KLS Vishwanathrao Deshpande Institute of Technology

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(Approved by AICTE, New Delhi, Affiliated to VTU, Belagavi)
(Recognized Under Section 2(f) by UGC, New Delhi)
Udyog Vidya Nagar, Haliyal - 581 329, Dist.: Uttara Kannada
www.klsvdit.edu.in | principal@klsvdit.edu.in | hodece@klsvdit.edu.in

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	Prof. Rajeshwari P.
Course Name	:	Digital Communication
Course Code	:	BEC503
Year of Question Paper	:	June/July 2025
Date of Submission	:	15/7/25.

Faculty Member HoD 15-07-2025. Dean (Acad.)

Head of the Department
Dept. of Electronic & Communication Engg.
KLS V.D.I.T., HALIYAL (U.K.)

CBCS SCHEME

USN 2VD22EC049

BEC503

Fifth Semester B.E./B.Tech. Degree Examination, June/July 2025

Digital Communication

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

			M	L	C
		Module – 1			
Q.1	a.	List the properties of Hilbert transform.	6	L2	CO1
	b.	Describe preenvelope of low pass signal.	4	L2	CO1
	c.	Outline the steps for deriving and reconstructing the band pass signal from in-phase and quadrature components.	10	L2	CO1
		OR			
Q.2	a.	Discuss correlation receiver of AWGN channel.	7	L2	CO1
	b.	Describe the matched filter with a necessary diagram	8	L2	CO1
	c.	Relate signal representation of 2 BIQ code	5	L2	CO1
		Module – 2			
Q.3	a.	Illustrate BPSK using coherent detection with transmitter and receiver and deriving expression for error probability function.	10	L2	CO2
	b.	Interpret the working of coherent generation and detection of QPSK. Draw QPSK w/fm for I/P binary sequence 01101000	10	L2	CO2
		OR			
Q.4	a.	Demonstrate M-ary QAM M = 4 with signal space diagram.	10	L2	CO2
	b.	Discuss the working of FSK coherent receiver and transmitter with block diagram.	10	L2	CO2
		Module – 3			
Q.5	a.	International Morse code uses a sequence of dots and dashes to transmit letters of English alphabet. The dash is represented by a current pulse that has a duration of 3 units and the dot has a duration of 1 unit. The probability of occurrence of a dash is 1/3 of probability of occurrence of a dot. i) Calculate information content of dot and dash. ii) Calculate average information in the dot-dash-code. iii) Assume that dot last 1m sec, which is the same-time interval as the pause between symbols. Find average rate of information transmission.	8	L3	CO3

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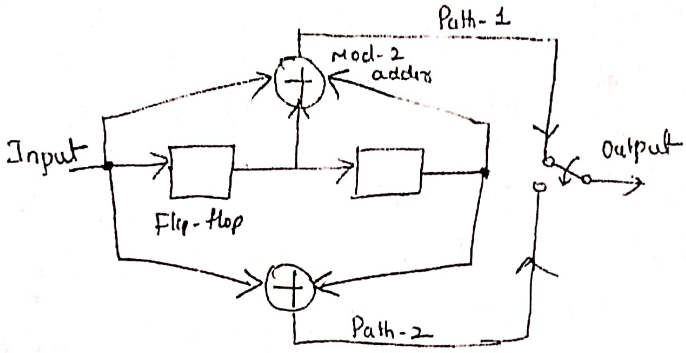
	b.	A binary source is emitting an independent sequence of 0's and 1's with probabilities P and 1-P outline the entropy of source, with a diagram.	5	L2	CO3
	c.	Show that entropy of 2 MS is given by $H(s^2)=2H(s)$ considering $P_0=\frac{1}{2}$, $P_1=\frac{1}{4}$, $P_2=\frac{1}{4}$.	7	L2	CO3
		OR			
Q.6	a.	Summarize properties of mutual information.	6	L2	CO3
	b.	Construct Huffman tree with symbols $\{S_0, S_1, S_2, S_3, S_4\}$ having probabilities {0.4, 0.2, 0.2, 0.1, 0.1}.	7	L3	CO3
	c.	A binary symmetric channel has matrix $P(Y/X)=\begin{bmatrix}0.8 & 0.2\\0.2 & 0.8\end{bmatrix}$ Also $P(X_1)=0.6$ $P(X_2)=0.4$. Calculate $I(X, Y)$ C_S, η_{ch}.	7	L3	CO3
		Module – 4			
Q.7	a.	Illustrate different error correcting codes.	5	L2	CO4
	b.	Outline the procedure of syndrome decoding.	6	L2	CO4
	c.	Illustrate encoding procedure (n, k) cyclic code steps considering linear feedback shift register with (n-k) stages.	9	L2	CO4
		OR			
Q.8	a.	Define G and H matrix show that $C.H^T=0$.	5	L2	CO4
	b.	For (6, 3) linear block code the parity matrix is $P=\begin{bmatrix}1 & 0 & 1\\0 & 1 & 1\\1 & 1 & 0\end{bmatrix}$ i) Calculate the generator matrix ii) Compute all possible code words.	10	L3	CO4
	c.	A (15, 5) cyclic code has the generator polynomial given by $g(x)=1+x+x^2+x^4+x^5+x^8+x^{10}$. Construct the block diagram of encoder and syndrome calculator.	5	L3	CO4
		Module – 5			
Q.9	a.	Consider a (3, 1, 2) convolutions encoder with $g^{(1)}=110$, $g^{(2)}=101$, $g^{(3)}=111$ i) Build the encoder diagram. ii) Compute the code word for message sequence (11101).	14	L3	CO5

	b.	Consider convolutional encoder shown in Fig.Q.9(b). Compute the generator polynomial, output polynomial for path1 and path2. Also compute encoded sequence.  Fig.Q.9(b)	6	L3	CO5
		OR			
Q.10	a.	Interpret optimum decoding of convolutional codes.	6	L3	CO5
	b.	Apply viter bi decoder algorithm steps considering all-zero sequence (0100010000).	14	L3	CO5

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BEC 503

Fifth Semester B.E Degree Examination June/July 2025

Digital Communication — Max. Marks: 100

Q1 a) List the properties of Hilbert Transform. — (6 mks)

Soln: The properties of Hilbert Transform (HT)

1> Linearity: The HT is a linear operator

$H\{a x_1(t) + b x_2(t)\} = a H\{x_1(t)\} + b H\{x_2(t)\}$ a & b are const.

2> Time Invariance: The HT is time invariant

$H\{x(t-t_0)\} = \hat{x}(t-t_0)$

3> Frequency Domain Representation: The HT introduces a phase shift of $\pm 90°$ ($\pm \pi/2$) depending on the sign of frequency

$F\{\hat{x}(t)\} = -j \,\mathrm{sgn}(f)\, X(f)$

4> Phase Shift Property: The HT shifts the phase of all +ve frequency components by $-\pi/2$ & all -ve frequencies by $+\pi/2$

5> Energy Preservation: The HT preserves the energy of a signal

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |\hat{x}(t)|^2 dt$$

6> Involution (Double HT): Applying the HT twice yields the -ve of the original signal:

$$H\{H\{x(t)\}\} = -x(t)$$

b> Describe preenvelope of low signal (4 mks)

Soln: The preenvelope, also known as the analytic signal, of a real valued signal $x(t)$ is a complex valued signal defined as

$x_a(t) = x(t) + \hat{x}(t)$, where $\hat{x}(t)$ is HT of $x(t)$

Its primary function is to eliminate the -ve frequency components of the original signal's spectrum.

