

	c.	The kinetic energy of an electron is equal to the energy of a photon with a wave length of 560 nm. Calculate the de Broglie wave length of the electron.	5	L3	CO2
Module – 3					
Q.5	a.	State the Pauli matrices and apply Pauli matrices on the states $ 0\rangle$ and $ 1\rangle$.	9	L2	CO2
	b.	Discuss the CNOT gate and its operation on four different input states.	6	L2	CO2
	c.	Given $ \Psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ and $ \phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$, Prove that $\langle \Psi \phi \rangle = \langle \phi \Psi \rangle^*$.	5	L3	CO2
OR					
Q.6	a.	Explain the representation of qubit using Bloch sphere.	7	L2	CO3
	b.	Describe the working of controlled – Z gate mentioning its matrix representation and truth table.	8	L3	CO3
	c.	A linear operator 'X' operates such that $X 0\rangle = 1\rangle$ and $X 1\rangle = 0\rangle$. Find the matrix representation of 'X'.	5	L3	CO3
Module – 4					
Q.7	a.	Enumerate the assumptions of Quantum free electron theory of metals and mention the failures and classical free electron theory.	7	L2	CO3
	b.	Describe Meissner effect and hence classify superconductors into Soft and Hard super conductors using $M - H$ graphs.	8	L2	CO3
	c.	Calculate the probability of occupation of an energy level 0.2eV above Fermi level at temperature 27°C.	5	L3	CO3
OR					
Q.8	a.	Define Fermi factor and discuss the variation of Fermi factor with temperature and energy.	7	L2	CO3
	b.	Explain the phenomenon of superconductivity and discuss qualitatively the BCS theory of super conductivity for negligible resistance of metal at temperature close to absolute zero.	8	L2	CO3
	c.	A superconductivity Tin has a critical temperature of 3.7K at zero magnetic field and a critical field of 0.0306 Tesla at 0°K. Find the critical field at 2K.	5	L3	CO3
Module – 5					
Q.9	a.	Elucidate the importance of size and scale, weight and strength in animations.	8	L2	CO4
	b.	Discuss the salient features of normal distribution using bell curves.	7	L2	CO4

	c.	A slowing object in an animations has a first frame distance 0.5m and the first slow in frame 0.35m. Calculate the base distance and the number of frames in sequence.	5	L3	CO4
OR					
Q.10	a.	Describe Jumping and parts of Jump.	8	L2	CO4
	b.	Discuss modeling the probability for proton decay.	7	L2	CO4
	c.	In a diffraction grating experiment the laser light undergoes second order diffraction for diffraction angle 1.48° . The grating constant $d = 5.05 \times 10^{-5}\text{m}$ and the distance between the grating and screen is 0.60m. Find the wavelength of LASER light.	5	L3	CO5

Q.1 a. Obtain the expression for energy density equation using Einstein's coefficients at thermal equilibrium condition.

⇒ Let the system be at thermal equilibrium which means that the total energy of the system remains unchanged inspite of the interaction that takes place inside the system.

Under such condition, the number of photons absorbed by the system per second is equal to number of photons it emits per second by both stimulated and spontaneous emission process.

At thermal equilibrium

Rate of Int. absorption = Rate of spont. emission + Rate of stimulated emission.

$$\frac{dN_{12}}{dt} = \frac{dN_{21}}{dt} + \frac{dN_{21}}{dt} \quad (\text{Stm})$$

$$B_{12} N_1 E_\nu = A_{21} N_2 + B_{21} N_2 E_\nu \rightarrow (1)$$

$$B_{12} N_1 E_\nu = B_{21} N_2 E_\nu = A_{21} N_2$$

$$E_\nu (B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$E_\nu = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

$$E_\nu = \frac{A_{21}}{B_{21}} \cdot \frac{N_2}{\left[\frac{B_{12}}{B_{21}} \cdot N_1 - N_2 \right]}$$

$$E_\nu = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\left[\frac{B_{12}}{B_{21}} \cdot \frac{N_1}{N_2} - 1 \right]} \rightarrow (2)$$

From Boltzmann law we have

$$N_1 = e^{-E_1/KT} \quad N_2 = e^{-E_2/KT}$$

$$\Rightarrow \frac{N_1}{N_2} = e^{\frac{(E_2-E_1)}{KT}}, \quad \frac{N_1}{N_2} = e^{\frac{\Delta E}{KT}}$$

$$\frac{N_1}{N_2} = e^{h\nu/KT} \rightarrow (3)$$

Substituting equation (3) in eqⁿ (2) we get

$$E_\nu = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\left[\frac{B_{12}}{B_{21}} \cdot e^{h\nu/KT} - 1 \right]} \rightarrow (4)$$

From Plank's law of radiation

$$E_\nu = \frac{8\pi h\nu^3}{c^3} \left[\frac{1}{e^{h\nu/KT} - 1} \right] \rightarrow (5)$$

Comparing equation (5) and eqⁿ (4)

$$\Rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \rightarrow (6)$$

$$B_{21} = B_{12} = 1 \rightarrow (7)$$

Equation (6) & (7) represents the Einstein's coefficients.

For a system to be at thermal equilibrium

$$B_{21} = B_{12} = B \quad \text{and} \quad A_{21} = A$$

Substituting for B_{21} , B_{12} , A_{21} in Equation (4) we get.

$$E_{\nu} = \frac{A}{B} \left[\frac{e^{h\nu/kT}}{e^{h\nu/kT} - 1} \right] \rightarrow (8)$$

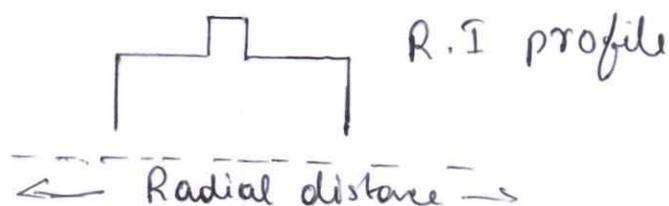
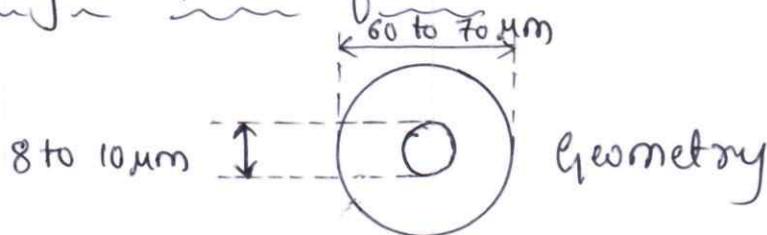
Equation (8) represents the expression for energy density of incident radiation for a system to be at equilibrium.

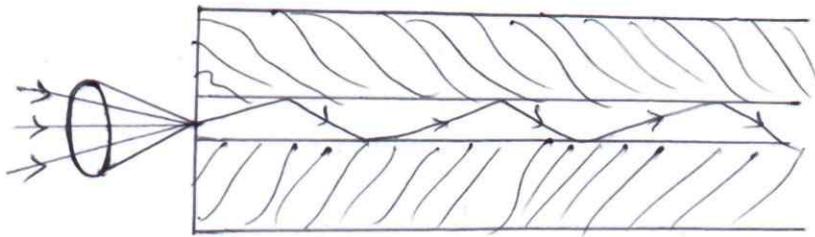
b. Discuss the types of optical fibers based on modes of propagation and Refractive Index profile.

⇒ Types of optical fibers Based on the refractive index profile and mode of propagation.

- ① Single mode fiber
- ② Step index multimode fiber
- ③ Graded index multimode fiber.

