KLS Vishwanathrao Deshpande Institute of Technology
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(Approved by AICTE, New Delhi, Affiliated to VTU, Belagavi)
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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	Dr. Nagaraj Bhat
Course Name	:	Network Analysis
Course Code	:	BEC304
Year of Question Paper	:	Dec 2024- Jan 2025
Date of Submission	:	

Faculty Member

HoD

Head of the Department
Dept. of Electronic & Communication Engg.
KLS V.D.I.T. HALIYAL (U.K.)

Dean (Acad.)

CBCS SCHEME

USN

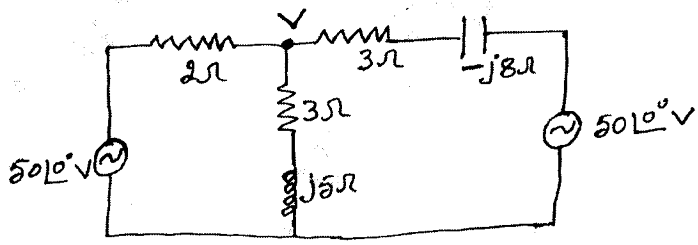
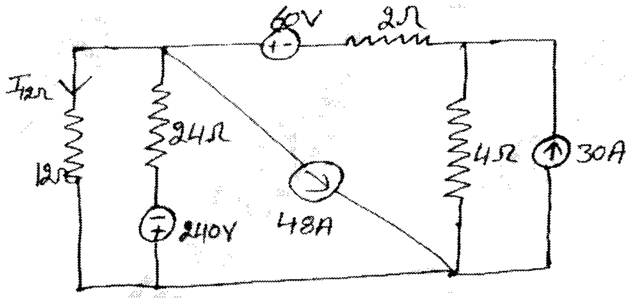
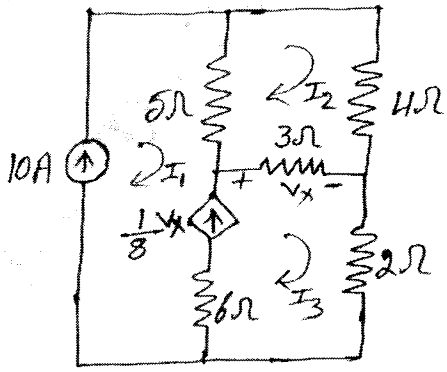
BEC304

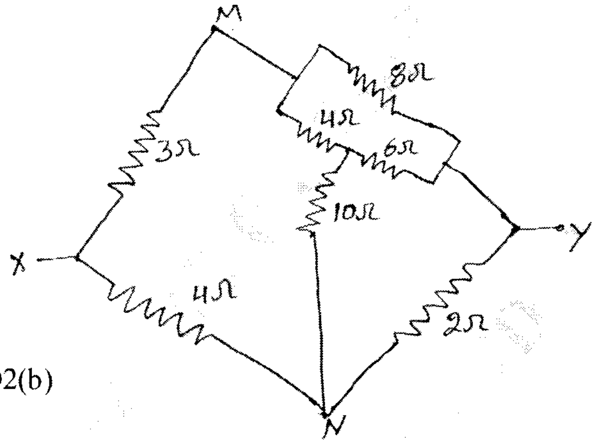
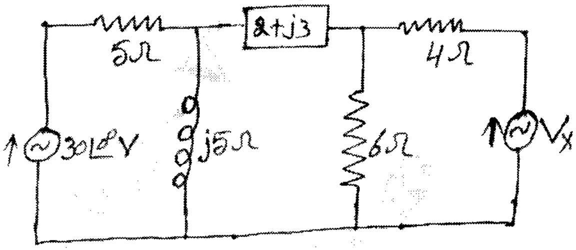
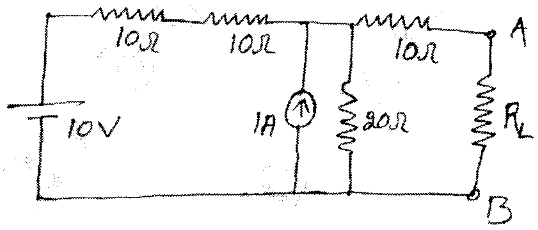
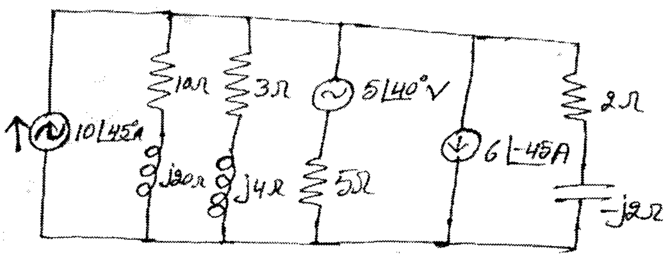
Third Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025

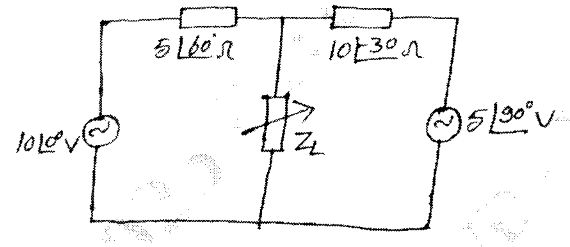
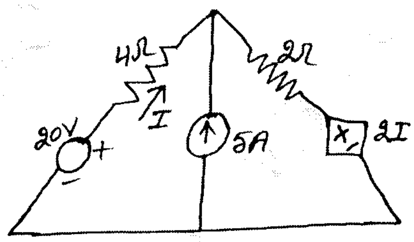
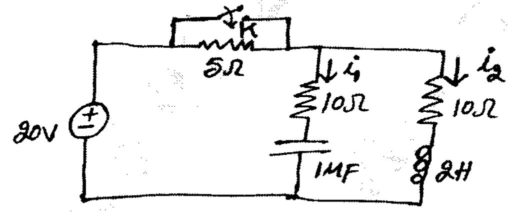
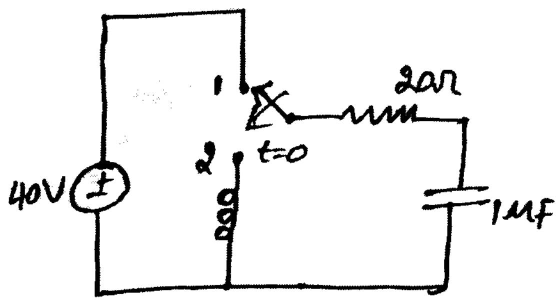
Network Analysis

