CBCS SCHEME

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BMATE101

First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

Mathematics - I for EEE Stream

Time: 3 hrs. Max. Marks: 100

***Note:** 1. Answer any FIVE full questions, choosing ONE full question from each module.*
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. VTU Formula Handbook is permitted.

		Module – 1	M	L	C
Q.1	a.	With usual notations, prove that $\tan\phi = r\cdot \frac{d\theta}{dr}$.	6	L2	CO1
	b.	Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersects orthogonally.	7	L2	CO1
	c.	Find the radius of curvature for the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.	7	L3	CO1
		OR			
Q.2	a.	Find the angle of intersection between curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$.	7	L2	CO1
	b.	Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$.	8	L2	CO1
	c.	Using modern mathematical tool, write a program to plot the curve $r = 2\lvert\cos 2\theta\rvert$.	5	L3	CO1
		Module – 2			
Q.3	a.	Expand $\sqrt{1+\sin 2x}$ using Maclaurin's series expansion upto terms containing x^6.	6	L2	CO1
	b.	If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.	7	L2	CO1
	c.	Show that the function $f(x,y) = x^3 + y^3 - 3xy + 1$ is minimum at the point (1, 1).	7	L3	CO1
		OR			
Q.4	a.	If $u = \frac{xy}{z}$, $v = \frac{yz}{x}$ and $w = \frac{xz}{y}$ then show that $J\left(\frac{u,v,w}{x,y,z}\right) = 4$.	7	L2	CO1
	b.	Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.	8	L3	CO1
	c.	Using modern tool write a program to evaluate $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$.	5	L3	CO5

		Module – 3			
Q.5	**a.**	Solve $x\frac{dy}{dx} + y = x^3y^6$.	**6**	**L2**	**CO2**
	b.	Find the orthogonal trajectories of a family of curves $\frac{2a}{r} = 1 - \cos\theta$.	**7**	**L3**	**CO2**
	c.	Solve $xy(p^2) - (x^2 + y^2)p + xy = 0$.	**7**	**L2**	**CO2**
		OR			
Q.6	**a.**	Solve $(x^2 + y^2 + x)dx + xy\,dy = 0$.	**6**	**L2**	**CO2**
	b.	A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t.	**7**	**L3**	**CO2**
	c.	Solve $(px - y)(py + x) = a^2p$ by reducing into Clairaut's form using the substitution $X = x^2$ and $Y = y^2$.	**7**	**L2**	**CO2**
		Module – 4			
Q.7	**a.**	Evaluate $\int_{-1}^{+1}\int_{0}^{z}\int_{x-z}^{x+z} (x + y + z)\,dy\,dx\,dz$.	**6**	**L2**	**CO3**
	b.	Change the order of integration and evaluate $\int_{0}^{1}\int_{x}^{\sqrt{x}} xy\,dy\,dx$.	**7**	**L2**	**CO3**
	c.	Prove that $\beta(m, n) = \frac{\sqrt{m}.\sqrt{n}}{\sqrt{m+n}}$.	**7**	**L2**	**CO3**
		OR			
Q.8	**a.**	Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dxdy$ by changing into polar coordinates.	**6**	**L2**	**CO3**
	b.	Evaluate $\int_{0}^{\pi/2}\sqrt{\cot\theta}\,d\theta$ by expressing in terms of gamma functions.	**7**	**L2**	**CO3**
	c.	Using double integration find the area between the curves $y^2 = 4ax$ and $x^2 = 4ay$.	**7**	**L3**	**CO3**

		Module – 5			
Q.9	**a.**	Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$.	**6**	**L2**	**CO4**
	b.	Investigate the values of λ and μ such that the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ may have i) Unique solution ii) Infinite solution iii) No solution.	**7**	**L3**	**CO4**
	c.	Using Rayleigh's power method find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking $[1, 1, 1]^T$ as initial eigen vector.	**7**	**L3**	**CO4**
		OR			
Q.10	**a.**	Solve by using Gauss – Jordan method $x + y + z = 9$, $x - 2y + 3z = 8$ and $2x + y - z = 3$.	**7**	**L2**	**CO4**
	b.	Solve by using Gauss – Siedel method $20x + y - 2z = 17$, $3x + 20y - z = -18$ and $2x - 3y + 20z = 25$.	**8**	**L2**	**CO4**
	c.	Using modern mathematical model, write a program to test the consistency of the equations $x + 2y - z = 1$, $2x + y + 4z = 2$ and $3x + 3y + 4z = 1$.	**5**	**L3**	**CO5**

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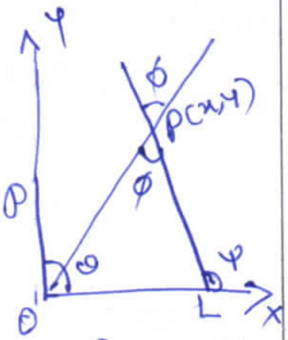
AY: 2025-26

Department: Electrical & electronics engineering
Semester/Division/Branch: II /EE(A,B,C Div)
Subject with Sub. Code: Mathematics I for Electronics &electrical engineering Stream(BMATE101)
Name of Faculty: Prof. Akshata B Patil.