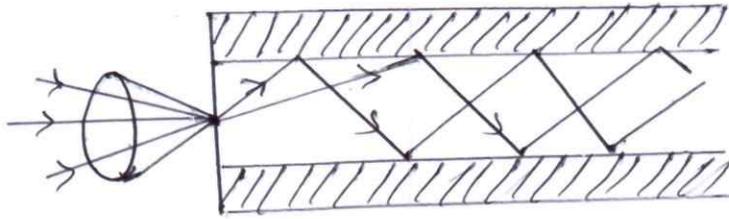
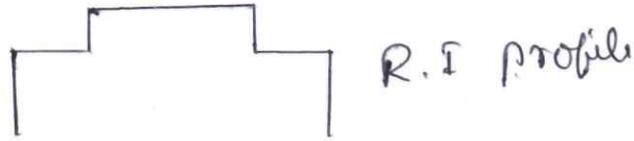
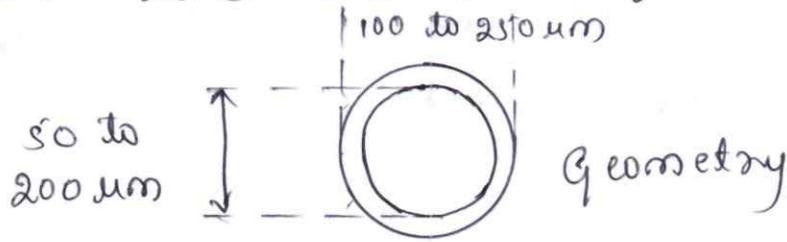
① Single mode fiber :-





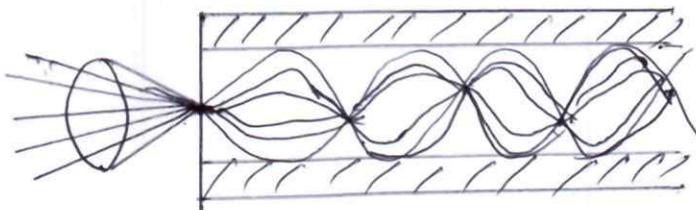
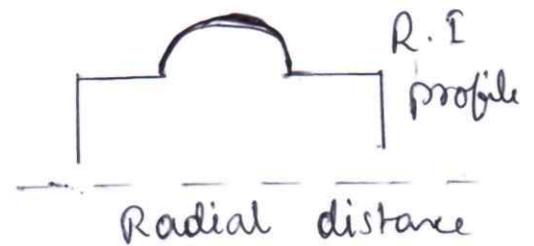
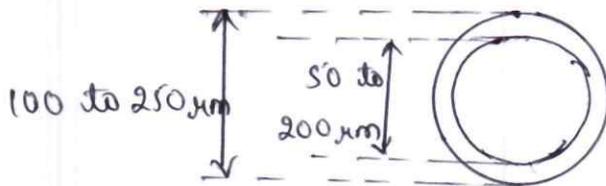
- Single mode fibers have a core materials of uniform refractive index value.
- cladding material also has a uniform refractive index that of lesser value than that of core.
- Thus its refractive index profile take a shape of a step. the diameter of the core is about $8-10\ \mu\text{m}$ and the diameter of the cladding is about $60-70\ \mu\text{m}$
- Because of its narrow core, it can guide just a single mode as shown in figure.
- single mode fibers are extensively used ones they are less expensive. They LASER as the source of light.

② step index multimode fiber:



→ A step index multimode fiber is very much similar to the single mode fiber except that its core is of large diameter. A typical core has a diameter of 50 to 200 μm and a cladding about 100 to 250 μm outer diameter.

③ Graded index multimode fiber:



c. Given the numerical aperture 0.30 and RI of core 1.49, calculate the critical angle for the core - cladding interface.

Solution: Given $NA = 0.30$

$$n_1 = 1.49$$

$$\theta_c = ?$$

$$NA = \sqrt{n_1^2 - n_2^2} = \sin \theta_c$$

$$0.30 = \sqrt{(1.49)^2 - n_2^2}$$

$$(1.49)^2 - n_2^2 = (0.30)^2$$

$$n_2^2 = (1.49)^2 - (0.30)^2$$

$$\boxed{n_2 = 1.46}$$

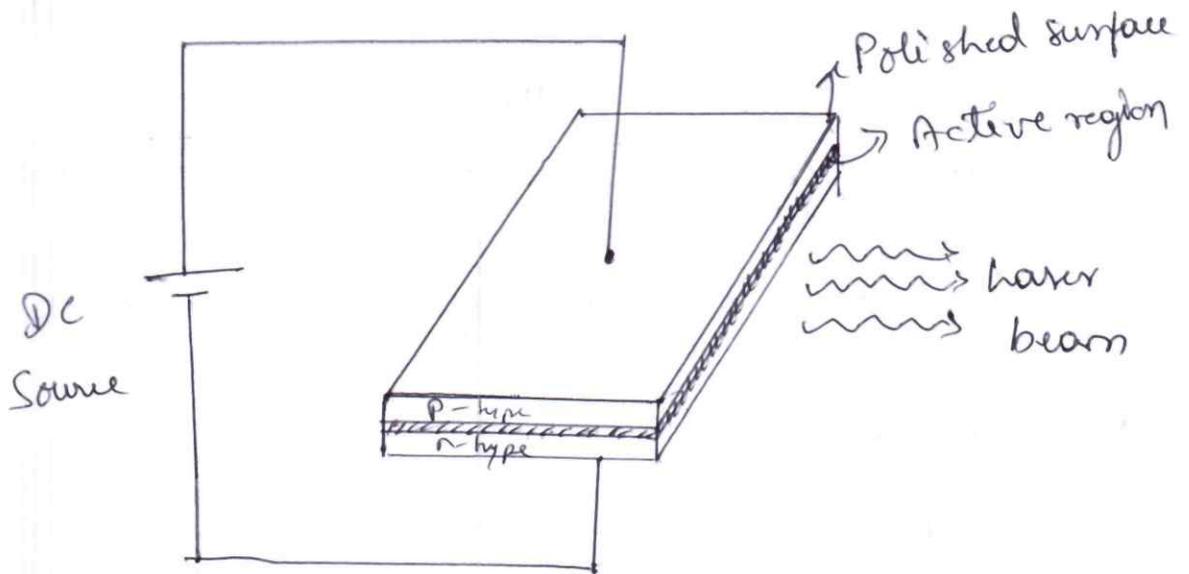
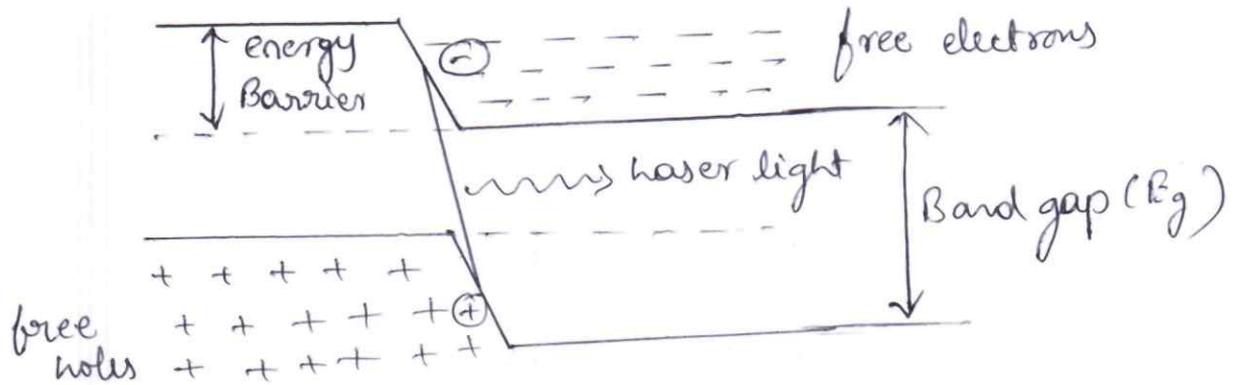
$$\theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$$

$$\theta_c = \sin^{-1} \left(\frac{1.46}{1.49} \right)$$

$$\boxed{\theta_c = 78.48^\circ}$$

Q.2 a. Illustrate the construction and working of semiconductor laser with a neat sketch with energy level diagram.

⇒



* Specification :

Ga - As → semiconductor - Cube 0.4 mm
 Thickness 0.1 μm

Ga - As + Tellurium → n - type

Ga - As + Zinc → p - type

* In 1962 by Kaul.

* Working:

Forward Biased

Holes + Electron recombination

$$\lambda = 840 \text{ nm}$$

* Characteristics:

- It is a solid state laser
- Active medium: The P-N junction
- Pumping method: The direct conversion method
- Power output: $\pm \text{mW}$
- Nature of output: both continuous and pulsed
- Wavelength of the output: 8300 \AA to 8500 \AA

* Advantages:

- It has excellent efficiency
- The o/p can be modulated
- It produces both continuous wave & pulsed wave o/p
- It is highly economical.
- It is simple and compact and very small in dimension
- It exhibits high efficiency
- The output can be easily increased by controlling junction current
- It is operated with lesser power than ruby & CO_2

* Applications!

