CBCS SCHEME

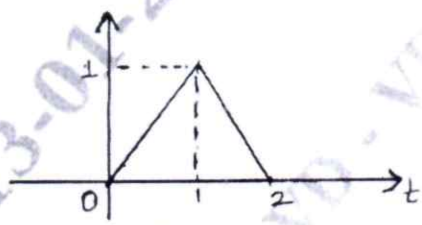
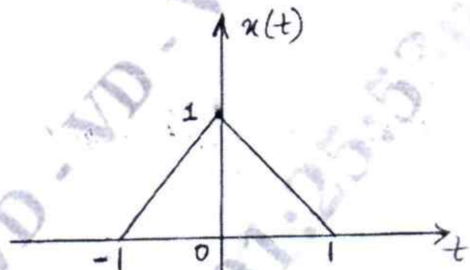
USN | | | | | | | | | | |

BEE502

Fifth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Signals and DSP

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Explain the classification of signals with examples.	06	L2	CO1
	b.	Determine and sketch the even and odd parts of the signal shown in Fig.Q1(b).  Fig.Q1(b)	06	L3	CO1
	c.	For the continuous time single x(t) shown in Fig.Q1(c), sketch the signal: (i) $y_1(t) = x(3t+2)$ (ii) $y_2(t) = x(3t) + x(3t+2)$  Fig.Q1(c)	08	L3	CO1
		OR			
Q.2	a.	Check whether the following signals are periodic or not. If periodic, solve the fundamental period: (i) $x_1(n) = (-1)^n$ (ii) $x_2(n) = \cos\left(\frac{\pi}{3}n\right) + \cos\left(\frac{\pi}{4}n\right)$	06	L3	CO1
	b.	Determine the following signal y(n) = 2x(n) + 3 is linear, time variant, causal, memory and invertible.	06	L3	CO1
	c.	Evaluate the continuous time convolution integral given as $y(t) = e^{-at}u(t) * u(t)$.	08	L3	CO1
		Module – 2			
Q.3	a.	State and prove the following properties of DFT: (i) Linearity (ii) Circular time shift (iii) Symmetry of real valued sequences	08	L2	CO2
	b.	For the sequences $x_1(n) = \cos\left(\frac{2\pi n}{N}\right)$, $x_2(n) = \sin\left(\frac{2\pi n}{N}\right)$, $0 \le n \le N-1$, solve for N-point circular convolution $x_1(n) \circledast_N x_2(n)$.	06	L3	CO2

	c.	Determine the 4-point DFT of the sequence, $x(n) = (1, -1, 1, -1)$. Also, using time shift property, find the DFT of the sequence. $y(n) = x((n-2))_4$.	06	L3	CO2
		OR			
Q.4	a.	Define DFT and IDFT and compute 4-point DFT of a single $x(n) = (1, 2, 1, 0)$ using DFT matrix.	08	L3	CO2
	b.	The 5-point DFT of a complex sequence $x(n)$ is given as $X(K) = (j,\ 1+j,\ 1+j2,\ 2+j2,\ 4+j)$ Compute $Y(K)$, if $y(n) = x^*(n)$.	06	L3	CO2
	c.	Using DFT, IDFT method, compute circular convolution of the sequences $x_1(n) = (1, 1, 1)$ and $x_2(n) = (1, -2, 2)$.	06	L3	CO2
		Module – 3			
Q.5	a.	Compute 8 point DFT of the sequence $x(n) = (1, 2, 3, 4, 4, 3, 2, 1)$ using radix-2 DIT-FFT algorithm.	12	L3	CO3
	b.	Determine the 4-point real sequence $x(n)$, if its 4-point DFT samples are $X(0) = 6$, $X(1) = -2 + j2$, $X(2) = -2$. Use DIF-FFT algorithm.	08	L3	CO3
		OR			
Q.6	a.	Given the sequence $x_1(n)$ and $x_2(n)$ below, compute the circular convolution $x_1(n) \circledast_N x_2(n)$ for $N = 4$. Use DIT-FFT algorithm.	10	L3	CO3
	b.	Solve for the 4-point circular convolution of $x(n)$ and $h(n)$ using radix-2 DIF-FFT algorithm. Given $X(n) = (1, 1, 1, 1)$, $h(n) = (1, 0, 1, 0)$.	10	L3	CO3
		Module – 4			
Q.7	a.	Design a Butterworth analog highpass filter that will meet the following specifications: (i) Maximum passband attenuation = 2 dB (ii) Passband edge frequency = 200 rad/sec (iii) Minimum stopband attenuation = 20 dB (iv) Stopband edge frequency = 100 rad/sec	10	L3	CO4
	b.	Obtain the direct form I and direct form II of the following transfer function: $H(z) = \dfrac{8z^3 - 4z^2 + 11z + 2}{\left(z - \frac{1}{4}\right)\left(z^2 - z + \frac{1}{2}\right)}$	10	L3	CO4
		OR			
Q.8	a.	Design a Chebyshev I filter to meet the following specifications: (i) Passband ripple : ≤ 2 dB (ii) Passband edge : 1 rad/sec (iii) Stopband attenuation : ≥ 20 dB (iv) Stopband edge : 1.3 rad/sec	10	L3	CO4
	b.	The system function of an analog filter is given by $H(s) = \dfrac{s + 0.1}{(s + 0.1)^2 + 9}$. Obtain the system function of IIR digital filter by using impulse invariant method.	10	L3	CO4

			Marks	Level	CO
		Module – 5			
Q.9	**a.**	A filter is to be designed with the following desired frequency response: $$H_d(\omega) = \begin{cases} 0 & -\frac{\pi}{4} < \omega < \frac{\pi}{4} \\ e^{-j2\omega} & \frac{\pi}{4} < \lvert\omega\rvert < \pi \end{cases}$$ Compute the frequency response of the FIR filter designed using a rectangular window defined below: $$\omega_R(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$$	**10**	**L3**	**CO5**
	b.	Determine the filter coefficients h(n) obtained by sampling $H_d(\omega)$ given by, $$H_d(\omega) = \begin{cases} e^{-j3\omega} & 0 < \omega < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < \lvert\omega\rvert < \pi \end{cases}$$ Also, obtain the frequency response, $H(\omega)$. Take N = 7.	**10**	**L3**	**CO5**
		OR			
Q.10	**a.**	The desired frequency response of a lowpass filter is given by $$H_d(e^{j\omega}) = H_d(\omega) = \begin{cases} e^{-j3\omega} & \lvert\omega\rvert < \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} < \lvert\omega\rvert < \pi \end{cases}$$ Determine the frequency response of the FIR filter if Hamming window is used with N = 7.	**10**	**L3**	**CO5**
	b.	The frequency response of an FIR filter is given by $H(\omega) = e^{-j3\omega}(1 + 1.8\cos 3\omega + 1.2\cos 2\omega + 0.5\cos\omega)$ Determine the coefficients of the impulse response h(n) of the FIR filter.	**10**	**L3**	**CO5**

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Solution of VTU QP Dec. 2024 | Jan. 2025

SIGNALS AND DSP [BEE502]

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Max. Marks : 100

Module-1

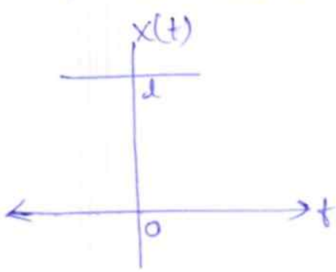
[Q1a.] Explain the classification of signals with examples. (06 M)

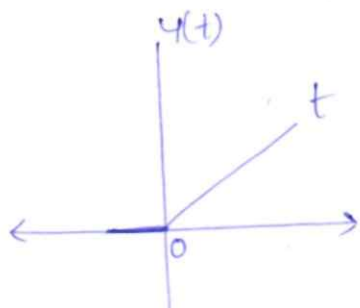
→ ① Continuous time and discrete time signals.
② Even and odd signals.
③ Periodic and Non Periodic signals.
④ Energy signals & power signals.
⑤ Deterministic and Random signals.

① Continuous time and Discrete time signals.

Continuous time signals:

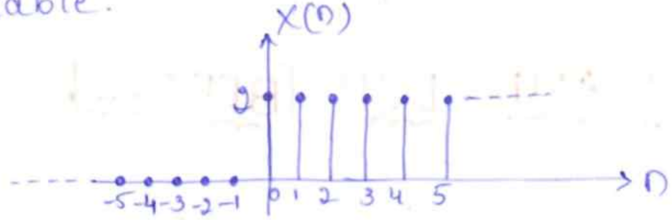
A signal $x(t)$ is said to be continuous if it is defined for all values of the independent variable 't'.
where 't' is the independent variable which represent time.





Discrete time signals:

A signal $x(n)$ is said to be discrete if it is defined at discrete instant of time, where 'n' is the independent variable.



② Even and Odd signals.