Q.No.	Solution and Scheme	Marks
	MODULE - 01	
Q.1 a)	Let $P(r,\theta)$ be any point on the curve $r=f(\theta)$ $X\hat{O}P=\theta$ and $OP=r$ Let PL be the tangent to the curve at P subtending an angle ψ with the positive direction of the initial line (x-axis) and ϕ be the angle between the radius vector OP and the tangent PL.  That is, $O\hat{P}L=\phi$ From the figure $\psi=\phi+\theta$ $\tan\psi=\tan(\phi+\theta)$ $\tan\psi=\dfrac{\tan\phi+\tan\theta}{1-\tan\phi\tan\theta}$ ----- (1) Let (x,y) be the Cartesian co-ordinates of P. So that, $x=r\cos\theta$, $y=r\sin\theta$ since r is a function of θ. we also know from the geometrical meaning of the derivative that. $\tan\psi=\dfrac{dy}{dx}$ = slope of the tangent PL. $\tan\psi=\dfrac{dy/d\theta}{dx/d\theta}$ since x & y are function of θ.	(2) (2)

Q.No.	Solution and Scheme	Marks
	$\tan\varphi = \dfrac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \dfrac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$	
	where, $r' = dr/d\theta$	
	Dividing both the Nr & Dr by $r'\cos\theta$ we have,	
	$\tan\varphi = \dfrac{\frac{r\cos\theta}{r'\cos\theta} + \frac{r'\sin\theta}{r'\cos\theta}}{\frac{-r\sin\theta}{r'\cos\theta} + \frac{r'\cos\theta}{r'\cos\theta}}$	(2)
	$\tan\varphi = \dfrac{\frac{r}{r'} + \tan\theta}{1 - \frac{r}{r'}\tan\theta}$ ----- (2)	
	comparing eqn (1) and (2)	
	$\tan\phi = r/r' = \dfrac{r}{(dr/d\theta)}$	
	$\tan\phi = r\left(\dfrac{d\theta}{dr}\right)$	
	Equivalently we can write it in the form	
	$\dfrac{1}{\tan\phi} = \dfrac{1}{r}\left(\dfrac{dr}{d\theta}\right)$ or $\cot\phi = \dfrac{1}{r}\left(\dfrac{dr}{d\theta}\right)$	
	$\boxed{\tan\phi = r \cdot \dfrac{d\theta}{dr}.}$	6M
1b]	we have, $r = a(1 + \cos\theta)$; $r = b(1 - \cos\theta)$	
	Taking log on both sides	
	$\log r = \log a + \log(1 + \cos\theta)$; $\log r = \log b + \log(1 - \cos\theta)$	(1)
	Differentiating w.r.t 'θ'	
	$\dfrac{1}{r}\dfrac{dr}{d\theta} = \dfrac{-\sin\theta}{1+\cos\theta}$; $\dfrac{1}{r}\dfrac{dr}{d\theta} = \dfrac{\sin\theta}{1-\cos\theta}$	(1)

Q.No.	Solution and Scheme	Marks
	$\cot\phi_1 = \frac{-2\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2)}$; $\cot\phi_2 = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2\theta/2}$	(1)
	$\cot\phi_1 = \cot\left(\frac{\pi}{2}+\frac{\theta}{2}\right) \Rightarrow \phi_1 = \pi/2 + \theta/2$; $\cot\phi_2 = \cot(\theta/2)$	(2)
	$\therefore \lvert\phi_1-\phi_2\rvert = \lvert \pi/2 + \frac{\theta}{2} - \frac{\theta}{2}\rvert = \pi/2$	(1)
	Thus the curves intersect each other orthogonally	7M.
1c]	Let, $x^3+y^3 = 3axy$ Differentiate wrt 'x' $3x^2+3y^2\cdot\frac{dy}{dx} = 3a\left(x\cdot\frac{dy}{dx}+y\right)$ $3y^2\cdot\frac{dy}{dx} - 3ax\cdot\frac{dy}{dx} = 3ay-3x^2$ $3(y^2-ax)\frac{dy}{dx} = 3(ay-x^2)$ $\frac{dy}{dx} = y_1 = \frac{ay-x^2}{y^2-ax}$ at $\left(\frac{3a}{2},\frac{3a}{2}\right)$ $y_1 = \frac{\frac{3a^2}{2}-\frac{9a^2}{4}}{\frac{9a^2}{4}-\frac{3a^2}{2}} = -1$	(2)
	Next: $\frac{d^2y}{dx^2} = y_2 = \frac{(y^2-ax)(ay_1-2x)-(ay-x^2)(2yy_1-a)}{(y^2-ax)^2}$ at $\left(\frac{3a}{2},\frac{3a}{2}\right)$ $y^2-ax = \frac{9a^2}{4}-\frac{3a^2}{2} = \frac{3a^2}{4} \Rightarrow \frac{9a^2-6a^2}{4} = \frac{3a^2}{4}$	(2)
	$ay-x^2 = \frac{3a^2}{2}-\frac{9a^2}{4} = \frac{-3a^2}{4}$ Hence, $\left(\frac{3a}{2},\frac{3a}{2}\right)$ $y_2 = \frac{(3a^2/4)(-a-3a)-(-3a^2/4)(-3a-a)}{(3a^2/4)^2}$.	

