $I(X;Y)$. It is calculated in bits/symbol. The average mutual information is defined as the amount of source information gained per received symbol. Average mutual information is different from entropy

$$I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i\, y_j)\, I(x_i\, y_j)$$

$I(x_i, y_j)$ is weighted by joint probabilities $P(x_i, y_j)$ over all the possible joint events.

From (A),

$$I(X;Y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i, y_j) \log_2 \frac{P(x_i/y_j)}{P(x_i)}$$

Properties of Mutual Information:

(i) The mutual information of the channel is symmetric

$$I(X;Y) = I(Y;X)$$

(ii) The mutual information can be expressed in terms of entropies of channel input or output and conditional entropies.

$$I(X;Y) = H(X) - H(X/Y) = H(Y) - H(Y/X)$$

$H(X/Y)$ & $H(Y/X)$ are conditional entropies.

(iii) Mutual information is always positive. $I(X;Y) \geq 0$

(iv) Mutual information is related to joint entropy $H(X,Y)$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

6 b. Expression for the channel capacity of binary symmetric channel.

Channel capacity can be expressed interms of mutual information. Mutual information is given as,

$$I(x;y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(x_i;y_j) \log_2 \frac{P(x_i|y_j)}{P(x_i)}$$

$$P(x_i;y_j) = P(y_j|x_i) P(x_i)$$

$$\frac{P(x_i|y_j)}{P(x_i)} = \frac{P(y_j/x_i)}{P(y_j)}$$

$$\therefore I(x;y) = \sum_{i=1}^{n} \sum_{j=1}^{m} P(y_j|x_i) P(x_i) \log_2 \frac{P(y_j|x_i)}{P(y_j)}$$

$$P(y_j) = \sum_{i=1}^{n} P(y_j/x_i) P(x_i)$$

$$\therefore I(x;y) = \sum_{i=1}^{m} \sum_{j=1}^{n} P(y_j|x_i) P(x_i) \log_2 \frac{P(y_j/x_i)}{\sum_{i=1}^{n} P(y_j/x_i) P(x_i)}$$

Mutual information is obtained from transition probabilities $P(y_j|x_i)$ and $P(x_i)$

Transition probabilities $P(y_i|x_i)$ are characteristic of the channel, $P(x_i)$ are independent of the channel.

The channel capacity of the discrete memoryless channel is given as maximum average mutual information the maximization is taken with respect to input probabilities $P(x_i)$

$$C = \max_{P(x_i)} I(x;y)$$

Binary Symmetric channel.

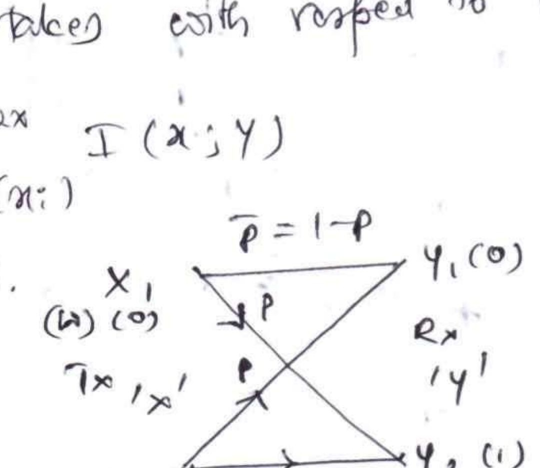



fig. 1. Binary Symmetric Channel

$$C = I(x,y) \cdot r_s$$

6a. Consider the binary symmetric channel, shown in fig 1.

Let $P(x_1) = \omega$

$P(x_2) = \bar{\omega} = 1-\omega$.

Let $P$ be the probability of Error ie Probability of receiving '1' when '0' is transmitted or receiving '0' when 1 is transmitted.

The channel matrix is given by,

$$P(Y/x) = \begin{bmatrix} \bar{P} & P \\ P & \bar{P} \end{bmatrix}$$

$$\therefore H(Y/x) = \sum_j P_j \log_2 \frac{1}{P_j} = h$$

$$\therefore H(Y/x) = \bar{P} \log_2 \frac{1}{\bar{P}} + P \log_2 \frac{1}{P} \quad \text{—— (1)}$$

To find $H(Y) = \sum_j P(Y_j) \log_2 \frac{1}{P(Y_j)}$

$$P(Y) = [\ \bar{P}\omega + P\bar{\omega} \ : \ P\omega + \bar{P}\bar{\omega}\ ]$$

$$P(x, Y) = P[Y/x][P(x)]$$

$$= \begin{bmatrix} \bar{P} & P \\ P & \bar{P} \end{bmatrix} [\omega \ \bar{\omega}]$$

$$P(x, Y) = \begin{bmatrix} \bar{P}\omega & P\omega \\ P\bar{\omega} & \bar{P}\bar{\omega} \end{bmatrix}$$

$$P(Y) = [\ \bar{P}\omega + P\bar{\omega} \quad P\omega + \bar{P}\bar{\omega}\ ]$$

$$H(Y) = (\bar{P}\omega + P\bar{\omega}) \log_2 \frac{1}{(\bar{P}\omega + P\bar{\omega})} + (P\omega + \bar{P}\bar{\omega}) \log_2 \frac{1}{(P\omega + \bar{P}\bar{\omega})} \quad \text{—— (2)}$$