1> Low frequency signal (Baseband): If "low" implies a signal with frequency content centered near DC, the preenvelope still forms a complex representation, but its envelope might be less pronounced compared to BP signal.

If 'low' refers to Noisy small amplitude signal possibly buried in noise.

(c) Outline the steps for deriving and reconstructing the BP signal from in phase & quadrature components. (concepts)

Soln: A real BP signal can be expressed in its canonical form as

$$s(t) = A(t)\cos(2\pi f_c t + \phi(t))$$

where $A(t)$ = instantaneous amp & $\phi(t)$ is instantaneous phase.

Alternatively, the BP signal can be expressed interms of its in-phase $I(t)$ and quadrature $Q(t)$ components as

$$s(t) = I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t)$$

Here, $I(t)$, $Q(t)$ are real valued low pass signal that contain the information modulated onto the carrier.

To obtain $I(t)$ & $Q(t)$ from $s(t)$:

→ for $I(t)$ multiply $s(t)$ by a cosine wave at the carrier freq & then LPF the result

$$s(t)\cdot\cos(2\pi f_c t) = [I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t)]$$

$$= I(t)\cos^2(2\pi f_c t) - Q(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

Using Trignometric Identities

$$\left(\cos^2 x = \frac{1+\cos(2x)}{2},\ \sin x\cos x = \frac{\sin(2x)}{2}\right);$$

$$= I(t)\frac{(1+\cos(4\pi f_c t))}{2} - Q(t)\frac{\sin(4\pi f_c t)}{2}$$

Applying LPF high freq $4\pi f_c t$ are removed.

$$\therefore = \frac{1}{2}I(t) + \frac{1}{2}I(t)\cos(4\pi f_c t) - \frac{1}{2}Q(t)\sin(4\pi f_c t)$$

$$\therefore I(t) = 2\cdot LPF[s(t)\cos(2\pi f_c t)]$$

for $Q(t)$: Multiply $s(t)$ by a $-ve$ sinewave (or a cosine wave with a $-90°$ phase shift) at the carrier freq & then LPF

$$s(t)\cdot(-\sin(2\pi f_c t)) = [I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t)]$$

$$= -I(t)\cos(2\pi f_c t)\sin(2\pi f_c t) + Q(t)\sin^2(2\pi f_c t)$$

$$= -I(t)\frac{\sin(4\pi f_c t)}{2} + Q(t)\frac{1-\cos(4\pi f_c t)}{2} \quad \left(\because \sin^2 x = \frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{2}Q(t) - \frac{1}{2}I(t)\sin(4\pi f_c t) - \frac{1}{2}Q(t)\cos(4\pi f_c t)$$

Applying LPF remove high freq $(4\pi f_c t)$

$$\therefore Q(t) = 2\cdot LPF\{s(t)(-\sin(2\pi f_c t))\}$$

Reconstructing the BP signal from I & Q Components is by multiplying $I(t)$ by cos & $Q(t)$ by sine carriers.

$$\therefore s(t) = I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t) \,//.$$

Q(2)(a) Discuss Correlation receiver of AWGN channel. (7mks)

Soln: The correlation receiver is a fundamental & optimal receiver str of dc systems operating over a additive white Gaussian Noise (AWGN) channel. The core principle of correlation receiver is a matched filtering. For each transmitted signal the receiver have a replica.

→ If $S_i(t)$ is the ith possible transmitted signal waveform for $0 \le t \le T$ & $r(t)$ is received signal $r(t) = s_i(t) + n(t)$ ($n(t)$ → AWGN)

→ The correlation receiver calculate correlation integral for each possible signal $s_j(t)$: $Z_j = \int_0^T r(t) s_j(t)\, dt$

→ The correlation receiver is considered optimal for AWGN channel coz it implements the max likelihood (ML) decision rule, which minimizes the prob of symbol error.

→ Each correlator in the bank is "tuned" to one of the possible transmitted waveforms. Each branch multiplies incoming rec. signal $r(t)$ ē a locally generated replica of one of the possible transmitted signals $s_j(t)$.

→ The symbol is then integrated & sampled at the end of the symbol duration T.

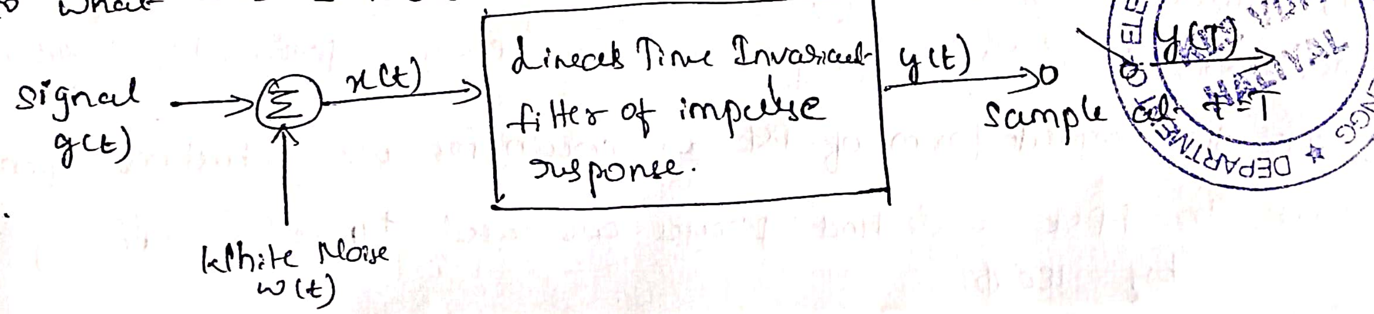
→ These sampled values (Z_j) are fed to decision device that selects the largest value & declares the corresponding symbol.

→ The correlation receiver is mathematically eqvt to the matched filter receiver.

b) Describe the matched filter with a necessary diagram.

Soln: The matched filter is the optimal linear filter for maximizing the SNR in the presence of additive stochastic noise.

They are commonly used in radars, in which a signal is sent out & we measure the reflected signal looking for something similar to what was sent out.



- The filter I/P $x(t)$ consists of a pulse signal $g(t)$ corrupted by additive channel noise $w(t)$ as shown by

$$x(t) = g(t) + w(t), \quad 0 \le t \le T.$$

- The $w(t)$ is the sample function of a white noise process of zero mean & power spectral density $N_0/2$.
- Since the filter is linear, the resulting output $y(t)$ may be expre-ssed as
- where $g_0(t)$ & $n(t)$ are produced by the signal & noise components of I/P $x(t)$ respectively.