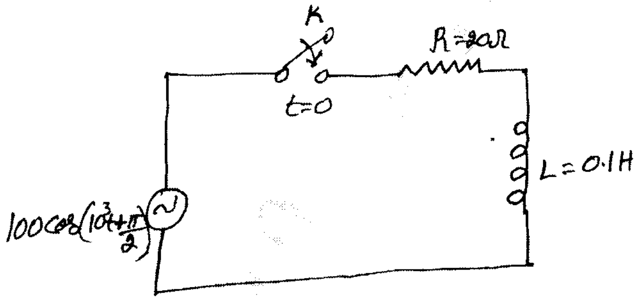
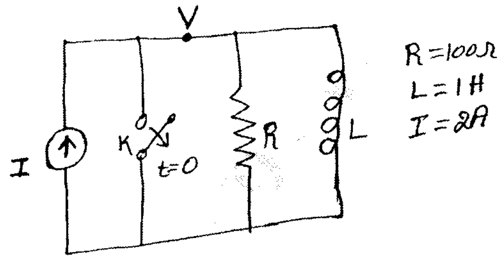
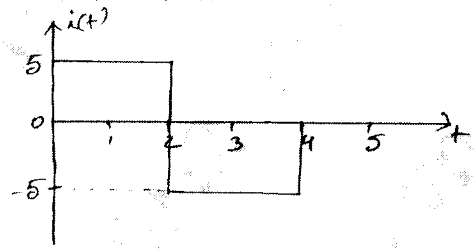
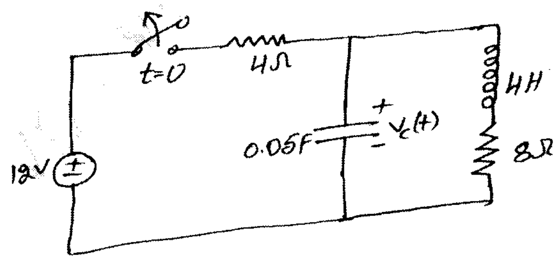
Time: 3 hrs. Max. Marks: 100

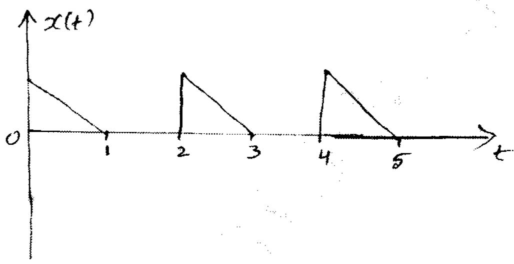
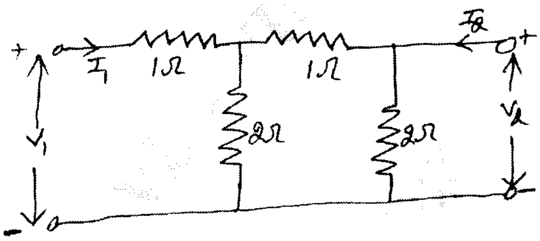
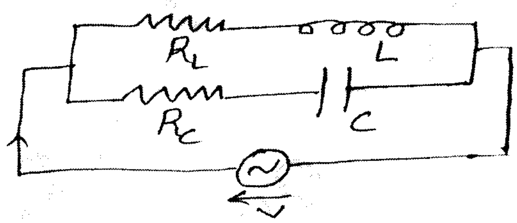
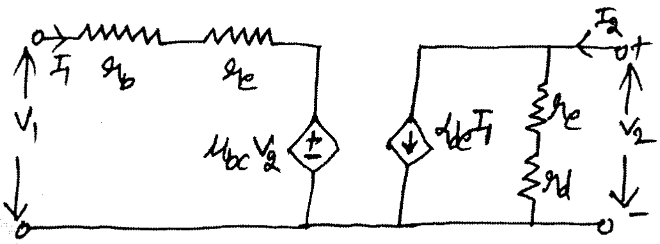
Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Three impedances are connected in Delta. Obtain the star equivalent of the network.	7	L3	CO1
	b.	For the circuit shown in Fig. Q1(b). Find the voltage 'V' at node by using nodal analysis. Fig. Q1(b) 	6	L3	CO1
	c.	Determine the current in 12Ω resistor shown in Fig. Q1(c) using source transformation method. Fig. Q1(c) 	7	L3	CO1
		OR			
Q.2	a.	Find the loop currents I_1, I_2, and I_3 in the circuit shown in Fig. Q2(a). Fig. Q2(a) 	7	L3	CO1

	b.	Determine the resistance between the terminals X, Y using star delta transformation in the network shown in Fig. Q2(b).  Fig. Q2(b)	**6**	**L3**	**CO1**
	c.	Use the nodal analysis to find the value of V_X and the circuit shown in Fig. Q2(c). Such that the current through (2 + j3) Ω Impedance is zero.  Fig. Q2(c)	**7**	**L3**	**CO1**
		Module – 2			
Q.3	**a.**	Sate and prove Superposition theorem.	**7**	**L2**	**CO2**
	b.	For the circuit shown in Fig. Q3(b), obtain the Thevenin's equivalent circuit.  Fig. Q3(b)	**7**	**L3**	**CO2**
	c.	Using Millman's theorem, find current flowing through (3 + j4) Ω impedance for the circuit shown in Fig. Q3(c).  Fig. Q3(c)	**6**	**L3**	**CO2**
		OR			

Q.4	**a.**	State and prove Norton's theorem.	**7**	**L2**	**CO2**
	b.	Find the value of Z_L for Maximum Power transfer and the value of Maximum power for the circuit shown in Fig. Q4(b). Fig. Q4(b) 	**6**	**L3**	**CO2**
	c.	Find current 'I' using Super position theorem for the circuit shown in Fig. Q4(c). Fig. Q4(c) 	**7**	**L3**	**CO2**
		Module – 3			
Q.5	**a.**	Use the concepts of initial condition to illustrate the voltage behavior in inductor circuit for DC supply.	**6**	**L3**	**CO3**
	b.	In the circuit steady state is reached with switch 'K' open. The switch is closed at t = 0. Compute i, di/dt and d^2i/dt^2 at $t = 0^+$. Fig. Q5(b) 	**7**	**L3**	**CO3**
	c.	The switch is moved from position (1) to position (2) at t = 0. The steady state has been reached before switching. Computer i, di/dt and d^2i/dt^2 at $t = 0^+$ for Fig. Q5(c). Fig. Q5(c) 	**7**	**L4**	**CO3**
		OR			

Q.6	a.	In the circuit shown in Fig. Q6(a), determine complete solution for current when switch 'K' is closed at t = 0. Fig. Q6(a) 	10	L3	CO3
	b.	Compute v , dv/dt , d^2v/dt^2 at t = 0^+ for the circuit shown in below Fig. Q6(b), when the switch K is opened at t = 0. Fig. Q6(b) 	10	L4	CO3
		Module – 4			
Q.7	a.	Using waveform synthesis method to express the voltage pulse interms of unit step. Find i) L{i(t)} ii) L{∫ i(t).dt}. Fig. Q7(a) 	8	L3	CO4
	b.	State and prove initial value and final value theorem for Laplace transform.	6	L2	CO4
	c.	Obtain the Laplace transform of step and ramp function with relevant expressions.	6	L3	CO4
		OR			
Q.8	a.	Determine $i_L(t)$ for t ≥ 0 using Laplace transform for circuit shown in Fig. Q8(a). Fig. Q8(a) 	10	L3	CO4

	b.	Find the Laplace transform of the periodic signal x(t) as shown in Fig. Q8(b).  Fig. Q8(b)	10	L3	CO4
		Module – 5			
Q.9	a.	Define Z – parameters. Determine Y parameters interms if Z – parameters.	6	L3	CO5
	b.	Show that resonant frequency is geometric mean of cut off frequency in series R – L – C circuit.	7	L3	CO5
	c.	Apply the two – port network analysis technique to determine ABCD – parameters of the network shown in Fig. Q9(c).  Fig. Q9(c)	7	L3	CO5
		OR			
Q.10	a.	Derive the expression for the resonant frequency of the circuit shown in Fig. Q10(a). Also show that the circuit resonate at all frequency if $R_L = R_C = \sqrt{\frac{L}{C}}$.  Fig. Q10(a)	10	L3	CO5
	b.	The model of a transistor in the CE mode is shown in Fig. Q10(b). Determine the h – parameters.  Fig. Q10(b)	10	L3	CO5