If $r_s = 1$,

channel capacity $C = \log_2(m) - h$ //

for Binary channel, $m=2$ $C = 1-h$ //

$$\therefore C = 1 - \left[\bar{p}\log_2 \frac{1}{\bar{p}} + p\log_2 \frac{1}{p}\right] \quad \text{——} ③$$

Let $w = \bar{w} = 1/2$

$\therefore$ Eqn ② changes to

$$H(Y) = \frac{1}{2}\left[(\bar{p}+p)\log_2 \frac{2}{(\bar{p}+p)} + (p+\bar{p})\log_2 \frac{2}{(p+\bar{p})}\right]$$

$$= \frac{1}{2}[1+1] = 1$$

$$I(X,Y) = H(Y) - H[Y/X]$$

$$= 1 - \left[\bar{p}\log_2\left(\frac{1}{\bar{p}}\right) + p\log_2 \frac{1}{p}\right]$$

Channel capacity $C$: for $r_s = 1$

$$\therefore C = 1 - \left[\bar{p}\log_2 1/\bar{p} + p\log_2 \frac{1}{p}\right]$$

The above equation gives channel capacity for Binary symmetric channel, which is same as Eqn ③.

7(a) Error Control coding is the calculated coding of redundancy. The error control coding improves the data quality to a great extent and also reduction in $E_b/N_o$ for a fixed bit error rate.

The disadvantages of error control coding are

(i) Increased bandwidth

(ii) System becomes more complex due to implementation of decoding operation in the receiver.

The channel encoder at the transmitter systematically adds bits to the transmitted message, these additional bits carry no information, but make it possible for the channel decoder to detect and correct errors in the information bits. This reduces the overall probability of Error Pe. These additional bits are called redundant bits.

| Linear Block codes | Convolution code |
| --- | --- |
| Takes k input bits and produces n output bits. | Takes small number of input bits and produces output for each period. |
| The information bits are followed by parity bits | Information bits are spread along sequence without immediate parity bit |
| No memory and current state, encoding is independent of previous state. | Has memory of current state Encoding depends on previous state |
| Systematic form with defined position for parity bits. | Non systematic form, no defined position for parity bits. |
| Useful for detecting and preventing random errors | Useful for detecting and preventing burst errors. |
| Ex: cyclic codes<br>BCH Codes<br>Reed Solomon codes | Ex: Turbo codes<br>Trellis code. |

7b. $c$ is a valid code vector

Show $cH^T = 0$, $H$ is parity check matrix of code.

$$X = [M]_{1\times k}[G]_{k\times n}$$

$$XH^T = [m]_{1\times k}[G]_{k\times n}[H^T]_{n\times q}$$

We know,

$$[G]_{k\times n} = [I_{k\times k} \mid P_{k\times q}]_{k\times n}$$

and $$[H]_{q\times n} = [P^T_{q\times k} \mid I_{q\times q}]_{q\times n}$$

Consider matrix product $HG^T$,

$$HG^T = [P^T_{q\times k} \mid I_{q\times q}]\begin{bmatrix} I_{k\times k} \\ \hline P^T_{q\times k}\end{bmatrix}$$

$$= [P^T \oplus P^T]_{q\times k} = 0$$

III$^{ly}$ by identity $GH^T = 0$,

$$GH^T = [I_{k\times k} \mid P_{k\times q}]\begin{bmatrix} P_{k\times q} \\ \hline I_{q\times q}\end{bmatrix}$$

$$= [P \oplus P] = 0$$

$\therefore$ $$XH^T = [m]_{1\times k} \times 0$$

$$= 0 //$$

7c) (7, 4) Binary cyclic code

$g(x) = 1 + x + x^3$

Message vector [1001]

Design an Encoder.

$$g(x) = g_0 + g_1 x + g_2 x^2 + \cdots\cdots + g_{n-k} x^{n-k} \quad \text{—— (1)}$$

$$g(x) = 1 + x + x^3 \quad \text{—— (2)}$$

Compare (1) & (2)

