CBCS SCHEME

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BME502

Fifth Semester B.E./B.Tech. Degree Examination, June/July 2025
Turbo Machines

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.
3. Use of Steam table is permitted.

		Module – 1	M	L	C
Q.1	a.	Draw and explain the parts of general turbomachines.	6	L2	CO1
	b.	Distinguish between turbomachines and positive displacement machines.	6	L2	CO1
	c.	1/5 scale model of a pump was tested in a laboratory at 1000 rpm. The head developed and power input at the best efficiency point were found to be 8 m and 30 KW. If the prototype pump has to work against a head of 25 m, determine its working speed, power required to drive it and the ratio of flow rates handled by the two pumps.	8	L3	CO1
		OR			
Q.2	a.	Define the static and stagnation state of fluid.	4	L2	CO1
	b.	Define the following with the help of h-s diagram for power absorbing and power generating machine. i) Total to total efficiency ii) Total to static efficiency iii) Static to total efficiency iv) Static to static efficiency.	8	L2	CO1
	c.	Show that the polytrophic efficiency during expansion process is given by $\eta_p = \dfrac{\ln(T_2/T_1)}{\left(\dfrac{\gamma-1}{\gamma}\right)\ln(P_1/P_2)}$	8	L3	CO1
		Module – 2			
Q.3	a.	Define degree of reaction and utilization factor. Establish relation between them.	10	L2	CO2
	b.	Draw the velocity triangle at inlet and outlet of turbo machines and derive the Euler turbine equation with usual notations.	10	L2	CO2
		OR			
Q.4	a.	Derive head-capacity relationship for centrifugal pump and explain the effect of discharge angle on it.	10	L2	CO2

	b.	An inward flow radial vane turbine has the following data, power = 150 kW, speed = 32000 rpm, out diameter of the impeller = 20 cm, inner diameter of the impeller 8 cm, absolute velocity of gas at entry = 387 m/sec. Absolute velocity of gas at exit = 193 m/sec and radial in direction. Construct the velocity triangles at entry and exit of the impeller and determine: i) Mass flow rate ii) Percentage energy transfer due to change of radius.	10	L3	CO2
		Module – 3			
Q.5	a.	Prove that maximum blade efficiency of a single impulse turbine is given by $\eta_b = \cos^2\alpha_1$ with combined velocity diagram.	10	L2	CO3
	b.	The nozzle of a D-laval turbine delivers 2 kg /sec of steam at a speed of 2400 m/sec. The nozzle are inclined at an angle of 16 degree to the plane of the wheel. The blade velocity is 600 m/sec. Allowing a blade velocity coefficient of 0.72, calculate: i) Blade efficiency ii) Power developed by the blades iii) Energy lost in the blades. The blade angle at outlet may be taken as 25°.	10	L3	CO3
		OR			
Q.6	a.	Prove the condition for maximum efficiency of a reaction turbine using a combined velocity diagram.	10	L2	CO3
	b.	The following particulars refer to a stage of an impulse reaction turbine. Outlet angel of fixed blade = 20°, outlet angle of moving blades = 30°, radial height of fixed and moving blades = 10 cm, mean blade velocity = 138 m/sec, blade speed ratio = 0.625, specific volume of steam at fixed blade outlet = 1.235 m^3/kg, specific volume of steam at moving blade out = 1.305 m^3/kg, speed of the rotor = 3000 rpm, calculate the degree of reaction, the adiabatic heat drop in pair of blade rings and gross stage efficiency, Given the following coefficient which are same for both fixed and moving blades, $\eta = 0.9$, carry over coefficient = 0.86.	10	L3	CO3
		Module – 4			
Q.7	a.	Define and write mathematical equation. i) Hydraulic efficiency ii) Mechanical efficiency iii) Overall efficiency iv) Volumetric efficiency.	10	L2	CO4
	b.	In a power station, a pelton wheel produce 15000 KW under a head of 350 m, while running at 500 rpm. Assume a turbine efficiency of 0.84, coefficient of velocity for Nozzle as 0.98, speed ratio 0.46 and bucket velocity coefficient 0.86. Calculate: i) Number of jet ii) Diameter of each jet iii) Tangential force exerted on the buckets if the bucket deflect the jet through 165°.	10	L3	CO4

		OR			
Q.8	**a.**	Explain with a next sketch working of hydro electric power plant.	**6**	**L1**	**CO4**
	b.	With a neat sketch. Explain the working of draft tube and list out the application.	**4**	**L2**	**CO4**
	c.	The following data is given for a Francis turbine. Net head = 70 m, speed = 600 rpm, power at the shaft = 367.5 kW, overall efficiency = 85%, hydraulic efficiency = 95%, flow ratio = 0.25, width ratio = 0.1, outer diameter to inner diameter ratio = 2.0. The thickness of vanes occupies 10% of the circumferential area of runner, velocity of flow is constant at inlet and discharge is radial at outlet. Determine: i) Guide blade angle ii) Runner vane angle at inlet and outlet iii) Width of the wheel at inlet iv) Diameter of runner at inlet and outlet.	**10**	**L3**	**CO4**
		Module – 5			
Q.9	**a.**	Derive an expression for a minimum starting speed of a centrifugal pump.	**5**	**L2**	**CO5**
	b.	Derive an expression for the static pressure rise in the impeller of a centrifugal pump with inlet and outlet velocity diagram.	**5**	**L2**	**CO5**
12	**c.**	A centrifugal pump running at 1000 rpm. The outlet angle of vane is 45° and the velocity of flow at outer let is 2.5 m/sec, the discharge through the pump is 200 lit/sec, when the pump is working against the total head of 20 m, if the nanometric efficiency of the pump is 80%, determine : i) Diameter of the impeller ii) Width of the impeller at outlet.	**10**	**L3**	**CO5**
		OR			
Q.10	**a.**	Explain with a neat sketch working of centrifugal compressor.	**5**	**L2**	**CO5**
	b.	Explain the surging and choking in centrifugal compressor.	**5**	**L2**	**CO5**
	c.	An axial flow compressor stage draws air from with the stagnation conditions 1.013 bar and 308 K. Assuming 50% reaction stage with a flow coefficient of 0.52 and the ratio $\Delta V_{wh} = 0.25$, find the rotor blade angle at the inlet and exit as well as the mean rotor speed. The total to total efficiency of the stage is 0.87 when the stage produces a total to total pressure ratio of 1.23. Find also pressure coefficient and the power input to the system, assuming the work input factor to be 0.86. The mass flow rate is 12 kg/sec.	**10**	**L3**	**CO5**

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VTU Question paper Solution		
Examination June/July, 2025.		
Scheme : 2022	Turbomachines	BME502.
Staff Name : Chandrakanth M.T	Signature	

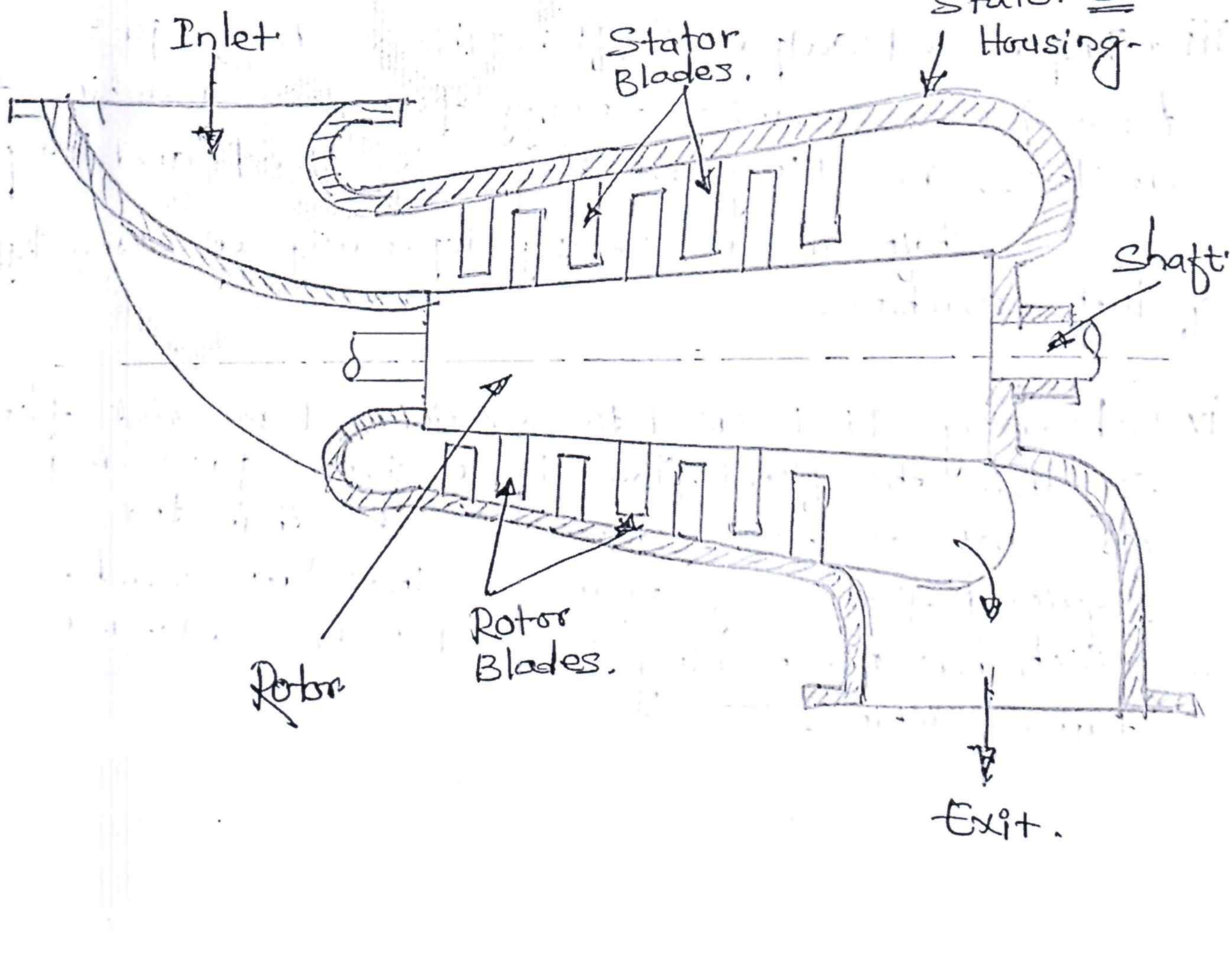
Module - 1

Q1 (a)

Definition of turbo machine.

A turbo machine is a device in which energy transfer occurs between a flowing fluid and rotating element due to dynamic action resulting in a pressure and movementum of the fluid.

Parts of a Turbomachine



The principal components of turbo machines are

(i) Rotating element: It carries vanes or blades and operates in a a stream of fluid. Energy transfer occurs between the flowing fluid and this rotating element due to momentum exchange between the two.

* Rotating element of water turbine is called as runner.
* Rotating element of gas & steam turbine called as rotor.
* Rotating element of pump is called as impeller.

(ii) Stationary element: stationary element may be called as guide blades or nozzle depending on the particular machine. It acts as guide part for the proper control of flow direction and the energy conversion process.

(iii) Input and Output shaft: These shafts just transfer mechanical energy from one machine to another, resulting in change of speed and torque.

Shaft are necessary depending upon the type of turbo machines.

(iv) Housing: It is used to contain and restrict fluid so that the fluid flows in a given space and does not escape in directions other than those required for energy transfer. The turbo machine which has no housing is said to be extended turbo machine.

Q.1 (b)

Comparision between Positive displacement machine and Turbomachine.

Positive Displacement Machine	Turbo machines.
I. Action	
i) Creates thermodynamic and mechanical action between a nearly static fluid and a slow moving surface.	(i) Creates thermodynamic and dynamic action between a flowing fluid and a rotating element.
ii) Involves a volume change or displacement of fluid and a slow moving surface.	(ii) Involves energy transfer with pressure and movement change.
II. Operation.	
i) commonly involves a reciprocating motion and unsteady flow of fluid	(i) Involves in principle, a steady flow of fluid and pure rotary motion of the mechanical element.
ii) Stopping positive displacement machine results in trapping of the working fluid.	(ii) Stopping of the machine will let the fluid state change rapidly and become the same as that of surroundings.
III Mechanical Features.	
i) commonly works at low speeds. ii) Complex in design. iii) Due to reciprocating motion vibration are more and hence heavier foundations are needed.	(i) Works at high rotational speed. (ii) Relatively simple in design. (iii) Due to rotary motion, vibration problems are less.