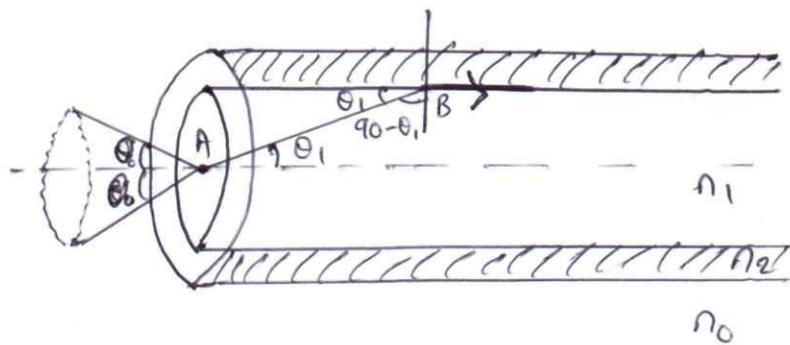
- It is used in OFC
- It is used in commercial CD recording and Reading
- It is used in Bar code reading, laser printing and laser cooling.
- It is used to heal the wound by IR radiation

b. Define Acceptance angle and Numerical aperture and hence derive an expression for Numerical aperture in terms of Refractive indices of core, cladding and surrounding.

⇒ Defⁿ :-

Acceptance angle is defined as "The maximum angle that a light ray can take relative to the axis of the fiber to propagate through the fiber. Since the acceptance angle of an optical fiber is called as "NA"

Expression for Numerical Aperture and condition for propagation!



→ Consider a light ray entering into the core of an optical fiber with an angle of incidence (θ_0)

→ Let $n_0, n_1,$ and n_2 are the refractive indices of the surrounding medium, core and cladding respectively

→ Now, applying Snell's law at the point of entry of the ray we get at A

$$\frac{\sin \theta_0}{\sin \theta_1} = \frac{n_1}{n_0}$$

$$\sin \theta_0 = \frac{n_1}{n_0} \sin \theta_1 \rightarrow (1)$$

applying Snell's law at B,

$$\frac{\sin (90^\circ - \theta_1)}{\sin 90^\circ} = \frac{n_2}{n_1}$$

$$\Rightarrow \cos \theta_1 = \frac{n_2}{n_1} \rightarrow (2)$$

From Expression (1)

$$\sin \theta_0 = \frac{n_1}{n_2} \sqrt{1 - \cos^2 \theta_1}$$

Substitute for θ_1 from (2)

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin \theta_0 = \frac{n_1}{n_0} \sqrt{\frac{n_1^2 - n_2^2}{n_1^2}}$$

$$\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

$$\text{i.e. NA} = \sqrt{n_1^2 - n_2^2}$$

→ Condition for propagation

If θ_i is the angle of incidence of the incident ray, then the ray will be able to propagate, if $\theta_i < \theta_0$

$$\Rightarrow \text{If } \sin \theta_i < \sin \theta_0$$

$$\text{or } \sin \theta_i < \sqrt{n_1^2 - n_2^2}$$

$$\boxed{\text{i.e. } \sin \theta_i < \text{N.A}}$$

- c. In an optical fiber experiment the LASER light propagating through optical fiber cable of 1.5 m. made a spot diameter of 8 mm on the screen. The distance between the end of the optical fiber cable and the screen is 0.031 m. Calculate angle of contact and N.A of given optical fiber.

Solution :-

Given :- $D = 8 \times 10^{-2} \text{ m}$
 $L = 0.031 \text{ m}$

$$\theta_0 = \tan^{-1} \left(\frac{D}{2L} \right)$$

$$\theta_0 = \tan^{-1} \left(\frac{8 \times 10^{-2}}{2 \times 0.031} \right)$$

$$\theta_0 = \tan^{-1} \left(\frac{8 \times 10^{-2}}{0.062} \right)$$

$$\theta_0 = \tan^{-1} (129 \times 10^{-3})$$

$$\theta_0 = \tan^{-1} (0.129)$$

$$\boxed{\theta_0 = 7.35^\circ}$$

MODULE - 02

Q.3. a. Derive an expression for de-Broglie wavelength by analogy and hence discuss the significance of de-Broglie waves.

⇒ The wave associated with the moving particle is called matter waves or de-Broglie wave (Pilot wave)

The wavelength associated with particles with mass ' m ' and moving with certain velocity ' v ' and momentum ' p ' is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

Consider a photon with frequency ' ν ' and wavelength ' λ '. Its energy according to Planck's theory is $E = h\nu$

$$E = h\nu = \frac{hc}{\lambda}$$

where, $h \rightarrow$ Planck's constant

If a photon is considered as a particle of mass ' m ' moving with a velocity ' c ', then energy of photon

$$E = mc^2 \Rightarrow h\nu = mc^2$$

$$\frac{h\nu}{\lambda} = mc^2$$

$$\text{momentum} = mc = p = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} \rightarrow \text{de Broglie Equation}$$

de-Broglie wavelength of an electron
Consider an electron of mass 'm' which is at rest is subjected to a potential difference of V . The electrical work done (exv) will appear as k.E of the electron

$$\text{we } E = eV \text{ and } E = E = \frac{1}{2}mv^2$$

$$m^2v^2 = 2mE$$

$$\Rightarrow mv = \sqrt{2mE} = p$$

Wavelength of the electron wave, $\lambda = \frac{h}{\sqrt{2mE}}$

Instead of an electron, if a particle of charge 'q' is accelerated through a potential difference V . then

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

* Significance of de-Broglie waves

→ Matter waves are associated only with particles in motion.

→ They are not electromagnetic in nature

→ Group velocity is associated with matter waves.

→ As a result of superposition of large number of component waves which slightly differ in frequency, matter waves are localized.

→ The phase velocity has no physical meaning for matter waves

→ The amplitude of the matter wave at a given point is associated with the probability density of finding the particle at the point.

→ The wave length of matter wave is given by $\lambda = \frac{h}{mv}$

b. Set-up Schrodinger time independent wave equation in one dimension.

⇒ The wave equation which has variations only with respect to position and describes the steady state is called Time-Independent Schrodinger wave equation.

Consider a particle of mass 'm' moving with velocity 'v' along +ve x-axis. The de-Broglie wave length λ is given by,

$$\lambda = \frac{h}{mv} \rightarrow (1)$$

The wave equation for one dimensional propagation of wave is given by

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \rightarrow (2)$$

The wave function is given by

$$\psi = \psi_0 e^{i(kx - \omega t)} \rightarrow (3)$$

Here ψ_0 is the amplitude at the point of consideration, ω is angular frequency and k is the wave number. Differentiating ψ twice with respect to t , we get

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0 e^{i(kx - \omega t)} \rightarrow (4)$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \rightarrow (5)$$

Substituting for ω and v we get
for eqⁿ (5) is eqⁿ (2)

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} (-\omega^2 \psi) \rightarrow (6)$$

Substituting for ω and v we get

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{(f\lambda)^2} (-(2\pi f)^2 \psi) \rightarrow (7)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2} \psi \rightarrow (8)$$

Substituting for λ from equation (1)

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{4\pi^2}{\left(\frac{h}{mv}\right)^2} \psi \rightarrow (9)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{4\pi^2 (mv)^2}{h^2} \psi \rightarrow (10)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{8\pi^2 m \left(\frac{1}{2}mv^2\right)}{h^2} \psi \rightarrow (11)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{8\pi^2 m (E - U)}{h^2} \psi \rightarrow (12)$$

$$\text{Hence, } \frac{1}{2}mv^2 = E - U \rightarrow (13)$$

Here E is the total and U is potential energy of the particle.

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m (E - U)}{h^2} \psi = 0} \rightarrow (14)$$

This can be

c. Calculate the energy of the first three energy states for an electron in one dimensional potential well of width 0.1 nm.

Solutions :-

Given :- $a = 0.1 \text{ nm} = 10^{-10} \text{ m}$
 $m = 9.1 \times 10^{-31}$

$$E_n = \frac{n^2 h^2}{8ma^2}$$

$\Rightarrow n = 1$

$$E_{n1} = \frac{1^2 \times (6.628 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$= ~~39.06 \times 10^{-18}~~ 6.029 \times 10^{-18} \text{ J}$$

$$= 37.68 \text{ eV}$$

\Rightarrow Second excited state energy $n=2$

$$E_{n2} = 4E_1 = 24.115 \times 10^{-18} \text{ J} = 150.72 \text{ eV}$$

\Rightarrow Third excited state energy $n=3$

$$E_{n3} = 9E_1 = 54.260 \times 10^{-18} \text{ J} = 339.12 \text{ eV}$$

$$E_{n1} = 37.68 \text{ eV}$$

$$E_{n2} = 150.72 \text{ eV}$$

$$E_{n3} = 339.12 \text{ eV}$$

Q4 a. State and explain Heisenberg's uncertainty principle and show that electron does not exist inside the nucleus using Heisenberg's uncertainty principle.