(c) Relate signal representation of 2 BIQ code. (5 marks)

Soln: 2BIQ is a line coding scheme that is widely used in DC. particularly for ISDN. The name 2BIQ directly describes its function 2B (Two binary), it takes 2 binary bits, 1Q (1 Quaternary) It converts each dbit into single quaternary symbol. 2BIQ uses a gray code mapping, where adjacent vtg levels differ by only one bit. This ensures that if noise causes a received symbol to be misinterpreted as an adjacent level, it results in only 1 bit error in the decoded data rather than 2 bit error.

The standard mapping is as follows.

Dbit	Quaternary symbol (vtg level)
10	+450mV
11	+150mV
01	-150mV
00	-450mV.

5

Module 2

Q3) a) Illustrate BPSK using coherent detection with transmitter & receiver & deriving expression for error probability function. (10 marks)

gral 3 1p

Soln: Its the simplest form of PSK is known for its robustness against noise. In BPSK 2 distinct phases are used typically differing by 180° ϕ.

3(a). Let carrier signal be $\cos(2\pi f_c t)$

for binary '1' transmitted signal $s_1(t) = \sqrt{E_b/T_b}\cos(2\pi f_c t)$

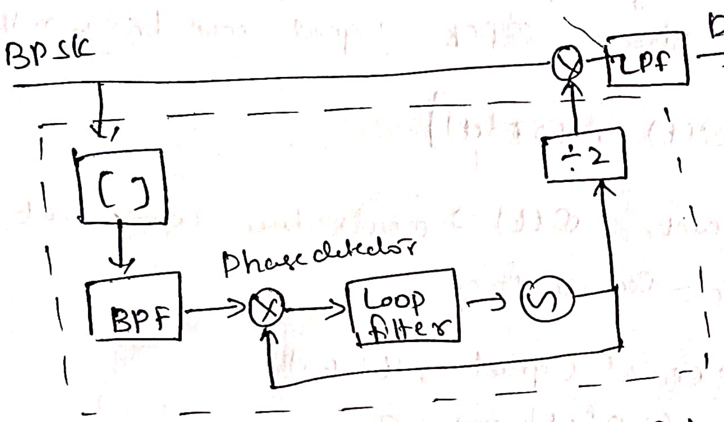
" " '0' " " $s_0(t) = \sqrt{E_b/T_b}\cos(2\pi f_c t + \phi)$

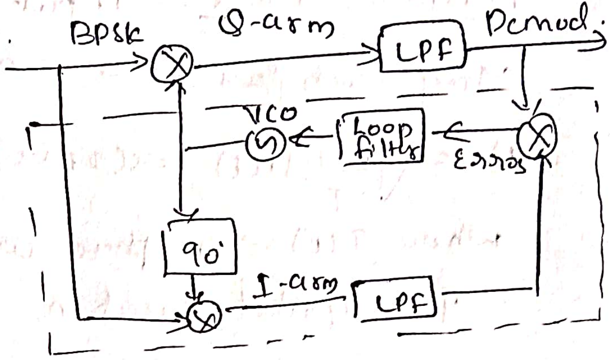
$= -\sqrt{E_b/T_b}\cos(2\pi f_c t)$

where $E_b \to$ Energy/bit, $T_b \to$ the bit duration

$f_c \to$ carrier freq.

BPSK Transmitter & Receiver. Carrier Recovery





Derivation of Error Prob (Pe) for BPSK. Coherent Detection

Assume the AWGN channel 2 sided power spectral Density (PSD) of $N_0/2$.

$\phi_1(t) = \sqrt{2/T_b}\cos(2\pi f_c t)$

$s_1(t) = s_0(t) = -\sqrt{E_b/T_b}\cos(2\pi f_c t)$.

$\therefore$ received signal $r(t) = s(t) + n(t)$.

$$r(t) = \int_0^T r(t)\cdot\sqrt{2/T}\cos(2\pi f_c t)\,dt$$

sub $r(t) = s(t) + n(t)$

$$r = \int_0^T s(t)\sqrt{2/T}\cos(2\pi f_c t)\,dt + \int_0^T n(t)\sqrt{2/T}\cos(2\pi f_c t)\,dt$$

$$= \pm\sqrt{E_b} + n \qquad (N(0, N/2))$$

$$\therefore \boxed{r = \pm\sqrt{E_b} + n}$$

$P_e = P(r < 0 \mid \text{bit 1 sent}) = P_r(n < -\sqrt{E_b})$

Since $n \sim N(0, N/2)$ we can write

$$P_e = Q\left(\frac{\sqrt{E_b}}{\sqrt{N_0/2}}\right) = Q\sqrt{\frac{2E_b}{N_0}}$$

3(b) Interpret working of coherent generation and detection of QPSK. Draw QPSK wlfm for I/P binary sequence 01101000. (10 marks)

Soln: QPSK transmits 2 bits per symbol. Each symbol modulates the phase of a carrier. Four possible phase values 00 → 45°, 01 → 135°, 11 → 225°, 10 → 315°

Coherent Detection of QPSK.

① Transmitter Block. Input bits grouped into 2 bit symbols ex 00,01 Map each pair to a unique phase. QPSK signal can be written as

$$s(t) = \sqrt{\frac{2E_b}{T}}\left[I(t)\cos(2\pi f_c t) - Q(t)\sin(2\pi f_c t)\right]$$

where $I(t)$ → in-phase component, $Q(t)$ → Quadrature component.
T → symbol duration, f_c → Carrier freq.

② Receiver Block: Multiply received signal $r(t)$ with
$\cos(2\pi f_c t)$ → S_I, $\sin(2\pi f_c t)$ → S_Q.

③ QPSK Waveforms for input 01101000.

→ group bit into pairs. 01, 10, 10, 00.

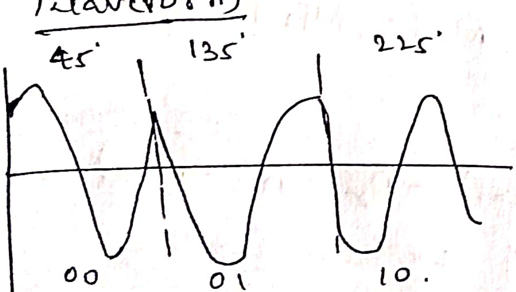
→ Phase Mapping.

Bits	Phase	I	Q
00	45°	+1	+1
10	135°	−1	+1
10	225°	−1	−1
00	315°	+1	−1

Our sequence.

01 → 135° → (−1, +1)
10 → 315° → (+1, −1)
10 → 315° → (+1, −1)
00 → 45° → (+1, +1)

Waveform



Q4 a) Demonstrate M-ary QAM M=4 with signal space diagram. (10marks)

Soln: QAM is a modulation technique that conveys data by changing the amplitude of 2 carrier waves using in phase (I) & Quadrature (Q) components.