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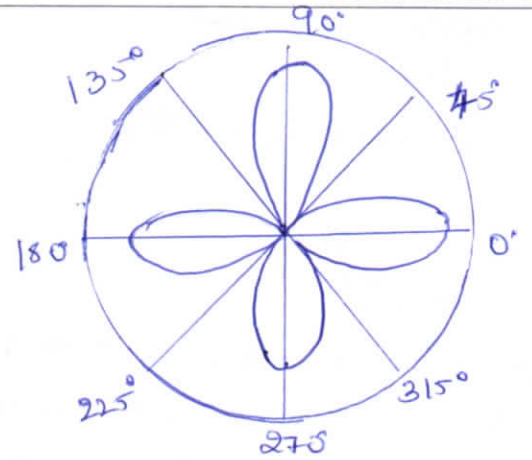
Q.No.	Solution and Scheme	Marks
	$y_2 = \frac{-3a^3-3a^3}{9a^4/16} = \frac{16(-6a^3)}{9a^4} = \frac{-32}{3a}$	
	$\rho = \frac{[1+y_1^2]^{3/2}}{y_2} = \frac{(1+1)^{3/2}}{-32/3a}$	(2)
	$\rho = \frac{2\sqrt{2}\cdot 3a}{-32} = \frac{3a\sqrt{2}}{-16} = \frac{-3a\sqrt{2}}{8\sqrt{2}\sqrt{2}}$	(1)
	$\lvert\rho\rvert = \frac{+3a}{8\sqrt{2}}$	7M
Q2a	$r^n = a^n \cos n\theta$; $r^n = b^n \sin n\theta$	
	Taking log on both sides	
	$n\log r = n\log a + \log(\cos n\theta)$; $n\log r = n\log b + \log(\sin n\theta)$	(1)
	Diff wrt 'θ'	
	$\frac{n}{r}\frac{dr}{d\theta} = -n\frac{\sin n\theta}{\cos n\theta}$; $\frac{n}{r}\frac{dr}{d\theta} = \frac{n\cos n\theta}{\sin n\theta}$	(2)
	$\frac{1}{r}\frac{dr}{d\theta} = -\tan n\theta$; $\frac{1}{r}\frac{dr}{d\theta} = \cot n\theta$	
	$\cot\phi_1 = \cot(\pi/2 + n\theta)$; $\cot\phi_2 = \cot n\theta$	(2)
	$\phi_1 = \pi/2 + n\theta$; $\phi_2 = n\theta$	
	$\lvert\phi_1 - \phi_2\rvert = \lvert\pi/2 + n\theta - n\theta\rvert = \pi/2$	(2)
	Thus the curvy intersect each other orthogonally.	$\cancel{7}$M.

Q.No.	Solution and Scheme	Marks
2b.	$r^m = a^m(\cos m\theta + \sin m\theta)$ Taking log on both sides $m\log r = m\log a + \log(\cos m\theta + \sin m\theta)$	(1)
	Diff wrt 'θ' $\frac{m}{r}\, dr/d\theta = \frac{-m\sin m\theta + m\cos m\theta}{\cos m\theta + \sin m\theta}$ $\frac{1}{r}\frac{dr}{d\theta} = \frac{\cos m\theta - \sin m\theta}{\cos m\theta + \sin m\theta} = \frac{\cos m\theta(1-\tan m\theta)}{\cos m\theta(1+\tan m\theta)}$	(2)
	$\cot\phi = \cot(\pi/4 + m\theta) \Rightarrow \phi = \pi/4 + m\theta$ Consider, $p = r\sin\phi$ $p = r\sin(\pi/4 + m\theta)$ $p = r[\sin(\pi/4)\cos m\theta + \cos(\pi/4)\sin m\theta]$ $p = r/\sqrt{2}[\cos m\theta + \sin m\theta]$	(2)
	Now, we have, $r^m = a^m(\cos m\theta + \sin m\theta)$ ------ (1) $p = r/\sqrt{2}(\cos m\theta + \sin m\theta)$ ------ (2)	(2)
	Using (2) in (1) we get, $r^m = a^m \frac{p\sqrt{2}}{r}$ or $r^{m+1} = \sqrt{2}a^m p$	(1)
	Thus, $r^{m+1} = \sqrt{2}a^m p$ is the required pedal equation	8M
2c	# Plot Four Leaved Rose $r = 2\lvert\cos 2\theta\rvert$ from pylab import * theta = linspace (0, 2* pi, 1000)	(2)
	r = 2* abs (cos (2* theta)) polar (theta, r, 'r') show ()	(2)

Q.No.	Solution and Scheme	Marks
		(1) 5M.
	MODULE-02	
Q3a.	we have, $y(x) = y(0) + x y_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0) + \frac{x^4}{4!} y_4(0) + \frac{x^5}{5!} y_5(0) + \frac{x^6}{6!} y_6(0)$	(1)
	$y = \sqrt{1+\sin 2x} = \sqrt{\cos^2 x + \sin^2 x + 2\sin x \cdot \cos x}$ $y = \sqrt{(\cos x + \sin x)^2} = \cos x + \sin x$ $y = \cos x + \sin x$; $y(0) = 1$ $y_1 = -\sin x + \cos x$; $y_1(0) = 1$ $y_2 = -\cos x - \sin x$; $y_2(0) = -1$ $y_2 = -y$; $y_3(0) = -1$ $y_3 = -y_1$; $y_4(0) = 1$ $y_4 = -y_2$; $y_5(0) = -1$ $y_5 = -y_3$; $y_6(0) = 1$ $y_6 = -y_4$	(4)
	Thus by substituting these values in the expansion of $y(x)$ $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \cdots$	(1) 6M.