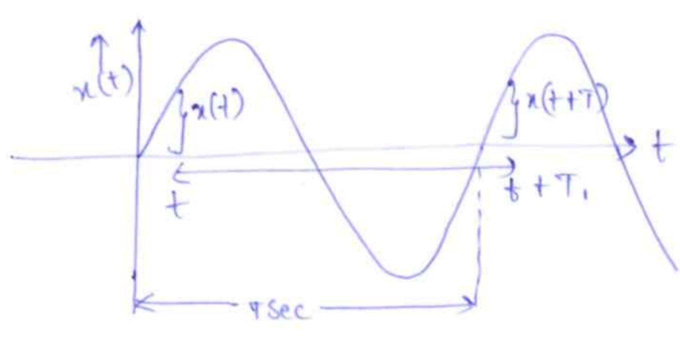
Even signals : If the signal is symmetric about y-axis, mathematically if $x(-t) = x(t)$ or $x(-n) = x(n)$ then the signal is even signal.

Odd signals : If the signal is antisymmetric about y-axis, then the signal is called as odd signal.
Mathematically if $x(-t) = -x(t)$ or $x(-n) = -x(n)$ then the signal is odd signal.

③ Periodic and Non Periodic signals.

Periodic signal : The signal $x(t)$ is said to be periodic if it satisfies the condition $x(t+T) = x(t)$, $\forall t$
where T is +ve constant, then T is called fundamental period & $\frac{1}{T} = f$ is called fundamental frequency.

$\omega = 2\pi f = \frac{2\pi}{T}$ is a angular frequency rad/sec.



Non-Periodic : A signal $x(t)$ is Non-periodic if their doesn't exists a T such that $x(t+T) = x(t)$.

(4) Energy & Power signals

<u>Energy signals</u> : A signal is said to be energy signal if its normalized energy is non zero & finite.

i.e $0 < E < \infty$

<u>Power signals</u> : A signal is said to be power signal if its normalized power is non-zero & finite.

i.e $0 < P < \infty$

Energy of CT & DT signal.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \text{ for CT signal}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \text{ for DT signal.}$$

(5) Deterministics & Random Signals.

<u>Deterministics signals</u> : A signal is said to be deterministics if there is no uncertainity about its value before its occuranc

i.e $x(n) = \cos \omega n$.

<u>Random signals</u> : A signal is said to be random if there is uncertainty about its value before its occurance.

Ex : Noise generated in electronic components, cables & channels etc.

[Q1b.] Determine and sketch the even and odd parts of the signal shown in Fig. Q1(b) [06 M]

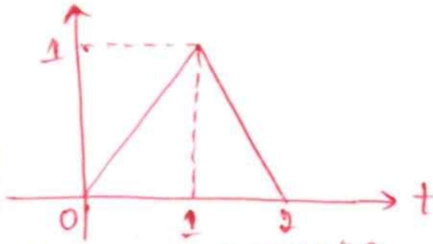


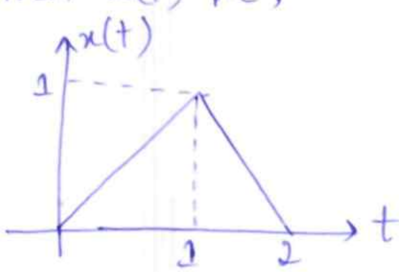
Fig. Q1(b)

→ WKT,

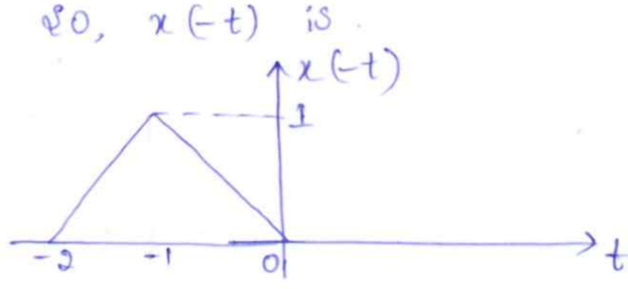
Even signal, $x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

& Odd signal, $x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$

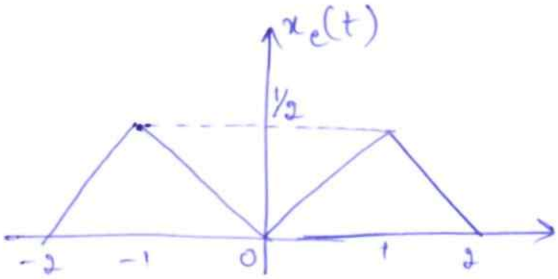
Given $x(t)$ i.e,

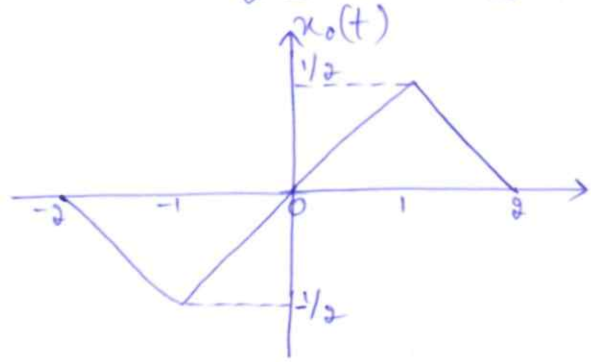


so, $x(-t)$ is



$\therefore \; x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] \quad \& \quad x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$





[Note : Given amplitude is 1 $\therefore$ $1 \times \frac{1}{2} = \frac{1}{2}$] amplitude for even + odd signals

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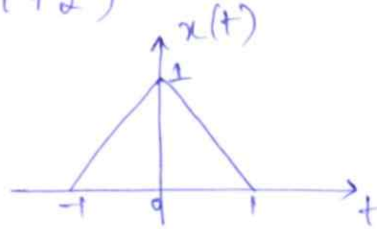
[Q1C] For the continuous time signal $x(t)$ shown in Fig. Q1(c), sketch the signal : (i) $y_1(t) = x(3t+2)$ (ii) $y_2(t) = x(3t) + x(3t+2)$ [08 M]

$\rightarrow$ (i) $y_1(t) = x(3t+2)$

$x(t)$
$\downarrow$
$x(t+2)$
(Time shifting)
$\downarrow$
$x(3t+2)$
(Time scaling)



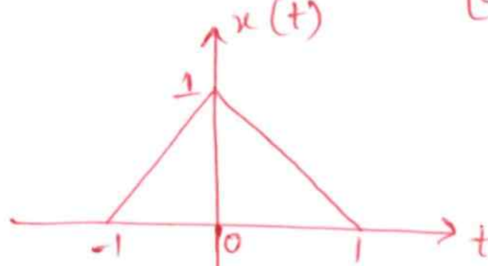
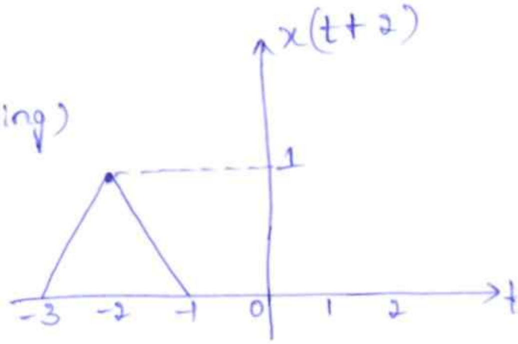
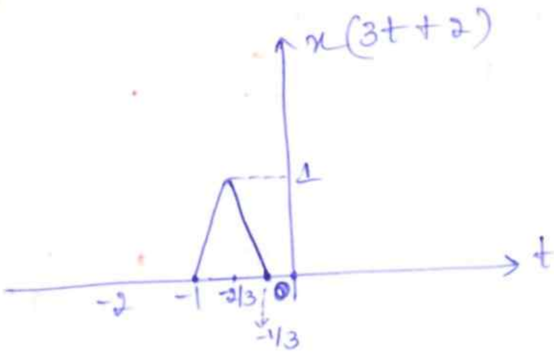


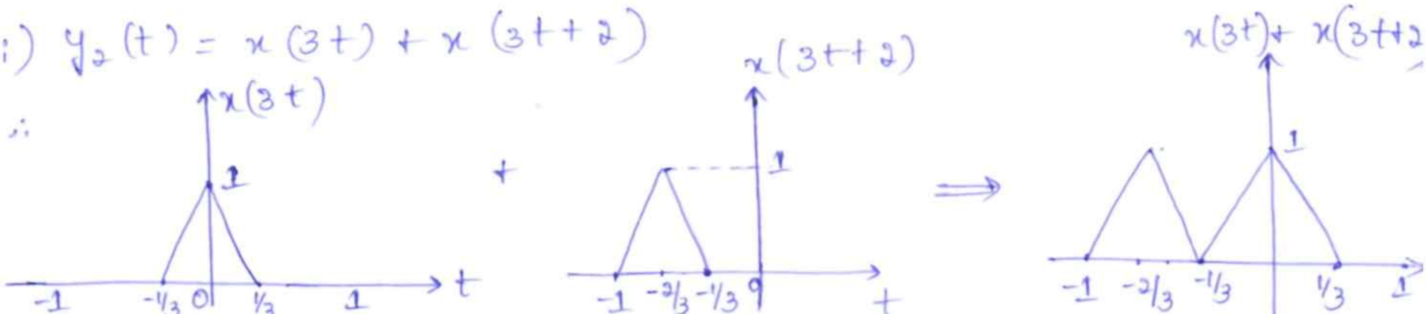
Fig. Q1(c).