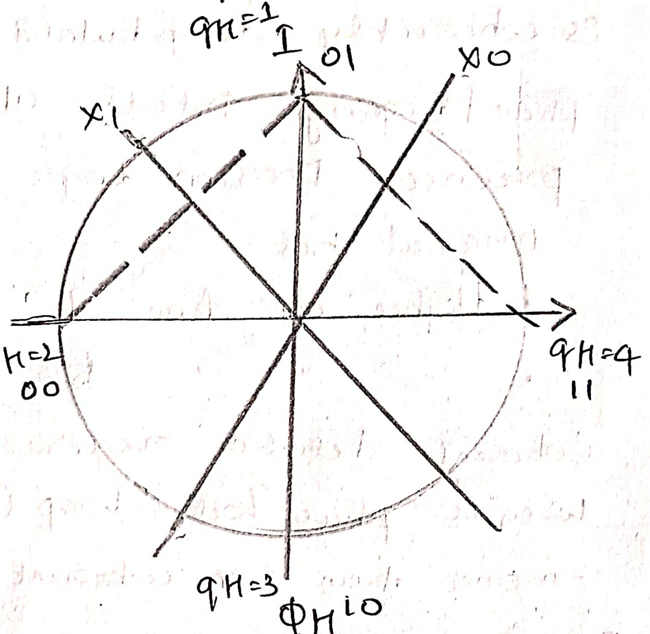
When M=4, its called 4-QAM & it conveys 2 bits/symbol.

$\therefore$ $\log_2(4) = 2$ bits/symbol.

4-QAM symbol Mapping: Each symbol in 4-QAM corresponds to a unique pair of amplitude values in the IQ plane.

Gray coded Mapping.

Bits	Symbol	I	Q
00	S_1	-1	-1
01	S_2	-1	+1
11	S_3	+1	+1
10	S_4	+1	-1

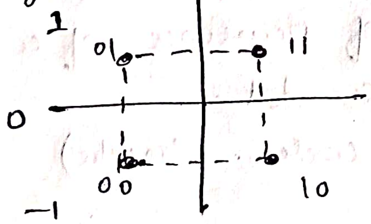


Mathematical Representation

$S_k(t) = I_k \cdot \cos(2\pi f_c t) + Q_k \sin(2\pi f_c t)$

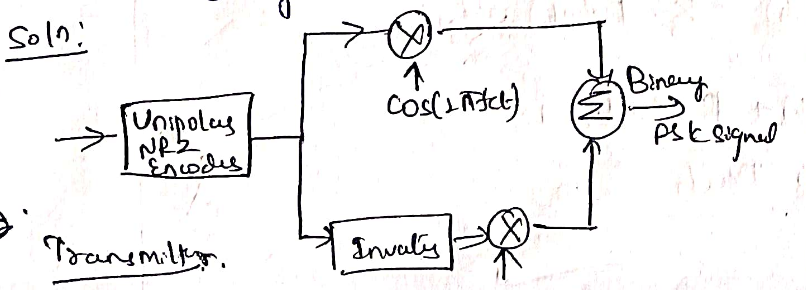
where $I \in \{-1, +1\}$, $Q_k \in \{-1, +1\}$, f_c is carrier freq.

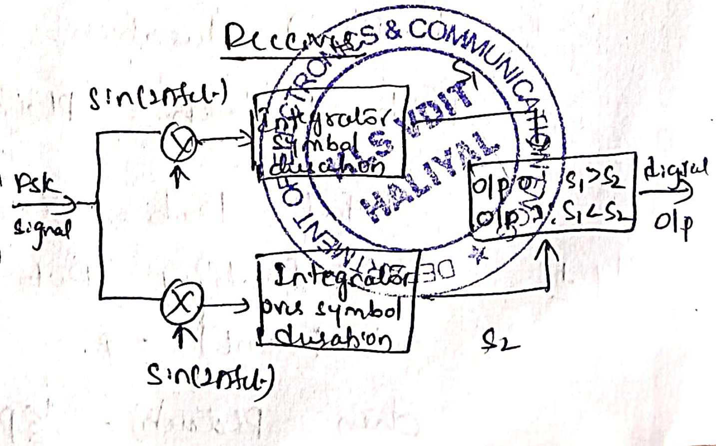
Signal space diagram



Q4(b) Discuss working of FSK coherent receiver & transmitter with block diagram. (10marks)

Soln:





8 In coherent FSK, the receiver uses a reference carrier that is phase locked to the received signal enabling accurate demodulation.

(1) FSK Transmitter: Binary data 0 or 1 is input. Data is fed to oscillator 1 for freq f_1 (if bit is 1), Oscillator 2 for frequency f_2 (if bit is 0). A Mux selects appropriate freq based on i/p.

The o/p is an FSK signal where Bit 1 → f_1, Bit 0 → f_2.

(2) FSK Receiver: Two band pass filters (BPFs) extracts the components

- BPF 1 passes freq f_1
- BPF 2 passed freq f_2. Each filtered signal is coherently demodulated using a reference signal matched in phase/frequency. Detectors o/p analog vtgs proportional to signal presence. Decision logic compares o/ps and reconstructs the original bit.
  - Higher o/p from f_1 → Bit 1
  - " " " f_2 → " 0.

Coherent detection requires carrier synchronization usually achieved using a phase locked loop (PLL). Provides better bit error performance than non coherent receivers.

Q5(a) International Morse code use a sequence of dots & dashes to transmit letters of English alphabet. The dash is represented by a current pulse that has a duration of 3 units & the dot has a duration of 1 unit. The Prob of occurrence of a dash is 1/3 of probability of occurrence of a dot.

i) Calculate information in the dot dash code. (8marks)

Soln Given: Dot (.) duration = 1 unit

Dash (–) duration = 3 units

Prob of dash = $P(dash) = \frac{1}{3} \cdot P(dot)$

Total Prob = $P(dot) + P(dash) = 1$.

Step 1: Find Prob of Dot & Dash

Let $P(dot) = p$.

then $P(dash) = \frac{1}{3}p$

5(a)

$P + \frac{1}{3}P = 1 \Rightarrow \frac{4}{3}P \Rightarrow 1 \Rightarrow P = \frac{3}{4}$.

$P(\text{dot}) = \frac{3}{4}$, $P(\text{dash}) = 1/4$.

Step 2: Calculate Inforⁿ (in bits) of Dot & Dash

Inforⁿ content of a symbol with prob p is

$$I = -\log_2(p)$$

Inforⁿ in dot: $I_{dot} = -\log_2\left(\frac{3}{4}\right) = \log_2\left(\frac{4}{3}\right) \approx 0.4150$ bits

Inforⁿ in dash:

$$I_{dash} = -\log_2(1/4) = \log_2(4) = 2 \text{ bits}$$

Step 3: Calculate Avg inforⁿ / symbol (Entropy).