$g_0 = 1$, $g_1 = 1$, $g_2 = 0$ and $g_3 = 1$

Flipflops : 3, $R_0$, $R_1$ $R_2$

Modulo-2 adders : 2

Connection from the output of the gate to $R_0$ and to the first modulo-2 adder

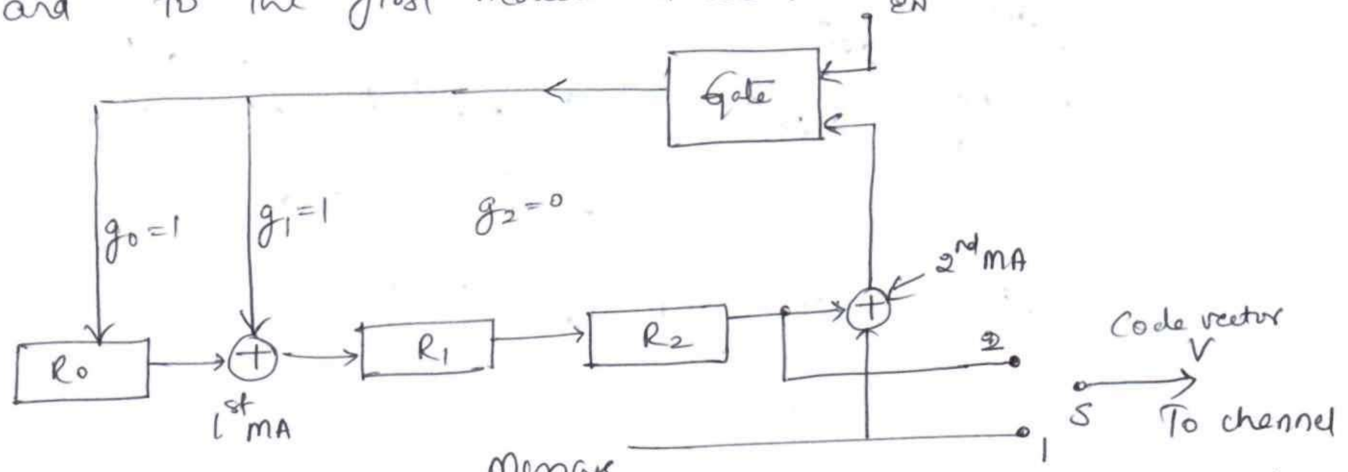



Fig 1. Encoder for (7, 4) cyclic code.

D = [1001], the shift register contents are

| Number of shifts | Input D | Shift Register contents $R_0$ | $R_1$ | $R_2$ | Remainder bits → R |
|---|---|---|---|---|---|
| | | 0 | 0 | 0 | – |
| 1 | 1 | $\phi$ | $\phi$ | 0 | – |
| | 0 | $\phi$ | 1 | $\phi$ | – |
| 2 | 0 | $\phi$ | 1 | 1 | – |
| 3 | 1 | 0 | 1 | 1 | – |
| 4 | | | | | |
| 5 | x | 0 | 0 | 1 | 1 |
| 6 | x | 0 | 0 | 0 | 1 |
| 7 | x | 0 | 0 | 0 | 0 |

8 @ Block Diagram of generator and Parity Check Matrix with Equations. Syndrome equation and its properties.

Let message block of k(code words) bits be represented as row vector called message vector,

$$[D] = \{d_1, d_2 \ldots, d_k\}$$

where $d_1, d_2 \ldots d_k$ are either '0's or 1's.

The channel encoder systematically adds $(n-k)$ number of check bits to form a $(n, k)$ linear block code.

Then the $2^k$ code vectors can be,

$$C = \{c_1, c_2 \ldots c_n\}$$

In systematic linear block code, message bits appear at the beginning of the code vector,

$$\therefore \; c_i = d_i \text{ for all } i = 1, 2 \ldots k$$

$$[C] = \{\underbrace{c_1\, c_2 \ldots, c_k}_{k \text{ message bits}},\ \underbrace{c_{k+1}, c_{k+2}, \ldots c_n}_{(n-k) \text{ check bits}}\}$$

$(n-k)$ number of check bits $c_{k+1}, c_{k+2} \ldots c_n$ are derived from $k$ message bits using rule,

$$c_{k+1} = p_{11} d_1 + p_{21} d_2 + \ldots + p_{k1} d_k$$

$$c_{k+2} = p_{12} d_1 + p_{22} d_2 + \ldots + p_{k2} d_k$$

$$\vdots$$

$$c_n = p_{1,n-k} d_1 + p_{2,n-k} d_2 + \ldots + p_{k,n-k} d_k$$

$p_{11}, p_{21}, p_{12}, p_{22}$ are either '0' or '1'

$$[c_1\, c_2 \ldots c_k\, c_{k+1}\, c_{k+2} \ldots c_n] = [d_1\, d_2\, d_k] \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & p_{11} & p_{12} & \ldots & p_{1,n-k} \\ 0 & 1 & 0 & \ldots & 0 & p_{21} & p_{22} & \ldots & p_{2,n-k} \\ \vdots \\ 0 & 0 & 0 & \ldots & 1 & p_{k1} & p_{k2} & \ldots & p_{k,n-k} \end{bmatrix}$$

$$[C] = [D][G]$$

$[G]$ is called as Generator matrix, order $k \times n$

$$[G] = [I_k : P]_{k \times n}$$

where $I_k$ = unit matrix of order '$k$'

$[P]$ = arbitrary matrix called 'Parity matrix', order $k \times (n-k)$

and | = denotes the demarcation between unit matrix $I_k$ and parity matrix P.

$$[G] = [P : I_k]$$

Parity Check matrix: [H]:

Generator matrix given by,

$$[G] = [I_k | P] = \left[\begin{array}{c} \overbrace{\begin{matrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 1 \end{matrix}}^{k \text{ columns}} \end{array} \begin{array}{c} \overbrace{\begin{matrix} P_{11} & P_{12} & \cdots & P_{1,n-k} \\ P_{21} & P_{22} & & P_{2,n-k} \\ & & & \\ P_{k1} & P_{k2} & & P_{k,n-k} \end{matrix}}^{n-k \text{ columns}} \end{array}\right]$$

Parity check matrix H given by,

$$[H] = [P^T : I_{n-k}]$$

$$\therefore H = \left[\begin{array}{c} \overbrace{\begin{matrix} P_{11} & P_{21} & \cdots & P_{k1} \\ P_{12} & P_{22} & & P_{k2} \\ P_{1,n-k} & P_{2,n-k} & & P_{k,n-k} \end{matrix}}^{k \text{ columns}} \end{array} \begin{array}{c} \overbrace{\begin{matrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{matrix}}^{n-k \text{ columns}} \end{array}\right]$$

[H] matrix is a $(n-k) \times (n)$ matrix and this matrix is used in error correction.