⇒ Statement :- The simultaneous determination of the exact position and momentum of a moving particle is impossible.

Explanation :- According to this principle if Δx is the error involved in the measurement of position and Δp_x is the error involved in the measurement of momentum during their simultaneous measurement, then the product of the corresponding uncertainties is given by.

$$\Delta x \Delta p_x \geq \frac{h}{4\pi} \rightarrow (1)$$

$$\Delta E \Delta t \geq \frac{h}{4\pi} \rightarrow (2)$$

$$\Delta \theta \Delta L \geq \frac{h}{4\pi} \rightarrow (3)$$

The product of the errors is the order of Planck's constant. If one quantity is measured with high accuracy then the simultaneous measurement of the other quantity becomes less accurate.

* Non-existence electron inside the nucleus

Beta rays are emitted by the nucleus.

When it was first observed it was believed that electrons exist inside the nucleus and are emitted at certain instants.

If the electron can exist inside the atomic nucleus then uncertainty in its position must not exceed the diameter of the nucleus. The diameter of the nucleus is of the order of Δx_{\max} is 10^{-14} m. Applying Heisenberg's uncertainty principle for an electron expected to be inside the nucleus we get,

$$\Delta x_{\max} \Delta p_{\min} \geq \frac{h}{4\pi} \rightarrow (1)$$

$$\Delta p_{\min} \geq \frac{h}{4\pi \Delta x_{\max}} \rightarrow (2)$$

$$\Delta p_{\min} \geq \frac{6.625 \times 10^{-34}}{4 \times 3.142 \times 10^{-14}} = 5.276 \times 10^{-21} \text{ kg ms}^{-1} \rightarrow (3)$$

\therefore The electron should possess momentum

$$p_{\min} \approx \Delta p_{\min} = 5.276 \times 10^{-21} \text{ kg ms}^{-1} \rightarrow (4)$$

Non-relativistic equations of energy of the electron is given by

$$E = \frac{(p_{\min})^2}{2m_e} = 1.52 \times 10^{-11} \text{ J} \rightarrow (5)$$

Here, m_e is the rest mass of the electron

$$E_{\text{min}} = \frac{1.53 \times 10^{11}}{1.6 \times 10^{19}} = 9.5 \text{ MeV} \rightarrow (6)$$

According to experiments, the energy associated with the beta ray (electron) emission is around 3 MeV which is much lesser than the energy of the electron expected to be inside the nucleus 9.5 MeV. Hence electrons do not exist inside the nucleus.

b. Explain Eigen values and Eigen functions and Hence derive the Eigen functions of a particle inside a infinite potential well of width "a" using the method of Normalization.

⇒ The Schrodinger wave equation is a second order differential equation. Thus solving the Schrodinger wave equation to a particular system we get many expressions for wave function (ψ). However, all wave functions are not acceptable. Only those wave functions which satisfy certain conditions are acceptable. Such wave functions are called Eigen functions for the system. The energy values

Corresponding to the eigen functions are called eigen values. The wave functions are acceptable if they satisfy the following conditions.

- ψ must be finite everywhere (cannot be infinite)
- ψ must be single valued which implies that solution is unique for a given position in space.
- ψ and its first derivatives with respect to its variables must be continuous everywhere.

Normalization of wave function

The wave function for a particle in one dimensional potential well of infinite height is given by the equation

$$\psi(x) = A \sin\left(\frac{n\pi x}{a}\right) \rightarrow (1)$$

In this eqⁿ 'A' is an arbitrary constant and it can take any value. The process of determination of value of the arbitrary constant is called normalization of wave function.

The particle has to exist somewhere inside the potential the probability of finding the particle inside the potential well is given by

$$\int_0^a |\psi(x)|^2 dx = \int_0^a P dx = 1 \rightarrow (2)$$

Substituting for the wave function in the integral

$$\int_0^a A^2 \sin^2 \left(\frac{n\pi x}{a} \right) dx = 1 \rightarrow (3)$$

from trigonometry $\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$.

Therefore the above equation could be written as

$$\int_0^a \frac{A^2}{2} \left[1 - \cos \left(\frac{2n\pi x}{a} \right) \right] dx = 1 \rightarrow (4)$$

integrating the above equation we get

$$\frac{A^2}{2} \left[x - \frac{a}{2n\pi} \sin \left(\frac{2n\pi x}{a} \right) \right]_0^a = 1 \rightarrow (5)$$

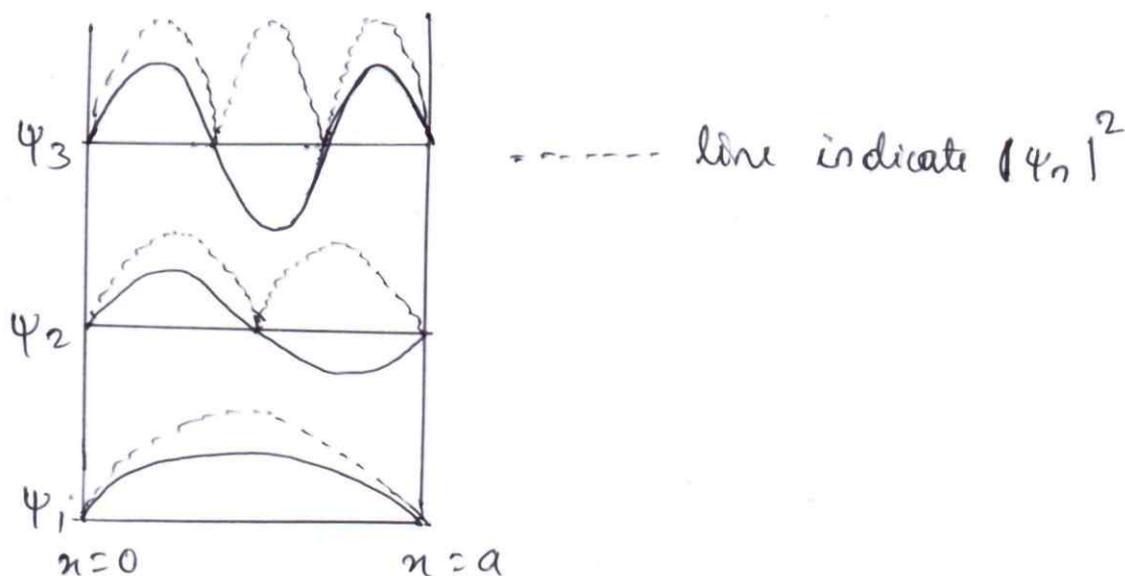
$$\Rightarrow A = \sqrt{\frac{2}{a}} \rightarrow (6)$$

Substituting this in equation $\psi(x) = A \sin \left(\frac{n\pi x}{a} \right)$

the normalized wave function or eigen function for a particle in one dimensional potential well of infinite height is given by

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \left(\frac{n\pi x}{a} \right) \rightarrow (7)$$

The wave functions and the probability densities for the first three values of n are as shown below



Thus for ground state ($n=1$). The probability of finding the particle at the walls is zero and at the center $\frac{a}{2}$ is maximum. The first excited state has three nodes and the second excited state has four nodes.

c. The kinetic energy of an electron is equal to the energy of a photon with a wave length of 560nm . calculate the de-Broglie wave length of the electron.

⇒ Solution :- Given: $\lambda = 560\text{nm}$

kinetic Energy of the electron = Energy of the photon with λ

$$E = \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{560 \times 10^{-9}}$$

$$J = 3.549 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 3.549 \times 10^{-19}}}$$

$$|\lambda = 0.24 \times 10^{-10} \text{ m}|$$

MODULE - 03

Q.5 a. state the Pauli matrices and apply Pauli matrices on the states $|0\rangle$ and $|1\rangle$.

⇒ Pauli Matrices and their operation on $|0\rangle$ and $|1\rangle$ states, There are four extremely useful matrices called Pauli matrices. The Pauli matrices of the following form

$$\sigma_0 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow (1)$$

This is an identity matrix

$$\sigma_1 = \sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \rightarrow (2)$$

$$\sigma_2 = \sigma_y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \rightarrow (3)$$

$$\sigma_3 = \sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow (4)$$

Pauli Matrices operation on $|0\rangle$ and $|1\rangle$ states

$$1. \quad \sigma_0 |0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$\sigma_0 |1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$2. \sigma_x |0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$\sigma_x |1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$3. \sigma_y |0\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i|1\rangle$$

$$\sigma_y |1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i|0\rangle$$

$$4. \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle$$

b. Discuss the CNOT gate and its operations on four different input states.