$H = \sum P_i \cdot I_i$, $H = P(\text{dot}) \cdot I_{dot} + P(\text{dash}) \cdot I_{dash}$

$$H = \frac{3}{4} \times 0.4150 + \frac{1}{4} \times 2 = 0.3113 + 0.5$$

$$\boxed{H = 0.8113 \text{ bits/symbol.}}$$

Step 4: Avg Rate of Inforⁿ (bits/ms)

rate = entropy/avg duration

• Dot lasts 1ms • Dash lasts 3ms.

Avg duration $= \frac{3}{4} \times 1 + \frac{1}{4} \times 3 = 0.75 + 0.75 = 1.5$ ms

Now, avg rate $\text{Rate} = \frac{0.8113 \text{ bits}}{1.5 \text{ ms}} = 0.5409$ bits/ms

$0.5409 \times 1000 = 540.9$ bits/sec

Q5(b) A binary source is emitting an independent sequence of 0's & 1's with probabilities P & 1-P. Outline the entropy of source with a diagram. (5 marks).

Soln: Entropy $H(P) = -P\log_2(P) - (1-P)\log_2(1-P)$

where P → Prob of 0, 1-P is Prob of 1.

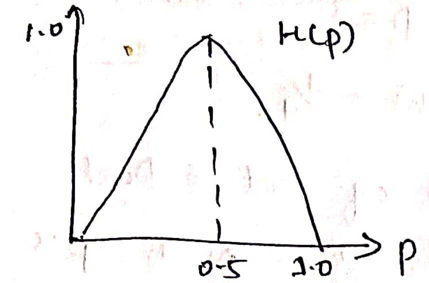
Behaviour of the Entropy function.

When P = 0 or P = 1.

H = 0 bits (no uncertainty - the source always emits the same symbol)

when P = 0.5; H = 1 bit (max uncertainty - Equal chance of 0 & 1)

Graph



Q5(b) A binary source is emitting an independent sequence of 0's & 1's with probabilities P & $1-P$ outline the entropy of source etc

Q5(c). Show that entropy of 2ms is given by $H(S^2) = 2H(S)$ considering $P_0 = 1/2$, $P_1 = 1/4$, $P_2 = 1/4$ (7 marks)

Soln: To show that the entropy of 2 memory less source (2ms) is given by

$$H(S^2) = 2H(S).$$

A memoryless source emits symbols independently

Given: $P_0 = 1/2$, $P_1 = 1/4$, $P_2 = 1/4$

$$\therefore H(S) = -\sum_{i=0}^{2} P_i \log_2 P_i$$

Substitute the given probabilities

$$H(S) = -\left(\frac{1}{2}\log_2 \frac{1}{2} + \frac{1}{4}\log_2 1/4 + 1/4 \log_2 1/4\right)$$

Evaluate each term.

- $1/2 \log_2 1/2 = 1/2 \cdot (-1) = -1/2$
- $1/4 \log_2 1/4 = 1/4 \cdot (-2) = \frac{-1}{2}$

So $H(S) = -\left(-\frac{1}{2}, \frac{-1}{2}, \frac{-1}{2}\right) = 1.5$ bits

Lets denote the pairs.

$$(0,0), (0,1), (0,2), (1,0) \ldots (2,2)$$

$$P(a,b) = P(a) \cdot P(b)$$

Entropy of 2MS - Denoted as $H(S^2)$

$$H(S^2) = -\sum_{a,b} P(a)\, P(b) \log_2 (P(a) P(b))$$

$$H(S^2) = -\sum_{a,b} P(a)\, P(b) \left[\log_2 P(a) + \log_2 P(b)\right]$$

distribute: $H(S_2) = -\sum P(a)\, P(b) \log_2 P(a) - \sum_{a,b} P(a)\, P(b) \log_2 P(b)$

5.c)

Summing over b of P(b) = 1 & likewise for a, we get.

$$H(S^2) = -\sum_a P(a)\log_2 P(a) - \sum_b P(b)\log_2 P(b)$$

$$\boxed{H(S^2) = H(S) + H(S) = 2H_S}$$

Since H(S) = 1.5 bits.

$$H(S^2) = 2 \times 1.5 = 3 \text{ bits}$$

Q6) a) Summarize properties of mutual information (6 marks)

Soln: Mutual Information $I(X;Y)$ quantifies how much info one random variable X reveals about another Y. It measures the reduction in uncertainty of one variable due to knowledge of the others

$$I[X;Y] = H(X) + H(Y) - H(X,Y)$$

$$I[X;Y] = H(X) - H(X|Y) = H(Y) - H(Y|X)$$

6 key Properties of Mutual Information.

No.	Property	Explanation.
1	Non negativity ⇒	$I(X;Y) \geq 0$; equality holds iff X & Y are independent.
2	Symmetry ⇒	$I(X;Y) = I(Y;X)$; mutual info is symmetric
3	Relation to Entropy ⇒	$(I(X;Y) = H(X) - H(X))$
4	Zero if independent ⇒	if $X \perp Y$, then $I(X;Y) = 0$; they share no information.
5	Additivity ⇒	for independent pairs; $I(X_1X_2);(Y_1,Y_2) = I(X_1;Y_1) + I(X_2;Y_2)$
6	Chain Rule :	$(I(X;Y,Z) = I(X;Y) + I(X;Z))$

(b) Construct Huffman code with symbols $\{S_0, S_1, S_2, S_3, S_4\}$ having probabilities $\{0.4, 0.2, 0.2, 0.1, 0.1\}$ (7 marks)

Soln: Construct a Huffman Tree for the symbols $\{S_1, S_2, S_3, S_4, S_5\}$ with prob $\{0.4, 0.2, 0.2, 0.1, 0.1\}$

Symbols & Prob.

(1) $S_1 \to 0.4$, $S_2 \to 0.2$, $S_3 \to 0.2$, $S_4 \to 0.1$, $S_5 \to 0.1$

Build the Huffman Tree.

Combine S_4 & S_5 $S_4(0.1) + S_5(0.1) = A(0.2)$

New list $S_1(0.4), S_2(0.2)$ $S_3(0.2)$ $A(0.2)$

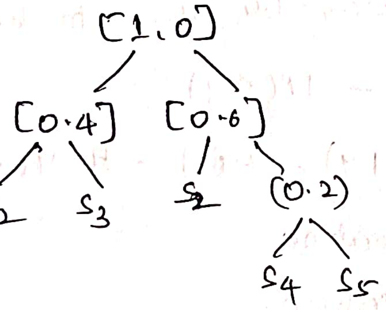
Combine S_2 & S_3 $S_2(0.2) + S_3(0.2) = B(0.4)$

New list S_1 $(0.4), A(0.2), B(0.4)$

Combine A & S_1 $A(0.2) + S_1(0.4) = C(0.6)$

Final combine $B(0.4) + C(0.6) = \text{Root } (1.0)$.

Draw the Tree



Binary codes. Go left=0, right=1

$S_2 \Rightarrow 000 \rightarrow L \rightarrow L \rightarrow L$

$S_3 \Rightarrow 001 \rightarrow L \rightarrow L \rightarrow R$

$S_1 \Rightarrow 10 \rightarrow R \rightarrow L$

$S_4 \Rightarrow 110 \Rightarrow R \rightarrow R \rightarrow L$

$S_5 \Rightarrow 111 \Rightarrow R \rightarrow R \rightarrow R$

Final Huffman Codes.