(ii) $y_2(t) = x(3t) + x(3t+2)$

$\therefore$



OR

[Q2a] Check whether the following signals are periodic or not. If periodic, solve the fundamental period.

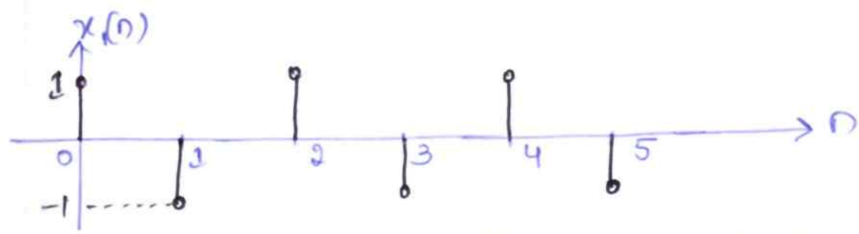
(i) $x_1(n) = (-1)^n$ (ii) $x_2(n) = \cos\left(\frac{\pi}{3}n\right) + \cos\left(\frac{\pi}{4}n\right)$ [06 M]

(i) $x_1(n) = (-1)^n$

Given signal $x_1(n)$ is not in the form of standard sin/cos.

$\therefore$ plot $x_1(n)$ for sample values of n.

$\therefore$ n	0	1	2	3	4	5
$x_1(n) = (-1)^n$	$(-1)^0$	$(-1)^1$	$(-1)^2$	$(-1)^3$	$(-1)^4$	$(-1)^5$
	1	-1	1	-1	1	-1



Given signal is repeating after every 2 samples.

$\therefore$ Given signal is periodic with period $N = 2$ samples/cycle.

(ii) $x_2(n) = \cos\left(\frac{\pi}{3}n\right) + \cos\left(\frac{\pi}{4}n\right)$

Let us consider the given signal as,

$x(n) = \cos\left(\frac{\pi}{3}n\right) + \cos\left(\frac{\pi}{4}n\right)$ which is in the form

$x(n) = x_1(n) + x_2(n)$.

where $x_1(n) = \cos\left(\frac{\pi}{3}n\right)$ & $x_2(n) = \cos\left(\frac{\pi}{4}n\right)$

$2\pi f_1 = \frac{\pi}{3}$

$f_1 = \frac{\cancel{\pi}}{3} \times \frac{1}{2\cancel{\pi}}$

$f_1 = 1/6 = \frac{k}{N}$

$\therefore$ $x_1(n)$ is periodic

$N_1 = 6$ samples/cycle

$2\pi f_2 = \frac{\pi}{4}$

$f_2 = \frac{\cancel{\pi}}{4} \times \frac{1}{2\cancel{\pi}}$

$f_2 = 1/8 = \frac{k}{N}$

$\therefore$ $x_2(n)$ is periodic

$N_2 = 8$ samples/cycle.

$\therefore x(n)$ is periodic if $\frac{N_1}{N_2} = \frac{n}{m}$ i.e ratio of integers.

$$\frac{N_1}{N_2} = \frac{6}{8} = \frac{3}{4} \quad \therefore x_2(n) \text{ is periodic.}$$

& period N of $x(n)$ is,

$N = LCM(N_1, N_2)$

$\therefore N = \lambda_1 N_1 = \lambda_2 N_2$

$\frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_1} = \frac{4}{3}$

$\lambda_1 = 4 \quad \lambda_2 = 3$

$\therefore N = \lambda_1 N_1$

$N = 4 \times 3 = 12$

$\therefore N = 12$ samples/cycle.

$\therefore x(n)$ is periodic with period $N = 12$ samples/cycle.

[Q2b] Determine the following signal $y(n) = 2x(n) + 3$ is linear, time variant, casual, memory and invertible. [06 M]

$\rightarrow$ $y(n) = 2x(n) + 3$

Linear:

$x_1(n) \; : \; y_1(n) = 2x_1(n) + 3$

$x_2(n) \; : \; y_2(n) = 2x_2(n) + 3$

$y'(n) \; : \; y'(n) = 2[x_1(n) + x_2(n)] + 6$

$x_1(n) + x_2(n) \; : \; y''(n) = 2[x_1(n) + x_2(n)] + 3$

$\therefore y'(n) \neq y''(n) \quad \therefore$ System is non-linear.

time variant : $y(n) = 2x(n) + 3$

Sub $n = n - n_0$ $\quad y(n) = 2x(n - n_0) + 3$

$\therefore y(n - n_0) = 2x(n - n_0) + 3$

$\therefore y(n) = y(n - n_0) \quad \therefore$ System is time variant.

Casual :

$$y(n) = 2x(n) + 3.$$

sub $n=0$

$$y(0) = 2x(0) + 3$$

$y(0)$: present o/p; $x(0)$: present i/p

$\therefore$ System is casual because output at n depends only on present & past values of the input.

Memory :

$$y(n) = 2x(n) + 3$$

sub $n=0$

$$y(0) = 2x(0) + 3$$

$y(0)$: present o/p; $x(0)$: Present i/p

The system is memoryless because the output depends only on the present i/p.

Invertible :

$$y(n) = 2x(n) + 3$$

$$\Rightarrow \quad x(n) = \frac{y(n) - 3}{2}$$

The system is invertible because we can recover $x(n)$ from $y(n)$.

[Q2c] Evaluate the continuous time convolution integral given as $y(t) = e^{-at}u(t) * u(t)$. [08 M]

$\rightarrow$ Given $y(t) = e^{-at} u(t) * u(t)$

in the form $y(t) = x(t) * h(t)$

WKT,

WKT, $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Let $x(\tau) = e^{-a\tau} u(\tau)$

& $h(t-\tau) = u(t-\tau)$

Then $y(t) = \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) \cdot u(t-\tau)\, d\tau$

To determine limits of Integration.

Since, $u(\tau) = 0$ for $\tau < 0$

$u(t-\tau) = 0$ for $\tau > t$

$\therefore$ limits are $\tau \in [0, t]$ (for $t \geq 0$)

$\therefore y(t) = \int_{0}^{t} e^{-a\tau}\, d\tau$

$$\therefore \int_{0}^{t} e^{-a\tau}\, d\tau = \left[\frac{-1}{a} e^{-a\tau}\right]_{0}^{t}$$

$$= \frac{-1}{a}\left(e^{-at} - 1\right)$$

$$= \frac{1}{a}\left(1 - e^{-at}\right)$$

$\therefore y(t) = \frac{1}{a}\left(1 - e^{-at}\right) u(t)$ //

Module - 2

[Q3a] State and prove the following properties of DFT: (08M)
(i) Linearity (ii) Circular time shift (iii) Symmetry of real valued sequences.

→ (i) **Linearity :**

Linearity property states that

$$DFT\{a x_1(n) + b x_2(n)\} = a X_1(k) + b X_2(k), \quad k = 0, 1, \ldots N-1$$

Proof :

$$\underset{n=0,1,\ldots N-1}{x(n)} \longrightarrow \boxed{DFT} \longrightarrow \underset{k=0,1,\ldots N-1}{X(k)}$$

Let $X_1(k)$ & $X_2(k)$ are N-point DFTs of $x_1(n)$ & $x_2(n)$

then PT, $DFT[a x_1(n) + b x_2(n)] = a X_1(k) + b X_2(k)$

WKT, $DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

Let $x(n) = a x_1(n) + b x_2(n)$,

we get,
$$DFT\{a x_1(n) + b x_2(n)\} = \sum_{n=0}^{N-1} [a x_1(n) + b x_2(n)] W_N^{kn}$$
$$= a \sum_{n=0}^{N-1} x_1(n) W_N^{kn} + b \sum_{n=0}^{N-1} x_2(n) W_N^{kn}$$
$$= a X_1(k) + b X_2(k), \quad 0 \le k \le N-1$$

$$\therefore \boxed{DFT\{a x_1(n) + b x_2(n)\} = a X_1(k) + b X_2(k)}$$ //

(ii) **Circular time shift :**

Let the DFT of $x(n)$ be $X(k)$ then

PT, $DFT\{x((n-m))_N = W_N^{km} X(k)\}$

Proof : WKT, $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn}$

Put $n = n - m$

$$x(n-m) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-k(n-m)}$$

$$x((n-m))_N = \frac{1}{N}\sum_{k=0}^{N-1}\left[X(k)\,W_N^{km}\right]W_N^{-kn}$$

$$\Rightarrow x((n-m))_N = IDFT\left[X(k)\,W_N^{km}\right]$$

or $\boxed{DFT\{x((n-m))_N\} = W_N^{km}\,X(k)}$

Above eqn can also be written as,

$$x((n-m))_N \xleftrightarrow{DFT} W_N^{km}\,X(k) //$$

(iii) <u>Symmetry of real valued sequences:</u>

Let $X(k)$ be the DFT of $x(n)$ where $x(n)$ is a real sequence then PT, $X(k) = X^*(N-k)$, $k = 0, 1, \ldots N-1$

<u>Proof</u>: $X(k) = \sum_{n=0}^{N-1} x(n)\,W_N^{kn}$

$$X^*(k) = \sum_{n=0}^{N-1} x(n)\,W_N^{-kn}$$

$$= \sum_{n=0}^{N-1} x(n)\,W_N^{-kn}\cdot W_N^{Nn} \qquad \because W_N^{Nn} = 1$$

$$= \sum_{n=0}^{N-1} x(n)\,W_N^{(N-k)n}$$

$$\boxed{X(k) = X^*(N-k)}$$

The above proof implise that $X(k)$ is symmetric about k value equal to $N/2$.