IV. Efficiency of conversion	
i) High efficiency because of static energy transfer ii) Compression & expansion efficiency are almost same.	(i) Low efficiency because of dynamic energy transfer. (ii) Efficiency of compression process is low.
V. Volumetric efficiency.	
Volumetric efficiency is lower due to frequent opening and closing of valves.	Volumetric efficiency is nearly 100% as the valves are open all the time.
VI. Fluid phase change and surging.	
i) Problems of phase change and surging are relatively minor.	(i) Phase change occuring during flow can cause serious problems to smooth operation. (ii) Surging or pulsation of fluid leads to vibrate excessively

Q 1 (c).

1/5 scale model of a pump was tested in a Laboratory at 1000 rpm. The head developed and power input at the best efficiency point were found to be 8 m and 30 kW. If the prototype pump has to work against a head of 25 m, determine its working speed, power required to drive it and the ratio of flow rates handled by the two pumps.

Sol^n: Given data:

$$\frac{D_m}{D_p} = \frac{1}{5}, \quad N_m = 1000 \text{ rpm}, \quad H_m = 8 \text{ m}, \quad P_m = 30 \text{ kW}$$

$H_p = 25$ m,

Determine: $N_p = ?$, $P_p = ?$ and $\frac{Q_p}{Q_m} = ?$

[i] Speed of prototype

From head coefficient.

$$\left(\frac{gH}{N^2D^2}\right)_m = \left(\frac{gH}{N^2D^2}\right)_p$$

$$N_p^2 = \frac{H_p}{H_m} \times \left(\frac{D_m}{D_p}\right)^2 \times N_m^2 = \frac{25}{8} \times \left(\frac{1}{5}\right)^2 \times (1000)^2$$

$$\therefore \boxed{N_p = 353.5 \text{ rpm}}$$

(ii) Power developed by the prototype.

From power coefficient.

$$\left(\frac{P}{N^3D^5}\right)_m = \left(\frac{P}{N^3D^5}\right)_p.$$

$$P_p = P_m \times \left(\frac{D_p}{D_m}\right)^5 \times \left(\frac{N_p}{N_m}\right)^3 = 30 \times 5^5 \times \left(\frac{353.5}{1000}\right)^3$$

$$\therefore \boxed{P_p = 4143 \text{ kW.}}$$

(iii) Ratio of flow rates

From flow coefficient

$$\left(\frac{Q}{ND^3}\right)_m = \left(\frac{Q}{ND^3}\right)_p$$

$$\frac{Q_p}{Q_m} = \left(\frac{D_p}{D_m}\right)^3 \times \left(\frac{N_p}{N_m}\right)$$

$$= 5^3 \times \frac{353.5}{1000}$$

$$\therefore \boxed{\frac{Q_p}{Q_m} = 44.187}$$

OR

Q 2 (a)

Static state and stagnation or Total state.

(i) Static state : If properties of fluid are measured with instruments or devices which are at rest relative to the fluid are called as static properties.

Example : For measuring temperature of any fluid particles moving with a given speed, the measuring thermometer should theoretically move with a same as that of fluid particles.

(ii) Stagnation or total state : It is defined as the terminal state of a fictitious, isentropic and work free process, during which kinetic and potential engeries are reduced respect to an arbitrary and pre-specified datum state.

Example : The properties measured by inserting instrument like pitot tube and thermocouples in flowing fluid stream are approximated to stagnation properties. Static properties are indicated by P (pressure), S (entropy), h (enthalpy), T (Temperature) etc. Where as stagnation properties are indicated by a subscript in front of the given property symbol or notation.

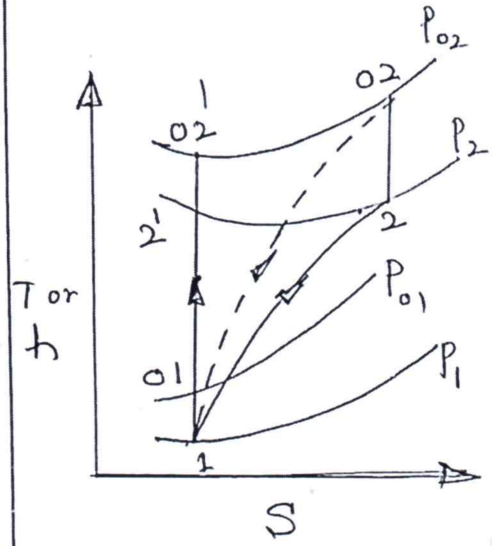
ie, P_o (pressure), S_o (entropy), h_o (enthalpy), T_o (Temperature) etc, Q 2(b)

Overall isentropic efficiency

(i) Total to - Total efficiency (η_{t-t}):

It is defined as the ratio of ideal work to the actual work between the stagnation states.

i.e. $$\eta_{t-t} = \frac{W_{ideal}}{W_{act.}}$$



$$= \frac{h_{02}' - h_{01}}{h_{02} - h_{01}} = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}}$$

for isentropic process 01 – 02'

$$\frac{T_{02}'}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore T_{02}' = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{02}' - T_{01} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - T_{01}$$

$$= T_{01}\left(P_{r0}^{\frac{\gamma-1}{\gamma}} - 1\right)$$

where, P_{r0} = stagnation pressure ratio

$$\therefore \boxed{\eta_{t-t} = \frac{T_{01}\left(P_{r0}^{\frac{\gamma-1}{\gamma}} - 1\right)}{T_{02}' - T_{01}}}$$

(ii) Static-to-static efficiency (η_{s-s})

It is defined as the ratio of ideal work to the actual work between the states. When gas velocities at entry and exit are neglected, the stagnation properties becomes equal to static properties.

$$\eta_{s-s} = \frac{W_{ideal}}{W_{act.}} = \frac{T_2' - T_1}{T_2 - T_1}$$

Now, $$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = P_r^{\frac{\gamma-1}{\gamma}}$$

$$T_2' = T_1 P_r^{\frac{\gamma-1}{\gamma}}$$

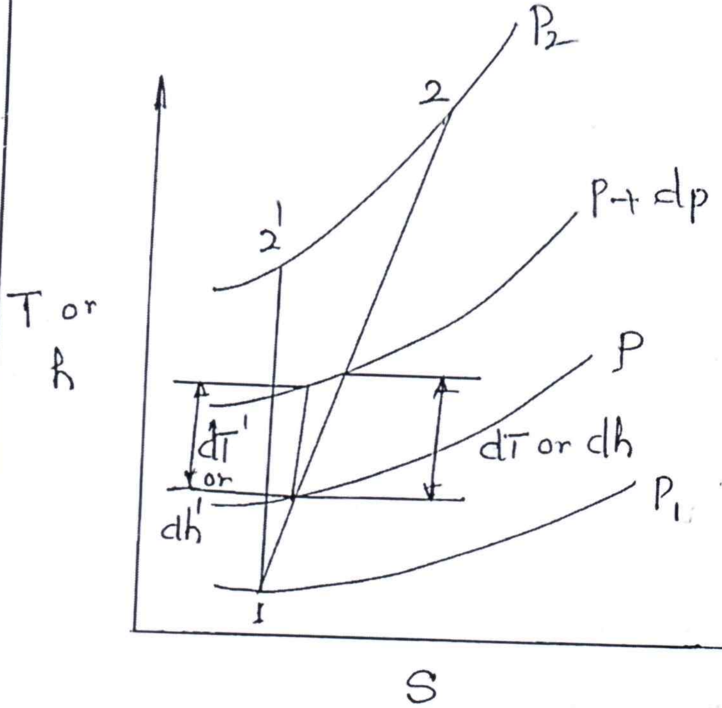
$$T_2' - T_1 = T_1 P_r^{\frac{\gamma-1}{\gamma}} - T_1 \quad \left[\text{Add } -T_1 \text{ for Both LHS \& RHS}\right]$$

$$= T_1\left[P_r^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\therefore \boxed{\eta_{s-s} = \frac{T_1\left(P_r^{\frac{\gamma-1}{\gamma}} - 1\right)}{(T_2' - T_1)}}$$

Q 2[C]

Stage efficiency or Polytropic efficiency.



Consider a single stage compression process working between the pressure P_1 & P_2

A finite compression stage is assumed to be made of infinitesimal no. of small stages. Each of these small stages has an efficiency η_p called polytropic efficiency.

$$\therefore \eta_p = \frac{\text{Isentropic Temperature rise.}}{\text{Actual temperature rise.}}$$

$$\eta_p = \frac{dT'}{dT} = \frac{dh'}{dh}$$

But, $dh = C_p dT$

$dh' = Tds - Vdp$.

For sentropic process $Tds = 0$

$\therefore dh' = Vdp$.

$$\eta_p = \frac{Vdp}{C_p dT}$$

But $V = \frac{RT}{P}$

$$\therefore \eta_p = \frac{RT}{P} \times \frac{dp}{C_p dT}$$

$\left[\because C_p = \frac{\gamma R}{\gamma - 1}\right]$

$$\frac{dT}{T} = \left(\frac{\gamma - 1}{\gamma}\right) \frac{1}{\eta_p} \frac{dp}{P} \quad \text{—— (1)}$$

Integrating

$$\log_e T = \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\eta_p}\log_e P + \log_e(\text{constant})$$

$$= \log_e P^{\frac{\gamma-1}{\gamma\cdot\eta_p}} + \log_e(\text{constant})$$

$$= \frac{T}{P^{\frac{\gamma-1}{\gamma\cdot\eta_p}}} = \text{constant}.$$

or $$\frac{T_1}{P_1^{\frac{(\gamma-1)}{\gamma\cdot\eta_p}}} = \frac{T_2}{P_2^{\frac{(\gamma-1)}{\gamma\cdot\eta_p}}}$$ or $$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma\cdot\eta_p}} \quad \text{---} \quad (2)$$

Integrating equation (1) between T_1 & T_2 & P_1 & P_2

$$\int_1^2 \frac{dT}{T} = \left(\frac{\gamma-1}{\gamma}\right)\eta_p \int_1^2 \frac{dp}{p}$$

$$\log_e \frac{T_2}{T_1} = \left(\frac{\gamma-1}{\gamma}\right)\eta_p \log_e \frac{P_2}{P_1}$$

$$\therefore \boxed{\eta_p = \frac{\log_e\left(\frac{T_2}{T_1}\right)}{\left(\frac{\gamma-1}{\gamma}\right)\log_e\left(\frac{P_2}{P_1}\right)}}$$

Module – 2.

Q 3 (a)

Relation between degree of reaction and Utilization factor.