$S_1 \rightarrow 0.4 \rightarrow 10$, $S_2 \rightarrow 0.2 \rightarrow 000$, $S_3 \rightarrow 0.2 \rightarrow 001$, $S_4 \rightarrow 0.1 \rightarrow 110$, $S_5 \rightarrow 0.1 \rightarrow 111$

6c) A binary symmetric channel has matrix $P(Y|X) = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$

Also $P(X_1) = 0.6$, $P(X_2) = 0.4$ Calculate $I(X, Y)$ C_s, η_{ch}. (7 marks)

Soln: Channel matrix $P(Y|X)$:

$$P(Y|X) = \begin{bmatrix} P(Y_1|X_1) & P(Y_2|X_1) \\ P(Y_1|X_2) & P(Y_2|X_2) \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

I/P Prob: $P(X_1) = 0.6$, $P(X_2) = 0.4$

Calculate Joint Prob $P(X, Y)$

using the formula: $P(X_i, Y_j) = P(X_i) \cdot P(Y_j | X_i)$

Pair	Formula	Value.
$P(X_1, Y_1)$	$0.6, 0.8$	0.48
$P(X_1, Y_2)$	$0.6, 0.2$	0.12
$P(X_2, Y_1)$	$0.4, 0.2$	0.08
$P(X_2, Y_2)$	$0.4, 0.8$	0.32

$P(Y_1) = P(X_1 Y_1) + P(X_2 Y_1) = 0.48 + 0.08 = 0.56$

$P(Y_2) = P(X_1, Y_2) + P(X_2 Y_2) = 0.12 + 0.32 = 0.44$

6(c) Entropy of o/p $H(Y) = -\sum P(y_j) \log_2 P(y_j)$

$= -(0.56 \log_2 0.56 + 0.44 \log_2 0.44)$

$\approx -(0.56(-0.485) + 0.44(-1.184)) \approx 0.973$ bits

$H(Y|X) = \sum P(x_i) \cdot H(Y|x_i) = -0.8\log_2 0.8 - 0.2 \log_2 0.2$

≈ 0.7219

$H(Y|X) = 0.6 \times 0.7219 + 0.4 \times 0.7219 = 0.7219$ bits

Mutual Inforⁿ : $I(X;Y) = H(Y) - H(Y|X) = 0.973 - 0.7219 = 0.2511$ bits

Channel Capacity $C_s = 1 - H(p)$

$H(p) = -0.8 \log_2 0.8 - 0.2 \log_2 0.2 \approx 0.7219$

$C = 1 - 0.7219 = 0.2781$ bits

Channel Efficiency $\eta_{ch} = \frac{I(X;Y)}{C_s} = \frac{0.2511}{0.2781} \approx 0.9026$ or 90.26%.

Q(7) a) Illustrate different Error correcting codes. (5 marks)

Soln: ① Repetition Code: It's a simple error correction code, which repeats each ~~word~~ bit multiple times (usually 3 or more)

Ex: Data : 1 → Sent as 111.

If received 101, majority vote = 1. Corrects 1-bit errors within each triplet and it is very inefficient.

② Hamming Code: It's a single error correction, Double error detection (SEC-DED). Adds multiple parity bits at positions that are powers of 2. For a data bits, 3 parity bits are added.

Ex: $D_3\ D_2\ D_1\ D_0$ → Data bits

$P_2\ P_1\ P_0$ → Parity bits placed at positions 1, 2, 4

Corrects: Any single-bit error, Detects - double bit errors.

③ Cyclic Redundancy Check (CRC): It's a error detection & very strong. It uses polynomial division (mod 2 arithmetic). Data sent with a CRC remainder appended.

Ex: Data: 110100

Generator: 1011, CRC bits are calculated & appended.

It can detect Burst errors up to the length of the generator polynomial.

(7a) (4) Block Codes: It's a general Error Correcting code which encodes data in fixed size blocks. given by (n,k)
n: total no of bits (data + redundancy), k: no of data bits.
n-k: no of check bits
Ex: Hamming (7,4) ⇒ 4 data bits Encoded into 7 bits.

(5) Convolutional Codes: It's streaming Error correction In which Input bits are passed through a shift register and combined using modulo 2 arithmetic. It gives output continous stream, each out-put bit depends on current & pre-vious inputs.

7(b) Outline the procedure of syndrome decoding. (6 marks)

Soln: Syndrome decoding is a method used in error detection and correction in digital comm systems, particularly with linear block codes (like Hamming codes, BCH codes etc). The key is to use the syndrome, a pattern calculated from the received codeword to determine the error pattern & correct it.

→ let the received vector be r. This may contain errors due to noise in the transmission channel

$r = c + e$, where $c \rightarrow$ is transmitted codeword,

$e \rightarrow$ is error vector.

→ The syndrome 'S' is computed using parity check matrix H.

$$S = H \cdot r^T$$

If $S = 0$, there is no error

If $S \neq 0$, it indicates the presence & nature of error(s).

→ Compare the computed syndrome S with syndrome table. Each syndrome corresponds to a specific error vector e.

Find e such that $S = H \cdot e^T$

→ correct the Errors: Once the error vector e is identified correct the received vector

$$c = r - e = r + e.$$

→ from the corrected codeword c, extract the original msg bits

7) C) Illustrate encoding procedure (n,k) cyclic code steps considering linear feedback shift register with (n-k) stages (9marks)

Soln: Cyclic codes are type of linear block codes where any cyclic shift of a codeword results in another codeword. These are commonly encoded using generator polynomial g(x) & implemented efficiently using a linear feedback shift Register (LFSR).

Given: (n,k) cyclic code ⇒ Codeword length = n, msg length = k.

Generator polynomial g(x) of degree n-k

Input msg polynomial : m(x), LFSR with (n-k) shift register stages (registers)

Let the msg bits be $m = (m_0, m_1 \ldots m_{k-1})$

Represent as polynomial $m(x) = m_0 + m_1 x + m_2 x^2 + \ldots + m_{k-1} x^{k-1}$

shift the msg polynomial by (n-k) positions to make room for parity bits : $x^{n-k} \cdot m(x)$.

This is equivalent to appending (n-k) zeroes to the msg bits

- Perform modulo-2 polynomial division of $x^{n-k} m(x)$ by $g(x)$
- The remainder $r(x)$ is the parity bits
- The system codeword is : $c(x) = x^{n-k} m(x) + r(x)$
- In vector form : Codeword = [Message Bits | Parity Bits]
- Use feedback connections based on the generator polynomial g(x)
- After all k msg bits are shifted in, the content of the (n-k) shift registers holds the parity bits.

Add the relevant diagram.

Q8 (a) Define G and H matrix, show that $C \cdot H^T = 0$. (5marks)

Soln: The generator matrix G is used to encode the msg vector m into a codeword C. in a linear block code.