⇒ The CNOT gate is a typical multi-qubit logic gate and the circuit is as follows. The matrix representation of CNOT



Gate is given by

$$U_{\text{CNOT}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow (1)$$

The Transformation could be expressed as

$$|A, B\rangle \rightarrow |A, B \oplus A\rangle \rightarrow (2)$$

Consider the operations of CNOT gate on the four inputs $|00\rangle, |01\rangle, |10\rangle$ and $|11\rangle$

→ Operation of CNOT Gate for input $|00\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|0\rangle$. Hence no change in the state of Target qubit $|0\rangle$

$$|00\rangle \rightarrow |00\rangle \rightarrow (1)$$

→ Operation of CNOT Gate for input $|01\rangle$

Here in the inputs to the CNOT Gate the control qubit is $|0\rangle$. Hence no change in the state of Target qubit $|1\rangle$

$$|01\rangle \rightarrow |01\rangle \rightarrow (2)$$

→ Operation of CNOT Gate for input $|10\rangle$

Here in the input to the CNOT Gate the control qubit is $|1\rangle$. Hence the state of Target qubit flips from $|0\rangle$ to $|1\rangle$

$$|10\rangle \rightarrow |11\rangle \rightarrow (3)$$

→ Operation of CNOT Gate for Input $|11\rangle$

Here is the input to the CNOT Gate the control qubit is $|1\rangle$. Hence the state of Target qubit flips from $|1\rangle$ to $|0\rangle$

$$|11\rangle \rightarrow |10\rangle \rightarrow (4)$$

The Truth Table of operation of CNOT gate is as follows.

Input	Output
$ 00\rangle$	$ 00\rangle$
$ 01\rangle$	$ 01\rangle$
$ 10\rangle$	$ 11\rangle$
$ 11\rangle$	$ 10\rangle$

c. Given $|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$

Prove that $\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$

Solution: $\langle\phi|\psi\rangle = (\beta_1^* \ \beta_2^*) \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \beta_1^* \alpha_1 + \beta_2^* \alpha_2$ — (1)

$$\langle\psi|\phi\rangle^* = (\beta_1 \alpha_1^* + \beta_2 \alpha_2^*)^* \Rightarrow \text{---} \quad \text{2}$$

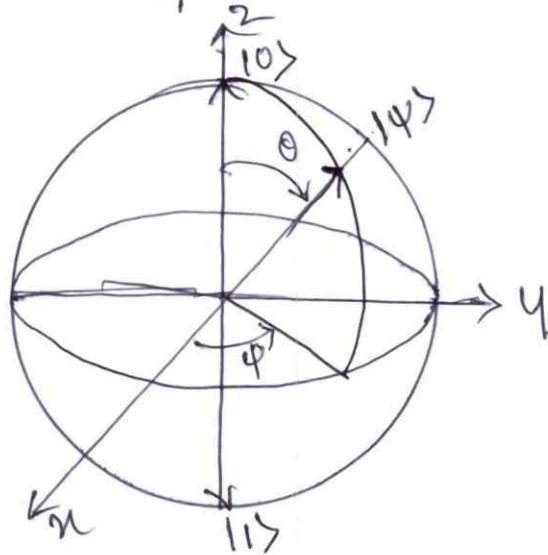
$$= \beta_1^* \alpha_1 + \beta_2^* \alpha_2 \text{ --- } \quad \text{(2)}$$

Thus from Eqn (1) & (2)

$$\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$$

Q. 6 a. Explain the representation of qubit using Bloch sphere.

⇒ The pure state space qubits (Two level Quantum Mechanical systems) can be visualized using an imaginary sphere called Bloch sphere. It has a unit radius



The Arrow on the sphere represents the state of the qubit. The north or south poles are used to represent the basis states $|0\rangle$ and $|1\rangle$ respectively. The other locations are superpositions of $|0\rangle$ and $|1\rangle$ states and represented by $\alpha|0\rangle + \beta|1\rangle$ with $\alpha^2 + \beta^2 = 1$. Thus a qubit can be any point on the Bloch Sphere.

The Bloch sphere allows the state of the qubit to be represented with spherical co-ordinates. They are the polar angle θ and the azimuth angle ϕ . The Bloch sphere is represented by the equation

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle \rightarrow (1)$$

Here $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$.

The normalization constraint given by

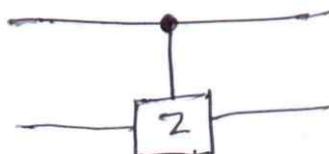
$$\left|\cos\frac{\theta}{2}\right|^2 + \left|\sin\frac{\theta}{2}\right|^2 = 1 \rightarrow (2)$$

b. Describe the working of controlled Z gate mentioning its matrix representation and truth table.

⇒ In controlled Z gate. The operation of Z gate is controlled by control qubit. If the control qubits is $|A\rangle = |1\rangle$ then only the Z gate transforms the target qubit $|B\rangle$ as per the Pauli Z operation. The action of controlled Z-gate could be specified by a matrix as follows.

$$U_Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \rightarrow (1)$$

The controlled Z-gate and the truth table as follows



Input	Output
$100\rangle$	$100\rangle$
$101\rangle$	$101\rangle$
$110\rangle$	$110\rangle$
$111\rangle$	$-111\rangle$

6. A linear operator 'X' operates such that $X|0\rangle = |1\rangle$ and $X|1\rangle = |0\rangle$. Find the matrix representation of 'X'.

Solution Given $X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$X|0\rangle = |1\rangle$$

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Multiplying the matrix on LHS and equating with the matrix on RHS we get

$$x_{11} = 0 \quad \& \quad x_{21} = 1$$

$$X|1\rangle = |0\rangle$$

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Multiplying the matrix on LHS and equating with on the RHS we get

$$x_{12} = 1 \quad \& \quad x_{22} = 0$$

Therefore $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ //

MODULE - 04

Q.7 a. Enumerate the assumptions of Quantum free electron theory of metals and mention the failures and classical free electron theory.

⇒ The failures of classical free electron theory led to the rise of Quantum Free electron theory and was proposed by Sommerfeld in the year 1928. The quantum free electron theory is based on the following assumptions.

Assumptions:

1. Unlike classical free electron theory, in quantum free electron theory, energy values of free electrons are quantized. The energy values of free electrons are discrete since their motion is confined within the bound areas of the metal.
2. Thus in a metal there exists large number of closely spaced energy levels for free electrons which form a band.

3. The distribution of free electrons in the energy levels is as per the Pauli's exclusion principle. Only a maximum of two electrons can occupy a given energy level. This also suggests the availability of two energy states for free electrons in an energy level corresponding to spin up and spin down states.
4. The potential setup by the lattice ions is assumed to be constant throughout the metal.
5. The mutual repulsion between electrons and the attraction between electrons and lattice ions are neglected.

* Failures of classical free electron theory of metals :-

Classical free electron theory of metal is successful in explaining the certain experimentally observed facts of electronic conduction in solids and thermal conductivity.

This theory fails to explain certain other experimental observations. The following are the failures of classical free electron theory of metals

1) Electronic specific heat of solids

According to the classical free electron theory metals the electronic specific heat is given by

$$C_v = \frac{3}{2} R = 12.5 \text{ J mole}^{-1} \text{ K}^{-1}$$

The experimental value of electronic specific heat is $C_v = 10^{-4} RT$. It is very small and also temperature dependent. Hence classical theory fails to explain the electronic specific heat of solids.

2) Dependence of σ on temperature

According to classical free electron theory of metals the electrical conductivity σ is inversely proportional to square root of temperature (\sqrt{T}). But experiments reveal that electrical conductivity (σ) is inversely proportional to temperature (T). Hence classical free electron theory fails to explain dependence of electrical conductivity

(σ) on the temperature (T).