We know that,

Degree of Reaction (R)

$$R = \frac{\text{Static enthalpy change in rotor}}{\text{Total enthalpy change in rotor.}}$$

$$= \frac{\cancel{\tfrac{1}{2}}\left[(u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]}{\cancel{\tfrac{1}{2}}\left[(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]}$$

$$R = \frac{(u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{\left[(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]}$$

By cross multiplying,

$$R\left[(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right] = (u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)$$

$$= \left[(u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right](1 - R) = R\,(V_{r_2}^2 - V_{r_1}^2)$$

$$(u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2) = \frac{R}{(1-R)}(V_1^2 - V_2^2) \quad \text{—— (1)}$$

But Utilization factor

$$\epsilon = \frac{\left[(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]}{\left[V_1^2 + (u_1^2 + u_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]}$$

— (2)

Substituting equation 1 in the eq^n 2. We get.

$$\epsilon = \frac{(V_1^2 - V_2^2) + \frac{R}{(1-R)}(V_1^2 - V_2^2)}{V_1^2 + \left(\frac{R}{1-R}\right)(V_1^2 - V_2^2)}$$

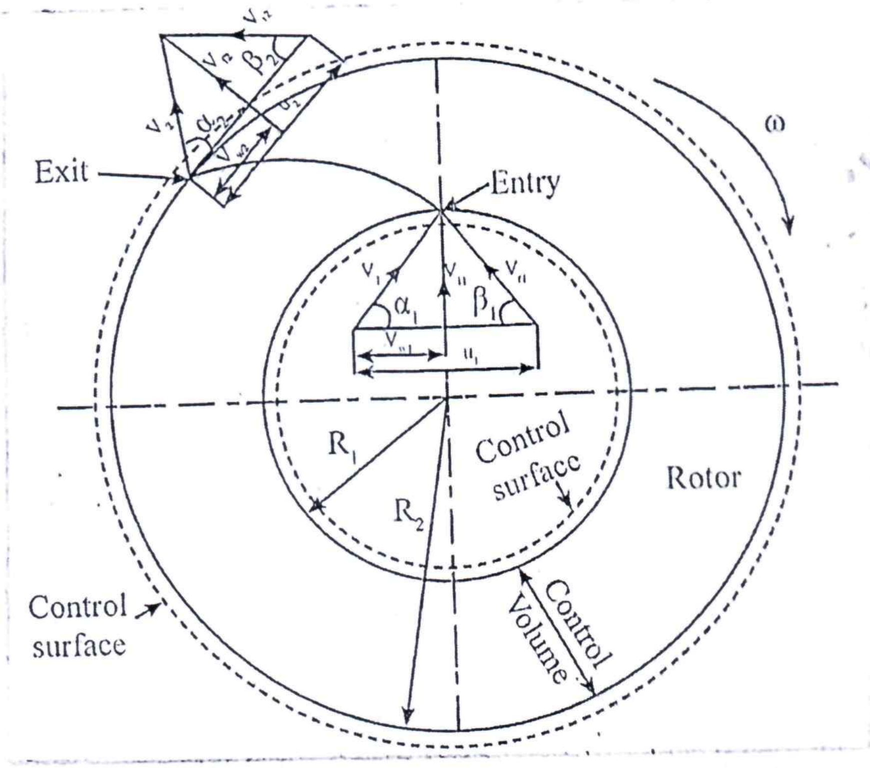
$$= \frac{(V_1^2 - V_2^2) + \left[1 + \frac{R}{1-R}\right]}{\frac{V_1^2 - RV_1^2 + RV_1^2 - RV_2^2}{1-R}}$$

$$= \frac{(V_1^2 - V_2^2)\left(\frac{1-\cancel{R}+\cancel{R}}{\cancel{1-R}}\right)}{\frac{V_1^2 - RV_2^2}{\cancel{(1-R)}}}$$

$$\boxed{\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}}$$

This is the general relation between ϵ and R for any type of turbine. It is valid under degree of reaction except $\boxed{R = 1}$

Q3(b)



let, V = Absolute Velocity of fluid (m/s).

N = Speed of the wheel (rpm)

r = Radius of the wheel (m)

ω = Angular Velocity of wheel (rad/s)

u = Linear Velocity of vane tip (m/s)

$\dot{m}$ = Mass flow rate of the fluid (kg/s)

Suffix '1' refers to the values at inlet and suffix '2' refers to the values at outlet.

Tangential momentum of fluid at entry = $V_{w_1} \dot{m}$ N

Momentum of momentum or angular momentum at entry = $V_{w_1} \dot{m} r_1$ Nm

Similary,
Angular momentum at outlet = $V_{w_2} \dot{m} r_2$ Nm

T = Torque on the wheel = change of angular momentum

$$T = (V_{w_1} r_1 - V_{w_2} r_2)\dot{m}$$

Work done = Rate of Energy transfered.

WD = Torque x Angular Velocity.

$$W.D = (V_{w_1} r_1 - V_{w_2} r_2)\omega \dot{m} \quad \frac{Nm}{s} \text{ or } W.$$

But, we have, $\omega r_1 = u_1$ and $\omega r_2 = u_2$.

$$\therefore \boxed{WD = (V_{w_1} u_1 - V_{w_2} u_2)\dot{m}} \quad W \quad \text{——} (1)$$

$$\boxed{\frac{W.D}{\text{unit mass flow rate}} = (V_{w_1} u_1 - V_{w_2} u_2)} \quad \frac{J}{kg}. \quad \text{——} (2)$$

Equation (1) & (2) are forms of the Euler turbine equation or Euler equation.

This is applied to all turbo machines like pumps, fans, blowers, turbines [Gas, steam, Water] and compressors.

* If $V_{w_1} u_1 > V_{w_2} u_2$ of eq^n (2) is positive, then machine is turbine.

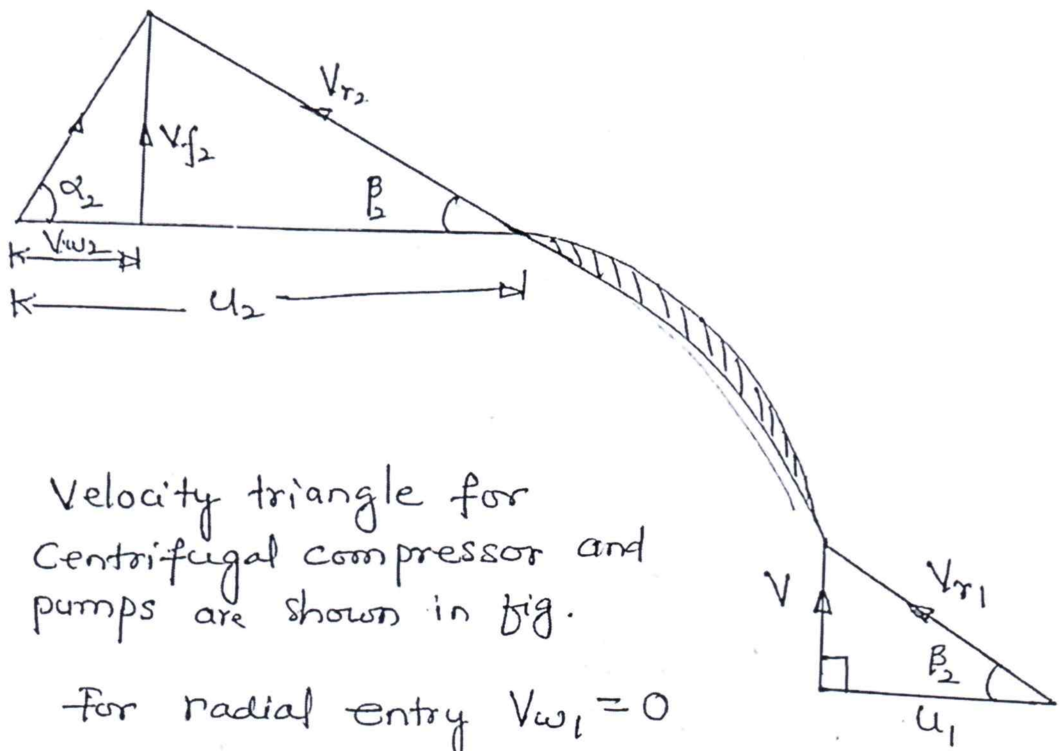
* If $V_{w_2} u_2 > V_{w_1} u_1$ of eq^n (2) is negative, then machine is called pump, fan, compressor and blower.

N.F.

[OR]

Q4 (a)

Heat-capacity relationship for centrifugal pump.



Velocity triangle for Centrifugal compressor and pumps are shown in fig.

For radial entry $V_{w_1} = 0$

$\therefore$ Work done $W = V_{w_2} u_2$

From outlet triangle.

$$\tan \beta_2 = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan\beta_2} \quad \text{or} \quad V_{w_1} = u_2 - \frac{V_{f_2}}{\tan\beta_2}$$

$$\therefore \text{W.D} \quad W = \left(u_2 - \frac{V_{f_2}}{\tan\beta_2}\right) u_2. \qquad \text{—— (1)}$$

But energy transfer / unit mass of fluid.

$$= gH \qquad \text{—— (2)}$$

Equating ① & ②.

$$gH = \left(u_2 - \frac{V_{f2}}{\tan\beta_2}\right) u_2$$

∴ Head generated $H = \dfrac{\left(u_2 - \frac{V_{f2}}{\tan\beta_2}\right) u_2}{g}$

or $H = \dfrac{u_2}{g}\left[u_2 - V_{f2}\cot\beta_2\right]$

The volume flow rate on capacity Q is equal to the product of radial velocity V_{f2} and discharge area A_2 normal to V_{f2}.

i.e., $Q = A_2 V_{f2}$

where $A_2 = \pi D_2 B_2$ | D_2 = diameter, B_2 = Breadth } at exit.

or $V_{f2} = \dfrac{Q}{A_2}$

∴ Head generated $H = \dfrac{u_2}{g}\left(u_2 - \dfrac{Q\cot\beta_2}{A_2}\right)$

$$H = \frac{u_2^2}{g} - \frac{u_2\cot\beta_2}{A_2 g} Q$$

For given a machine at a constant speed u_2, A_2 and β_2 are fixed.

∴ $H = K_1 - K_2 Q$.

where, $K_1 = \dfrac{u_2^2}{g}$, $K_2 = \dfrac{u_2\cot\beta_2}{A_2 g}$

where, K_2, determines wheather the slope of H V/s Q line is positive or Negative.

Q4(b) 2. An inward flow radial turbine has the following data. Power = 150 kW, speed = 32000 rpm. outer diameter of the impeller = 20cm, inner diameter of the impeller = 8 cm. Absolute velocity of gas at entry = 387 m/s. Absolute velocity of gas at exit = 193 m/s and is radial in direction. Constructed the velocity triangle at entry and exit of the impeller and determine.

i) Mass flow rate

ii) Percentage entry transfer due to change of radius.

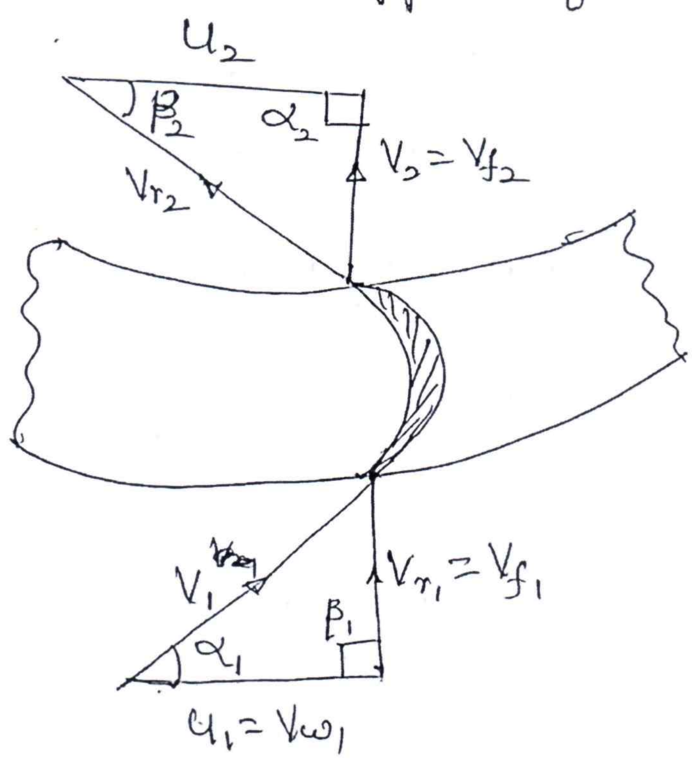
Sol^n: Given data : $P = 150\,kW$, $N = 32000$ rpm.

$D_1 = 26\,cm = 0.26\,m$

$D_2 = 8\,cm = 0.08\,m$

$V_1 = 387\,m/s$, $V_2 = 193\,m/s$, $\alpha_2 = 90^\circ$

$m = ?$, % energy transfer = ?



Impeller Velocity at inlet and exit.