Q.No.	Solution and Scheme	Marks
Q3b)	Let, $u = f(p, q, r)$ where $p = x/y$, $q = y/z$, $r = z/x$	
	$\therefore \{u \to (p, q, r) \to (x, y, z)\} \Rightarrow u \to x, y, z$	(1)
	$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\cdot\frac{\partial p}{\partial x} + \frac{\partial u}{\partial q}\cdot\frac{\partial q}{\partial x} + \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x}$	(2)
	$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p}\cdot\frac{1}{y} + \frac{\partial u}{\partial q}\cdot 0 + \frac{\partial u}{\partial r}\cdot\left(-z/x^2\right)$	
	Here, $x\frac{\partial u}{\partial x} = x/y\,\frac{\partial u}{\partial p} - \frac{z}{x}\frac{\partial u}{\partial r}$ ----- (1)	(2)
	||ly $y\frac{\partial u}{\partial y} = \frac{y}{z}\frac{\partial u}{\partial q} - \frac{x}{y}\frac{\partial u}{\partial p}$ ---- (2)	
	$z\frac{\partial u}{\partial z} = \frac{z}{x}\frac{\partial u}{\partial r} - \frac{y}{z}\frac{\partial u}{\partial q}$ ----- (3)	(2)
	Thus by adding (1), (2) & (3) we get,	
	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$	7M.
3c)	$z(x, y) = x^3 + y^3 - 3xy + 1$	
	$z_x = 3x^2 - 3y$; $z_y = 3y^2 - 3x$	(1)
	Let, $A = z_{xx}$, $B = z_{xy}$, $C = z_{yy}$ $\therefore A = 6x$, $B = -3$, $C = 6y$	(1)
	Now, at $(1,1)$, $z_x = 0$ and $z_y = 0$	
	Also, $A = 6$, $B = -3$, $C = 6$ $\therefore AC - B^2 = 27 > 0$	(2)
	Now, at $(1,1)$, $z_x = 0$ and $z_y = 0$, $AC - B^2 > 0$, $A = 6 > 0$	(2)
	$\therefore z(x,y)$ at $(1,1)$ satisfy the necessary & sufficient condition for minima	(1)
	Thus, $z(x,y)$ is Minimum at $(1,1)$.	7M

Q.No.	Solution and Scheme	Marks
4a	By the data, $u = xy/z$, $v = yz/x$, $w = xz/y$	
	$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \partial w/\partial z \end{vmatrix} = \begin{vmatrix} y/z & x/z & -\frac{xy}{z^2} \\ -\frac{yz}{x^2} & z/x & y/x \\ z/y & -\frac{zx}{y^2} & x/y \end{vmatrix}$$	(3)
	$$= \frac{y}{z}\left\{(z/x)(x/y) - (y/x)\left(-\frac{zx}{y^2}\right)\right\} - x/z\left\{[x/y]\left(-\frac{yz}{x^2}\right) - (y/x)(z/y)\right\}$$	(2)
	$$-\frac{xy}{z}\left\{\left(-\frac{yz}{x^2}\right)\left(-\frac{zx}{y^2}\right) - (z/x)(z/y)\right\}$$	
	$= 0 + 1 + 1 + 1 + 1 = 4$	(2)
	Thus, $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$.	7M
b]	$f(x,y) = x^3 + y^3 - 3x - 12y + 20$	
	$f_x = 3x^2 - 3$, $f_y = 3y^2 - 12$	
	we shall find points (x,y) such that $f_x = 0$ and $f_y = 0$	(1)
	i.e. $3x^2 - 3 = 0$ and $3y^2 - 12 = 0$	
	or $x^2 - 1 = 0$ and $y^2 - 4 = 0$	(2)
	i.e. $x = \pm 1$, $y = \pm 2$	
	$\therefore$ $(1,2)(1,-2)(-1,2)(-1,-2)$ are the stationary pts	
	Let, $A = f_{xx}$, $B = f_{xy}$, $C = f_{yy}$.	

	$(1,2)$	$(1,-2)$	$(-1,2)$	$(-1,-2)$
$A = 6x$	$6 > 0$	6	-6	$-6 < 0$
$B = 0$	0	0	0	0
$C = 6y$	12	-12	12	-12
$AC - B^2$	$72 > 0$	$-72 < 0$	$-72 < 0$	$72 > 0$
Conclusion	min. pt	Saddle pt	Saddle pt	Max. pt

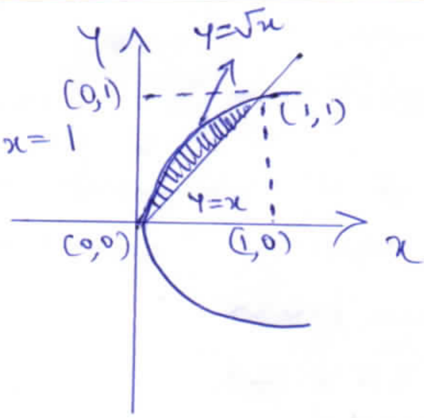
(2)