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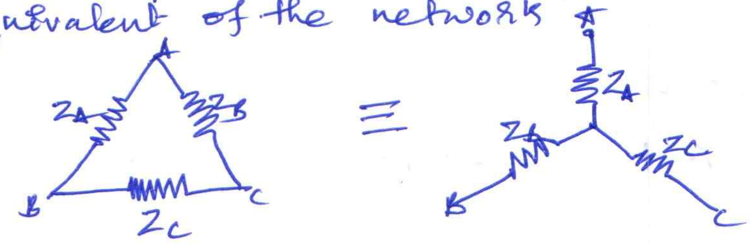
Course: Networks Analysis   Course Code: BEC304

1 a) Three impedances are connected in Delta. obtain the star equivalent of the network



—— (1M)

7M

Impedances Between A & B with C open in Δ & Y is

$$Z_A + Z_B = \frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}} \quad \text{—— (1)}$$

similarly $$Z_B + Z_C = \frac{Z_{BC}(Z_{AB} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}} \quad \text{—— (2)}$$

similarly $$Z_C + Z_A = \frac{Z_{CA}(Z_{AB} + Z_{BC})}{Z_{AB} + Z_{BC} + Z_{CA}} \quad \text{—— (3)}$$

3 M

Solving Equation (1) - (2) + (3)

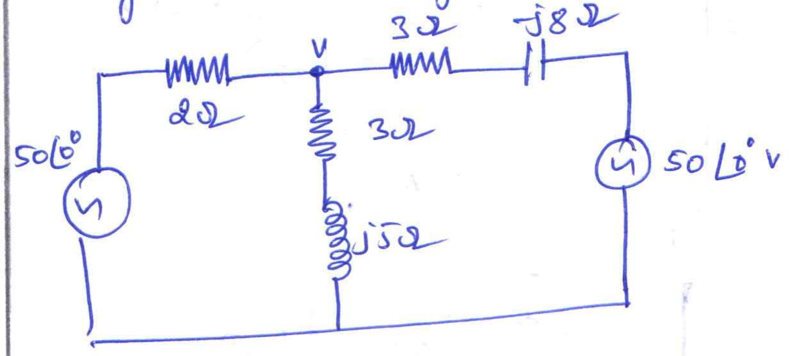
we get $$Z_A = \frac{Z_{AB} Z_{CA}}{\Sigma Z_{AB}} \quad \text{—— (4)}$$

similarly $$Z_B = \frac{Z_{AB} Z_{BC}}{\Sigma Z_{AB}} \quad \text{—— (5)}$$

similarly $$Z_C = \frac{Z_{BC} Z_{CA}}{\Sigma Z_{AB}} \quad \text{—— (6)}$$

4 M

1b) For the circuit shown find the voltage 'V' at node by using nodal Analysis



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Applying Node Analysis to the above circuit we get

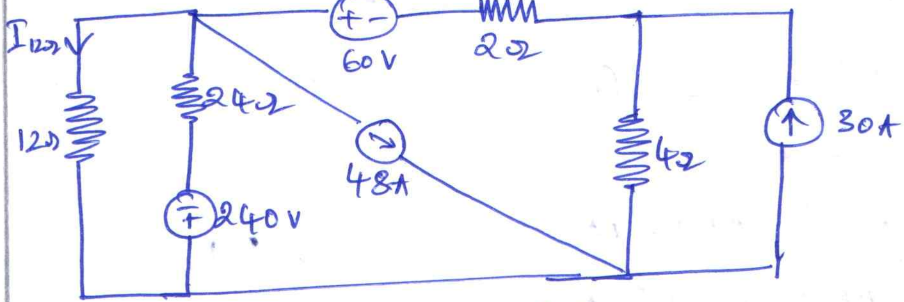
$$\frac{V_1 - 50\angle 0^\circ}{2} + \frac{V_1 - 50\angle 0^\circ}{(3 - j8)} + \frac{V_1}{3 + j5} = 0 \quad \text{—— 2M}$$

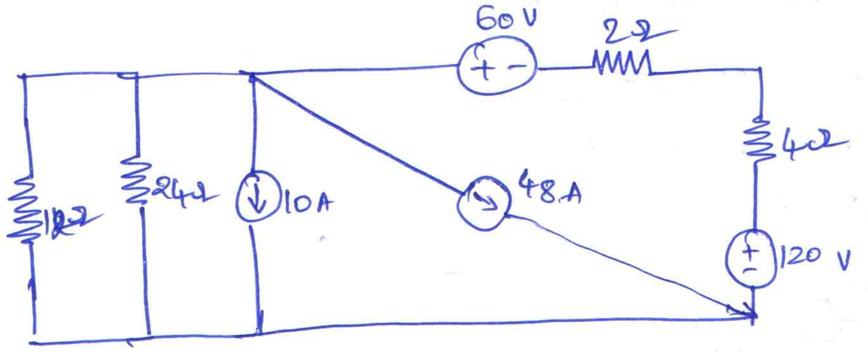
$$\frac{V_1}{2} + \frac{V_1}{3-j8} + \frac{V_1}{3+j5} - \frac{50\angle 0^\circ}{2} + \frac{50\angle 0^\circ}{(3-j8)} = 0 \quad \text{—— (2M)}$$

on simplification of above

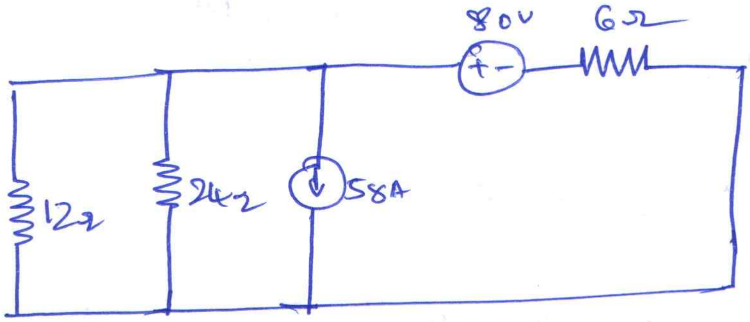
$$V_1 = 44.44 \angle 13.03 \text{ Volts} \quad \text{—— (2M)}$$