3) Dependence of σ on n , the number density

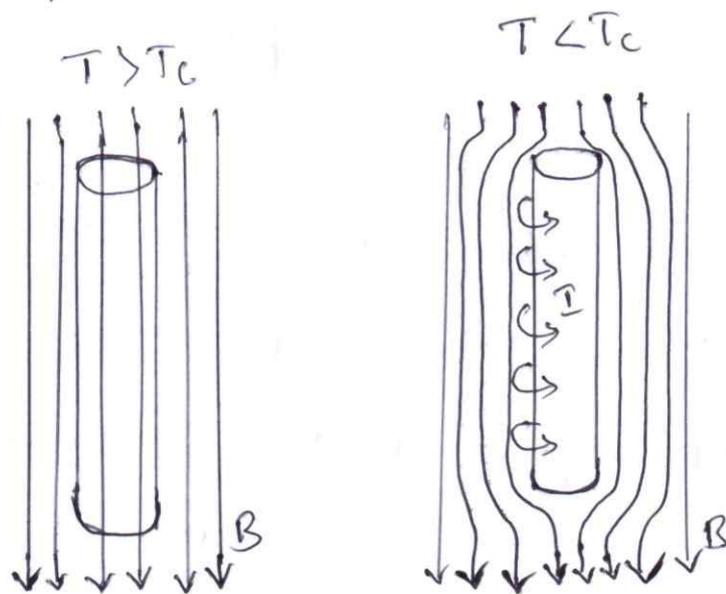
The theory predicts that direct dependence of electrical conductivity (σ) on number of free electrons per unit volume (n) called number density. But experiments have revealed different with $\sigma_{Cu} > \sigma_{Al}$ even though the number density $n_{Cu} < n_{Al}$. Hence it fails to explain the dependence of electrical conductivity σ on the number free electrons per unit volume n . The experimental observations are as in the table

metal	σ ($\Omega^{-1} m^{-1}$)	n (m^{-3})
Copper	5.88×10^7	8.45×10^{28}
Aluminium	3.65×10^7	18.06×10^{28}

b. Describe Meissner effect and hence classify superconductors into soft and hard superconductors using $M-H$ graphs.

\Rightarrow In 1933, Meissner and Ochsenfeld showed that when a superconducting material is placed in magnetic field it allows magnetic lines of force to pass through, if its temperature

is above T_c . If the temperature is reduced below the critical temperature T_c then it expels all the flux lines completely out of the specimen and exhibits perfect diamagnetism. This is known as Meissner's effect. Since superconductor exhibits perfect diamagnetism below the critical temperature T_c , magnetic flux density inside the material is zero.



The expression for magnetic flux density is given by

$$B = \mu_0 (M + H)$$

Here, B is Magnetic Flux Density, M is Magnetization and H is the applied magnetic field strength. For a superconductor, $B = 0$ at $T < T_c$. Thus we get,

$$M = -H$$

Thus Meissner's Effect signifies the negative magnetic moment associated with superconductors.

→ Types of Superconductors

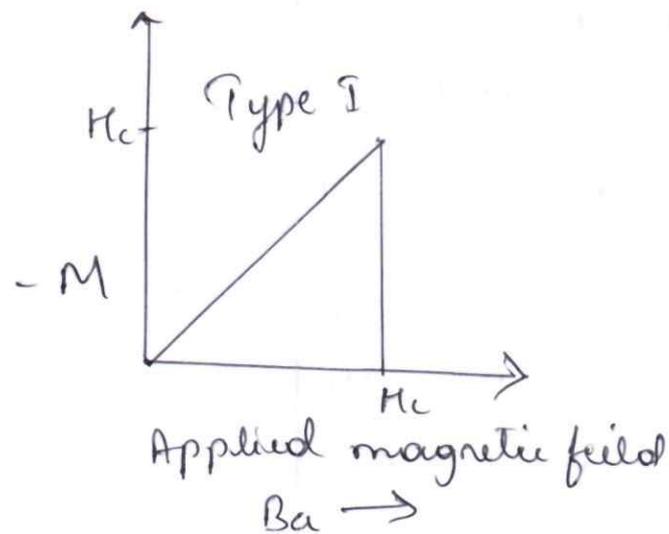
superconductors are classified into two types:

- 1) Type I superconductor or soft superconductors
- 2) Type II superconductor or hard superconductors

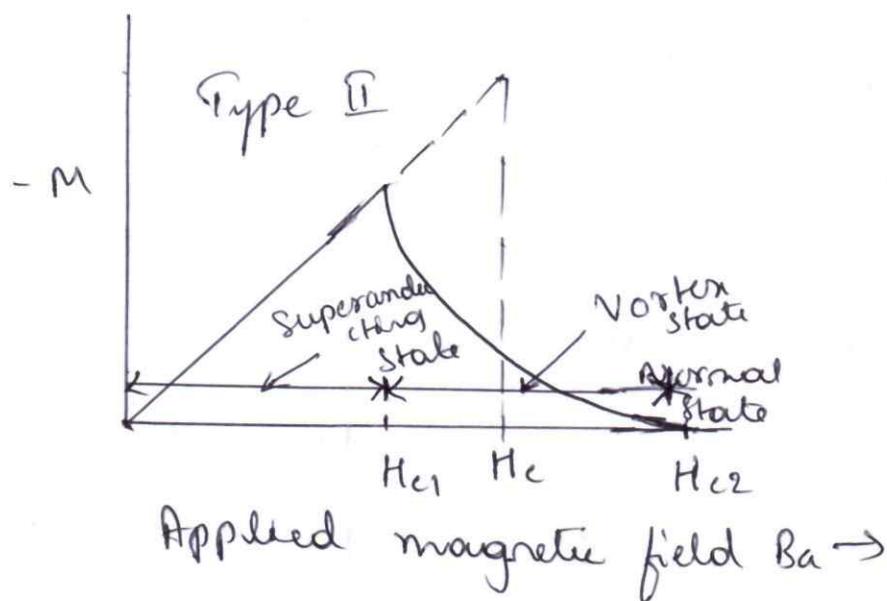
1) Type I Superconductor (soft superconductors)

Type I superconductor exhibit complete Meissner's effect and possess a single value of critical field. The graph of magnetic moment Vs magnetic field is as shown below. As the field strength increases the material becomes more and more diamagnetic until H becomes equal to H_c . above H_c the material allows the flux lines to pass through and exhibits normal conductivity. The value of H_c is very small for soft superconductors. Therefore soft superconductors cannot

withstand high magnetic fields. Therefore they cannot be used for making superconducting magnet magnets. Ex: Hg, Pb & Zn



2) Type II Superconductors (Hard Superconductor) superconducting materials, which can withstand high value of critical magnetic fields, are called Hard Superconductors.



The graph of magnetic moment vs magnetic field is as shown above.

Hard superconductors are characterized by two critical fields H_{c1} and H_{c2} . When applied magnetic field is less than H_{c1} partial flux penetrates and the diamagnetism. Beyond H_{c1} partial flux penetrates and the material is said to be Vortex state. Thus flux penetration occurs through small-channelized regions called filaments. As the strength of the field increases further, more and more flux fills the body and thereby decreasing the diamagnetic property of the material. At H_{c2} flux fills the body completely and material loses its diamagnetic property as well as superconducting property completely.

The value H_{c2} is hundreds of times greater than H_c of soft superconductors. Therefore they are used for making powerful superconducting magnets.

Exp:- NbTi, Nb₃Sn

c. Calculate the probability of occupation of an energy level 0.2 eV above Fermi level at temperature 27°C.

Solution

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1}$$

$$E - E_F = 0.2 \times 1.6 \times 10^{-19} \text{ J}$$

Substitution $f(E) = \frac{1}{e^{\frac{0.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}} + 1}$

$$f(E) = \frac{1}{e^{\frac{0.32 \times 10^{-19}}{414 \times 10^{-23}} + 1}}$$

$$= \frac{1}{e^{7.7} + 1}$$

$$= \frac{1}{e^{7.7} + 1}$$

$$f(E) = 4.395 \times 10^{-4}$$

Q. 8 a. Define Fermi factor and discuss the variation of Fermi factor with temperature and energy.

⇒ Defⁿ :- The probability of occupation of an energy level of energy (E) at temperature (T) under thermal equilibrium is evaluated using an expression called Fermi Factor.

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{KT}\right)} + 1}$$

* Fermi factor on energy and Temperature

As described, the Fermi factor is a function of energy and temperature. This dependence could be explained for energy levels below and above Fermi level at absolute zero and higher temperatures.

→ Probability of occupation of levels with energy $E < E_F$ and at $T = 0K$

The Fermi factor or Fermi function is given by

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{KT}\right)} + 1} \rightarrow (1)$$

Here $E - E_f$ is negative. Substituting the value for $T = 0$

$$f(E) = \frac{1}{e^{\frac{(E - E_f)}{k \times 0}} + 1}$$

$$f(E) = \frac{1}{e^{-\infty} + 1}$$

$$f(E) = \frac{1}{0 + 1}$$

$$f(E) = 1$$

Therefore $f(E) = 1$. Hence, at $T = 0K$, all energy levels below the Fermi level are completely filled.