(06M)

Q3b) For the sequences $x_1(n) = \cos\left(\frac{2\pi n}{N}\right)$, $x_2(n) = \sin\left(\frac{2\pi n}{N}\right)$, $0 \le n \le N-1$, solve for N-point circular convolution $x_1(n) \circledast_N x_2(n)$

→ Let us consider $N = 4$. & $y(n) = x_1(n) \circledast_4 x_2(n)$ then $0 \le n \le 3$

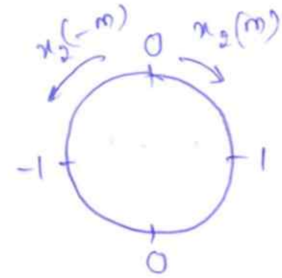
$\therefore$

	$\cos\left(\frac{2\pi n}{N}\right)$	$\sin\left(\frac{2\pi n}{N}\right)$
$n=0$	1	0
$n=1$	0	1
$n=2$	-1	0
$n=3$	0	-1

$\therefore x_1(n) = [1, 0, -1, 0]$ & $x_2(n) = [0, 1, 0, -1]$

WKT, $y(n) = x_1(n) \circledast_N x_2(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2((n-m))_N$

n	$x_1(m)$	$x_2((n-m))_4$	$y(n)$
0	[1, 0, -1, 0]	[0, -1, 0, 1]	0+0+0+0 = 0
1	[1, 0, -1, 0]	[1, 0, -1, 0]	1+0+1+0 = 2
2	[1, 0, -1, 0]	[0, 1, 0, -1]	0+0+0+0 = 0
3	[1, 0, -1, 0]	[-1, 0, 1, 0]	-1+0-1+0 = -2



$\therefore y(n) = [0, 2, 0, -2]$ //.

[Q3c]. Determine the 4-point DFT of the sequence, $x(n) = (1, -1, 1, -1)$. Also, using time shift property, find the DFT of the sequence, $y(n) = x((n-2))_4$. [06 M]

→ Time shift property is given by,

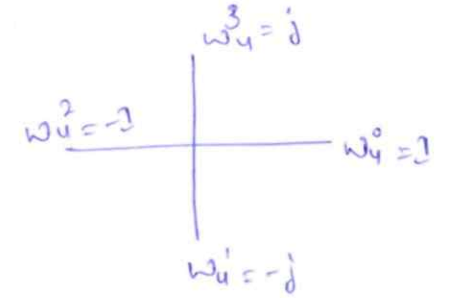
If $x(n) \underset{DFT}{\overset{N}{\longleftrightarrow}} X(k)$ then $x((n-m))_N \overset{DFT}{\longleftrightarrow} W_N^{km} X(k) = Y(k)$

Here $m = 2$ & $N = 4$.

$\therefore$ DFT of $x(n) = (1, -1, 1, -1)$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1-1+1-1 \\ 1+j-1-j \\ 1+1+1+1 \\ 1-j-1+j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

$\therefore X(K) = [0\ 0\ 4\ 0]$



$\therefore Y(k) = W_N^{km} X(k)$

for $k = 0$, $Y(0) = W_4^{0\times 2} X(0) = 1 \times 0 = 0$

for $k = 1$, $Y(1) = W_4^{1\times 2} X(1) = j \times 0 = 0$

for $k=2$, $Y(2) = W_4^{2\times 2} x(2) = +1 \times 4 = +4$

for $k=3$, $Y(3) = W_4^{3\times 2} x(3) = -1 \times 0 = 0$

$\therefore Y(k) = [0, 0, +4, 0]$ //

OR

[Q4a] Define DFT and IDFT and Compute 4-point DFT of a single $x(n) = (1, 2, 1, 0)$ using DFT matrix. [08M]

→ Definition of DFT :

The N-point DFT of a finite duration sequence $x(n)$ of length L, where $N \geq L$, is defined as

$$DFT\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} \quad ; \text{ for } k = 0, 1, 2, \ldots, (N-1)$$

Definition of IDFT

The Inverse Discrete Fourier Transform (IDFT) of the sequence $X(k)$ of length N is defined as

$$IDFT\{X(k)\} = x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}} \quad ; \text{ for } n = 0, 1, \ldots (N-1)$$

$$x(n) \xleftrightarrow[N]{DFT} X(k)$$

Given, $x(n) = (1, 2, 1, 0)$ $\therefore N = 4$

WKT, $X = W_N x$.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 4 \\ -j2 \\ 0 \\ j2 \end{bmatrix}$$

Hence $X(k) = (4, -j2, 0, j2)$ //

[Q4b] The 5-point DFT of a complex sequence $x(n)$ is given ~~by~~ as $X(K) = (j,\ 1+j,\ 1+j2,\ 2+j2,\ 4+j)$ Compute $Y(K)$, if $y(n) = x^*(n)$. [06 M]

→ Property of DFT for Complex conjugate

if $x(n) \xrightarrow{DFT} X(k)$,

then $x^*(n) \xrightarrow{DFT} Y(k) = X^*(-k) = X^*(N-k)$

Given $N = 5$

$\therefore\ Y(k) = X^*(N-k)$ for $k = 0, 1, 2, 3, 4$

$$\therefore\ Y(0) = X^*(5-0) = X^*(0) = -j$$
$$Y(1) = X^*(5-1) = X^*(4) = (4+j)^* = 4-j$$
$$Y(2) = X^*(5-2) = X^*(3) = (2+2j)^* = 2-2j$$
$$Y(3) = X^*(5-3) = X^*(2) = (1+2j)^* = 1-2j$$
$$Y(4) = X^*(5-4) = X^*(1) = (1+j)^* = 1-j$$

$$\therefore\ Y(K) = (-j,\ 4-j,\ 2-2j,\ 1-2j,\ 1-j)$$

[Q4c] Using DFT, IDFT method, compute circular convolution of the sequences $x_1(n) = (1, 1, 1)$ and $x_2(n) = (1, -2, 2)$. [06 M]

→ Using DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}, \quad k = 0, 1, 2.$$

For $x_1(n) = (1, 1, 1)$

$$X_1(0) = \sum_{n=0}^{2} x_1(n) \cdot e^{-j\frac{2\pi 0 n}{3}} = x_1(0) + x_1(1) + x_1(2) = 1+1+1 = 3$$

$$X_1(1) = 1 + 1 \cdot e^{-j\frac{2\pi}{3}} + 1 \cdot e^{-j\frac{4\pi}{3}} = 1 + e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}$$

$$\because\ e^{-j\frac{2\pi}{3}} = \cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3} = -1/2 - j\sqrt{3}/2$$
$$e^{-j\frac{4\pi}{3}} = \cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3} = -1/2 + j\sqrt{3}/2$$

$$\therefore\ X_1(1) = 1 + (-1/2 - j\sqrt{3}/2) + (-1/2 + j\sqrt{3}/2) = 1 - 1 = 0$$

$$X_1(2) = 1 - 1 = 0$$

$$\therefore\ X_1(k) = (3, 0, 0)$$

For $x_2(n) = (1, -2, 2)$

$X_2(0) = 1 + 2 + 2 = 1$

$X_2(1) = 1 + (-2)e^{-j2\pi/3} + 2e^{-j4\pi/3} = 1 + j2\sqrt{3}$

$X_2(2) = 1 - j2\sqrt{3}$ [$\because X_2(2)$ is Conjugate of $X_2(1)$]

$\therefore X_2(k) = (1,\ 1+j2\sqrt{3},\ 1-j2\sqrt{3})$

$Y(k) = X_1(k) \cdot X_2(k)$

$Y(0) = 3 \times 1 = 3$

$Y(1) = 0.(1+j2\sqrt{3}) = 0$ $\therefore Y(k) = (3, 0, 0)$

$Y(2) = 0.(1-j2\sqrt{3}) = 0$

IDFT of $Y(k)$ is

$$y(n) = \frac{1}{3}\sum_{k=0}^{2} Y(k) \cdot e^{j\frac{2\pi kn}{3}} = \frac{1}{3} \cdot 3 = 1 \text{ for all } n.$$