1 c) Determine the current in 12Ω resistor using source Transformation 7





— 2M



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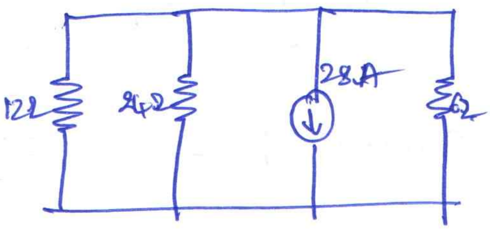
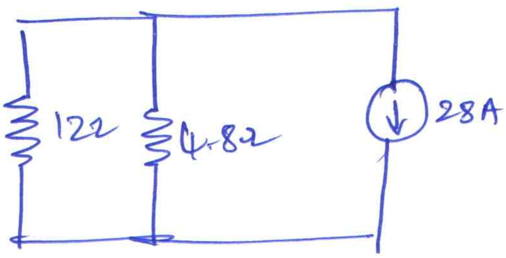
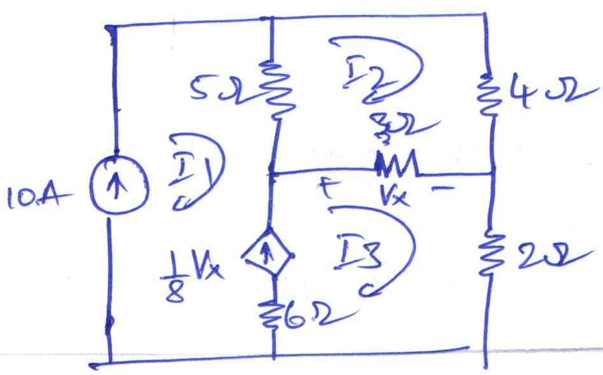




$$\Rightarrow \quad I_{12\Omega} = \frac{4.8}{(4.8+12)} \times 28\text{ A}$$

$$\boxed{I_{12\Omega} = 8\text{ A}}$$

2 a) Find the loop currents I_1, I_2 & I_3 in the circuit shown 7M



2 a) From the circuit it can be seen that $\boxed{I_1 = 10A}$ — 1M. — (1)

Applying KVL to loop 2

$$5(I_2 - I_1) + 4I_2 + 3(I_2 - I_3) = 0 \quad \text{— (2)} \quad \text{— 2 M}$$

from the circuit, across loop 3

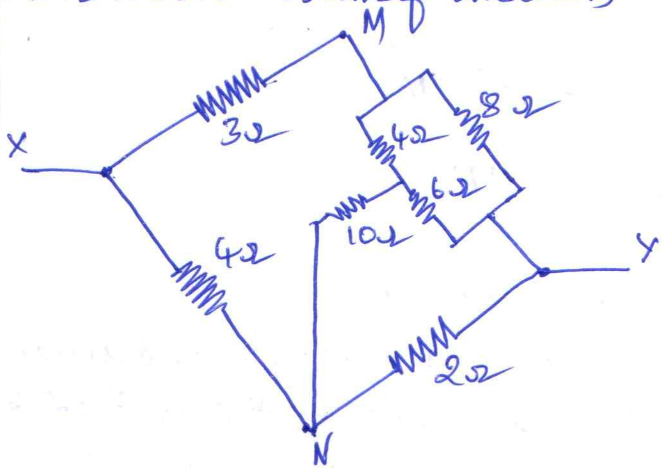
$$\frac{1}{8} V_x = (I_3 - I_1) = \frac{1}{8}(3I_3 - 3I_2)$$

$$\Rightarrow 3I_2 + 5I_3 = 80 \quad \text{— (3)} \quad \text{— 1 M}$$ 7

on solving Equations (1), (2) & (3)

$\boxed{I_1 = 10A}$, $\boxed{I_2 = 7.10A}$, $\boxed{I_3 = 11.73A}$ — 3 M

2 b) Determine the resistance between the terminals x, y using Star Delta Transformation



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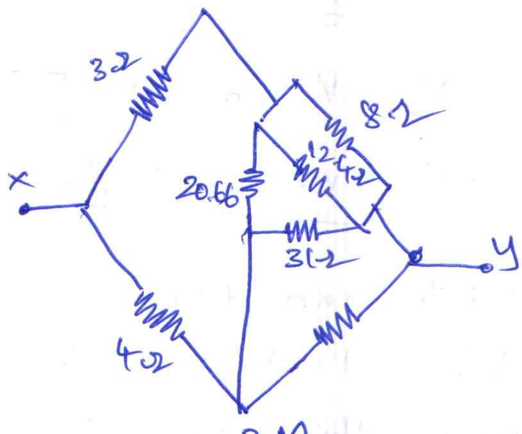
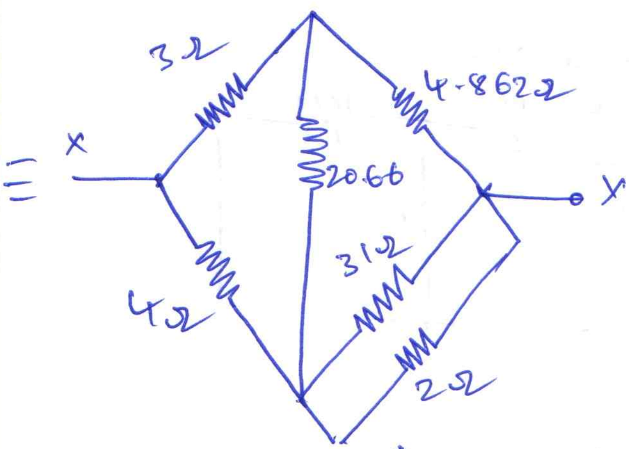


2 M 6



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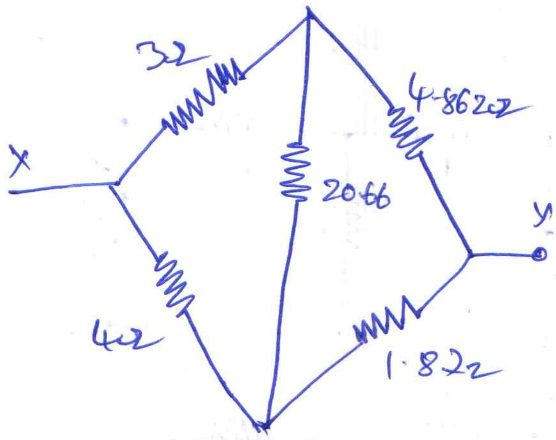

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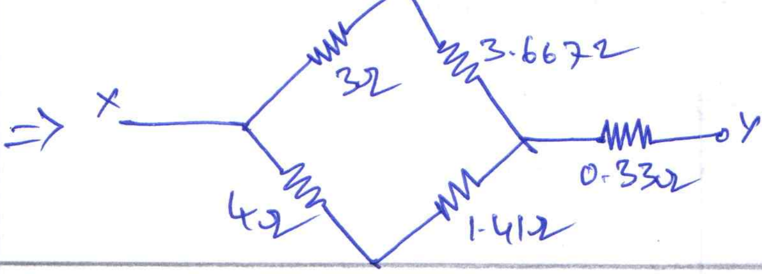
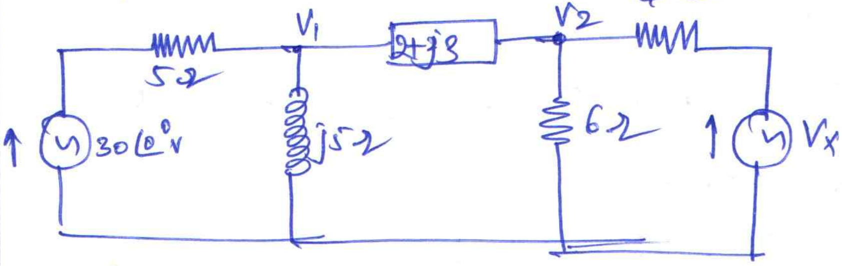


$\boxed{R_{xy} = 3.31\Omega}$

2c Use Nodal Analysis to find the value of V_x and the current through $(2+j3)\Omega$ Impedence is zero 7



Applying KCl at Node V_1

$$\frac{V_1 - 30}{5} + \frac{V_1}{j5} + 0 = 0 \quad \because (2+j3)\Omega = 0 \text{ [given]}$$

$\Rightarrow V_1 = 21.21 \angle 45$ Volts. — 4 M.