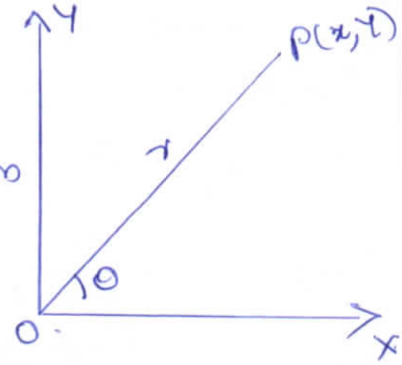
Q.No.	Solution and Scheme	Marks
	Maximum value of $f(x,y)$ is $f(-1,-2) = -1-8+3+24+20 = 38$ minimum value of $f(x,y)$ is $f(1,2) = 1+8-3-24+20$ $= 2$ Thus, maximum value is 38 and minimum value is 2	(2) (1) 8M
4C	from Sympy import * from math import inf x = Symbol('x') l = Limit((1+1/x)**x, x, inf).doit() display(l). out put: $x = 2.71828046$.	(3) (2) 5M.
	MODULE-03.	
Q5a.	$\frac{dy}{dx} + y/x = x^2y^6$ $\frac{1}{y^6}\frac{dy}{dx} + \frac{1}{x} \times \frac{1}{y^5} = x^2$ $\frac{1}{y^5} = t \Rightarrow \frac{1}{y^6}\frac{dy}{dx} = -\frac{1}{5}\frac{dt}{dx}$ $\therefore \frac{dt}{dx} - 5/x\, t = -5x^2$ I.F $e^{-\int 5/x\, dx} = e^{-5\log x}$ I.F $= \int Q(IF)dx + C \Rightarrow \frac{1}{y^5x^5} = \int -5x^2 \times \frac{1}{x^5} dx + C$ $\Rightarrow \frac{1}{x^5y^5} = \frac{5}{2x^2} + C.$	(2) (1) (2) (1) 6M.
Q5b.	we have, $\frac{2a}{r} = 1 - \cos\theta$ $\log 2a - \log r = \log(1-\cos\theta)$ Diff w.r.t 'θ' $-\frac{1}{r}\, dr/d\theta = \frac{\sin\theta}{1-\cos\theta}$ Replacing $\frac{dr}{d\theta}$ by $-r^2\frac{d\theta}{dr}$ & Simplifying RHS.	(1) (1)

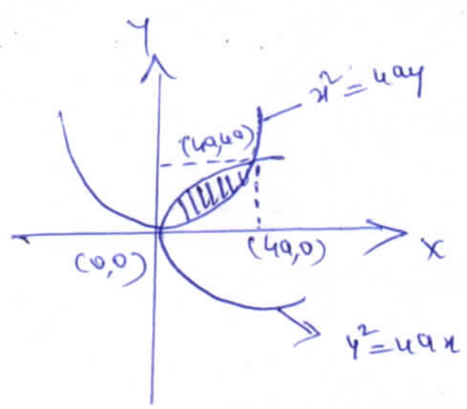
Q.No.	Solution and Scheme	Marks
	$-\frac{1}{r}\left(-r^2\frac{d\theta}{dr}\right) = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)}$	(2)
	or $r\frac{d\theta}{dr} = \cot(\theta/2)$	
	$\therefore \tan(\theta/2)\,d\theta = dr/r$	
	$\therefore \int \frac{dr}{r} - \int \tan(\theta/2)\,d\theta = C$	
	$\log r - \frac{\log \sec(\theta/2)}{(1/2)} = C.$	(2)
	$\log\left[r/\sec^2(\theta/2)\right] = \log b$	
	or $r^2/\sec^2(\theta/2) = b$ or $r\cos^2(\theta/2) = b.$	(1)
	Thus, $r(1+\cos\theta) = 2b$ or $2b/r = 1+\cos\theta$ is the required orthogonal trajectory.	7M
05 c]	$xy(p^2) - (x^2+y^2)p + xy = 0.$	
	$\Rightarrow xyp^2 - x^2p - y^2p + xy = 0$	(2)
	$\Rightarrow (py - x)(xp - y) = 0$	
	$py - x = 0 \Rightarrow p = x/y \Rightarrow \frac{dy}{dx} = x/y$	(2)
	$\Rightarrow (y^2 - x^2 - c) = 0$	
	$xp - y = 0 \Rightarrow p = y/x \Rightarrow \frac{dy}{dx} = y/x$	(2)
	$\Rightarrow (y - cx) = 0$	
	$\therefore (y^2 - x^2 - c)(y - cx) = 0.$	(1) 7M
Q 6 a]	Let, $M = x^2 + y^2 + x$ and $N = xy$	
	$\frac{\partial M}{\partial y} = 2y$ and $\frac{\partial N}{\partial x} = y$	(1)
	$\therefore \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - y = y$ ---- near to N	
	Now, $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{y}{xy} = \frac{1}{x} = f(x)$	(1)

Q.No.	Solution and Scheme	Marks
	Hence, $I.F = e^{\int f(x)dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$	
	multiplying the given equation by x,	
	$M = x^3 + xy^2 + x^2$ and $N = x^2 y$	(2)
	$\frac{\partial M}{\partial y} = 2xy$ and $\frac{\partial N}{\partial x} = 2xy$	
	The solution is,	
	$\int M\,dx + \int N(y)\,dy = C$	(2)
	ie $\int (x^3 + xy^2 + x^2)\,dx + \int 0.\,dy = C$	
	Thus, $\frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{x^3}{3} = C$ is the required solution.	6M.
Q.6b]	The given DE is	
	$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$ ----- (1)	(1)
	This is of the form $\frac{dy}{dx} + Py = Q$	
	whose solution is given by,	(1)
	$y.\,e^{\int P dx} = \int Q.\,e^{\int P.dx}\,dx + C.$	
	Applying for the form of DE as in (1)	
	ie $i e^{\int R/L.dt} = \int \frac{E}{L} e^{\int R/L\,dt} \int dt + C.$	
	That is,	(2)
	$i e^{Rt/L} = \frac{E}{L} \int e^{Rt/L}\,dt + C.$	
	$i e^{Rt/L} = \frac{E}{L} \cdot \frac{e^{Rt/L}}{(R/L)} + C.$	
	$i e^{Rt/L} = \frac{E}{R} \cdot e^{Rt/L} + C.$	
	or $i = \frac{E}{R} + Ce^{-(Rt/L)}$ ------- (2).	(1)
	This is the general solution of the equation & we shall apply the initial condition that $i = 0$ when $t = 0$	