$\therefore y(n) = (1, 1, 1)$ //

Module - 3

[Q5a]. Compute 8 point DFT of the sequence $x(n) = (1, 2, 3, 4, 4, 3, 2, 1)$ using radix-2 DIT-FFT algorithm. [12 M]

→ The scale factors

$W_8^0 = 1$

$W_8^1 = 1/\sqrt{2} - j\,1/\sqrt{2}$

$W_8^2 = -j$

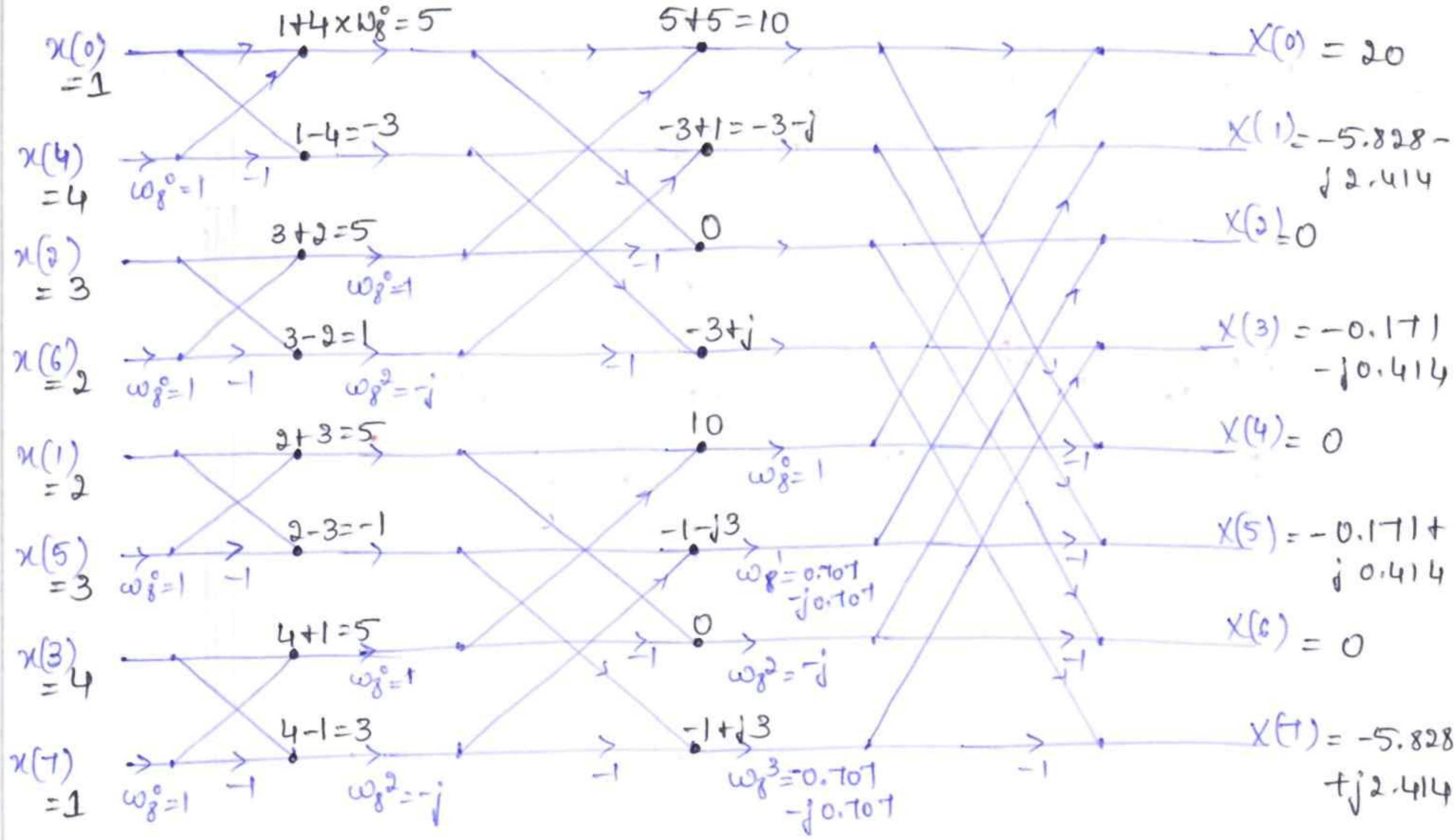
$W_8^3 = -1/\sqrt{2} - j\,1/\sqrt{2}$

$W_8^4 = -1$

$W_8^5 = -1/\sqrt{2} + j\,1/\sqrt{2}$

$W_8^6 = +j$

$W_8^7 = 1/\sqrt{2} + j\,1/\sqrt{2}$



$\therefore X(k) = (20, -5.828 - j2.414, 0, -0.171 - j0.414, 0, -0.171 + j0.414, 0, -5.828 + j2.414)$

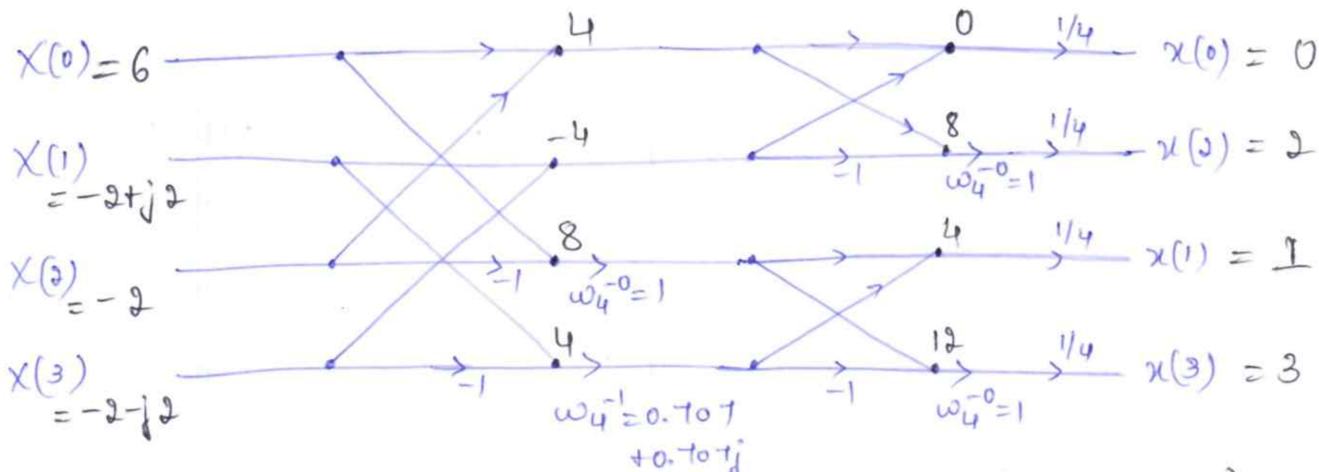
[Q5 b] Determine the 4-point real sequence $x(n)$, if its 4-point DFT samples are $X(0) = 6$, $X(1) = -2 + j2$, $X(2) = -2$. Use DIF-FFT algorithm. [08 M]

→ Since $x(n)$ is a real sequence, it has to satisfy symmetry condition

$$X(k) = X^*(4-k)$$

$$\Rightarrow X(3) = X^*(1) = -2 - j2$$

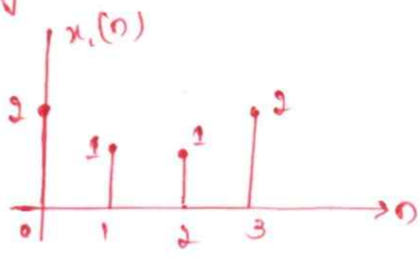
$$\therefore X(k) = (6, -2+j2, -2, -2-j2)$$

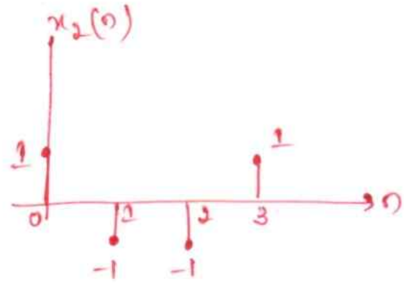


$\therefore x(n) = (0, 1, 2, 3)$

OR

[Q6a] Given the sequence $x_1(n)$ & $x_2(n)$ below, compute the circular convolution $x_1(n) \circledast_N x_2(n)$ for $N=4$. Use DIT-FFT algorithm. [10 M]