Applying KCl at Node 2

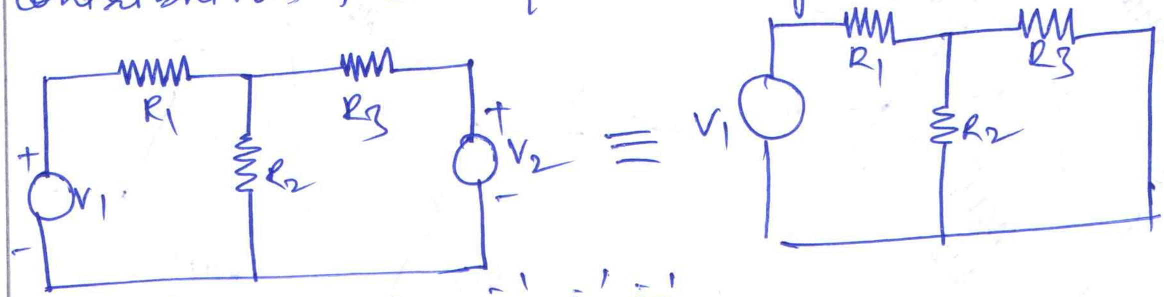
$$0 + \frac{V_2}{6} + \frac{V_2 - V_x}{4} = 0$$

$V_2 - V_1 = 0 \Rightarrow \boxed{V_2 = V_1}$ — 2 M

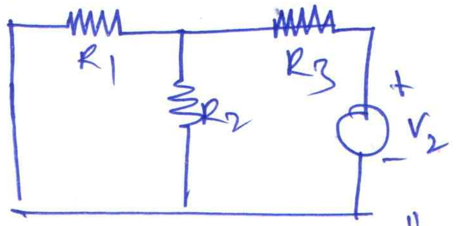
$\therefore \boxed{V_x = 35.33 \angle 45^\circ \text{ V}}$ — 1M

3 a) State and Prove Superposition Theorem.

Statement :- In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as Algebraic sum of individual contributions of each source acting Alone.



Calculate currents i_1', i_2', i_3'



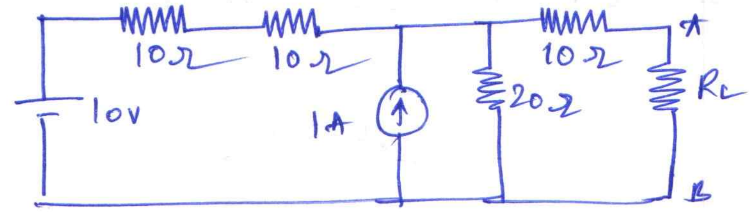
$I_3 = i_3' + i_3''$

$I_2 = i_2' + i_2''$

$I_1 = i_1' + i_1''$

Calculate currents i_1'', i_2'', i_3''

3 b) Obtain the Thevenin's Equivalent circuit



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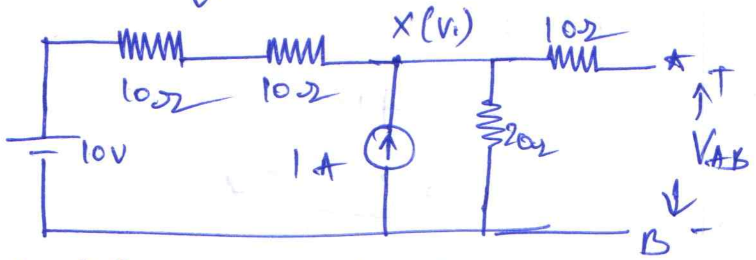
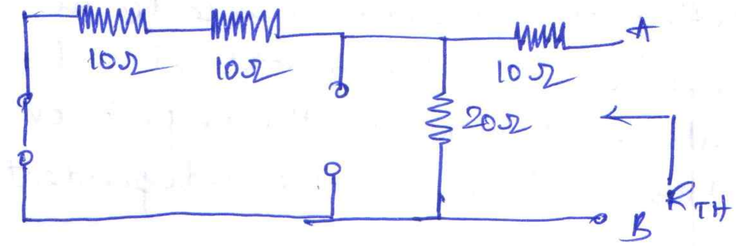


Applying KCL at Node X: $\frac{V_1 - 10}{20} - 1 + \frac{V_1}{20} = 0$

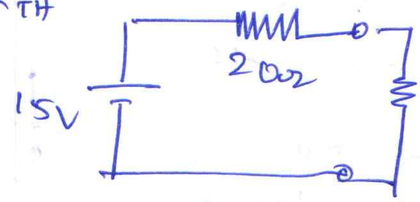
$\Rightarrow$ $\boxed{V_1 = 15\ V = V_{AB} = V_{TH}}$ — 3 M



$R_{TH} = (20 \| 20)\Omega + 10\Omega$

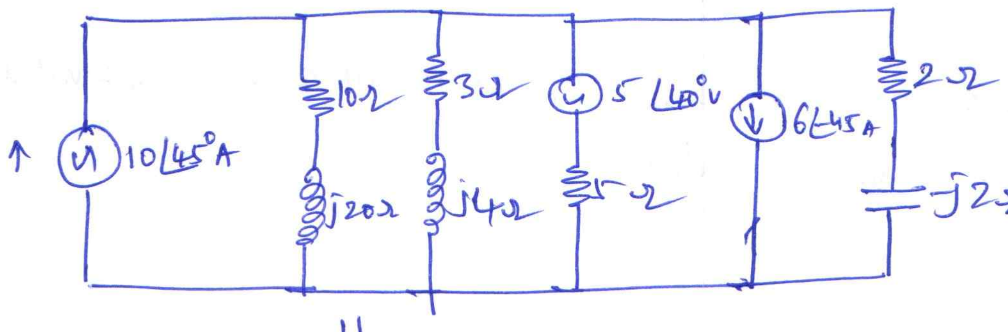
$\Rightarrow R_{TH} = 20\Omega$ — 2 M

Thevenin's Equivalent circuit



— 2M

3 c) Using Millman's Theorem find the current through $(3+j4)\Omega$ impedence for the circuit shown.



Applying Millman's Theorem

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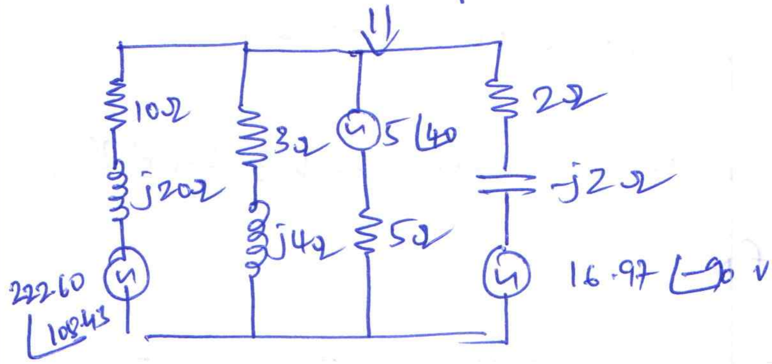
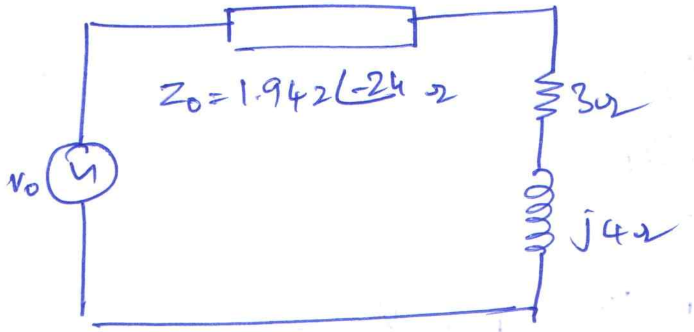