Q.No.	Solution and Scheme	Marks
	Hence eq (2) becomes, $0 = E/R + C$ or $C = -(E/R)$ Substituting this value in C in (2) we obtain the current i at any time t.	(2)
	Thus, $i = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$	7M.
Q6c]	$X = x^2 \Rightarrow \frac{dX}{dx} = 2x$, $Y = y^2 \Rightarrow \frac{dY}{dy} = 2y$	(1)
	Now, $p = \frac{dy}{dx} = \frac{dy}{dY} \cdot \frac{dY}{dX} \cdot \frac{dX}{dx}$	(2)
	$p = \frac{\sqrt{X}}{\sqrt{Y}} \cdot p'$	(2)
	$\therefore \left[\frac{\sqrt{X}}{\sqrt{Y}} p' \sqrt{X} - \sqrt{Y}\right]\left[\frac{\sqrt{X}}{\sqrt{Y}} p' \sqrt{Y} + \sqrt{X}\right] = a^2 \frac{\sqrt{X}}{\sqrt{Y}} \cdot p'$	(2)
	$\Rightarrow Y = Xp' - \frac{a^2 p'}{p'+1} \Rightarrow y^2 = cx^2 - \frac{a^2 c}{c+1}$	7M.
	MODULE-04	
Q7a]	$I = \int_{z=-1}^{1} \int_{x=0}^{z} \int_{y=x-z}^{x+z} (x+y+z)\, dy\, dx\, dz$	
	$= \int_{z=-1}^{1} \int_{x=0}^{z} \left[xy + \frac{y^2}{2} + zy\right]_{y=x-z}^{x+z} dx\, dz$	(2)
	$I = \int_{z=-1}^{1} \int_{x=0}^{z} \left\{x(\overline{x+z} - (\overline{x-z})) + \frac{1}{2}\left[(x+z)^2 - (x-z)^2\right] + z(\overline{x+z} - \overline{x-z})\right\} dx\, dz$	(1)
	$= \int_{z=-1}^{1} \int_{x=0}^{z} (2xz + 2xz + 2z^2)\, dx\, dz$	(1)
	$= \int_{z=-1}^{1} \int_{x=0}^{z} (4xz + 2z^2)\, dx\, dz$	(1)
	$= \int_{z=-1}^{1} (2z^3 + 2z^3)\, dz \Rightarrow \int_{z=-1}^{1} 4z^3\, dz = \left[z^4\right]_{-1}^{1} = 0 \Rightarrow \boxed{I = 0}$	(1) 6M.

Q.No.	Solution and Scheme	Marks
7b]	$I = \int_{x=0}^{1} \int_{y=x}^{\sqrt{x}} xy \, dy \, dx.$  $y=x$, $y=\sqrt{x}$ betn the lines $x=0, x=1$ we shall find the points of intersection of $y=x$ & $y=\sqrt{x}$ by equating their RHS.	2+1
	$x=\sqrt{x} \Rightarrow x^2 = x$ or $x(x-1)=0$ or $x=0,1$ ∴ This will give us $y=0$, $y=1$ and hence the points of intersection are $(0,0)$ and $(1,1)$ $y=\sqrt{x}$ or $y^2=x$ is a parabola symmetrical about the x-axis.	(1)
	we have on changing the order of Integration $I = \int_{y=0}^{1} \int_{x=y^2}^{y} xy \, dx dy$ $= \int_{y=0}^{1} y\left[\frac{x^2}{2}\right]_{x=y^2}^{y} dy = \frac{1}{2}\int_{y=0}^{1} y(y^2-y^4)\,dy$	(2)
	$I = \frac{1}{2}\int_{0}^{1}(y^3-y^5)\,dy = \frac{1}{2}\left[\frac{y^4}{4}-\frac{y^6}{6}\right]_0^1$	(1)
	$I = \frac{1}{2}\left(\frac{1}{4}-\frac{1}{6}\right) = \frac{1}{24}$ $\boxed{I = 1/24}$	7M.
7c]	we have by the definition of Beta & Gamma function. $\beta(m,n) = 2\int_{\theta=0}^{\pi/2} \sin^{2m-1}\theta . \cos^{2n-1}\theta \, d\theta$ ------ (1) $\Gamma n = 2\int_0^\infty e^{-x^2} . x^{2n-1} dx$ ------- (2) $\Gamma m = 2\int_0^\infty e^{-y^2} . y^{2m-1} . dy$ ----- (3) $\Gamma(m+n) = 2\int_0^\infty e^{-r^2} . r^{2(m+n)-1} . dr$ ---- (4)	(2)