- If the code is an (n,k) then G is k×n matrix

$\therefore C = m \cdot G$.

- Parity check matrix ~~C·HT=0~~ (H). is given by

① $H \cdot C^T = 0$.

for an (n,k) code, H is of size (n-k)×n

8(a) condition $C \cdot H^T = 0$

let $C = m \cdot G$, where m is msg vector.

Then $C \cdot H^T = (m \cdot G) \cdot H^T = m \cdot (G \cdot H^T)$

for a valid linear code: $G \cdot H^T = 0 \Rightarrow C \cdot H^T = 0$ //

(5) For (6,3) linear block code the parity matrix is $P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

i) Calculate the generator matrix ii) Compute all possible codewords. (Geometry)

Soln: Given: It is a (6,3) linear block code.

So: $n = 6$ (codeword length), $k = 3$ (msg length)

$n-k = 3$ (no of parity bits)

→ Parity submatrix P: $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

→ Generator matrix 'G' is of the form $G = [I_k][P]$

where • I_k is the $k \times k$ identity matrix

• P is the $k \times (n-k)$ parity matrix.

So: $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\therefore G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$ //

There are $2^k = 2^3 = 8$ possible msg vectors

$C = m \cdot G$

where m is the msg vector of length 3 & G is (3×6) matrix

msg (m)	codeword $C = m \cdot G$.
[0 0 0]	[0 0 0 0 0 0]
[0 0 1]	[0 0 1 1 1 0]
[0 1 0]	[0 1 0 0 1 1]
[0 1 1]	[0 1 1 1 0 1]
[1 0 0]	[1 0 0 1 0 0]
[1 0 1]	[1 0 1 0 1 0]
[1 1 0]	[1 1 0 1 1 1]
[1 1 1]	[1 1 1 0 0 1]

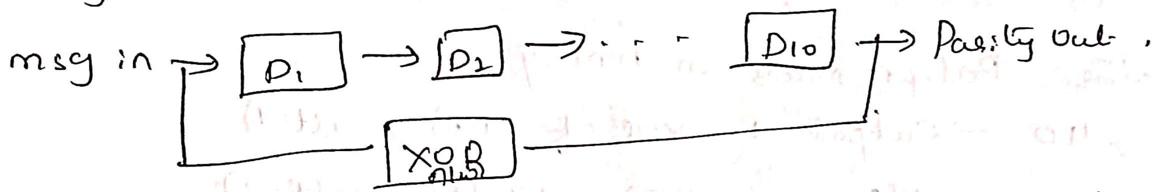
8 (c) A (15, 5) cyclic code has the generator polynomial given by $g(x) = 1+x+x^2+x^4+x^5+x^8+x^{10}$. Construct the block diagram of encoder & syndrome calculator. (6 marks)

Soln: $(n,k) = (15,5)$

$$g(x) = 1+x+x^2+x^4+x^5+x^8+x^{10}$$

degree of generator polynomial $\deg(g(x)) = 10 = n-k$

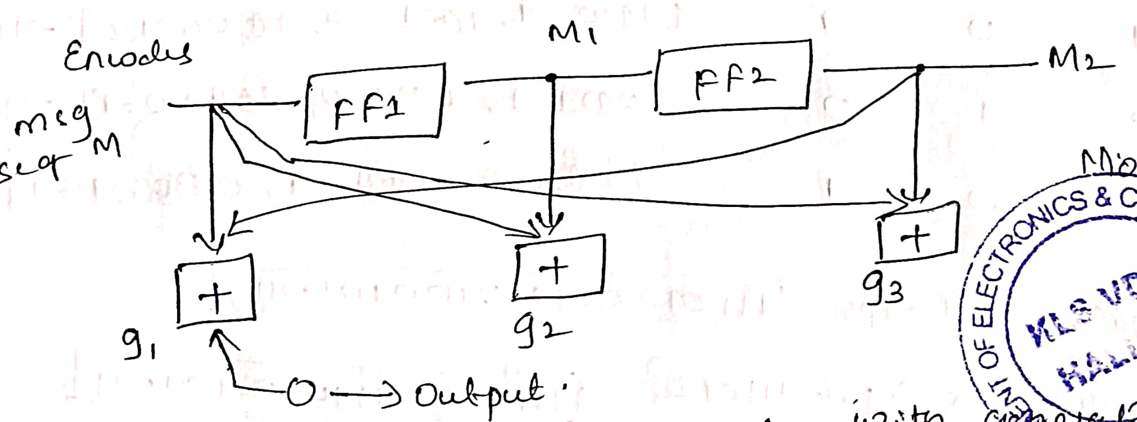
Block diagram.



- It has 5 bit message $m(x)$, shifts the msg into encoder one bit at a time. Use modulo-2 division compute the remainder. Taps are placed at $x^0, x^1, x^2, x^4, x^5, x^8$,
- Input the 5 bit msg serially. The LFSR performs modulo 2 division. The contents of the shift registers after input gives the 10 bit parity. The same LFSR
- To detect errors in received 15-bit codeword. The same LFSR is used, the entire 15-bit shifts the remainder in the shift register is the syndrome. if syndrome = 0 ⟹ No errors.
  if syndrome ≠ 0 ⟹ Errors detected.

9 (a) Consider a (3,1,2) convolutions encoder with $g^{(1)} = 110$, $g^{(2)} = 101$, $g^{(3)} = 111$ i) Build the encoder diagram ii) Compute the code word for msg sequence (11101). (14 marks).

Soln: Encoder



Consider a (3,1,2) convolutional encoder with generator polynomials.

① $g^1 = 110$, $g^{(2)} = 101$, $g^{(3)} = 111$

Given $(n, k, m) = (3, 1, 2)$ • i/p bits per time = 1, o/ps bits / time = 3

memory = 2 F/F.

The encoder uses 2 shift registers (D_1 D_2) & 1 input (current bit)

The output is generated using XOR connections defined by two generator polynomials.

- Current input bit = $u(t)$
- shift registers store previous inputs.

$D_1 = u(t-1)$, $D_2 = u(t-2)$

Generator Polynomials in binary

- $g^{(1)} = 110$ → Output 1 : XOR of $u(t)$ & $u(t-1)$
- $g^{(2)} = 101$ → O/P 2 : XOR of $u(t)$ & $u(t-2)$
- $g^{(3)} = 111$ → O/P 3 : XOR of $u(t)$, $u(t-1)$ & $u(t-2)$.

Each output bit is computed from taps (XOR) as follows.