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 32000}{60} = 335.1 \text{ m/s}.$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.08 \times 32000}{60} = 134.04 \text{ m/s}.$$

Energy transfer

$$W = E = V_{\omega_1} u_1 - V_{\omega_2} u_2$$

$$= V_{\omega_1} u_1$$

$$= u_1^2 = 335.1^2$$

$$E = 112.29 \text{ kJ/kg}.$$

$V_{\omega_2} = 0$ $\because \alpha_2 = 90°$

& $u_1 = V_{\omega_1}$ for $\beta_1 = 90°$

Power $P = m \times E$

$$150 = m \times 112.29$$

$$\therefore \boxed{m = 1.335 \text{ kg/s.}}$$

Energy transfer due to change of radius.

$$= \frac{1}{2}(u_1^2 - u_2^2) = \frac{1}{2}(335.1^2 - 134.04^2)$$

$$= 47.16 \text{ kJ/kg}.$$

percentage of energy transfer $= \dfrac{47.16}{E}$

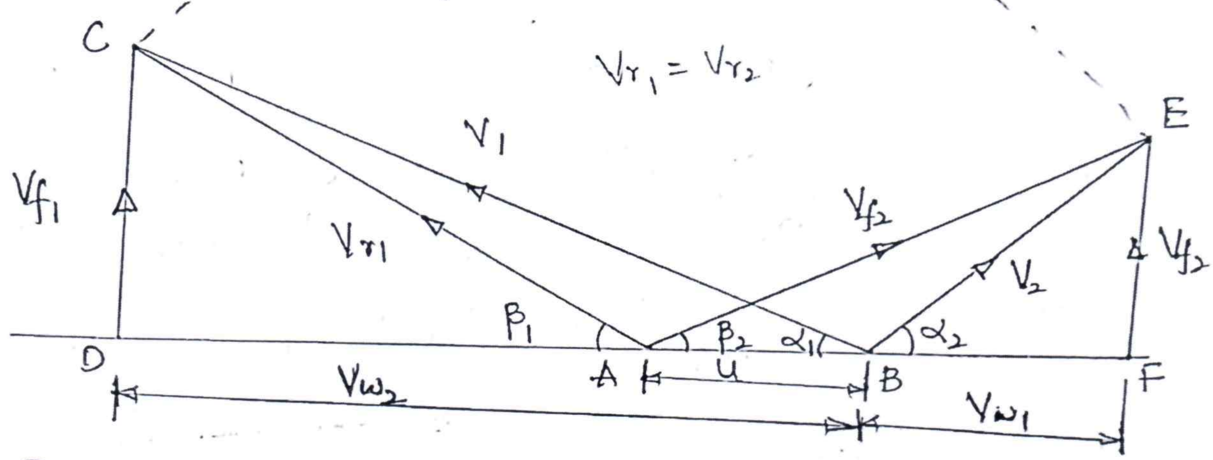
$$= \frac{47.16}{112.29} \times 100$$

$$\boxed{\% \text{ E transfer} = 42\%}$$

Q 5(a)

Combined Velocity diagram for Delaval turbine.

To make the solution of problems connected with turbines easier, the inlet and outlet triangles are combined by super-imposing one over the other.



Procedure:

1) Draw AB = u to a certain scale.
2) Draw BC = V_1 at an angle α_1 to AB.
3) Join AC, AC = V_{r_1}
4) Draw vertical line DC which meet AB at D projected DC = V_{f_1}, BD = V_{w_1}
5) Draw a line from a A subtending at an angle β_2
6) Draw an arc equal to AC, with A as centre ie $V_{r_1} = V_{r_2}$.
7) joint EB, EB = V_2. It makes an angle α_2 with AB
8) Draw Vertical EF to AB projected at E, EF = V_{f_2}, BF = V_{w_2}.

Now Blade efficiency (η_b)

$$\eta_b = \frac{2u(V_{w_1} + V_{w_2})}{V_1^2} = \frac{2u[V_1\cos\alpha_1 - u][1+K_1 Z]}{V_1^2} \quad \text{—— (1)}$$

$$= 2\left[\frac{uV_1\cos\alpha_1}{V_1^2} - \frac{u^2}{V_1^2}\right][1+K_1 Z]$$

$$= 2\left[\frac{u}{V_1}\cos\alpha_1 - \frac{u^2}{V_1^2}\right][1+K_1 Z]$$

$$\eta_b = 2[\phi\cos\alpha_1 - \phi^2][1+K_1 Z]$$

where,
ϕ = Speed ratio
$\phi = \frac{u}{V_1}$

for maximum efficiency

$$\frac{d\eta_b}{d\phi} = 0$$

$$\frac{d}{d\phi}\left[2(\phi\cos\alpha_1 - \phi^2)\right][1+K_1 Z]$$

$$\cancel{2}\cos\alpha_1 - \cancel{4}\phi = 0$$

$$\therefore \boxed{\phi = \frac{\cos\alpha_1}{2}} \quad \text{—— (2)}$$

This is the condition for maximum efficiency.

$$\therefore \eta_{max} = 2\left[\frac{\cos\alpha_1}{2} \times \cos\alpha_1 - \frac{\cos^2\alpha_1}{4}\right][1+K_1 Z]$$

$$= \frac{\cos^2\alpha_1}{2}[1+K_1 Z]$$

$$\frac{\cancel{2}\cos^2\alpha_1}{\cancel{2}} - \frac{\cancel{2}\cos^2\alpha_1}{\cancel{4}_2}$$

$$\frac{2\cos^2\alpha_1 - \cos^2\alpha_1}{2}$$

$$= \frac{\cos^2\alpha_1}{2}$$

For symmetrical blading, $\beta_1 = \beta_2$

$$\therefore Z_1 = \frac{\cancel{\cos\beta_2}}{\cancel{\cos\beta_1}} = 1$$

For no friction in the blade, $V_{r_1} = V_{r_2}$

$$\therefore K = 1$$

$$\therefore \boxed{\eta_{b\,max} = \cos^2\alpha_1}$$

(5)(b)

1. The nozzle angle of De-Laval turbine delivers 2 kg/s of steam at a speed of 2400 m/s. The nozzles are inclined at an angle of 16° to the plane of the wheel. The blade velocity is 600 m/s. Allowing a blade velocity coefficient of 0.72.

Calculate: (i) Blade efficiency

(ii) power developed by the blades.

(iii) Energy lost in the blades/s.

The blade angle at outlet may be taken as 25°.

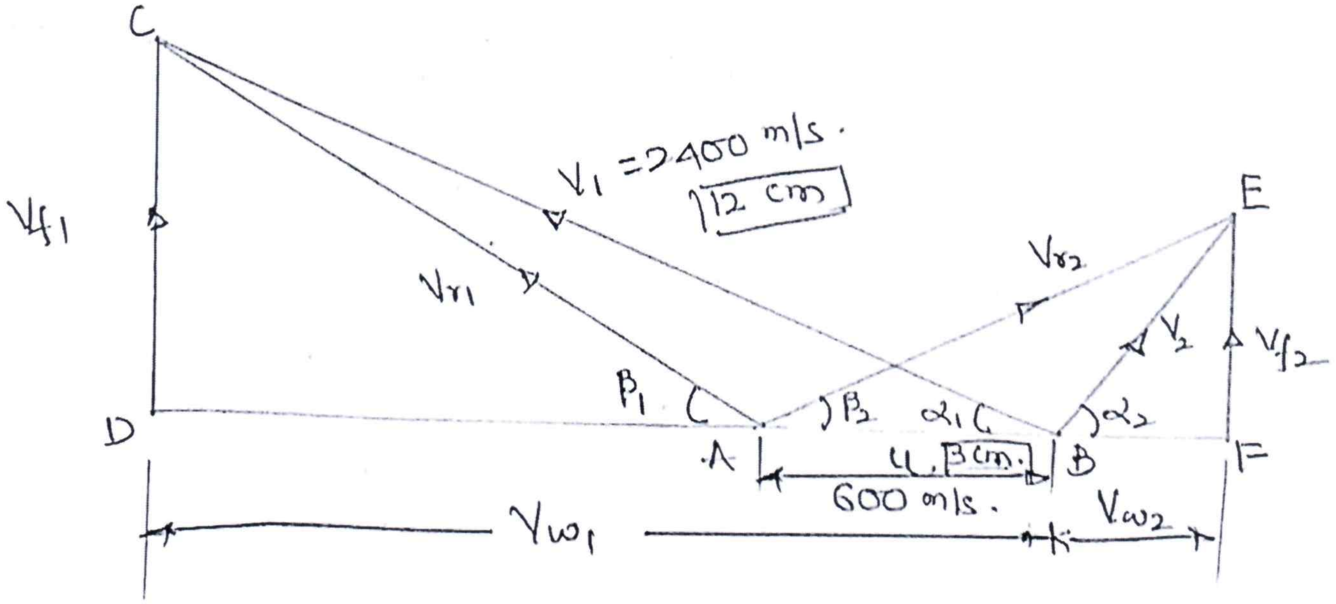
Solⁿ: Given data:

$m = 2$ kg/s. $V_1 = 2400$ m/s, $\alpha_1 = 16°$, $u = 600$ m/s.

$K = 0.72$. $\eta_b = ?$, $P = ?$, Energy lost $= ?$

$\beta_2 = 25°$.

$\boxed{\text{scale} = 200 \text{ m/s} = 1 \text{ cm}}$



From inlet Δ^{le}

$V_{r_1} = 9 \text{ cm} = 9 \times 200 = 1800 \text{ m/s}.$

$V_{r_2} = 0.72 \times 1800 = 1296 \text{ m/s}. = \boxed{6.5 \text{ cm}}$

We have, $\beta_2 = 25°$

with this data, outlet Δ^{le} is connected [constructed]

From combined velocity triangle.

$V_{w_1} = 11.63 \text{ cm} = 11.6 \times 200 = 2320.0 \text{ m/s}.$

$V_{w_2} = 2.7 \text{ cm} = 2.7 \times 200 = 540 \text{ m/s}.$

i) Blade efficiency

$$\eta_b = 2u\left(\frac{V_{w_1} + V_{w_2}}{V_1^2}\right) \quad \text{for } \alpha_2 < 90°.$$

$$= \frac{2 \times 600 (2320 + 540)}{2400^2} = 0.5958$$

$$\therefore \boxed{\eta_b = 59.58 \%}$$

ii) Power developed by the blades.

$$P = \frac{F_1 \times u}{1000} = \frac{m(V_{w_1} + V_{w_2})u}{1000} = \frac{2(2320 + 540) \times 600}{1000}$$

$$\therefore \boxed{P = = 3432 \text{ kW}}$$

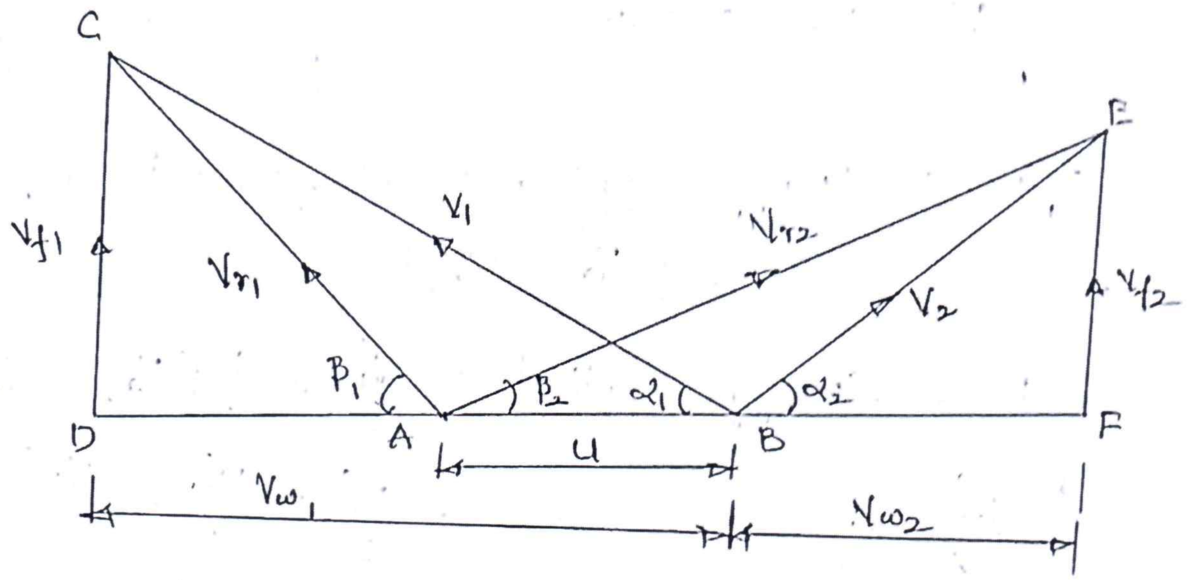
iii) Energy lost in the blades/s.

$$= \frac{1}{2} m [V_{r_1}^2 - V_{r_2}^2] = \frac{1}{2} \times 2 [1800^2 - 1296^2]$$

$$= 1560 \text{ Nm/s}.$$

[OR]

Q 6 (a) condition for maximum efficiency of a Reaction Turbine.



Assumptions:

(i) 50% Reaction turbine, ie., $\Delta h_m = \Delta h_s$.