7

$$E_0 = \frac{E_1/Z_1 + E_2/Z_2 + E_3/Z_3}{1/Z_1 + 1/Z_2 + 1/Z_3} \Rightarrow E_0 = 21.11 \angle 54.98 \text{ volts}$$

$$Z_0 = \frac{1}{1/Z_1 + 1/Z_2 + 1/Z_3} \Rightarrow Z_0 = 1.942 \angle -24.07 \,\Omega.$$

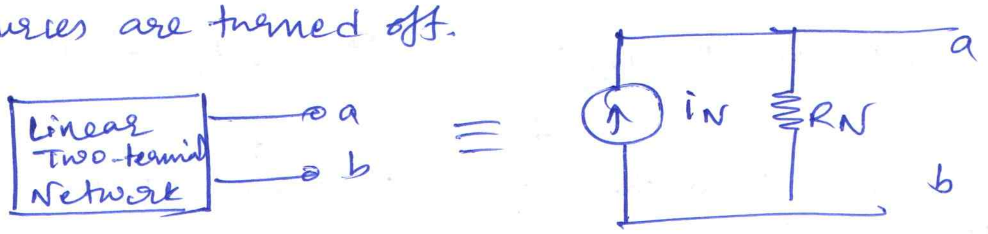


$$I = \frac{E_0}{Z_0 + 3 + j4}$$

$$I = 3.67 \angle 21.08 \text{ A}$$

4a) State and Prove Norton's Theorem.

Statement :- A linear two-terminal network can be replaced by an equivalent circuit consisting of a current source i_N in parallel with resistor R_N, where i_N is the short circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



1) If the n/w contains Resistors, independent sources & dependent sources

→ Determine short circuit current i_N with all sources activated

→ find open-circuit voltage V_{oc}

→ $R_N = \frac{V_{oc}}{i_N}$

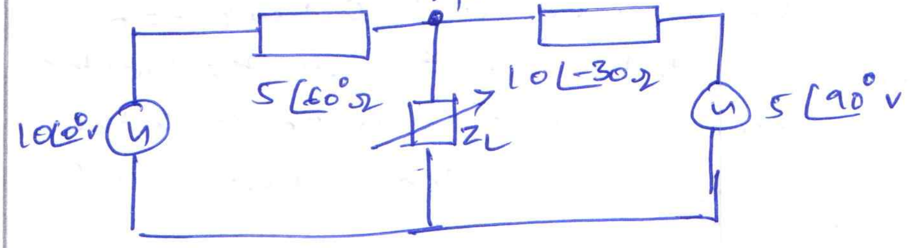
2) If the n/w contains resistors, dependent sources

→ Note that $i_N = 0$

→ Connect 1 A current source to terminals a-b & find V_{ab}

→ $R_t = \frac{V_{ab}}{1} = \frac{V_{oc}}{I_{sc}} = R_N.$

4b Find the value of Z_L for Maximum Power Transfer and the Value of Maximum power for the circuit. 6



Applying KCL at Node

$$\frac{V_1 - 10\angle 0^\circ}{5\angle 60^\circ} + 0 + \frac{V_1 - 5\angle 90^\circ}{10\angle -30} = 0$$

$\Rightarrow$ $V_1 = 6.708 \angle -26.5^\circ$ —— 2M

$\Rightarrow$ $V_0 = 6.708 \angle -26.5^\circ$

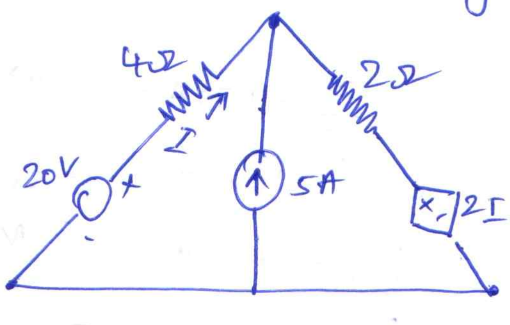
$$\Rightarrow Z_0 = \frac{(5\angle 60)(10\angle -30)}{(5\angle 60 + 10\angle -30)} = (3.732 + j2.4641)\Omega \text{ — 2M}$$

the condition for Maximum P_{max} $Z_L = Z_0^*$ $(3.732 - j2.464)$

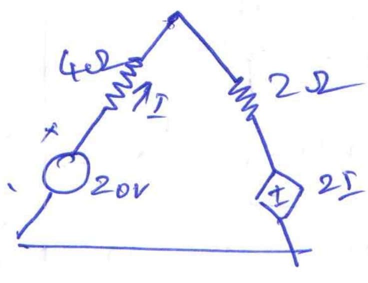
$$\therefore I_L = \frac{V_0}{Z_0 + Z_0^*} = \frac{6.708\angle -26.5}{2 \times 3.732} = 0.898 \angle -26.5 A$$

$P_{max} = I_L^2 \times R_L = (0.898)^2 \times 3.732 = 3.0142 W.$ —— 2M

4c) Find current I using superposition Theorem.



Case 2:

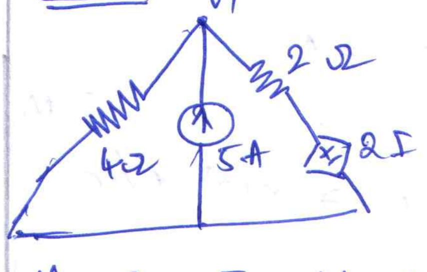


$-20 + 4I + 2I + 2I = 0$

$I' = 2.5 A$ 7

— 2M

Case 1:



$$\frac{V_1 - 0}{4} - 5 + \frac{V_1 - 2I}{2} = 0$$

$V_1 = -4I$ —— 2M

WKT from Case 2 $V_1 = -4I$

~~$5 - 4 \times 2.5$~~

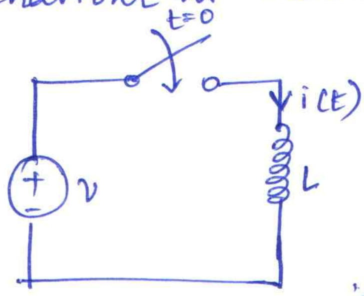
$I'' = -1.25 A$ — 1M

On Applying superposition Theorem

$I = I' + I''$

$= 2.5 - 1.25 = 1.25 A$ — 2M

5 a) Use the concept of initial condition to illustrate the voltage behaviour in inductor circuit for DC supply 6



* The switch is closed at $t=0$. Hence $t=0$ corresponds to the instant when the switch is just open and $t=0^+$ corresponds to the instant when the switch is just closed.