<u>Syndrome:</u>

$C = \{c_1, c_2, \cdots c_n\}$ valid code vector transmitted

$R = \{r_1, r_2 \cdots r_n\}$ received vector

$\therefore$ Error vector, $E = R - C = R + C$ //

The error vector 'e' can be represented as,

$$E = \{e_1, e_2 \cdots e_n\}$$

In order to find E & C, receiver does the decoding operation by determining a $(n-k)$ vector 'S' defined as

$$S = RH^T$$
$$= (S_1, S_2 \cdots S_{n-k})$$

$(n-k)$ vector S is called error syndrome of R

$$R = C + E$$

$$S = (C + E)H^T$$
$$= CH^T + EH^T$$

But $CH^T = 0$

$$\therefore S = EH^T \quad //$$

Syndrome S of the received vector will be zero if R is a valid code vector. When $R \neq C$, then $S \neq 0$ and the receiver then detects and corrects the error.

Syndrome provides information about the pattern in the received vector.

If the syndrome is zero $(S = 0)$, received vector is likely valid code word, and no errors are detected. or the errors are within the codes correcting capability

If the syndrome is nonzero, it indicates the presence of errors. Each nonzero syndrome corresponds to a specific error pattern and error can be corrected if its within the code's error-correcting capability

8 b. (7, 4) Linear Block code

Given $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$(n, k) = (7, 4)$

Length of code word = 7

Length of message = 4

$q = n - k$
$= 7 - 4$
$= 3$

find Generator Matrix

$$G = [I_k : P_{k \times q}]$$

$$= [I_4 : P_{4 \times 3}]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$C = M \cdot G$$

$$[c_1\ c_2\ c_3\ c_4\ c_5\ c_6\ c_7] = [m_1\ m_2\ m_3\ m_4] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$c_1 = m_1$

$c_2 = m_2$

$c_3 = m_3$

$c_4 = m_4$

$c_5 = m_1 \oplus m_2 \oplus m_4$

$c_6 = m_1 \oplus m_2 \oplus m_3$

$c_7 = m_2 \oplus m_3 \oplus m_4$

$\oplus$ indicates XOR operation

| $m_1$ | $m_2$ | $m_3$ | $m_4$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ | Hamming weight |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 3 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 4 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 3 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 3 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 4 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 3 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 4 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 3 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 4 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 3 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 4 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 4 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 3 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 3 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 |

Table 1. Possible Code words

Received code word is

$$Y = [1100010]$$

syndrome

$$S = YH^T$$

$$H = [P^T : I]$$

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\therefore S = [1100010] \overbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}^{H^T} \leftarrow \text{Ist place}$$

$$= 1 \quad 1 \oplus 1 + 1 \quad 1 \oplus 1$$

$$= 1 \quad 1 \quad 0$$

Error code $= [1000000]$

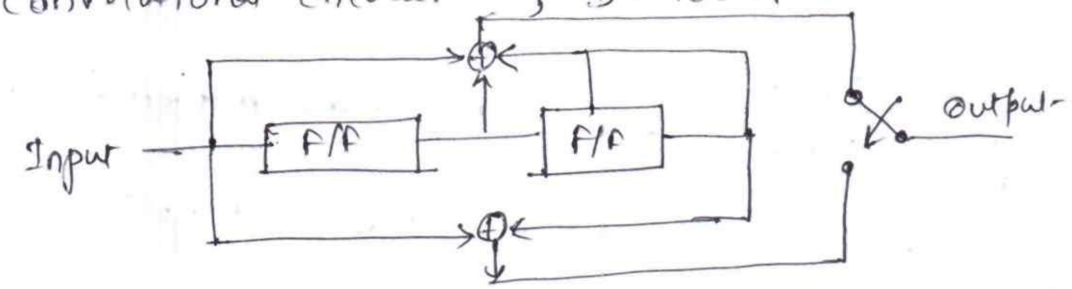
$\therefore$ Corrected Code word $= Y \oplus E$

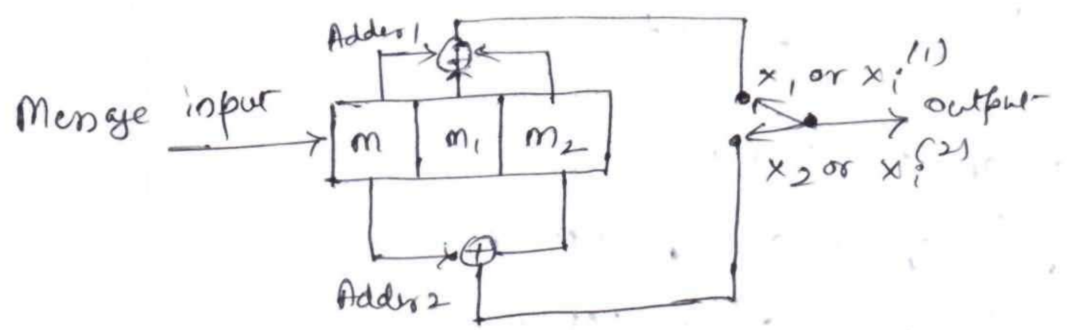
$$= [1100010] \oplus [1000000]$$

$$= [0100010]$$

1 (a) Computation of output sequence using Transform domain approach (Polynomial Multiplication)

Convolutional encoder, $D = 10011$




Encoder takes one message bit at a time

$k = 1$,

It generates two bits for every message bit

$\therefore n = 2$

Dimension $= (n, k) = (2, 1)$

$x_1$ ie $x_i^{(1)}$ is generated by adding all the three bits.