Q.No.	Solution and Scheme	Marks
	Now, $\Gamma m . \Gamma n = 4 \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} . x^{2m-1} . y^{2n-1} . dx\,dy$ ---- (5)	
	Let us evaluate RHS by changing into polars.	(1)
	putting $x = r\cos\theta$, $y = r\sin\theta$ we have, $x^2+y^2=r^2$	
	Also, $dx\,dy = r\,dr\,d\theta$. r varies from 0 to ∞, θ varies from 0 to $\pi/2$.	(1)
	We now have (5) in the form. $\Gamma m . \Gamma n = 4 \int_{r=0}^{\infty} \int_{\theta=0}^{\pi/2} e^{-r^2} (r\cos\theta)^{2m-1} (r\sin\theta)^{2n-1} . r\,dr\,d\theta$	(2)
	$= 4 \int_{r=0}^{\infty} \int_{\theta=0}^{\pi/2} e^{-r^2} . r^{2m+2n-1} . \sin^{2n-1}\theta \cos^{2m-1}\theta\, dr\, d\theta$	
	$= \left[2 \int_{r=0}^{\infty} e^{-r^2} . r^{2(m+n)-1} . dr \right] \left[2 \int_{\theta=0}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta \right]$	
	$\therefore \Gamma m . \Gamma n = \Gamma(m+n) . \beta(m,n)$ by using eqn (1) & (4)	(1)
	Thus, $\beta(m,n) = \dfrac{\Gamma m . \Gamma n}{\Gamma(m+n)}$.	7M.
Q8 a)	In polars we have $x = r\cos\theta$, $y = r\sin\theta$ and $dx\,dy = r\,dr\,d\theta$. $\therefore x^2+y^2 = r^2$	
	Since x, y varies from 0 to ∞. r also varies from 0 to ∞. In the first quadrant θ varies from 0 to $\pi/2$ 	2+1
	Hence, $I = \int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} e^{-r^2} . r\, dr\, d\theta$.	

Q.No.	Solution and Scheme	Marks
	Put, $r^2 = t \quad \because r\,dr = \frac{dt}{2}$; t also varies from 0 to ∞ $I = \int_{\theta=0}^{\pi/2} \int_{t=0}^{\infty} e^{-t} \cdot \frac{dt}{2} \, d\theta$	(2)
	$= \frac{1}{2} \int_{\theta=0}^{\pi/2} \left[-e^{-t}\right]_{t=0}^{\infty} d\theta = -\frac{1}{2} \int_{\theta=0}^{\pi/2} (0-1)\, d\theta = \frac{1}{2}\left[\theta\right]_0^{\pi/2} = \frac{\pi}{4}$	(1)
	Thus, $I = \pi/4$	6M.
8 b]	Let, $I = \int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta = \int_0^{\pi/2} \frac{\sqrt{\cos\theta}}{\sqrt{\sin\theta}} \cdot d\theta$	(2)
	$= \int_0^{\pi/2} \sin^{-1/2}\theta \cdot \cos^{1/2}\theta \, d\theta$	(1)
	Hence, $I = \frac{1}{2} \beta\left(\frac{-1/2+1}{2}, \frac{1/2+1}{2}\right)$ $I = \frac{1}{2} \beta\left(1/4, 3/4\right) = \frac{1}{2} \frac{\Gamma 1/4 \cdot \Gamma(3/4)}{\Gamma(1)} = \frac{1}{2} \cdot \frac{\pi\sqrt{2}}{1} = \frac{\pi}{\sqrt{2}}$	(2)
	Thus, $\boxed{I = \pi/\sqrt{2}}$	(2) 7M.
8 c]	Let, $A = \iint dy\,dx = \int_{x=0}^{4a} \int_{y=x^2/4a}^{2\sqrt{ax}} dy\,dx$	2+2
	$A = \int_{x=0}^{4a} \left(2\sqrt{ax} - x^2/4a\right) dx \Rightarrow$	(2)
	$\Rightarrow A = \frac{16a^2}{3}$ sq. units  	1 7M.

Q.No.	Solution and Scheme	Marks

MODULE-05

9 a] $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$R_1 \longleftrightarrow R_2$

$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ (2)

$R_2 \to -2R_1 + R_2$, $R_3 \to -R_1 + R_3$

$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ (2)

$R_2 \longleftrightarrow R_4$

$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & -5 & -9 & 1 \end{bmatrix}$

$R_3 \to 2R_2 + R_3$, $R_4 \to 5R_2 + R_4$ (2)

$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & -4 \end{bmatrix}$

$R_3 \longleftrightarrow R_4$ and $-1/4 R_3$

$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 6M.

The matrix A in the row echelon form is having three non-zero rows.