(ii) $\beta_1 = \alpha_2$, $\beta_2 = \alpha_1$

(iii) $V_{r_2} = V_1$ ie., Velocity of steam at exit from the preceeding stage is same as that of velocity of steam at the entrance to the succeeding stage.

Now, $W = (V_{w_1} + V_{w_2})u$.

From velocity Δ^{le}, $W = (DB + BF)u = [DB + (AF - AB)]u$

$= [V_1 \cos\alpha_1 + (V_{r_2}\cos\beta_2 - u)]u$

But, $\beta_1 = \alpha_2$, $\beta_2 = \alpha_1$ and $V_{r_2} = V_1$

$\therefore$ $W = [V_1\cos\alpha_1 + V_1\cos\alpha_1 - u]u$

$= [2V_1\cos\alpha_1 - u]u$

$W = 2uV_1\cos\alpha_1 - u^2$

Deviding and Multiplying by V_1^2

$\therefore$ $W = \dfrac{V_1^2[2uV_1\cos\alpha_1 - u^2]}{V_1^2}$

Efficiency of the turbine

$$\eta = \frac{WD/s}{\text{Enthalpy drop}} = \frac{\cancel{V_1^2}\left[2\phi\cos\alpha_1 - \phi^2\right]}{\frac{\cancel{V_1^2}}{2}\left[1 + 2\phi\cos\alpha_1 - \phi^2\right]}$$

$$\eta = \frac{2\left[2\phi\cos\alpha_1 - \phi^2\right]}{\left[1 + 2\phi\cos\alpha_1 - \phi^2\right]}$$

$$= \frac{2\left[2\phi\cos\alpha_1 - \phi^2 + 1 - 1\right]}{\left[1 + 2\phi\cos\alpha_1 - \phi^2\right]}$$

$$= \frac{2\left[1 + 2\phi\cos\alpha_1 - \phi^2\right] - 2}{\left[1 + 2\phi\cos\alpha_1 - \phi^2\right]}$$

$$\eta = 2 - \frac{2}{1 + 2\phi\cos\alpha_1 - \phi^2} \quad \text{—— (1)}$$

For maximum efficiency $1 + 2\phi\cos\alpha_1 - \phi^2$ should be maximum.

i.e., $\frac{d}{d\phi}\left[1 + 2\phi\cos\alpha_1 - \phi^2\right]$

$2\cos\alpha_1 - 2\phi = 0.$

$$\therefore \boxed{\phi = \cos\alpha_1} \quad \text{—— (2)}$$

this is the condition for maximum efficiency

Now substitute eq^n (2) in (1).

We get,

$$\eta_{max} = 2 - \frac{2}{1 + 2\times\cos\alpha_1\times\cos\alpha_1 - \cos^2\alpha_1}$$

$$= 2 - \frac{2}{1 + \cos^2\alpha_1} = \frac{\cancel{2} + 2\cos^2\alpha_1 - \cancel{2}}{1 + \cos^2\alpha_1}.$$

$$\boxed{\therefore \eta_{max} = \frac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1}}$$

Q6(b) 1. The following particular refer to a stage of an impulse reaction turbine. Outlet angle of fixed blade $= 20°$, outlet angle of moving blade $= 30°$, radial height of fixed and moving blades $= 1.0$ cm, mean blade velocity $= 138$ m/s, Blade speed ratio $= 0.625$, Specific volume of steam at fixed blade outlet $= 1.235$ m^3/kg, specific volume of steam at moving blade out $=$ 1.305 m^3/kg, speed of the rotor $= 3000$ rpm.

Calculate the degree of reaction, the adiabatic heat drop in pairs of blade rings and gross stages efficiency.

Given the following coefficient which are made same for both fixed and moving blades, $\eta = 0.9$, Carry over coefficient $= 0.86$.

Sol^n:

$\alpha_1 = 30°$, $\beta_2 = 30°$, $h_f = h_m = 10$ cm $= 0.1$ m,

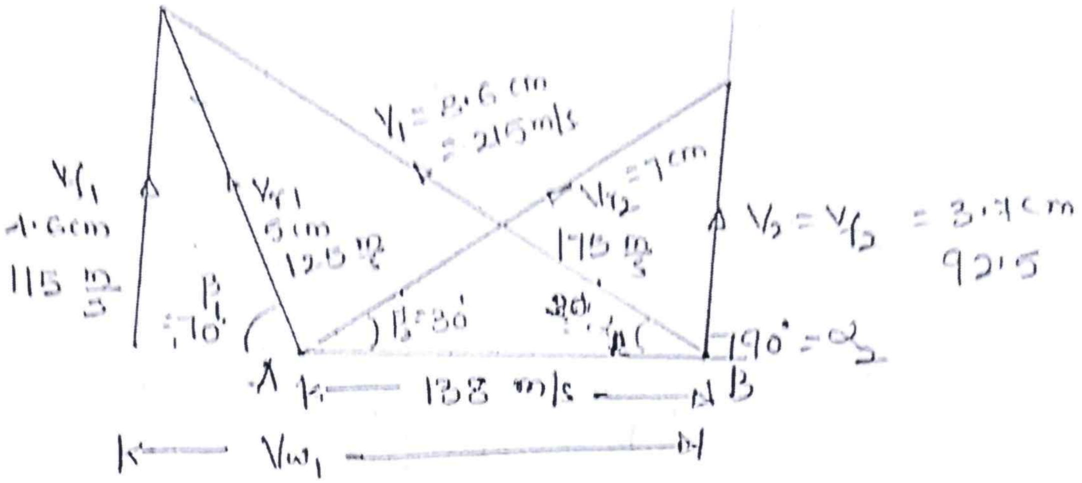
$u = 138$ m/s, $\frac{u}{V_1} = 0.625$, $V_{sf} = 1.235$ m^3/kg.

$V_{sm} = 1.305$ m^3/kg, $N = 3000$ rpm, $R = ?$, $\Delta H = ?$

$\eta_s = ?$ $\eta_n = 0.9$, $\psi = 0.86$.

Steam velocity, $V_1 = \frac{u}{0.625} = \frac{138}{0.625} = 220$ m/s.

Sol Scale 2.5 m/s = 1 cm.



from inlet Δ^{le}

V_{f_1} = 3cm × 2.5 = 75 m/s.

We have, $u = \frac{\pi D N}{60} = 138 = \frac{\pi \times D \times 3000}{60} \Rightarrow D = 0.878$ m.

fixed blade height, $h_f = \frac{m V_{sp}}{\pi D V_{f_1}}$

$$0.1 = \frac{m \times 1.235}{\pi \times 0.878 \times 75} \Rightarrow m = 16.76 \frac{kg}{s}.$$

Moving blade height, $h_m = \frac{m V_{sm}}{\pi D V_{f_2}}$

$$\therefore 0.1 = \frac{16.76 \times 1.805}{\pi \times 0.878 \times V_{f_2}}$$

$\therefore V_{f_2} = 99.29$ m/s = 3.2 cm.

from combined velocity diagram.

V_{w_1} = 4 cm × 2.5 = 100 m/s. // $V_{w_2} = 0$

V_{r_2} = 6.5 m × 2.5 = 165.5 m/s.

V_{w_1} = 8.5 cm × 2.5 = 212 m/s.

Module – 4.

Q7 (a)

Various efficiency

1] Hydraulic efficiency (η_h) : It is defined as the ratio of power given by water to the runner of turbine to the power supplied by the water at the inlet of the turbine.

ie $$\eta_h = \frac{\text{Workdone/s}}{\text{Kinetic energy/s}} = \frac{\rho a V_1 (V_{w_1} + V_{w_2}) u}{\frac{1}{2} \rho a V_1 \times V_1^2}$$

$$\boxed{\eta_h = \frac{2u [V_{w_1} \pm V_{w_2}]}{V_1^2}}$$

2. Mechanical Efficiency (η_m) : It is the ratio of the power available at the shaft of the turbine to the power delivered to the runner.

i.e., $$\eta_m = \frac{\text{Shaft work}}{\text{Water power at the runner}} = \frac{P}{WD/s.}$$

$$\therefore \eta_m = \frac{P}{\rho a V_1 (V_{w_1} \pm V_{w_2}) u}$$

3. Overall Efficiency (η_o) : It is the ratio of power available at the shaft of the runner to the power supplied by the water at the inlet of the turbine.

i.e., $$\eta_o = \frac{\text{Shaft power}}{\text{Water power at inlet of the turbine}} = \frac{P}{\frac{WQH}{1000}}.$$

4. <u>Volumetric Efficiency</u> (η_v):

It is the ratio of volumetric of water actually striking the runner to the volume of water supplied to the turbine.

i.e., $$\eta_v = \frac{Q - dQ}{Q}.$$

where, Q = Volume of water supplied from the reservoir.

dQ = Amount of water that ~~enenters~~ enters the tailrace without striking the turbine (runner).

Q7(b)

2] In a power station, a Pelton wheel produces 15000 kW under a head of 350 m, while running at 500 rpm. Assume a turbine efficiency of 0.84, coefficient of velocity for nozzle as 0.98, speed ratio 0.46 and bucket velocity-coefficient 0.86.

Calculate: i) No. of jets.

ii) Diameter of each jet.

iii) Tangential force exerted on the bucket deflect the jet through 165°.

Sol^n: $P = 15000$ kW, $H = 350$ m, $\eta_o = 0.84$, $C_v = 0.98$,

$\phi = 0.46$, $K = 0.86$, $\beta_2 = 180 - 165 = 15^\circ$.

$n = ?$, $d = ?$, $F = ?$

Jet Velocity

$$V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2\times 9.81\times 350} = 81.2 \text{ m/s}.$$

Bucket Velocity

$$u = \phi\sqrt{2gH} = 0.46\sqrt{2\times 9.81\times 350} = 38.11 \text{ m/s}.$$

Overall Efficiency

$$\eta_o = \frac{P}{\frac{WQH}{1000}} \Rightarrow 0.84 = \frac{15000}{\frac{9.81\times 1000\times Q\times 350}{1000}}$$

$$\therefore Q = 5.2 \text{ m}^3/\text{s}.$$

(i) Number of jets.

We know that,

Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{500\sqrt{15000}}{(350)^{5/4}} = 40.45 \text{ rpm}$

No. of jets $n = \frac{N_s \text{ for multi jet}}{N_s \text{ for single jet}}$

N_s for single jet pelton wheel is 30 rpm

$\therefore n = \frac{40.45}{30} = 1.34 \simeq 2$ jets.

$\boxed{n = 2 \text{ jets}}$

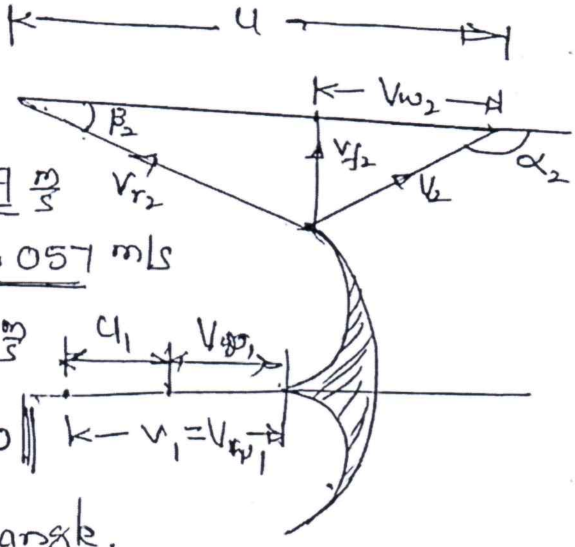
(ii) Diameter of each jet.

$Q = n \times aV_1 = n \times \frac{\pi}{4} d^2 V_1$

$5.2 = 2 \times \frac{\pi}{4} d^2 \times 81.2$

$\therefore \boxed{d = 0.202 \text{ m.}}$

(iii) Force exerted by the jet on buckets.



$V_{r_1} = V_1 - u = 81.2 - 38.11 = \underline{43.09 \frac{m}{s}}$

$V_{r_2} = 0.86 V_{r_1} = 0.86 \times 43.09 = \underline{37.057} \text{ m/s}$

$V_{r_2} \cos\beta_2 = 37.057 \times \cos 15 = 35.79 \frac{m}{s}$

Since $V_{r_2} \cos\beta_2 < u$, we get, $\alpha_2 > 90$

$\therefore$ From outlet velocity triangle.