$\therefore$ generating sequence $g_i^{(1)}$ is given as,

$$g_i^{(1)} = \{1, 1, 1\}$$

$\therefore$ Its polynomial, (i) Generating polynomial for adder 1

$$g^{(1)}(p) = g_0^{(1)} + g_1^{(1)} p + g_2^{(1)} p^2 + \dots + g_m^{(1)} p^m \quad \text{——} \ (1)$$

$$g^{(1)}(p) = 1 + 1 \times p + 1 \times p^2$$

$$= 1 + p + p^2$$

(ii) Generating polynomial for adder 2

Second sequence $g_i^{(2)} = \{1 0 1\}$

polynomial $g^{(2)}(p) = g_0^{(2)} + g_1^{(2)}(p) + g_2^{(2)} p^2 + \dots + g_m^{(2)} p^m$

$$g^{(2)}(p) = 1 + 0 \times p + 1 \times p^2$$
$$= 1 + p^2$$

(iii) Message Polynomial : $m(p) = m_0 + m_1 p + m_2 p^2 + \dots + m_{L-1} p^{L-1}$

$$m = \{10011\}$$
$$m(p) = 1 + 0 \times p + 0 \times p^2 + 1 \times p^3 + 1 \times p^4$$
$$= 1 + p^3 + p^4$$

(iv) Determine Output due to adder 1

$$x^{(1)}(p) = g^{(1)}(p) \cdot m(p)$$
$$x^{(2)}(p) = g^{(2)}(p) \cdot m(p)$$

$$x^{(1)}(p) = g^{(1)}(p) \cdot m(p)$$
$$= \{1 + p + p^2\} (1 + p^3 + p^4)$$
$$= 1 + p + p^2 + p^3 + p^6$$
$$x^{(1)}(p) = 1 + (1 \times p) + (1 \times p^2) + (1 \times p^3) + (0 \times p^4) + (0 \times p^5) + (1 \times p^6)$$

O/P Sequence $x_i^{(1)}$

$$x_i^{(1)} = \{1111001\}$$

(v) Determine, Output due to adder 2

$$x^{(2)}(p) = g^{(2)}(p) \cdot m(p)$$
$$= (1 + p^2)(1 + p^3 + p^4)$$
$$= 1 + p^2 + p^3 + p^4 + p^5 + p^6$$

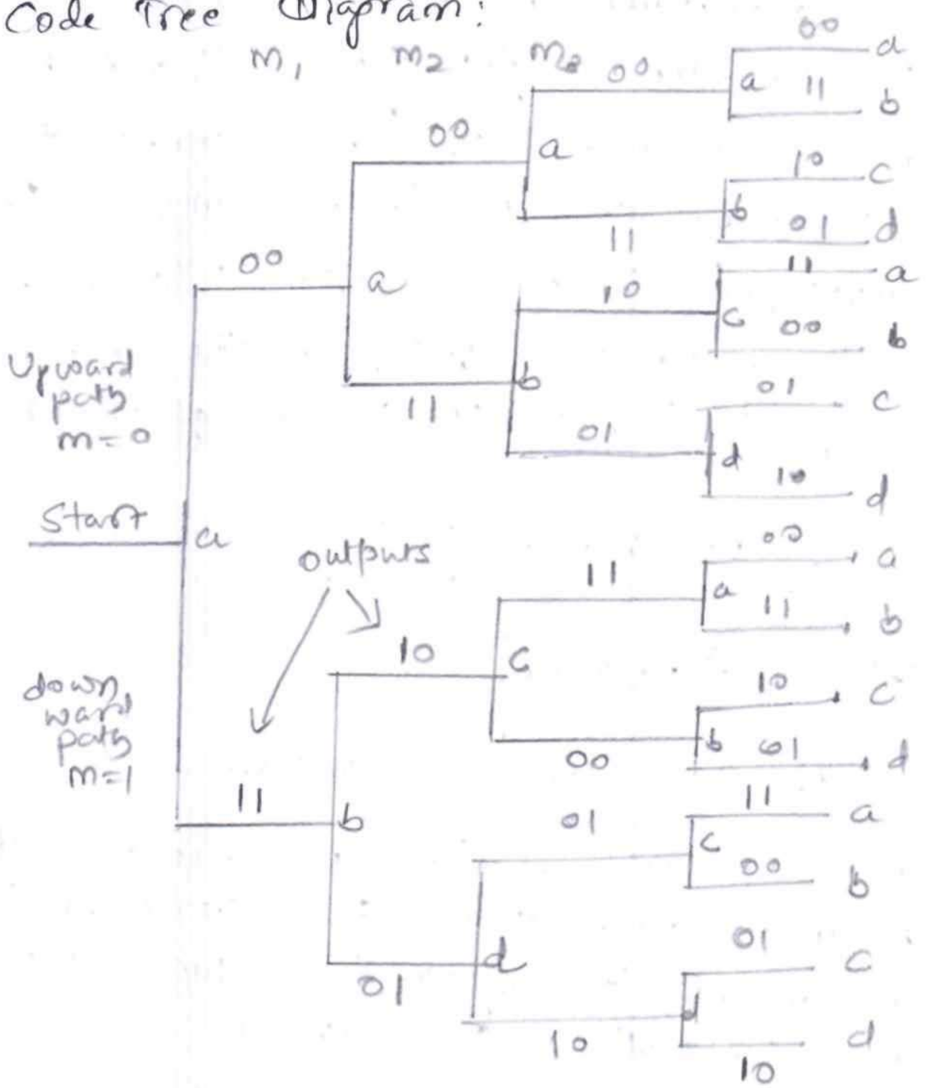
Output Sequence $x_i^{(2)}$ is,

$$x_i^{(2)} = \{1011111\}$$

(vi) Multiplexed O/P sequence

$$\{x_i\} = \{11, 10, 11, 11, 01, 01, 11\}$$

Code Tree Diagram:




Code Tree for convolutional Encoder

States

a = 00

b = 01

c = 10

d = 11

9 b. Recursive Systematic Convolutional code Encoder, Example

An RSC encoder is typically $r = 1/2$ and is termed a component encoder. The two component encoders are separated by an interleaver. Only one of the systematic outputs from the two component encoder is used, because the systematic output from the other component encoder is just a permuted version of the chosen systematic output

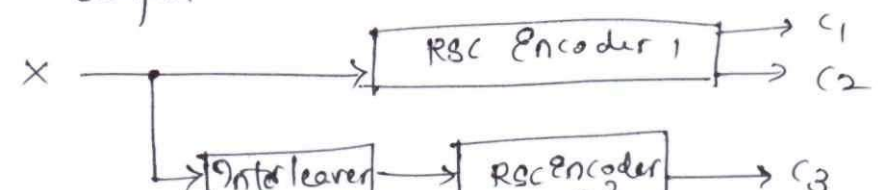



fig. 9b.1 Turbo code Encoder

figure shows,

$r = 1/3$ Turbo code Encoder. The first RSC encoder outputs the systematic $c_1$ and recursive convolutional $c_2$ sequences, while the second RSC encoder discards its systematic sequence and only outputs the recursive convolutional $c_3$ sequence.

The recursive systematic convolutional (RSC) encoder is obtained from the nonrecursive, nonsystematic (conventional) convolutional encoder by feeding back one of the its encoded output to its output.

Conventional convolutional Encoder with $r = 1/2$, $k = 3$

It is represented by two generator sequence

$g_1 = [1\,1\,1]$

$g_2 = [1\,0\,1]$

ie $g = [g_1, g_2]$

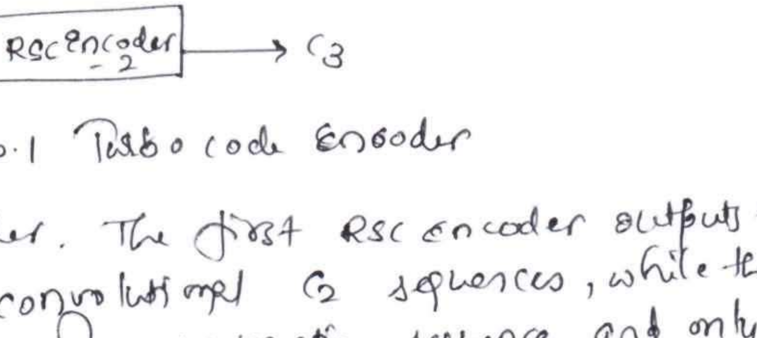
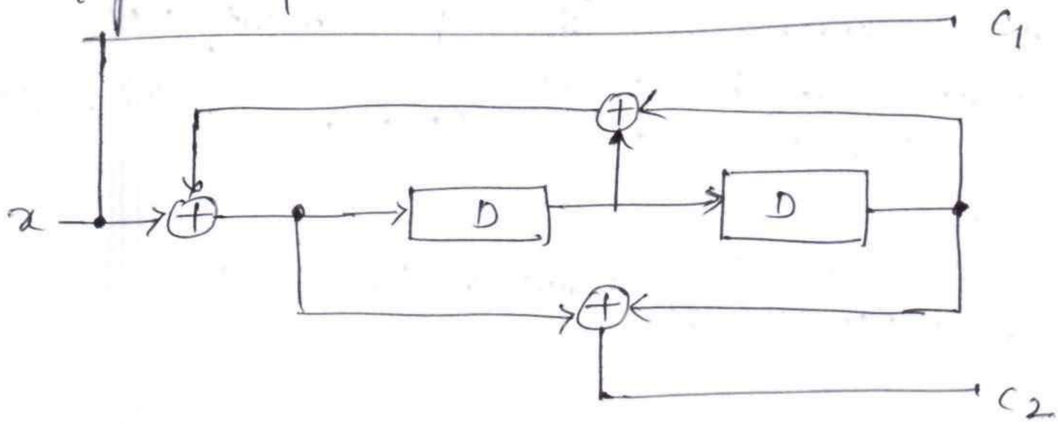



fig 2. Conventional Convolutional Encoder.

RSC Encoder of this conventional convolutional encoder is represented as $g = [1, g_2/g_1]$, where first output ($g_1$) is fed back to the input. 1 denotes systematic output, $g_2$ denotes feed forward output, $g_1$ is the feedback to the input of the RSC encoder.