Thus, $\boxed{\rho(A) = 3}$

Q.No.	Solution and Scheme	Marks
9b]	$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$ is the Augmented matrix.	
	$R_2 \to -R_1 + R_2,\ R_3 \to -R_1 + R_3$	
	$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$	(2)
	$R_3 \to -R_2 + R_3$	
	$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$	
	a) Unique solution: we must have $\rho(A) = \rho(A:B) = 3$. $\rho(A) = 3$ if $(\lambda-3) \neq 0$ since the other two entries in the last row of A are zero. If $(\lambda-3) \neq 0$ or $\lambda \neq 3$ then $\rho[A:B] = 3$. Hence the system will have unique solution if $\lambda \neq 3$.	(2)
	b] Infinite solution: we have $n=3$, $\rho(A) = \rho[A:B] = r < 3$, we must have $r=2$, $\therefore \rho(A) = \rho(A:B) = 2$. This is possible if $\lambda-3=0$, $\mu-10=0$. Hence the system will have infinite solution if $\lambda=3$ & $\mu=10$.	(2)
	c] No solution: we must have $\rho[A] \neq \rho[A:B]$. By case a] $\rho[A]=3$ if $\lambda \neq 3$ & hence if $\lambda=3$ we obtain $\rho[A]=2$. If we impose $(\mu-10) \neq 0$ then $\rho[A:B]$ will be 3. Hence the system has no solution if $\lambda=3$ and $\mu \neq 10$.	(1)
		7M

Q.No.	Solution and Scheme	Marks

9c) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking $X^{(0)} = [1, 1, 1]^T$

By the data,

$$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \lambda^{(1)} x^{(1)}$$ (1)

$$AX^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 7.34 \\ -2.67 \\ 4.01 \end{bmatrix} = 7.34\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \lambda^{(2)} x^{(2)}$$ (1)

$$AX^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 7.82 \\ -3.63 \\ 4.01 \end{bmatrix} = 7.82\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \lambda^{(3)} x^{(3)}$$ (1)

$$AX^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \begin{bmatrix} 7.94 \\ -3.89 \\ 3.99 \end{bmatrix} = 7.94\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \lambda^{(4)} x^{(4)}$$ (1)

$$AX^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 7.98 \\ -3.97 \\ 3.99 \end{bmatrix} = 7.98\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(5)} x^{(5)}$$ (1)

$$AX^{(5)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 4 \end{bmatrix} = 8\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix} = \lambda^{(6)} x^{(6)}$$ (1)

Thus the dominant eigen value is 8 and the corresponding eigen vector is $[1, -0.5, 0.5]'$ or $[2, -1, 1]'$ equivalently. (1)

7M

10a) The Augmented matrix of the system is,

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_2 \to -R_1 + R_2$, $R_3 \to -2R_1 + R_2$ (2)

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right]$$

we use the leading non-zero entry in second row (-3) to make the element above 1 and below 1 and -1 respectively zero. (1)

$R_1 \to R_2 + 3R_1$, $R_3 \to R_2 - 3R_3$

Q.No.	Solution and Scheme	Marks
	$[A:B] \sim \begin{bmatrix} 3 & 0 & 5 & : & 26 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 11 & : & 44 \end{bmatrix}$	
	$R_3 \to 1/11 \cdot R_3$ $\sim \begin{bmatrix} 3 & 0 & 5 & : & 26 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$	(2)
	we use the element 1 in the third row to make the two elements above 2 and 5 zero.	(1)
	$R_1 \to -5R_3 + R_1$, $R_2 \to -2R_3 + R_2$ $\sim \begin{bmatrix} 3 & 0 & 0 & : & 6 \\ 0 & -3 & 0 & : & -9 \\ 0 & 0 & 1 & : & 4 \end{bmatrix}$	(1)
	we have $3x = 6$, $-3y = -9$, $z = 4$ Thus, $x = 2$, $y = 3$, $z = 4$ is the required solution.	7M.
10b	The equations are diagonally dominant & hence we first write them in the form. $x = \frac{1}{20}[17 - y + 2z]$ $y = \frac{1}{20}[-18 - 3x + z]$ $z = \frac{1}{20}[25 - 2x + 3y]$	(1)
	we start with the trial solution $x = 0$, $y = 0$, $z = 0$.	
	Ist Iteration: $x^{(1)} = 17/20 = 0.85$ $y^{(1)} = \frac{1}{20}[-18 - 3(0.85)] = -1.0275$	(2)
	$z^{(1)} = \frac{1}{20}[25 - 2(0.85) + 3(-1.0275)] = 1.0109$	(2)
	IInd Iteration: $x^{(2)} = \frac{1}{20}[17 - (-1.0275) + 2(1.0109)] = 1.0025$ $y^{(2)} = \frac{1}{20}[-18 - 3(1.0025) + 1.0109] = -0.9998$ $z^{(2)} = \frac{1}{20}[25 - 2(1.0025) + 3(-0.9998)] = 0.9948$	(2)
	IIIrd Iteration: $x^{(3)} = \frac{1}{20}[17 - (-0.9998) + 2(0.9998)] = 0.9999 \simeq 1$ $y^{(3)} = \frac{1}{20}[-18 - 3(0.999) + 0.99] = -1.0005 \simeq -1$ $z^{(3)} = \frac{1}{20}[25 - 2(0.9997) + 3(-1.0005)] = 1.000 \simeq 1$	(1) 8M.

Thus,
$x = 1$, $y = -1$, $z = 1$ is the required solution.

Q.No.	Solution and Scheme	Marks
10c)	A = np. matrix([[1,2,-1],[2,1,4],[3,3,4]]) B = np. matrix([[1],[2],[1]]) AB = np. concatenate ((A, B), axis = 1)	(2)

	rA = np. linalg. matrix_rank(A)
	rAB = np. linalg. matrix_rank (AB)
	n = A. shape[1]
	if (rA = = rAB) :
	if (rA = = n) :
		(2)

	print ("The system has unique solution") print (np. linalg. solve (A, B)) else : print ("The system has infinitely many solution") else : print ("The system of equations is inconsistant")	(1)
		5M.

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