→ Probability of occupation of levels with energy $E > E_f$ and at $T = 0K$

The Fermi factor or Fermi function is given by Here $E - E_f$ is positive. Substituting the value for $T = 0$

$$f(E) = \frac{1}{e^{\frac{(E - E_f)}{k \times 0}} + 1} = \frac{1}{e^{\infty} + 1}$$

$$f(E) = \frac{1}{\infty + 1}$$

$$f(E) = 0$$

Therefore $f(E) = 0$. Hence, at $T = 0K$ all energy levels above the Fermi level are empty.

→ Probability of occupation of levels with energy $E = E_f$ and at $T > 0K$

Here $E - E_f = 0$. Substituting the values

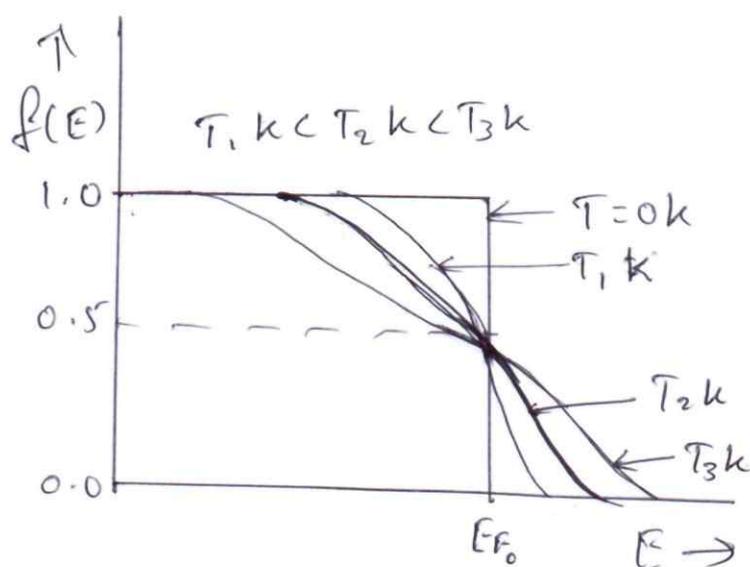
$$f(E) = \frac{1}{e^{\left(\frac{0}{kT}\right)} + 1}$$

$$f(E) = \frac{1}{1+1}$$

$$f(E) = \frac{1}{2} = 0.5$$

Thus for all temperatures above $0K$ the probability of occupation of Fermi level is $\frac{1}{2}$. Thus the variation of Fermi factor

with temperature is as shown in the graph



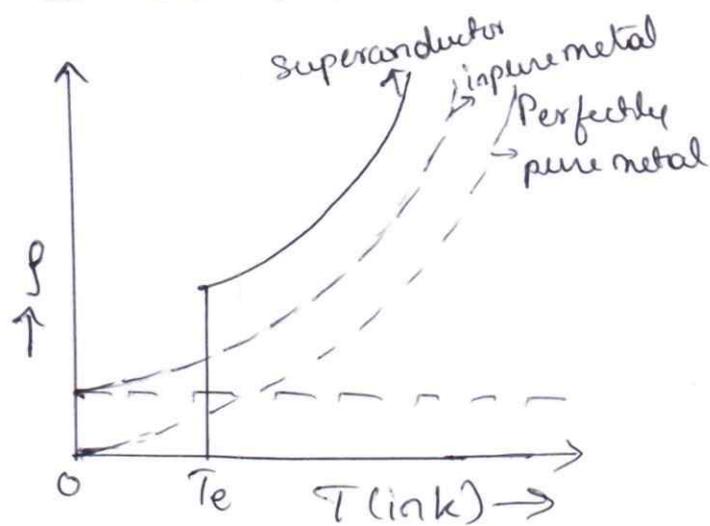
Variation of $f(E)$ as function of Temp & Energy

b. Explain the phenomenon of Superconductivity and discuss qualitatively the BCS theory of super conductivity for negligible resistance of metal at temperature close to absolute zero.

⇒ The phenomenon in which resistance of certain metals, alloys and compounds drops to zero abruptly, below certain temperature is called superconductivity.

Variation of Resistivity with Temperature:

The variation of the resistivity of a superconductor, pure and impure metals with temperature is as shown below



Critical Temperature: The temperature, below which materials exhibit superconducting property is called critical temperature, denoted by T_c . Critical temperature T_c is different for different substances. The

materials, which exhibit superconducting property, are called superconductors.

Above critical temperature materials is said to be in normal state and offers resistance for the flow of electric current. Below critical temperature material is said to be in superconducting state. Thus T_c is also called as transition temperature.

* BCS Theory of Superconductivity

Bardeen, Cooper and Schrieffer explained the phenomenon of superconductivity in the year 1957. The essence of the BCS theory is as follows.

Consider an electron approaching a positive ion core and suffers attractive Coulomb interaction. Due to this attraction ion core is set in motion and thus distorts that lattice. Let a second electron come in the way of distorted lattice and interaction between the two occurs which lowers the energy of the second electron. The two electrons therefore interact indirectly through the lattice distortion or the phonon field which lowers the energy of the electrons. The above interaction is interpreted as electron-lattice

C. A Superconductivity T is has a critical temperature of 3.7k at zero magnetic field and a critical field of 0.0306 Tesla at 0k. Find the critical field at 2k.

Solution

Given

$$T_c = 3.7K$$

$$H_0 = 0T$$

$$H_{c2} = 0.0306T$$

$$T_0 = 0K$$

$$H_c \text{ at } 2K = ?$$

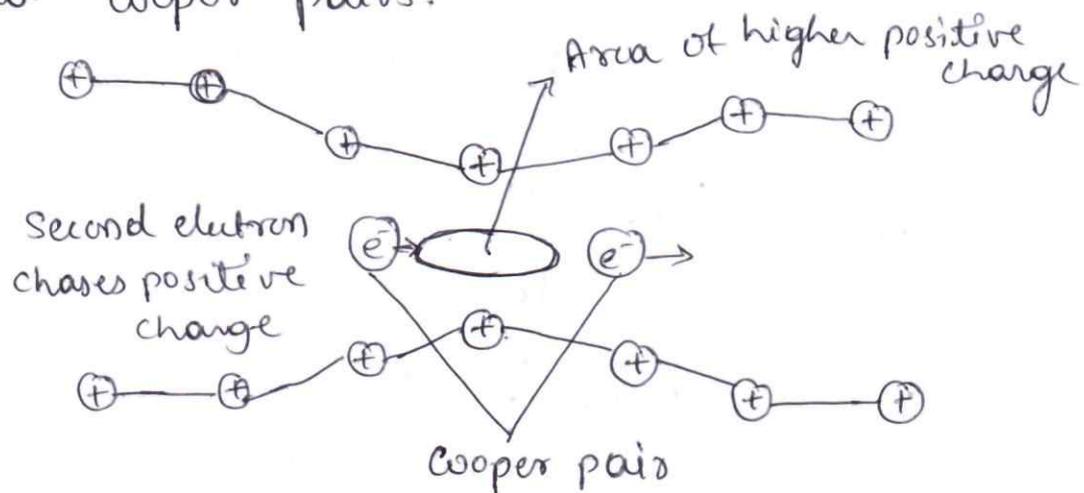
$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right]$$

$$H_c = 0.0306 \left(1 - \frac{2^2}{3.7^2} \right)$$

$$H_c = 0.0216T$$

- electron interaction through phonon field.

It was shown by Cooper that, this attractive force becomes maximum if two electrons have opposite spins and momentum. The attractive force may exceed Coulomb's repulsive force between the two electrons below the critical temperature, which results in the formation of bound pairs of electrons called Cooper pairs.



Below the critical temperature the dense cloud of Cooper pairs forms a collective state and the motion of all Cooper pairs is correlated resulting in zero resistance of the material.

MODULE - 05

Q.9 a. Elucidate the importance of size and scale, weight and strength in animations

⇒ Size:-

Size is simply how small or big an element is in relation to other objects within a design. Generally we use size to make a particular element stand out or to give it importance. However, size becomes a much more powerful design tool when it is considered alongside the scale.

Proportion and Scale:-

creating a larger or smaller character is not just a matter of scaling everything about the character uniformly.

Example: when you scale a cube, its volume changes much more dramatically than its surface area. Let us say each edge of the cube is 1 unit in length.

The area of the cube is 1 square unit, and the volume of the cube is 1 cubed

unit. If you double the size of the cube along each dimension, its height increases by 2 times, the surface area increases by 4 times and its volume increases by 8 times.