$V_1 = u(t) \oplus u(t-1)$, $V_2 = u(t) \oplus u(t-2)$

$V_3 = u(t) \oplus u(t-1) \oplus u(t-2)$

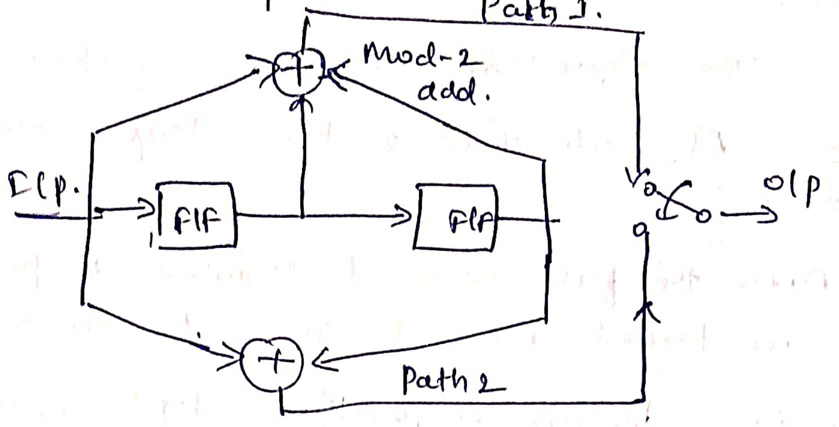
Message Sequence : $u = [1\,1\,1\,0\,1\,0\,0]$ $D_1 = D_2 = 0$

Step.	u(t)	D_1	D_2	Output (V_1 V_2 V_3)
1	1	0	0	$[1\oplus0 = 1,\ 1\oplus0 = 1,\ 1\oplus0\oplus0 = 1] \to 111$
2	1	1	0	$[1\oplus1 = 0,\ 1\oplus0 = 1,\ 1\oplus1\oplus0 = 0] \to 010$
3	1	1	1	$[1\oplus1 = 0,\ 1\oplus1 = 0,\ 1\oplus1\oplus1 = 1] \to 001$.
4	0	1	1	$[0\oplus1 = 1,\ 0\oplus1 = 1,\ 0\oplus1\oplus1 = 0] \to 110$
5	1	0	1	$[1\oplus0 = 1,\ 1\oplus1 = 0,\ 1\oplus0\oplus1 = 0] \to 100$
6	0	1	0	$[0\oplus1 = 1,\ 0\oplus0 = 0,\ 0\oplus1\oplus0 = 1] \to 101$
7	0	0	1	$[0\oplus0 = 0,\ 0\oplus1 = 1,\ 0\oplus0\oplus1 = 1] \to 011$

Concatenate all o/ps : $\boxed{111010001110100101011}$

so the codeword for input 11101 is $\boxed{111010001110100101011}$

b) Consider convolutional Encoder shown in fig Q.9(b). Compute the generator polynomial output polynomial for path 1 & path 2. Also compute encoded sequence. (6 marks).



Soln: For path 1 ($g_1(D)$): The o/p V_1 is generated by XORing the current i/p (I), the i/p delayed by one unit (D_1) and the i/p delayed by 2 units (D_2). ∴ the generator polynomial for path 1 is

$$g_1(D) = 1 + D^1 + D^2 \Rightarrow 111.$$

For path 2 ($g_2(D)$): The o/p V_2 is generated directly from the i/p delayed by 2 units (D_2) $\Rightarrow 001$.

Assume: let the i/p sequence 1000.

Initial state of F/Fs $D_1 = 0$, $D_2 = 0$.

Let's say o/p pair of (V_1, V_2).

I/P (I)	Previous D_1	Previous D_2	$V_1 = I \oplus D_1 \oplus D_2$	$V_2 = D_2$	New D_1	D_2	(V_1, V_2)
1	0	0	$1 \oplus 0 \oplus 0 = 1$	0	1	0	10
0	1	0	$0 \oplus 1 \oplus 0 = 1$	0	0	1	10
0	0	1	$0 \oplus 0 \oplus 1 = 1$	1	0	0	11
0	0	0	$0 \oplus 0 \oplus 0 = 0$	0	0	0	00

Q10 (a). Interpret optimum decoding of convolutional codes. (6 marks)

Soln: • Optimum decoding refers to the process of finding the most likely transmitted sequence given the received data.
• For convolutional codes this involves selecting the i/p sequence that most likely resulted in received codeword sequence.
• The Viterbi algorithm is the standard method for optimum decoding of convolutional codes. It performs max likelihood sequence estimation (MLSE).

Key features:

① Trellis Representation: The convolutional encoder is represented as a trellis diagram.

② Path Metrics: The algorithm calculates a metric

③ Survivor Path: At each state & time step only the most likely path is retained.

④ Traceback: After the full received sequence is processed the mostly likely path is traced back to give the decoded sequence.

(b) Apply viterbi decoder algorithm steps considering all-zero sequence (0100010000). (14 marks).

Soln: Let us assume a (2,1,2) convolutional encoder with Generator polynomials.

$$g_1 = (1,1,0) \Rightarrow g_1(x) +1+x$$

$$g_2 = (1,0,1) \Rightarrow g_2(x) = 1+x^2$$

This means: I/P = 1 bit, O/P = 2 bits, memory = 2.

Trellis Construction.

Present state	I/P	Next state	O/P
00	0	00	00
00	1	10	11
01	0	00	10
01	1	10	01
10	0	01	01
10	1	11	10
11	0	01	11
11	1	11	00

Viterbi Decoding Steps.

→ At time = 0; Start at state 00, path metric = 0,

All other states : metric = ∞

Time t = 1 | Received : 01.

from state 00, I/P 0 → next 00, O/P 00, Hamming dist = 1.

" " " " 1 → " 10, " 11, " " = 2

state 00: metric = 1

state 10: metric = 2

Time $t=2$ | Received : 00

From 00 (metric 1), i/p 0 → 00, o/p → 00, Hamming = 0; new metric → 1

from 00, i/p 1 → 10, o/p 11, Hamming = 2 → new metric = 3

from 10, (metric = 2), i/p 0 → 01, o/p → 01, Hamming = 1 → metric = 3

from 10, i/p 1 → 11, o/p 10, Hamming = 2 → metric = 4

Time $t=4$ | Received : 00

from (00) (1), i/p 0 → 00, o/p 00, Hamming = 0 → 2

from 00, i/p 1, → 10, o/p → 11, Hamming = 2 → 4

from (10) (3), i/p 0 → 01, o/p 01, Hamming = 1 → 4

from (10), i/p 1 → 11, o/p 10, Hamming = 2 → 5

from (01), " 0 → 00, o/p 10, 11 = 2 → 5

" (01), " 1 → 10, " 01, " = 1 → 4

from 11 (5), i/p 0 → 01, o/p 11, Hamming = 2 → 7

from 11, i/p 1, → 11, o/p 00, Hamming = 0 → 5

llly.

Time $t=5$ | Received = 00

- 00 metric = 2 (from 00)
- 10 : metric = 4 (from 00)
- 01 : metric = 5 (from 10)
- 11 : metric = 5 (from 11)

→ Final Decoded I/p sequence : → 00000

But recall, received was 0100010000 & we assumed all zero path, so the decoded msg sequence = 00000.

∴ received sequence ! 0100010000 using (2, 1, 2) convolutional encoder results in the decoded i/p sequence 00000.

END

Head of the Department
Dept. of Electronic & Communication Engg.
KLS V.D.I.T.. HALIYAL (U.K.)