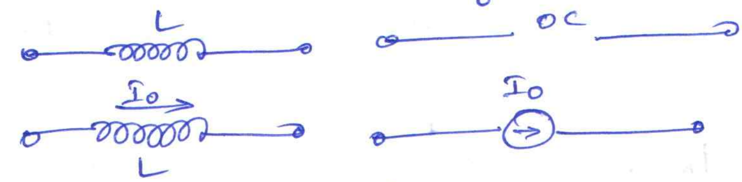
$$\therefore i = \frac{1}{L}\int_{-\infty}^{t} v \cdot d\tau$$

$$\therefore i = \frac{1}{L}\int_{-\infty}^{0} v \cdot d\tau + \frac{1}{L}\int_{0}^{t} v \cdot d\tau$$

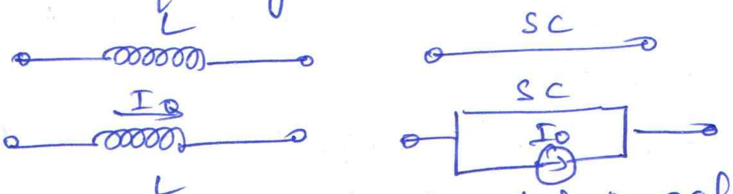
$$\Rightarrow i(t) = i(0^-) + \frac{1}{L}\int_{0^-}^{t} v \cdot d\tau$$

Putting $t=0^+$ on the both sides.

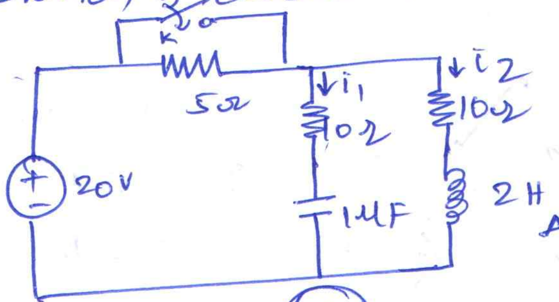
$$i(0^+) = i(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v \cdot d\tau \Rightarrow i(0^+) = i(0^-)$$



under steady condition



5 b) In the circuit the steady state is achieved with 'K' open. The switch is closed at $t=0$. Compute i, di/dt and d^2i/dt^2 at $t=0^+$ 7



(1M)

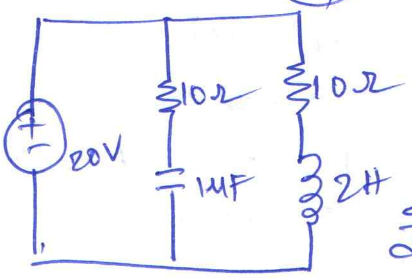
At $t=0^-$, $i_2(0^-) = \frac{20}{15} = 1.33A = i_2(0^+)$

$V_C(0^-) = 10\, i_2(0^-) = 10 \times 1.33 = 13.3V = V_C(0^+)$

2M

At $t=0^+$ the ckt reduces to

$$20 = 10\, i(0^+) + \underbrace{\frac{1}{C}\int_{0^+}^{t} i(\tau)\, d\tau}_{V_C(0^+)} \quad \text{—— (1)}$$



$$i_1(0^+) = \frac{20 - 13.3}{10} = 0.67A$$

Differentiating Equation (1) 2M

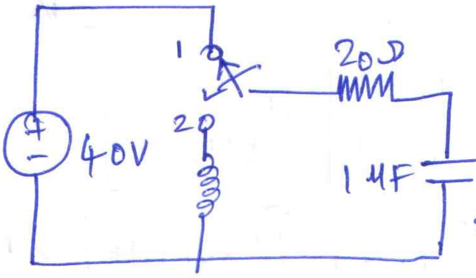
$$\frac{d}{dt} i_1(0^+) = \frac{-0.67}{10 \times 10^{-6}} = -0.67 \times 10^5 \text{ A/sec}$$

Applying KVL to loop 2

$$20 = 10\, i_2(0^+) + 2 \cdot \frac{d}{dt} i_2(0^+) \Rightarrow \frac{d}{dt}\{i_2(0^+)\} = 3.35 \text{ A/sec} \text{ — 2M}$$

8

5c) The switch is moved from position (1) to position (2) at $t=0$. The steady state has been reached before switching. Compute i, di/dt and d^2i/dt^2 at $t=0^+$ 7

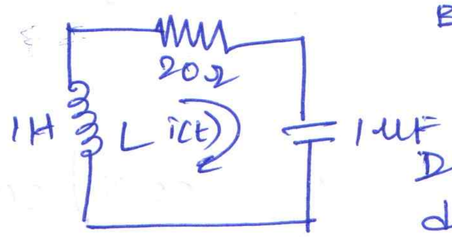


At $t=0^-$ the switch is in position 1

$\therefore V_C(0^-) = V_C(0^+) = 40V$

$\therefore i(0^-) = 0A = i(0^+)$] 2 M.

At $t=0^+$ the switch is in position 2.



By KVL

$$R\,i(t) + \frac{1}{C}\int_0^t i(t).dt + L.\frac{di(t)}{dt} = 0 \quad \text{—(1)} \quad -1M$$

Differentiating Equation (1)

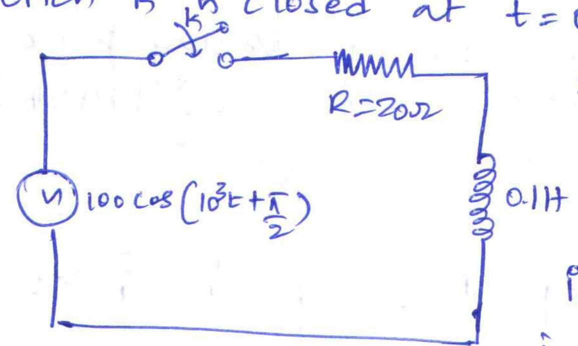
$$\frac{di(0^+)}{dt} = -40\ A/sec \quad \text{— 2 M}$$

Differentiating Eq^n

$$R.\frac{di(t)}{dt} + \frac{1}{C}\cancel{i(t)\,dt} + L\frac{d^2i(t)}{dt^2} \Rightarrow \frac{d^2i(t)}{dt^2} = 800\ A/sec^2$$

3M

6a) In the circuit determine the complete solution for current when switch 'k' is closed at $t=0$ 10



$$20i + 0.1\frac{di}{dt} = 100\cos(10^3t + \pi/2)$$

$$\therefore \frac{di}{dt} + 200i = 1000\cos(10^3t + \pi/2) \text{ -2M}$$

$$i_c = k.e^{-t/(L/R)} = k.e^{-200t}$$

$$i_{p_1} = \frac{V}{\sqrt{R^2+(\omega L)^2}}\cos\left(\omega t + \phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$i_{p_1} = \frac{100}{\sqrt{(20)^2 + (1000\times0.1)^2}}\cos\left[1000t + \frac{\pi}{2} - \tan^{-1}\left(\frac{1000\times0.1}{20}\right)\right] \text{ — 2M}$$

$$i_{p_1} = 0.9805\cos\left(1000t + \frac{\pi}{2} - 78.69^\circ\right) \text{ — 1M}$$

$$i = i_{p_1} + i_c = k.e^{-200t} + 0.98\cos\left(1000t + \frac{\pi}{2} - 78.6^\circ\right) \text{ — 2M}$$