Scaling Properties:

larger or heavier objects move slower while lighter or smaller objects move faster when designing characters, you can run into different situations having to do with size and scale, such as:

1. Human or animal-based characters that are much larger than we see in our everyday experiences, superheroes, Greek gods and monsters.
2. Human or animal-based characters that are much smaller than we are accustomed to, such as fairies and elves.
3. Characters that need to be noticeably larger, smaller, older, heavier, lighter, or more energetic than other characters.
4. Characters that are child versions of older characters.

Weight:

Two objects can appear to be different weights by manipulating their timing.

For example: If you were to hit a croquet ball and a balloon with a mallet, the result would be two different actions.

The ~~an~~ croquet ball would require more force to place it into motion, would go farther and need more force to stop it. On the other hand, the balloon would require far less force to send it flying and because of its low mass and weight, it wouldn't travel as far, and would require less force to stop it.

Strength

Strength is the maximum force a muscle or group of muscles can apply against a resistance in a push, pull or lift motion. Body weight is proportional to volume. The abilities of your muscles and bones, however, increase by area because their abilities depend

more on the cross-sectional area than volume. To increase a muscle or bone's strength, you need to increase its cross-sectional area.

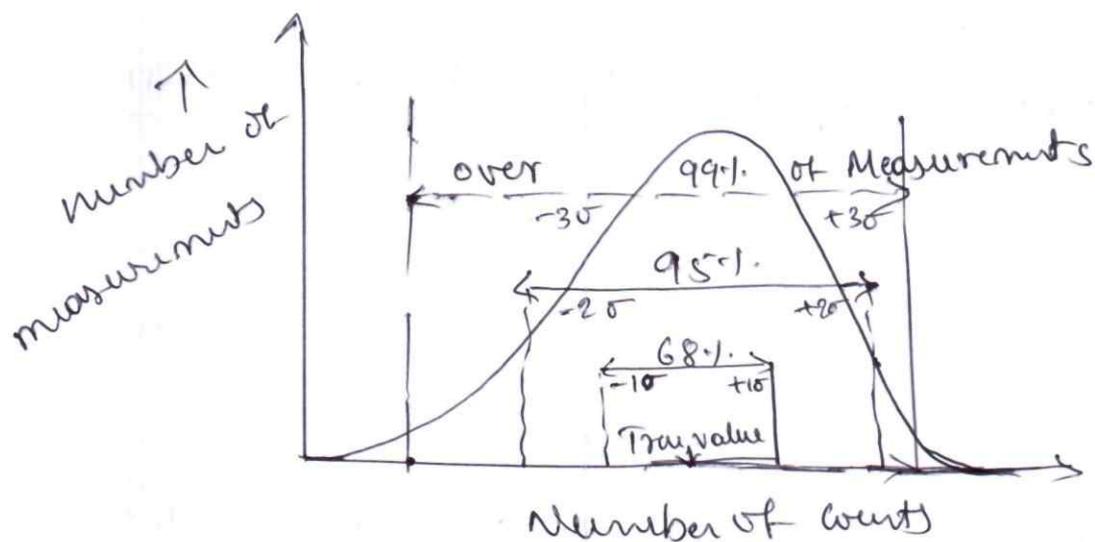
b. Discuss the salient features of normal distribution using bell curves.

⇒ A bell curve is a common type of distribution for a variable, also known as the normal distribution. The term "bell curve" originates from the fact that the graph used to depict a normal distribution consists of a symmetrical bell-shaped curve.

The highest point on the curve, or the top of the bell, represents the most probable event in a series of data (its Mean, Mode and Median in this case), while all other possible occurrences are symmetrically distributed around the mean, creating a downward-sloping curve on each side of the peak. The width of the bell curve is described by its standard deviation.

The term "bell curve" is used to describe a graphical depiction of a normal probability distribution,

whose underlying standard deviations from the mean create the curved bell shape. A standard deviation is a measurement used to quantify the variability of data dispersion, in a set of given values around the mean. The mean, in turn, refers to the average of all data points in the data set or sequence and will be found at the highest point of the bell curve.



Standard Deviations

The standard deviation is a measure of how spread out numbers are. 68% of values are within 1 standard deviation of the mean. 95% of values are within 2 standard deviations of the mean.

99.7% of values are within 3 standard deviations of the mean.

C. A slowing object in an animation has a first frame distance 0.5 m and the first slow is frame 0.35 m. Calculate the base distance and the number of frames in sequence.

Solution: Base distance =
$$\frac{\text{First Frame distance} - \text{First slow distance}}{2}$$

$$= \frac{(0.5 - 0.35)}{2}$$

$$\text{Odd multiplier} = \frac{\text{First Frame distance}}{\text{Base distance}}$$

$$= \frac{0.5}{0.075}$$

$$= 7$$

Thus the total number of frames = 7.

3.10. a. Describe jumping and parts of jump

⇒ Jumping:

A jump is an action where the character's entire body is in the air, and both the character's feet leave the ground at roughly the same time. A jump action includes a take off, free movement through the air, and a landing.

Parts of Jump

A jump can be divided into several distinct parts:

→ Crouch - A squattering pose taken as preparation for jumping

→ Takeoff - character pushes up fast and straightens legs with feet still on the ground. The distance from the character's center of gravity (CG) in the crouch to the CG when the character's feet are just about to leave the ground is called the push height. The amount of time (or number of frames) needed for the push is called the push time.

→ In the air - Both the character's feet are off the ground. and the character's center of gravity (CG) moves in a parabolic arc as any free-falling body would. First it reaches an apex, and then falls back to the ground at the same rate at which it rose. The height to which the character jumps, called the jump height, is measured from the CG at take off to the CG at the apex of the jump. The amount of time the character is in the air from take off to apex is called the jump time. If the take off pose and the landing pose are similar, then the jump height and jump time are about the same going up as they are going down.

→ Landing - character touches the ground and bends knees to return to a crouch. The distance from the character's CG when her feet hit to the ground to the point where the character stops crouching is called the stop height. The stop height is not always exactly the same as the push height.

b. Discuss modeling the probability for proton decay.

⇒ The experimental search for proton decay was undertaken because of the implications of the Grand unification theories.

The lower bound for the lifetime is now projected to be on the order of $\tau = 10^{33}$ years. The probability for observing a proton decay can be estimated from the nature of particle decay and the application of poisson statistics the number of protons N can be modeled by the decay equation

$$N = N_0 e^{-\lambda t}$$

Here $\lambda = \frac{1}{\tau} = 10^{-33} / \text{year}$

is the probability that any given proton will decay in a year. Since the decay constant λ is so small, the exponential can be represented by the first two terms of the exponential series.

$$e^{-\lambda t} = 1 - \lambda t,$$

Thus, $N \approx N_0 (1 - \lambda t)$

For a small sample, the observation of a proton decay is infinitesimal, but suppose we consider the volume of protons ~~are~~ represented by the Super Kameokande neutrino detector in Japan. The number of protons in the detector volume is reported by Ed Keams of Boston University to be 7.5×10^{33} protons. For one year of observation the number of expected proton decays is then

$$N - N_0 = \lambda N_0 t = (7.5 \times 10^{33} \text{ protons}) (10^{-33} / \text{year}) (1 \text{ year}) = 7.5$$

About 40% of the area around the detector tank is covered by photon-detector tubes, and if we take that to be the nominal efficiency of detection, we expect about three observations of proton decay events per year based on a 10^{33} year lifetime. So far, no convincing proton decay events have been seen. Poisson statistics provides a convenient means for assessing the implications of the absence of these observations. If we presume that $\lambda = 3$ observed decays per year is the mean, then the Poisson distribution

function tells us that the probability for zero observations of a decay is

$$p(k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad p(k) = \frac{3^0 e^{-3}}{0!} = 0.05^5$$

This low probability for a null result suggests that the proposed lifetime of 10^{33} years is too short. While this is not a realistic assessment of the probability of observations because there are a number of possible pathways for decay, it serves to illustrate in principle how even a non-observation can be used to refine a proposed lifetime.

- c. In a diffraction grating experiment the laser light undergoes second order diffraction for diffraction angle 1.48° . The grating constant $d = 5.05 \times 10^{-5} \text{ m}$ and the distance between the grating and screen is 0.60 m . Find the wavelength of LASER light.

Solution:

$$\lambda = \frac{d \sin \theta}{n}$$

$$n = 2$$

$$\lambda = \frac{5.05 \times 10^5 \sin(1.48)}{2} \text{ m}$$

$$\lambda = 6.522 \times 10^{-7} \text{ m.}$$

Solution Prepared by

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