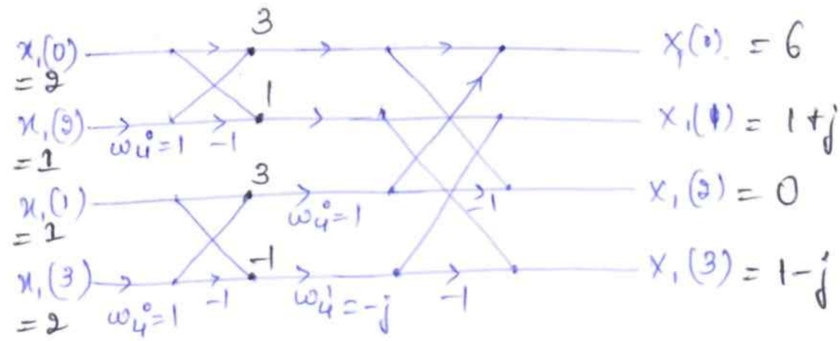


→ Given sequences $x_1(n) = (2, 1, 1, 2)$ & $x_2(n) = (1, -1, -1, 1)$

$\therefore y(n) = x_1(n) \circledast_N x_2(n)$

To find $X_1(k)$:

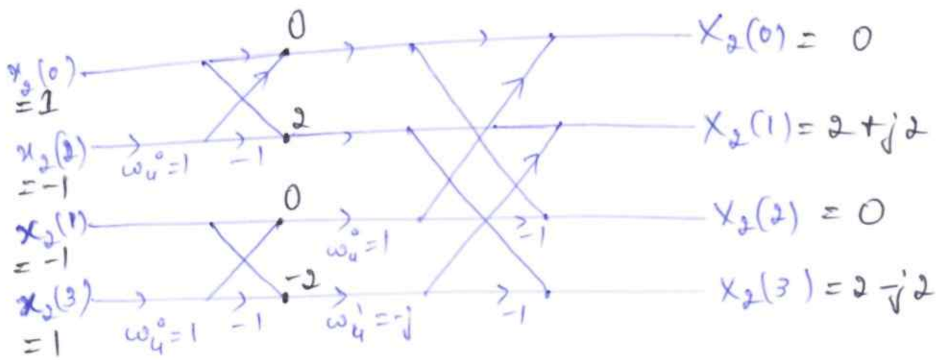
$x_1(n) = (2, 1, 1, 2)$



$\therefore X_1(k) = (6, 1+j, 0, 1-j)$

To find $X_2(k)$:

$x_2(n) = (1, -1, -1, 1)$

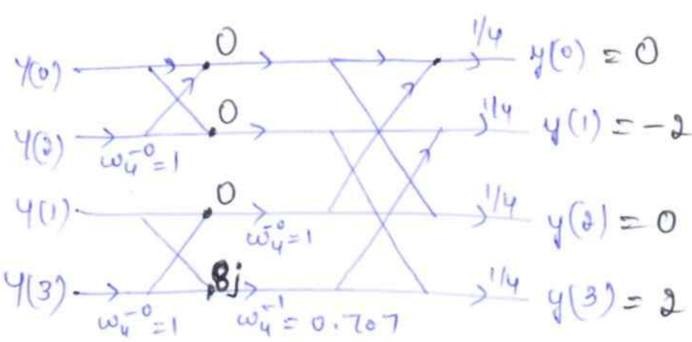


$\therefore X_2(k) = (0, 2+j2, 0, 2-j2)$

Hence $y(n) = x_1(n) \circledast_4 x_2(n)$

$Y(k) = X_1(k)\, X_2(k)$

$Y(k) = (0, 4j, 0, -4j)$



$\therefore y(n) = (0, -2, 0, -2)$

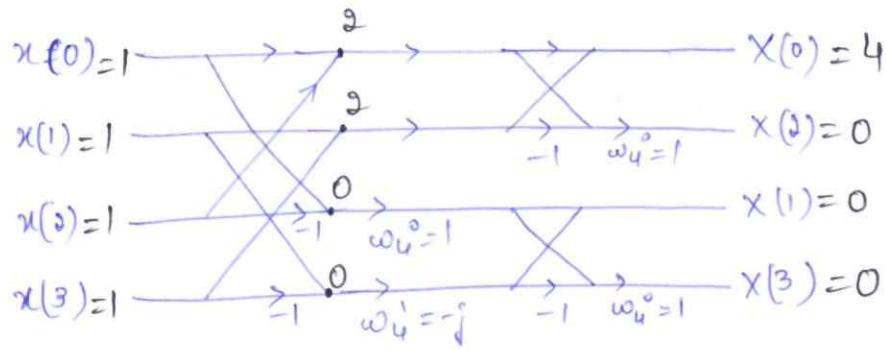
[Q6b] Solve for the 4-point circular convolution of $x(n)$ & $h(n)$ using radix-2 DIF-FFT algorithm. Given $x(n) = (1,1,1,1)$, $h(n) = (1,0,1,0)$. [10M]

$\rightarrow$ $y(n) = x(n) \circledast_4 h(n)$

$= \text{IFFT}\{X(k)\,H(k)\}$

<u>To find $X(k)$</u> :

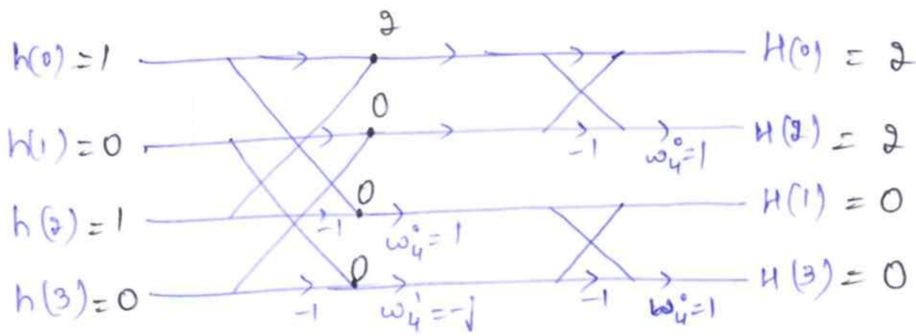
$x(n) = (1,1,1,1)$



$\therefore X(k) = (4,0,0,0)$

<u>To find $H(k)$</u> :

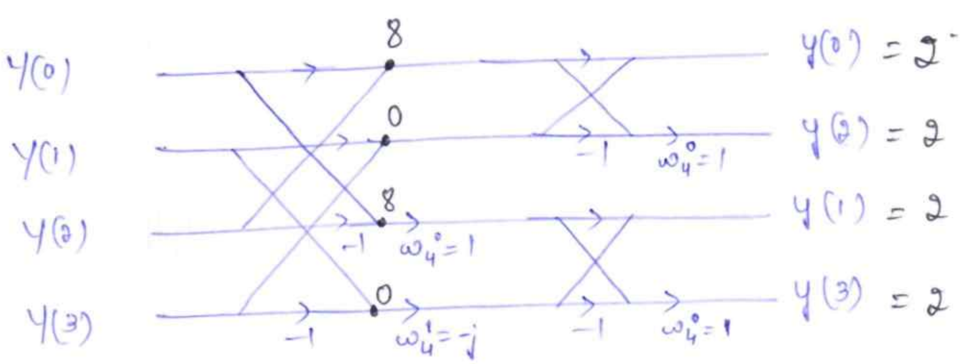
$h(n) = (1,0,1,0)$



$\therefore H(k) = (2,0,2,0)$

<u>To find $y(n)$</u> :

$Y(k) = X(k) \cdot H(k) = (8,0,0,0)$



Hence $y(n) = (2,2,2,2)$ //

Module - 4

[Q7a] Design a Butterworth analog highpass filter that will meet the following specifications:

(i) Maximum passband attenuation = 2dB

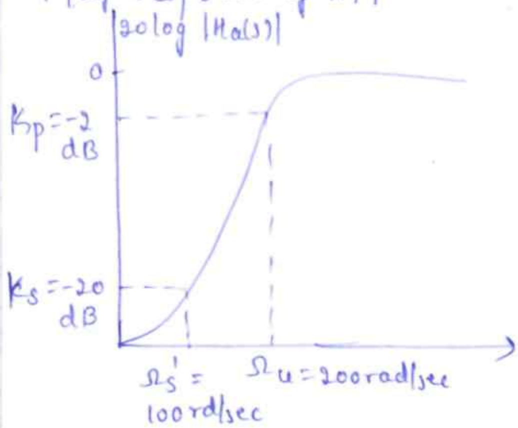
(ii) Passband edge frequency = 200 rad/sec

(iii) Minimum stopband attenuation = 20dB.