RSC Encoder obtained from fig 2, $r = 1/2$, $k = 3$

10a Convolution encoder has two flipflop with two states, three modulo-2 adders and as output multiplexer
Generator sequences of the encoder,

$g^{(1)} = (1,0,1)$ , $g^{(2)} = (1,1,0)$ $g^{(3)} = (1,1,1)$

(i) Generator Matrix [G]
(ii) Encoder block diagram
(iii) Code word for message input vector 11101

$g^{(1)} = (1,0,1)$

$g^{(2)} = (1,1,0)$

$g^{(3)} = (1,1,1)$

m = 3 which is length of generating sequence

M = m − 1 indicates 2 stage shift register

= 3 − 1

= 2 //

(ii)

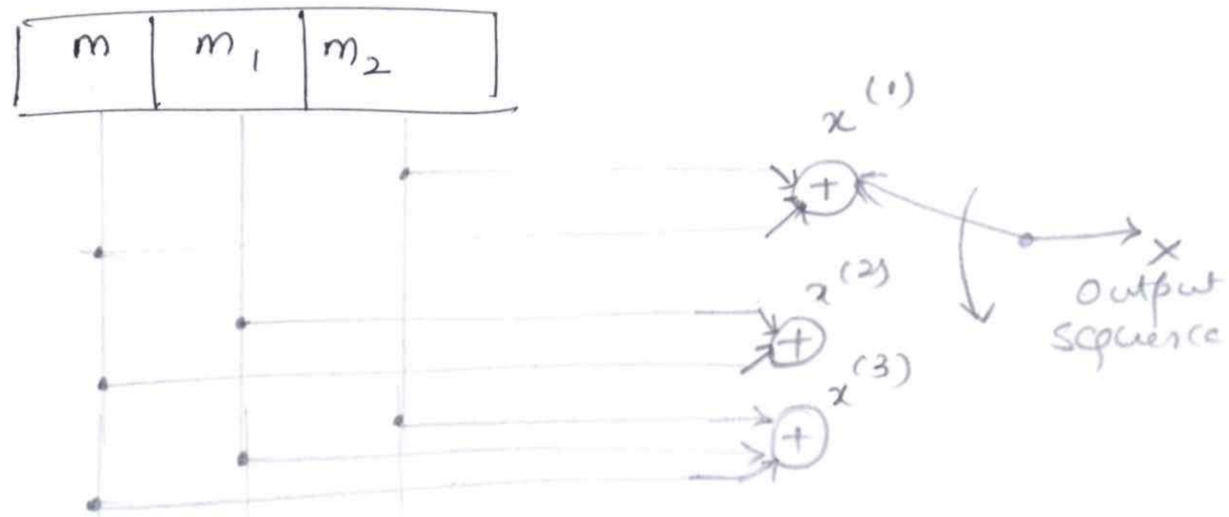



fig (ii) Convolutional Encoder.

(i) To obtain Generator matrix (G)

$g_i^{(1)} = \{ g_1^{(1)}\ g_2^{(1)}\ g_3^{(1)} \}$

$g_i^{(2)} = \{ g_1^{(2)}\ g_2^{(2)}\ g_3^{(2)} \}$

$g_i^{(3)} = \{ g_1^{(3)}\ g_2^{(3)}\ g_3^{(3)} \}$

The length of message sequence is,

$$M = \{m_1\ m_2\ m_3\ m_4\ m_5\} = \{1, 1, 1, 0, 1\}$$

$$\therefore\ L = 5$$

$n = 3$, due to three generating sequence, or 3 output bits for one message bit.

$L = 5$
$m = 3$
$M = m - 1 = 2$

Thus $n \times (L + m - 1) = 3 \times (5 + 3 - 1) = 21$ columns in the generator matrix.

$$G = \begin{bmatrix} g_1^1 & g_1^2 & g_1^3 & g_2^1 & g_2^2 & g_2^3 & g_3^1 & g_3^2 & g_3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_1^1 & g_1^2 & g_1^3 & g_2^1 & g_2^2 & g_2^3 & g_3^1 & g_3^2 & g_3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g_1^1 & g_1^2 & g_1^3 & g_2^1 & g_2^2 & g_2^3 & g_3^1 & g_3^2 & g_3^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_1^1 & g_1^2 & g_1^3 & g_2^1 & g_2^2 & g_2^3 & g_3^1 & g_3^2 & g_3^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g_1^1 & g_1^2 & g_1^3 & g_2^1 & g_2^2 & g_2^3 & g_3^1 & g_3^2 & g_3^3 \end{bmatrix}$$

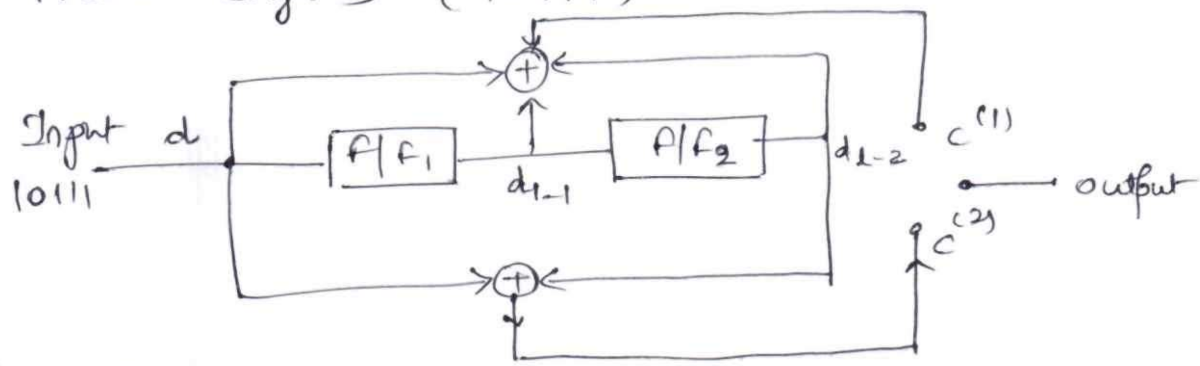
$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$X = M \cdot G$$