$\cos\beta_2 = \frac{u - V_{w_2}}{V_{r_2}} \Rightarrow \cos 15 = \frac{38.11 - V_{w_2}}{37.057}$

$V_{w_2} = 2.315$ m/s ..

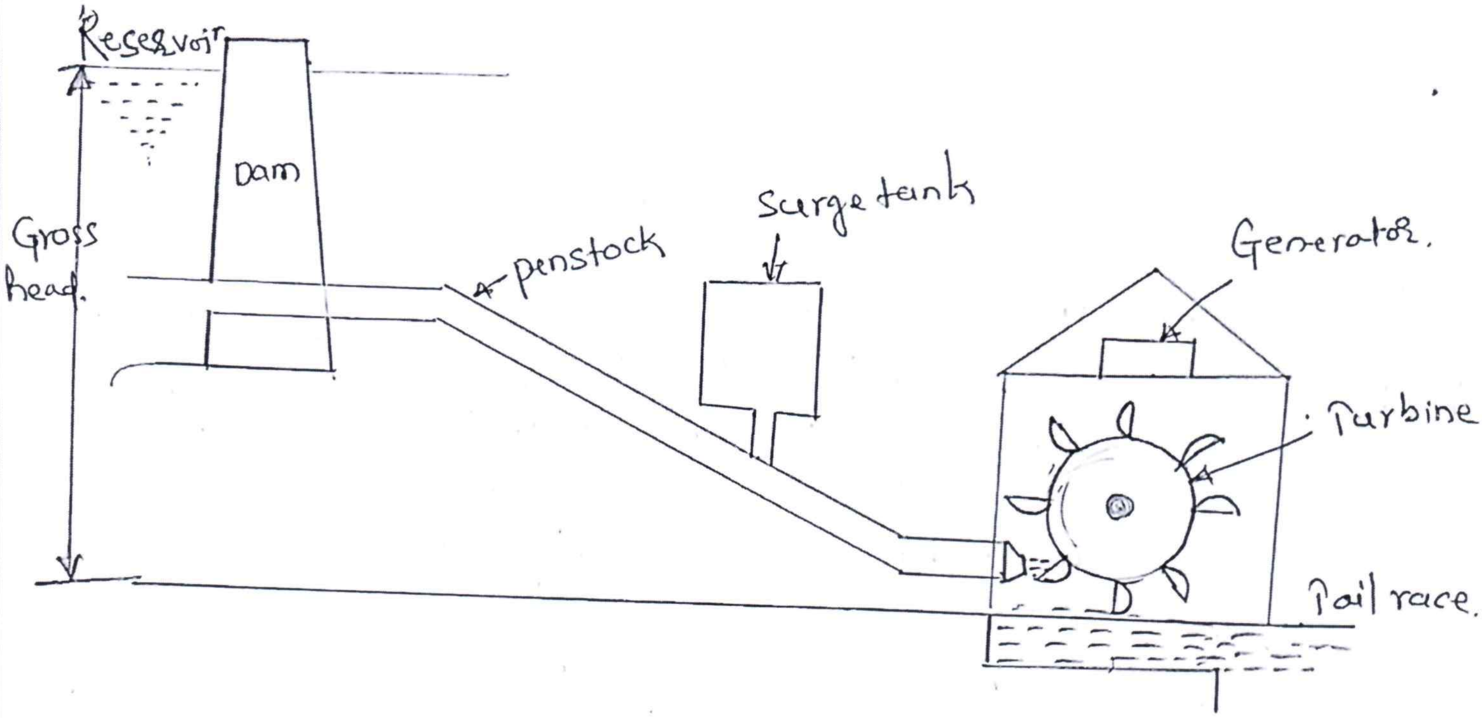
$\therefore F = \rho Q (V_{w_1} - V_{w_2}) = 1000 \times 5.2 \, (81.2 - 2.315)$

$\therefore \boxed{F = 410.2 \text{ N}}$

[OR]

Q 8 (a)

Hydroelectric power plant.



The main components of hydroelectric plant are

i) Dam to store water at a high level creating a reservoir.

ii) Penstock which are pressure pipes used to convey the water from the reservoir to the power house.

iii) Surge tank : to protect the pipe line from bursting due to sudden change in the discharge rate.

iv) Power house where turbine and generator are installed.

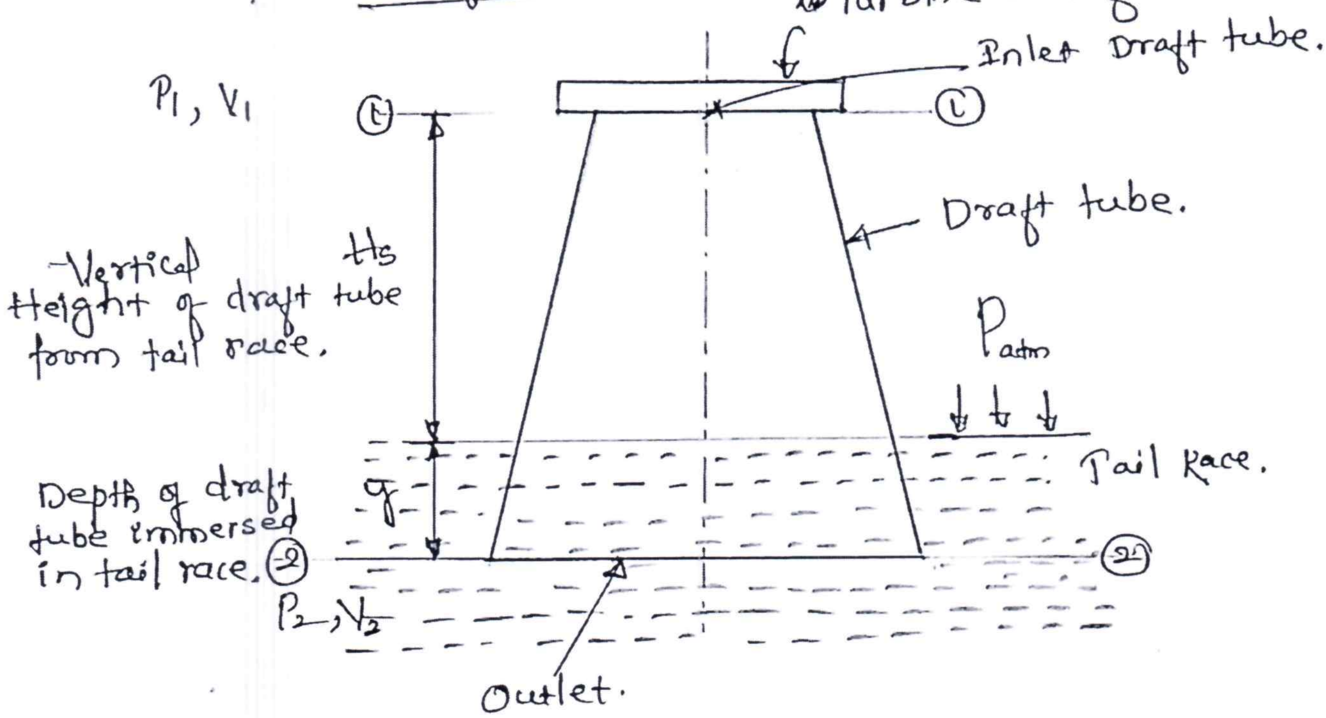
v) Head race : Free surface of water in the reservoir is called head race.

vi) Tail race : Free surface of water at the bottom of turbine is called tail race.

vii) Gross head : Height between head race and tail race is called as gross head.

Net head (H) = $H_g - H_f$ || where $h_f = \frac{4fLV^2}{2gD}$.

Q 8 (b). **Draft tube.**



Draft tube is a conical tube connected to the outlet of the turbine. In Reaction turbines at the exit of the runner, the pressure is less than atmospheric pressure. Due to this, water can't flow to the tail race effectively. To perform this function properly, a draft tube is connected at the exit of the runner. The other end of the tube is submerged below the level of the tail race.

The other functions of draft tube are,

(i) The turbine may be placed above the tail race and hence turbine may be inspected properly.

(ii) The kinetic energy rejected at the outlet of the turbine is converted into useful pressure energy.

Q(8) C

1) The following data is given for a Francis turbine. Net head = 70 m, Speed = 600 rpm, Power at the shaft = 367.5 kW, overall efficiency = 85%, hydraulic efficiency = 95%, flow ratio = 0.25 width ratio = 0.1, Outer diameter to inner diameter ratio = 2, The thickness of vanes occupies 10% of the circumferential area of runner, Velocity of flow is constant at inlet and discharge is radial at outlet. Determine:

(i) Guide blade angle (ii) Runner vane angle at inlet and outlet.

(iii) Width of the wheel at inlet.

(iv) Diameter of runner at inlet and outlet.

Solⁿ: Given data:

$H = 70\,m$, $N = 600\,rpm$, $P = 367.5\,kW$, $\eta_h = 0.95$,

$V_{f_1}/\sqrt{2gH} = 0.25$, $\frac{B_1}{D_1} = 0.1$, $\frac{D_1}{D_2} = 2.$

Actual area of flow (100% − 10%) = 90%.

$V_{f_1} = V_{f_2}$, $\alpha_1 = ?$, $\beta_1 = ?$, $\beta_2 = ?$, $D_1 = ?$, $B_1 = ?$

$D_2 = ?$

$$V_{f_1} = 0.25\sqrt{2gH} = 0.25\sqrt{2\times 9.81\times 70} = \underline{9.26\,m/s}.$$

$$\eta_0 = \frac{P}{\frac{W Q H}{1000}} \Rightarrow 0.85 = \frac{367.5}{\frac{9.81\times 1000\times Q\times 70}{1000}}$$

$$\therefore\ Q = 0.627\ \frac{m^3}{s}.$$

Runner diameter at inlet and outlet.

$$Q = 0.9\pi D_1 B_1 V_{f1}$$

$$0.625 = 0.9\pi \times D_1 \times 0.1 D_1 \times V_{f1}$$

$$= 0.9\pi \times D_1 \times 0.1 D_1 \times 9.26$$

$$\boxed{D_1 = 0.496\, m}$$

$$D_2 = \frac{D_1}{2} = \frac{0.496}{2} = 0.245\, m$$

$$\boxed{D_2 = 0.245\, m}$$

Width of wheel at inlet.

$$B_1 = 0.1 D_1 = 0.1 \times 0.496 = 0.0496\, m.$$

$$\boxed{B_1 = 0.0496\, m}$$

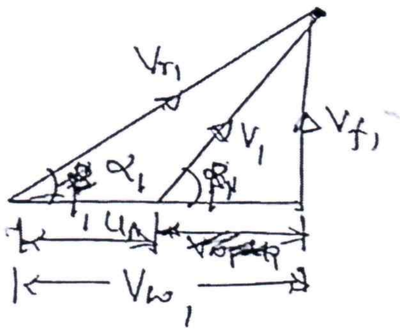
Guide blade angle.

$$\eta_h = \frac{V_{w1} u_1}{gH} \Rightarrow 0.95 = \frac{V_{w1} \times 15.39}{9.81 \times 70}.$$

$$\text{where, } u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.496 \times 600}{60}$$

$$\therefore u_1 = 15.39\ m/s,$$

From inlet Velocity Δ^{le}.



$$\tan\alpha_1 = \frac{V_{f1}}{V_{w1}} = \frac{12.02}{18.20}$$

$$\therefore \boxed{\alpha_1 = 33.44^\circ}$$

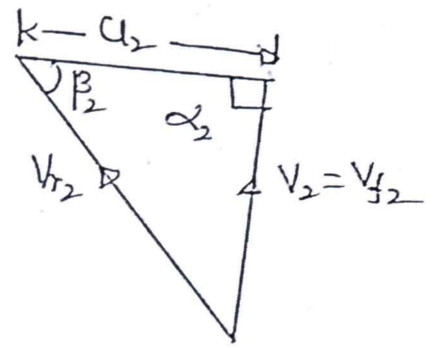
Vane angle at inlet.

Runner Vane angle at inlet and Outlet.

$$\tan\beta_1 = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.26}{42.38 - 15.39}$$

$$\beta_1 = 18.93^\circ$$

From outlet triangle



$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.245 \times 600}{60} = 7.6 \text{ m/s.}$$

$$\therefore \tan\beta_2 = \frac{V_{f_2}}{u_2} = \frac{9.26}{7.6}$$

$$\therefore \boxed{\beta_2 = 50.62^\circ}$$

Module - 5

Q9 (a) . Minimum starting speed of a centrifugal pump.