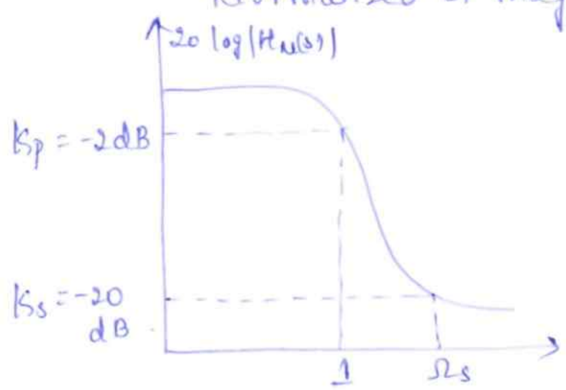
(iv) Stopband edge frequency = 100 rad/sec. [10 M]

→ Given, $K_p = -2\,dB$, $\Omega_p = 200$ rad/sec, $K_s = -20\,dB$, $\Omega'_s = 100$ rad/sec

Mag. response of HPF



Normalized LP mag. response



$$\Omega_s = \frac{\Omega_u}{\Omega'_s} = \frac{200}{100} = 2$$

$\Omega_p = 1$ $\quad K_p = -2\,dB$

$\Omega_s = 2$ $\quad K_s = -20\,dB$.

$$N = \frac{\log\left[\left(10^{-\frac{K_p}{10}} - 1\right) \Big/ \left(10^{-\frac{K_s}{10}} - 1\right)\right]}{20\log\left(\frac{\Omega_p}{\Omega_s}\right)} = 3.7 \approx 4 \quad \therefore \boxed{N = 4}$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{-\frac{K_p}{10}} - 1\right)^{1/2N}} = \frac{1}{\left(10^{0.2} - 1\right)^{1/8}} = 1.0693 \text{ rad/sec.}$$

$$H_4(s) = \frac{1}{(s^2 + 0.76543 + 1)(s^2 + 1.84788 + 1)} \longrightarrow (1)$$

Prototype of LP filter, $H_p(s) = H_4(s)\Big|_{s \to \frac{s}{\Omega_c}} = \frac{s}{1.0693}$

LP to HP freq transformation $s \to \frac{\Omega_p}{s} = \frac{200}{s}$

$$\therefore H_a(s) = H_p(s) = H_4(s)\Big|_{s \to \frac{\Omega_p}{s}} = \frac{200}{1.0693\,s} = \frac{187.031}{s}$$

Substitute $s \rightarrow \frac{187.031}{s}$ in eq^n ①.

We get,
$$H_a(s) = \frac{s^4}{(s^2 + 143.15s + 34980.75)(s^2 + 345.58s + 34980.75)}$$

[Q7b] Obtain the direct form I and direct form II of the following transfer function: [10 M]

$$H(z) = \frac{8z^3 - 4z^2 + 11z + 2}{\left(z - \frac{1}{4}\right)\left(z^2 - z + \frac{1}{2}\right)}$$

→ Divide N^r & D^r by z^3 we get,

$$H(z) = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - 5/4\, z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

Direct form I:

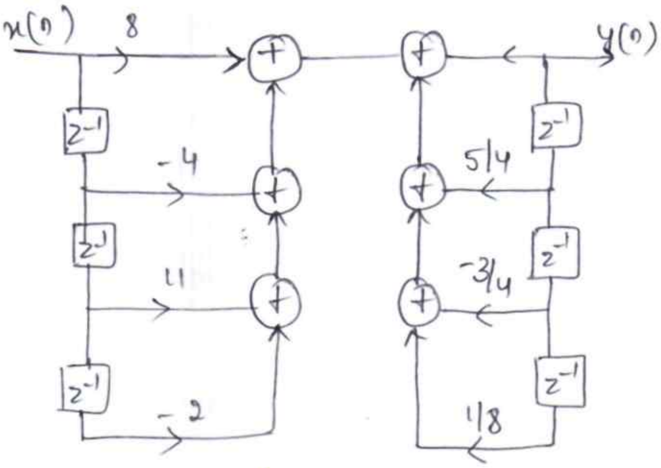
Let
$$H(z) = \frac{Y(z)}{X(z)} = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$\therefore Y(z) - \frac{5}{4}z^{-1}Y(z) + \frac{3}{4}z^{-2}Y(z) - \frac{1}{8}z^{-3}Y(z) = 8X(z) - 4z^{-1}X(z) + 11z^{-2}X(z) - 2z^{-3}X(z)$$

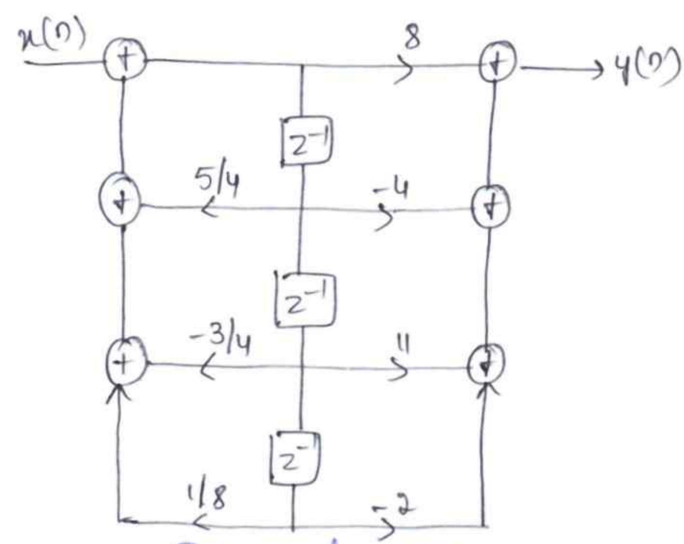
Taking z-inverse.

$$y(n) - \frac{5}{4}y(n-1) + \frac{3}{4}y(n-2) - \frac{1}{8}y(n-3) = 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3)$$

$$\therefore y(n) = 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3) + \frac{5}{4}y(n-1) - \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3)$$



Direct form I



Direct form II

OR

[Q 8a] Design a chebyshev I filter to meet the following specification
(i) Passband ripple : $\leq 2dB$.
(ii) Passband edge : 1 rad/sec
(iii) Stopband attenuation : $\geq 20dB$
(iv) Stopband edge : 1.3 rad/sec

[10 M]

→ WKT, $K_p = 20 \log \left[\frac{1}{\sqrt{1+\epsilon^2}}\right] = -2 \Rightarrow \epsilon = 0.76478$.

$$\& \ \delta_p = 1 - \frac{1}{\sqrt{1+\epsilon^2}} = 0.20567$$

$$K_s = 20 \log \delta_s = -20$$

$$\therefore \delta_s = 0.1$$

Discrimination factor, $d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = \frac{(1-0.20567)^{-2}-1}{(0.1)^{-2}-1} = 0.077$

Selectivity factor, $K = \frac{\Omega_p}{\Omega_s} = \frac{1}{1.3} = 0.769$

$$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)} = \frac{\cosh^{-1}\left(\frac{1}{0.077}\right)}{\cosh^{-1}\left(\frac{1}{0.769}\right)} = 4.3 \Rightarrow \boxed{N=5}$$

To find transfer function of the fifth order normalized lowpass Chebyshev I filter.

$$a = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{1/N} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{-1/N} = 0.21830398$$

$$b = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{1/N} + \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{-1/N} = 1.0235520$$

$$\sigma_k = -a \sin\left[(2k-1)\frac{\pi}{2N}\right] = -0.21830398 \sin\left[(2k-1)\frac{\pi}{10}\right]$$

$$\Omega_k = b \cos\left[(2k-1)\frac{\pi}{2N}\right] \quad = 1.0235520 \cos\left[(2k-1)\frac{\pi}{10}\right], k = 1, \dots 10$$

$$k = 1, 2, \dots 2N$$

$\therefore$ For the values of k from 1 to 5, we get the poles of $H_5(s)\,H_5(-s)$.

$\therefore$ k	σ_k	Ω_k
1	-0.06746	0.97346
2	-0.17662	0.60163
3	-0.21831	0
4	-0.17662	-0.60163
5	-0.06746	-0.97346

Hence $H_5(S) = \dfrac{K_N}{(S-S_1)(S-S_5)(S-S_2)(S-S_4)(S-S_3)}$

$$= \frac{K_N}{[(s+0.0674610-j0.9734557)(s+0.0674610+j0.9734557)(s+0.1766151-j0.6016287)(s+0.1766151+j0.6016287)(s+0.2183083)]}$$

$$= \frac{K_N}{s^5+0.70646s^4+1.4995s^3+0.6934s^2+0.459349s+0.08172}$$

Since N is odd, $K_N = b_0 = 0.08172$.