$$X = \underset{1\times 5}{\begin{bmatrix} 1 & 1 & 1 & 0 & 1 \end{bmatrix}} \underset{5\times 21}{\begin{bmatrix} 111 & 011 & 101 & 000 & 000 & 000 & 000 \\ 000 & 111 & 011 & 101 & 000 & 000 & 000 \\ 000 & 000 & 111 & 011 & 101 & 000 & 000 \\ 000 & 000 & 000 & 111 & 011 & 101 & 000 \\ 000 & 000 & 000 & 000 & 111 & 011 & 101 \end{bmatrix}}$$

$$= \begin{bmatrix} 1\,1\,1 & 1\,0\,0 & 0\,0\,1 & 1\,1\,0 & 0\,1\,0 & 0\,1\,1 & 1\,0\,1 \end{bmatrix}_{1\times 21}$$

10 b Convolutional Encoder fig. 10(b)
state Table, State Transition Table.
Trellis Diagram (10111)



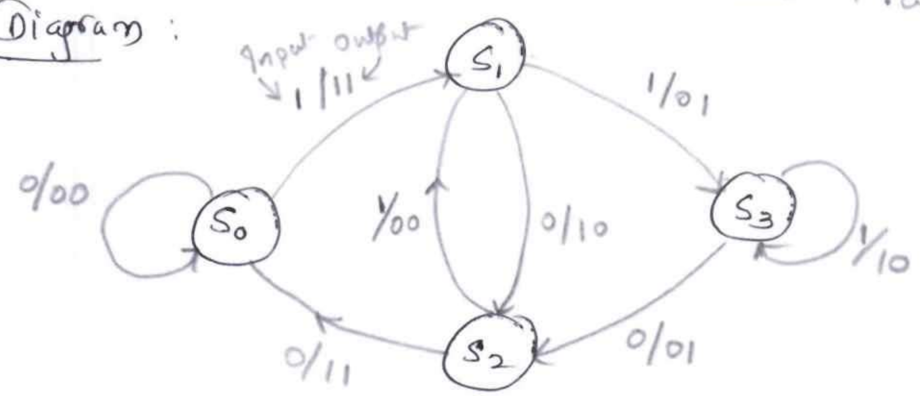

State Table

| state | $S_0$ | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|---|
| Binary Description | 00 | 10 | 01 | 11 |

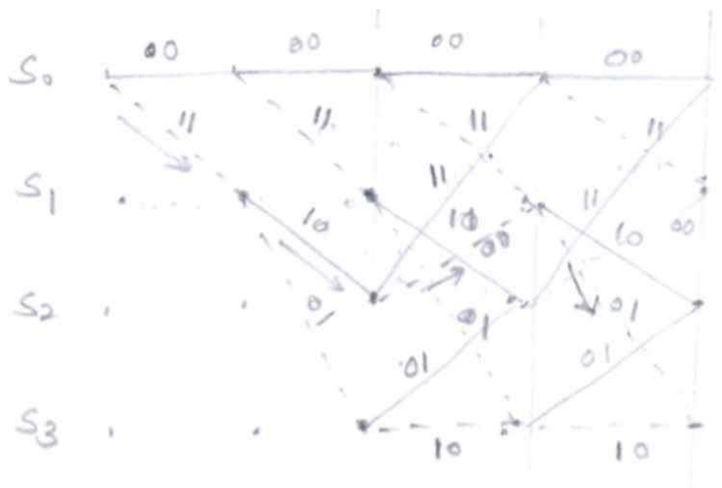
| Present state | Binary Description | Input | Binary Description Next state | Next state | $d$ | $d_{l-1}$ | $d_{l-2}$ | $c^{(1)}$ | $c^{(2)}$ |
|---|---|---|---|---|---|---|---|---|---|
| $S_0$ | 00 | 0 | 0 0 | $S_0$ | 0 | 0 | 0 | 0 | 0 |
| | | 1 | 1 0 | $S_1$ | 1 | 0 | 0 | 1 | 1 |
| $S_1$ | 10 | 0 | 0 1 | $S_2$ | 0 | 1 | 0 | 1 | 0 |
| | | 1 | 1 1 | $S_3$ | 1 | 1 | 0 | 0 | 1 |
| $S_2$ | 01 | 0 | 0 0 | $S_0$ | 0 | 0 | 1 | 1 | 1 |
| | | 1 | 1 0 | $S_1$ | 1 | 0 | 1 | 0 | 0 |
| $S_3$ | 11 | 0 | 0 1 | $S_2$ | 0 | 1 | 1 | 0 | 1 |
| | | 1 | 1 1 | $S_3$ | 1 | 1 | 1 | 1 | 0 |

State Transition Table

State Diagram :



Trellis Diagram : 10111




Input : 1011100

output [11, 10, 00, 01, 10, 01, 11]

44