Centrifugal pump will start to lift the water when the pressure rise in the impeller is more than or equal to manometer head. When the impeller rotates the water in contact with the impeller is also rotates.
This is the forced vortex. For this case, centrifugal head or head to pressure rise in the impeller is equal to.

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g}.$$

But for lifting of water
Head due to pressure rise $\geq H_m$.

ie., $\dfrac{u_2^2}{2g} - \dfrac{u_1^2}{2g} \geq H_m.$

For minimum speed.

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m \qquad \text{——— (1)}$$

But $\eta_{man} = \dfrac{g H_m}{V_{w_2} u_2}$ or $H_m = \eta_{man} \times \dfrac{V_{w_2} u_2}{g}$ ——— (2)

Equating (1) and (2),

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g}$$

$$\frac{\left(\frac{\pi D_2 N}{60}\right)^2}{2g} - \frac{\left(\frac{\pi D_1 N}{60}\right)^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g}.$$

$$= \eta_{man} \times \frac{V_{w_2}}{g} \times \frac{\pi D_2 N}{60}$$

Dividing by $\frac{\pi N}{g \times 60}$ on both sides, we get.

$$\frac{\pi D_2^2 N}{120} - \frac{\pi D_1^2 N}{120} = \eta_{man} V_{w_2} D_2$$

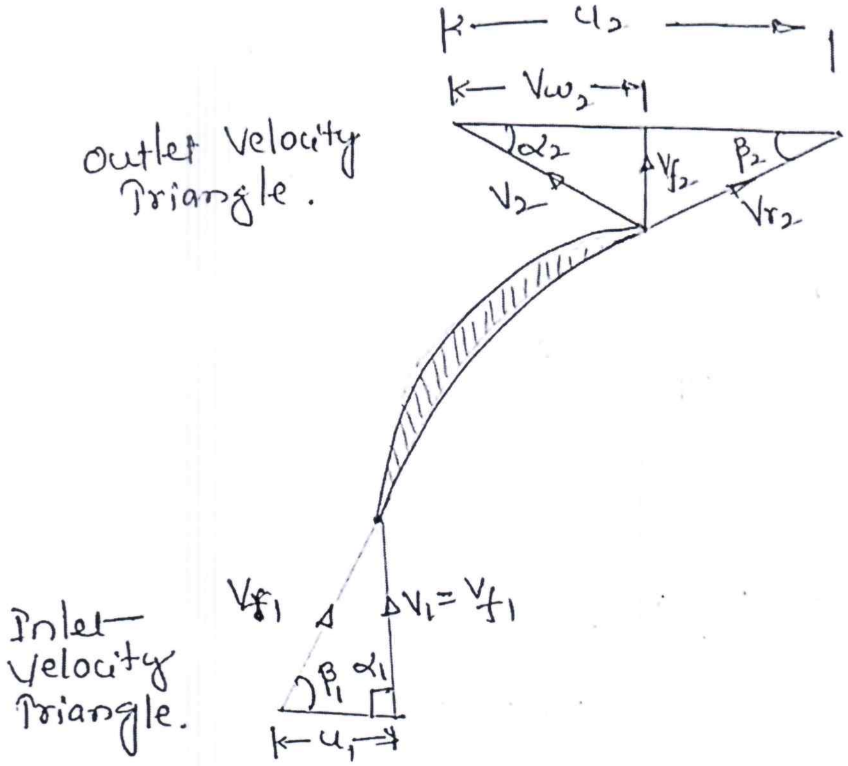
$$\frac{\pi N}{120}\left[D_2^2 - D_1^2\right] = \eta_{man} V_{w_2} D_2$$

$$\therefore \boxed{N = \frac{120 \times \eta_{man} \times V_{w_2} \times D_2}{\pi [D_2^2 - D_1^2]}}$$

It is the minimum speed for starting pump.

Q9(b)

Static Pressure Rise in the Impeller.



Applying Bernoulli's equation at inlet and outlet of the impeller.

$$\frac{P_i}{w} + \frac{V_i^2}{2g} + Z_i = \frac{P_o}{w} + \frac{V_o^2}{2g} + Z_o - \frac{V_{w_2} U_2}{g}$$

But $Z_i = Z_o$.

$$\therefore \frac{P_i}{w} + \frac{V_i^2}{2g} = \frac{P_o}{w} + \frac{V_o^2}{2g} - \frac{V_{w_2} U_2}{g}$$

$$\frac{P_o}{w} - \frac{P_i}{Pw} = \frac{V_i^2}{2g} - \frac{V_o^2}{2g} + \frac{V_{w_2} U_2}{g}$$

where, $\frac{P_o}{w} - \frac{P_i}{w}$ = pressure rise in impeller.

$\therefore$ Pressure rise in impeller.

$$= \frac{V_i^2}{2g} - \frac{V_o^2}{2g} + \frac{V_{w_2} U_2}{g}$$

From inlet velocity triangle.

$V_i = V_1 = V_f.$

From outlet velocity triangle., $V_o = V_2$.

$$\tan\beta_2 = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\beta_2} = u_2 - V_{f_2}\cot\beta_2$$

~~A~~lso, ~~V_f~~

$$V_2^2 = V_{f_2}^2 + V_{w_2}^2 = V_{f_2}^2 + (u_2 - V_{f_2}\cot\beta_2)^2$$

$$= V_{f_2}^2 + u_2^2 + V_{f_2}^2\cot^2\beta_2 - 2u_2 V_{f_2}\cot\beta_2$$

$$= V_{f_2}^2(1 + \cot^2\beta_2) + u_2^2 - 2u_2 V_{f_2}\cot\beta_2$$

$$= V_{f_2}^2 \operatorname{cosec}^2\beta_2 + u_2^2 - 2u_2 V_{f_2}\cot\beta_2$$

∴ Pressure ~~drop~~ rise (Δp)

$$\Delta p = \frac{V_{f_1}^2}{2g} - \frac{(V_{f_2}^2\operatorname{cosec}^2\beta_2 + u_2^2 - 2u_2 V_{f_2}\cot\beta_2)}{2g} + \frac{(u_2 - V_{f_2}\cot\beta_2)u_2}{2g}$$

$$= \frac{1}{2g}\left[V_{f_1}^2 - V_{f_2}^2\operatorname{cosec}^2\beta_2 - u_2^2 + \cancel{2u_2 V_{f_2}\cot\beta_2} + 2u_1^2 - 2u_2 \cancel{2V_{f_2}\cot\beta_2}\right]$$

$$\boxed{\therefore \text{Pressure Rise } (\Delta p) = \frac{1}{2g}\left[V_{f_1}^2 + u_2^2 - V_{f_2}\operatorname{cosec}^2\beta_2\right]}$$

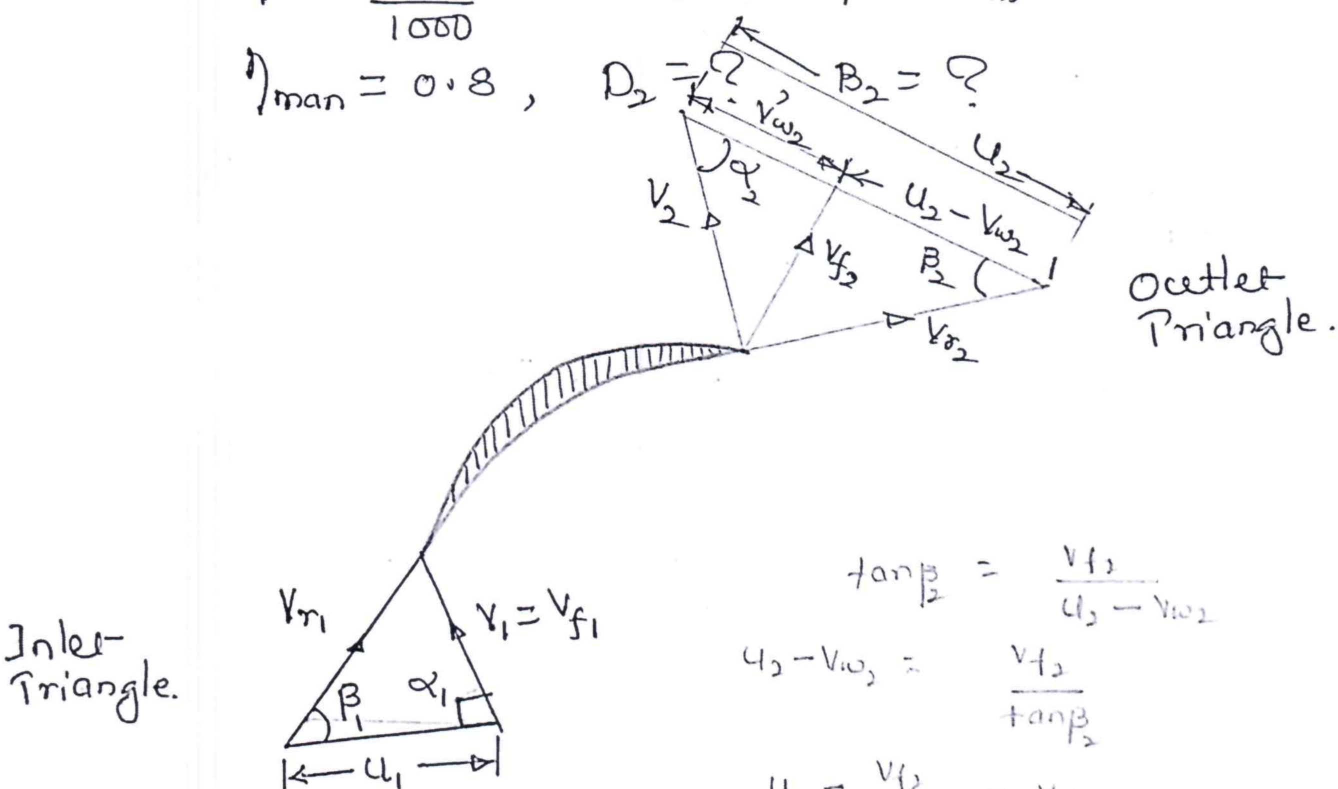
Q9(g). A centrifugal pump running at 1000 rpm. the outlet angle of vane is 45° and the velocity of flow at ~~outer~~ outlet is 2.5 m/s, the discharge through the pump is 200 lit/sec, when the pump is working against the total head of 20 m. If the manometric efficiency of the pump is 80%, Determine :

(i) Diameter of impeller (ii) Width of the impeller at outlet.

Solⁿ: Given data : $N = 1000$ rpm, $\beta_2 = 45^\circ$, $V_{f_2} = 2.5$ m/s.

$Q = \frac{200 \text{ lit/sec}}{1000} = 0.2 \text{ m}^3/\text{s}$. $H_m = 20$ m.

$\eta_{man} = 0.8$, $D_2 = ?$ $B_2 = ?$



$\tan\beta_2 = \frac{V_{f_2}}{u_2 - V_{w_2}}$

$u_2 - V_{w_2} = \frac{V_{f_2}}{\tan\beta_2}$

$u_2 - \frac{V_{f_2}}{\tan\beta_2} = V_{w_2}$

From outlet triangle.

$$\tan\beta_2 = \frac{V_{f_2}}{u_2 - V_{w_2}} \Rightarrow \tan(45) = \frac{2.5}{u_2 - V_{w_2}}$$

$$V_{w_2} = u_2 - 2.5 \quad \text{--- (1)}$$

We know that,

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} \Rightarrow 0.8 = \frac{9.81 \times 20}{V_{w_2} u_2}$$

$$\therefore V_{w_2} = \cancel{183.6}\ \frac{9.81 \times 20}{0.8 \times u_2} = \frac{245.25}{u_2} \quad \text{——— (2)}$$

Comparing eq^n (1) and (2).

$$u_2 - 2.5 = \frac{245.25}{u_2}$$

$$u_2^2 - 2.5u_2 = 245.25$$

$$u_2^2 - 2.5u_2 - 245.25 = 0.$$

By using quadratic eq^n.

$$ax^2 + bx + c = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

solving above equation using quadratic eq^n we get.