Hence, $H_5(S) = \dfrac{0.08172}{S^5+0.70646S^4+1.4995S^3+0.6934S^2+0.459349s+0.08172}$

[Q8b] The system function of an analog filter is given by $H(S) = \dfrac{S+0.1}{(S+0.1)^2+9}$. Obtain the system function of IIR digital filter by using impulse invariant method. [10 M]

→ $H(S) = \dfrac{S+0.1}{(S+0.1)^2+3^2}$ Consider $T = 1$ sec

WKT, $H(S) = \dfrac{S+a}{(S+a)^2+b^2} \Rightarrow H(z) = \dfrac{1-e^{-aT}\cos bT\, z^{-1}}{1-2\cos bT e^{-aT} z^{-1} + e^{-2aT} z^{-2}}$

$$\therefore H(z) = \frac{1-e^{-(0.1)T}\cos(3T)z^{-1}}{1-2e^{-(0.1)T}\cos(3T)z^{-1}+e^{-2(0.1)T}z^{-2}}$$

$$= \frac{1-e^{-0.1}\cos 3\, z^{-1}}{1-2e^{-0.1}\cos 3\, z^{-1}+e^{-0.2}z^{-2}} = \frac{1-0.9035z^{-1}}{1-1.807z^{-1}+0.818z^{-2}}$$

Module-5

[Q9a] A filter is to be designed with the following desired frequency response :
$$H_d(\omega) = \begin{cases} 0 & -\frac{\pi}{4} < \omega < \frac{\pi}{4} \\ e^{-j2\omega} & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

Compute the frequency response of the FIR filter designed using a rectangular window defined below: $\omega_R(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$

[10 M]

→ The inverse DTFT of $H_d(\omega)$ is
$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega$$
$$\Rightarrow h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{-j2\omega} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} e^{-j2\omega} e^{j\omega n}\, d\omega$$
$$= \frac{1}{\pi(n-2)}\left[\sin\left[\pi(n-2)\right] - \sin\left(\frac{\pi}{4}(n-2)\right)\right], \quad n \neq 2$$

also, $h_d(2) = \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^0\, d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\pi} e^0\, d\omega = \frac{1}{2\pi}\left[\frac{3\pi}{4} + \frac{3\pi}{4}\right] = \frac{3}{4}$

The impulse response of the FIR filter is given by .
$h(n) = h_d(n)\,\omega_R(n), \quad 0 \le n \le 4$ where $\omega_R(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$

The coefficients ~~for~~ $h_d(n)$ & $h(n)$ are

n	$h_d(n)$	$\omega_R(n)$	$h(n) = h_d(n)\,\omega_R(n)$
0	-0.159	1	-0.159
1	-0.225	1	-0.225
2	0.75	1	0.75
3	-0.225	1	-0.225
4	-0.159	1	-0.159

since N is odd, the frequency response is,
$$H(e^{j\omega}) = H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left(h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h(n)\cos\left[\omega\left(n - \left(\frac{N-1}{2}\right)\right)\right]\right)$$

since $N = 5$, we get
$$H(\omega) = e^{-j2\omega}\left(h(2) + 2h(0)\cos 2\omega + 2h(1)\cos\omega\right)$$
$$= e^{-j2\omega}(0.75 - 2\times 0.159\cos 2\omega + 2\times -0.225\cos\omega)$$
$$= e^{-j2\omega}(0.75 - 0.318\cos 2\omega - 0.45\cos\omega) \quad //$$

[Q9b] Determine the filter coefficients $h(n)$ obtained by sampling $H_d(\omega)$ given by, $H_d(\omega) = \begin{cases} e^{-j3\omega} & 0 < \omega < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < |\omega| < \pi \end{cases}$ [10M]

Also, obtain the frequency response, $H(\omega)$. Take $N = 7$.

→ $h(n) = \frac{1}{7}\left[1 + 2\cos\left(\frac{2\pi}{7}(n-3)\right)\right] = \frac{1}{7}\left[1 + e^{j\frac{2\pi}{7}(n-3)} + e^{-j\frac{2\pi}{7}(n-3)}\right]$

WKT, $H(\omega) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n} = \frac{1}{7}\sum_{n=0}^{6}\left[1 + e^{j\frac{2\pi}{7}(n-3)} + e^{-j\frac{2\pi}{7}(n-3)}\right] e^{-j\omega n}$

WKT, $\sum_{n=0}^{N-1} e^{-j\omega n} = e^{-j\frac{(N-1)\omega}{2}} \frac{\sin\left(\frac{N\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$

$\Rightarrow \sum_{n=0}^{6} e^{-j\omega n} = e^{-j3\omega} \frac{\sin\left(\frac{7\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$

Hence, $7H(\omega) = \sum_{n=0}^{6} e^{-jn\omega} + e^{-j\frac{6\pi}{7}} \sum_{n=0}^{6} e^{-jn\left(\omega - \frac{2\pi}{7}\right)} + e^{j\frac{6\pi}{7}} \sum_{n=0}^{6} e^{-jn\left(\omega + \frac{2\pi}{7}\right)}$

$$= e^{-j3\omega}\left[\frac{\sin\left(\frac{7\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} + e^{-j\frac{6\pi}{7}} e^{-j3\left(\frac{-2\pi}{7}\right)} \frac{\sin\left[\frac{7}{2}\left(\omega - \frac{2\pi}{7}\right)\right]}{\sin\left[\frac{1}{2}\left(\omega - \frac{2\pi}{7}\right)\right]} + e^{+j\frac{6\pi}{7}} e^{-j3\left(\frac{2\pi}{7}\right)} \frac{\sin\left[\frac{7}{2}\left(\omega + \frac{2\pi}{7}\right)\right]}{\sin\left[\frac{1}{2}\left(\omega + \frac{2\pi}{7}\right)\right]}\right]$$

$$\Rightarrow H(\omega) = \frac{1}{7} e^{-j3\omega}\left[\frac{\sin\left(\frac{7\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} + \frac{\sin\left[\frac{7}{2}\left(\omega - \frac{2\pi}{7}\right)\right]}{\sin\left[\frac{1}{2}\left(\omega - \frac{2\pi}{7}\right)\right]} + \frac{\sin\left[\frac{7}{2}\left(\omega + \frac{2\pi}{7}\right)\right]}{\sin\left[\frac{1}{2}\left(\omega + \frac{2\pi}{7}\right)\right]}\right] //$$

OR

[Q10a] The desired frequency response of a lowpass filter is given by

$H_d(e^{j\omega}) = H_d(\omega) = \begin{cases} e^{-j3\omega} & |\omega| < \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} < |\omega| < \pi \end{cases}$

Determine the frequency response of the FIR filter if Hamming window is used with $N = 7$. [10M]

→ By def^n, the inverse DTFT of $H_d(\omega)$ is,

$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-3\pi/4}^{\frac{3\pi}{4}} e^{-j3\omega} e^{j\omega n}\, d\omega$

$$h_d(n) = \frac{\sin\left[\frac{3\pi}{4}(n-3)\right]}{\pi(n-3)}, \quad n \neq 3$$

$$h_d(3) = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{\frac{3\pi}{4}} e^0\, d\omega = \frac{3}{4}$$

The impulse response of FIR filter is, $h(n) = h_d(n)\,\omega_{Ham}(n), \; 0 \le n \le 6$

where, $\omega_{Ham}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right), \quad 0 \le n \le N-1$

$$= 0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right), \quad 0 \le n \le 6$$

Hence,
$$h(n) = \begin{cases} \frac{\sin\left[\frac{3\pi}{4}(n-3)\right]}{\pi(n-3)} \times \left[0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right)\right], & n \neq 3 \\ \frac{3}{4} \times \left[0.54 - 0.46\cos\left(\frac{2\pi n}{6}\right)\right], & n = 3 \end{cases}$$

n	$h_d(n)$	$\omega_{Ham}(n)$	$h(n) = h_d(n)\,\omega_{Ham}(n)$
0	0.075	0.08	0.006
1	-0.159	0.31	-0.049
2	0.225	0.77	0.173
3	0.75	1	0.75
4	0.225	0.77	0.173
5	-0.159	0.31	-0.049
6	0.075	0.08	0.006

Since N is odd,

$$H(e^{j\omega}) = H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left(h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} 2h(n)\cos\left[\omega\left(n - \left(\frac{N-1}{2}\right)\right)\right]\right)$$

$$= e^{-j3\omega}\left[h(3) - 2h(0)\cos 3\omega + 2h(1)\cos 2\omega + 2h(2)\cos\omega\right]$$

$$= e^{-j3\omega}\left(0.75 + 2\times 0.006\cos 3\omega + 2\times -0.049\cos 2\omega + 2\times 0.173\cos\omega\right)$$

$$= e^{-j3\omega}\left(0.75 + 0.012\cos 3\omega - 0.098\cos 2\omega + 0.346\cos\omega\right)$$

[Q10b] The frequency response of an FIR filter is given by

$H(\omega) = e^{-j3\omega}(1 + 1.8\cos 3\omega + 1.2\cos 2\omega + 0.5\cos\omega)$

Determine the coefficients of the impulse response $h(n)$ of the FIR filter. [10M]

→ $\alpha = \frac{N-1}{2} = 3 \Rightarrow N = 7$

Since N is odd,

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left(h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} 2h(n)\cos\left[\omega\left(n - \left(\frac{N-1}{2}\right)\right)\right]\right)$$

$$= e^{-j3\omega}\left(h(3) + \sum_{n=0}^{2} 2h(n)\cos\left[\omega(n-3)\right]\right)$$

$$= e^{-j3\omega}\left[h(3) + 2h(0)\cos 3\omega + 2h(1)\cos 2\omega + 2h(2)\cos\omega\right]$$

Comparing the above equation with the frequency response given in the problem, we get

$h(3) = 1$, $h(0) = 0.9$, $h(1) = 0.6$ and $h(2) = 0.25$

$h(n) = h(N-1-n)$

$\Rightarrow h(n) = h(6-n)$, $n = 0, 1, \ldots 6$

$h(6) = h(0) = 0.9$

$h(5) = h(1) = 0.6$

$h(4) = h(2) = 0.25$ //

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