$a = 1$, $b = -2.5$, $c = -245.25$

$$\therefore u_2 = \frac{-(-2.5) \pm \sqrt{(-2.5)^2 - 4(1)(-245.35)}}{2(1)}$$

$$= 2.5 \pm \sqrt{6.25 + 245.35}$$

$$u_2 = 2.5 + 15.86 = 18.361$$

Also, $u_2 = 2.5 - 15.86 = -13.36.$

(i) Diameter of impeller.

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times D_2 \times 1000}{60} = 18.361$$

$$\therefore D_2 = \frac{18.36 \times 60}{\pi \times 1000} = 0.350 \text{ m.}$$

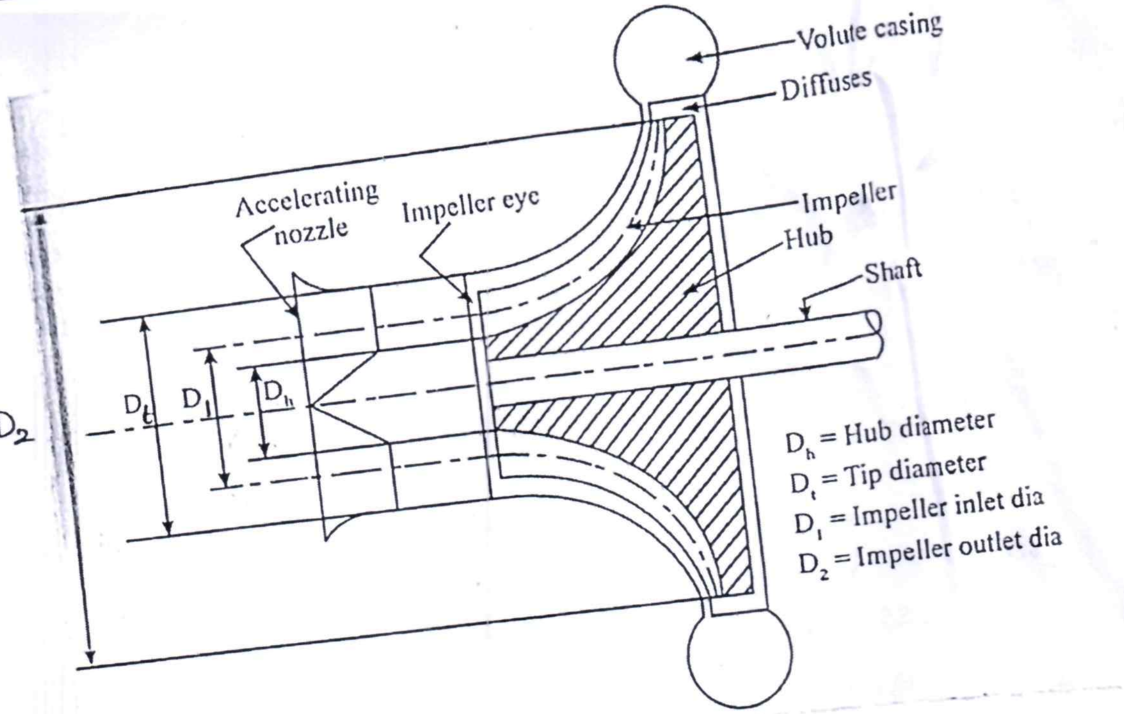
(ii) Width of the impeller.

$$Q = \pi D_2 B_2 V_{f_2} \Rightarrow 0.2 = \pi \times 0.350 \times B_2 \times 2.5$$

$$B_2 = 0.072 \text{ m}$$

OR

Q10 (a) Working of centrifugal compressors.



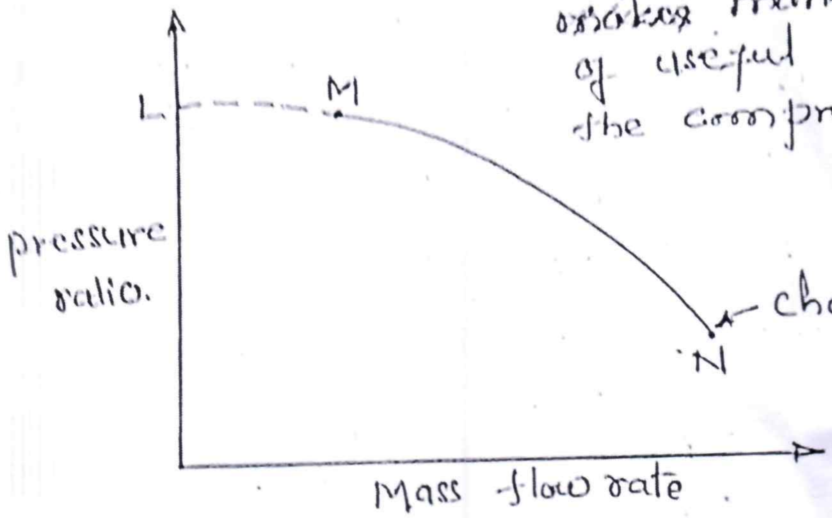
The gas (air or fluid) enters the impeller eye of a centrifugal compressor in an axial direction with absolute velocity, V_1. The gas then flows radially through the impeller passage due to centrifugal force. The impeller rotates at very high speed [20000 to 30000 rpm]. Energy is imparted to the gas by the rotating blades where it is converted into kinetic energy as it ~~is~~ moves from radius r_1 to r_2, along with there is a small pressure rise during its radial flow in the impeller. The impeller vanes at the eye are bent to provide shockless energy.

The gas leaving from the impeller blades turns through an angle, β_2 and leaves with an absolute Velocity, V_2 at an angle α_2. The gas then enters the diffuser. The diffuser surrounding the impeller converts the kinetic Energy into pressure energy. Hence there is raise in static pressure of the gas. Gas then, enters the casing, and the outlet pipe, where some more kinetic Energy into pressure Energy.

The Clearence between the impeller blades and inner wall of the casing must be kept as small as possible to reduce leakage. Modren compressors are designed in such a way that equal pressure rise takes place in impeller and diffuser.

The maximum pressure obtained in a single Stage compressor is in the rangle of 110 to 250 kPa. For Very high pressure of the order of 10 bar, multistage Centrifugal compressure are used.

Q 10
(b) Surging and Choking



The Point M on the curve ~~xxxxxx~~ marks the limit of useful operation of the compressor.

Surging is the phenomenon resulting in unstable periodic and reversal of flow due to a momentary increase in delivery pressure.

A point N the compressor is chocked and is processing the maximum mass flow rate. On the section MN of the curve the flow is stable. The fall in mass flow rate will result in a rise in pressure ratio. On the section LM of the curve, the flow is not stable. A fall in mass flow rate will be accompanied by a fall in pressure ratio.

In this situation, any small distrurbance causing a choke in mass flow will cause a fall in pressure ratio and the flow may reverse at some ~~time~~ point.

When the temporary disturbance is removed the flow will pick up and it is found that the small disturbance cause the flow to Oscillate rapidly.

The oscillation is noisy and if allowed to continue can cause structual damages in the compressor this Phenomenon is called Surging.

Q10 [I] (C) — An axial flow compressor stage drawn air from with the stagnation conditions 1.013 bar and 308 K. Assuming 50% reaction stage with a flow co-efficient of 0.52 and the ratio $\Delta V_{w}/u = 0.25$, find the rotor blade at an angle at the inlet and exit as well as the mean rotor speed. The total-to-total efficiency of the stage is 0.87 when the stage produces a total-to-total pressure ratio of 1.23. Find also the pressure coefficient and the power input in the system, assuming the work input factor to be 0.86. The mass flow rate is 12 kg/s.

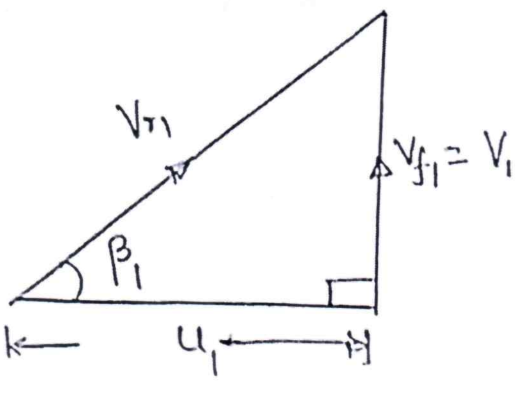
Sol^n: Given data: $V_1 = V_{f_1}$ (no inlet guide vane),

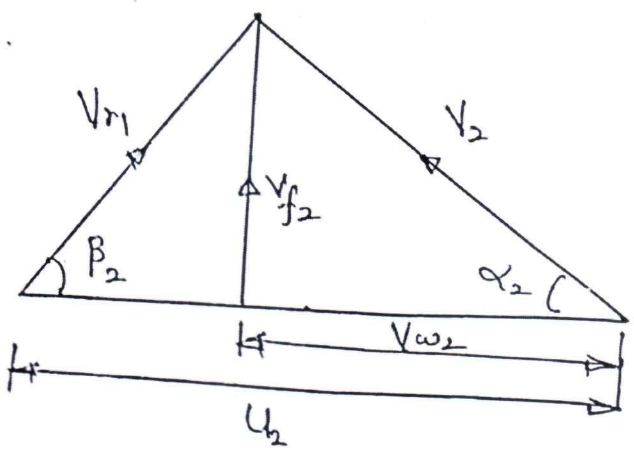
$R = 0.5$, $\psi = 0.52$, $\frac{\Delta V_w}{u} = 0.25$, $m = 12\ \frac{kg}{s}$,

$\frac{P_{02}}{P_{01}} = 1.23$, $\eta_{t-t} = 0.87$, $P_{01} = 1.013$ bar,

$T_{01} = 308\,K$, $\phi = 0.86$,

find: $u = ?$, $\beta_1 = ?$, $\beta_2 = ?$, $P = ?$





(i) Blade angles.

We know that,

$$W = V_f u\left[\cot\beta_1 - \cot\beta_2\right]$$

$$\cancel{u}\left[V_{w_2} - V_{w_1}\right] = V_f \cancel{u}\left[\cot\beta_1 - \cot\beta_2\right]$$

$$\Delta V_w = V_f\left[\cot\beta_1 - \cot\beta_2\right]$$

$$0.25\cancel{u} = 0.52\cancel{u}\left[\cot\beta_1 - \cot\beta_2\right]$$

$$\therefore \cot\beta_1 - \cot\beta_2 = 0.48 \quad \text{———} (1)$$

Again, $$R = \frac{V_f}{2u}\left[\tan\theta_1 + \tan\theta_2\right]$$

or $$R = \frac{V_f}{2u}\left[\cot\beta_1 + \cot\beta_2\right]$$

$$0.5 = \frac{0.52\cancel{u}}{2\cancel{u}}\left[\cot\beta_1 + \cot\beta_2\right]$$

$$\therefore (\cot\beta_1 + \cot\beta_2) = 1.922 \quad \text{———} (2)$$

Solving equation (1) and (2) we get.

$$\boxed{\beta_1 = 39.8^\circ, \quad \beta_2 = 54.2^\circ}$$

(ii) Mean rotor speed.

$$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{T_{02}}{302} = (1.23)^{\frac{1.4-1}{1.4}}$$

$$T_{02} = 321\ K.$$

$$\eta_{t-t} = \frac{T_{02} - T_{01}}{T'_{02} - T_{01}} \Rightarrow 0.87 = \frac{327 - 308}{T'_{02} - 308}.$$

$\therefore T'_{02} = 329.8\ K.$

$$\Delta h'_0 = C_p (T'_{02} - T_{01})$$
$$= 1005\ (329.8 - 308)$$
$$= 21909\ J/kg.$$

Work input factor.

$$\phi_w = \frac{\Delta h'_0}{u V_f (\cot\beta_1 - \cot\beta_2)} = \frac{\Delta h_0}{u\, \Delta h_w}$$

$$0.86 = \frac{21909}{u \times 0.25 \times u}$$

$$\therefore \boxed{u = 318.5\ m/s.}$$

(iii) Pressure Coefficient.

$$\phi_p = \frac{\Delta h'_0}{\frac{u^2}{2}} = \frac{21909}{\frac{318.5^2}{2}} = \underline{\underline{0.43}}$$

$$\therefore \boxed{\phi_p = 0.43}$$

(iv) power input

$$P = m \Delta h'_0 = 12 \times 21909$$

$$\boxed{P = 262.91\ kW}$$

[Chandrakanth].