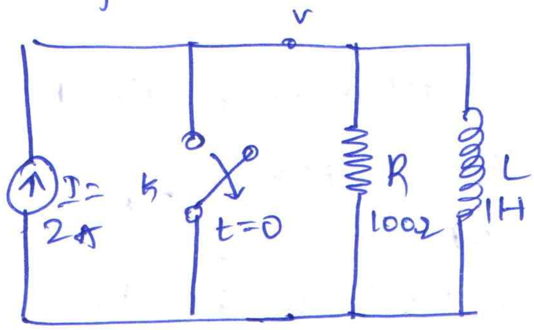
Now to find k, At $t=0$

$$0 = k + 0.98\cos(0 + \pi/2 - 78.6^\circ) \Rightarrow k = -0.98\cos(\pi/2 - 78.6^\circ)$$

$\therefore$ The complete solution is

$$i = \left[-0.98\cos(\pi/2 - 78.6^\circ)e^{-200t} + 0.98\cos\left[1000t + \pi/2 - 78.6^\circ\right]\right] \text{ 3M}$$

6 b) Compute v, dv/dt, d^2v/dt^2 at $t=0^+$ for the circuit when switch K is opened at $t=0$ 10



At $t=0^-$, switch is closed

$i(0^-) = 0 = i(0^+)$ 2M

$I = I_R + I_L$

$$2 = \frac{v(t)}{100} + \left[\frac{1}{1}\int_{-\infty}^{t} v(t).dt\right]$$

$$2 = \frac{v(t)}{100}\left[\int_{-\infty}^{0} v(t)dt + \int_{0^-}^{t} v(t).dt\right] \quad 2M$$

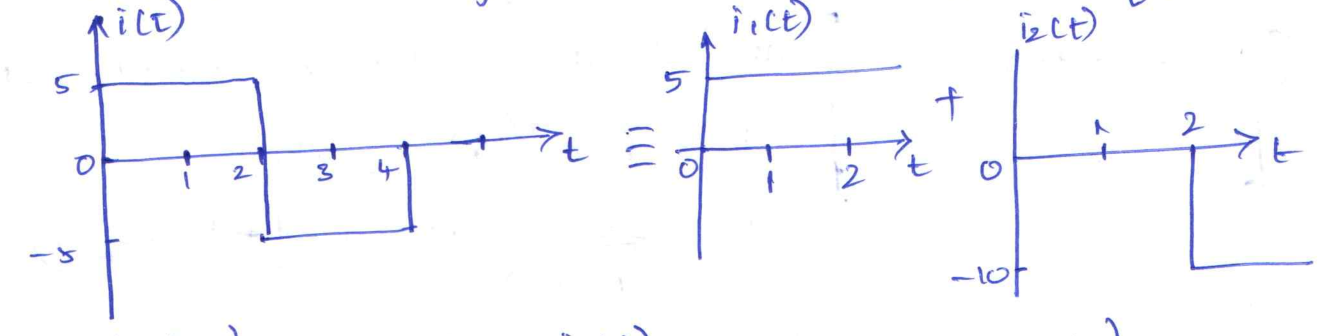
$$\int_{0^-}^{t} v(t).dt + \frac{v(t)}{100} = 2$$

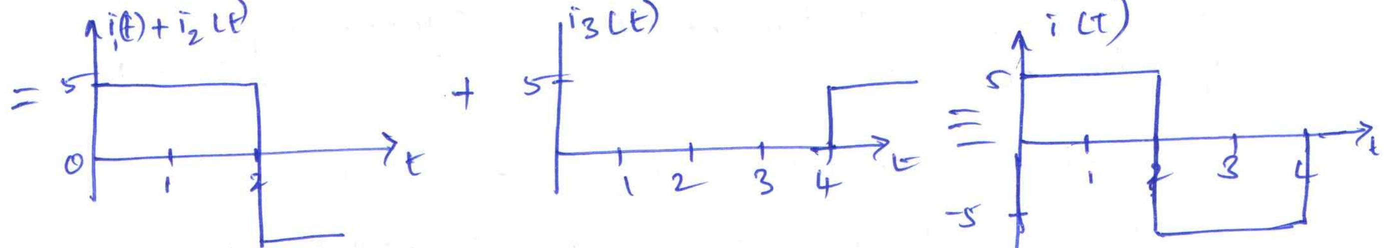
At $t=0^+ \Rightarrow \int_{0^-}^{0^+} v(t).dt + \frac{v(0^+)}{100} = 2 \Rightarrow \boxed{v(0^+) = 200\ v}$ 3M

$v(t) + \frac{1}{100}\frac{dv(t)}{dt} = 0 \Rightarrow \frac{dv}{dt}(0^+) = -200\ v/sec.$

$\frac{dv}{dt} + \frac{1}{100}\frac{d^2v(t)}{dt^2}(0^+) = 0 \Rightarrow \frac{d^2v(t)}{dt^2}(0^+) = 2\times10^5\ v/sec^2$ 3M

7a) Using waveform synthesis method express the voltage pulse interms of unit step & find i) $L\{i(t)\}$ ii) $L\{\int i(t)dt\}$





$i(t) = i_1(t) + i_2(t) + i_3(t) = 5u(t) - 10u(t-2) + 5u(t-4)$

Taking L.T of above Eq^n

$$I(s) = \frac{5}{s} - \frac{10}{s}e^{-2s} + \frac{5}{s}e^{-4s} = \frac{5}{s}\left[1 - e^{-2s} + e^{-4s}\right] = \frac{5}{s}\left[1-e^{-2s}\right]^2$$

$$f(t) = \int i(t).dt = \int\left[5u(t) - 10u(t-2) + 5u(t-4)\right]dt = 5\delta(t) - 10\delta(t-2) + 5\delta(t-4)$$

Course: Network Analysis   Course Code: BEC304

7b State and Prove Initial value and final value theorem for Laplace transform. 6

Initial Value Theorem :- This allows to find initial value $x(0)$ directly from its Laplace Transform.

If $x(t)$ is a causal signal then $x(0) = \lim_{s\to\infty} s.X(s)$.

$$L\left\{\frac{dx(t)}{dt}\right\} = s.X(s) - x(0) = \int_0^\infty \frac{dx}{dt} e^{-st} dt$$

If we let $s\to\infty$, then integral on RHS vanishes due to damping

$$\lim_{s\to\infty}[s.X(s) - x(0)] = 0 \Rightarrow x(0) = \lim_{s\to\infty} s.X(s) \quad \text{— 3 M}$$

Final Value Theorem :- This allows to find final value $x(\infty)$ directly from its Laplace Transform.

If $x(t)$ is a causal signal then $\lim_{t\to\infty} x(t) = \lim_{s\to 0} s.X(s)$

$$L\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0) = \int_0^\infty \frac{dx(t)}{dt} e^{-st} dt = s X(s) - x(0)$$

if $s\to 0$ then

$$\lim_{s\to 0}[s.X(s) - x(0)] = \lim_{s\to 0} \int_0^\infty \frac{dx}{dt} e^{-st} dt = \int_0^\infty \frac{dx(t)}{dt}\left(\lim_{s\to 0} e^{-st}\right) dt$$

$$= \int_0^\infty \frac{dx(t)}{dt} dt = x(t)\Big|_0^\infty = x(\infty) - x(0)$$

$$\lim_{s\to 0} s.X(s) - x(0) = x(\infty) - x(0) \qquad \boxed{x(\infty) = \lim_{s\to 0} s.X(s)}$$

7c) Obtain the Laplace Transform of Step and Ramp functions with relavant Expressions 6

$$f(t) = u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t \leq 0 \end{cases}$$

$$L\{f(t)\} = \int_0^\infty f(t).e^{-st}.dt$$

$$= \int_0^\infty u(t).e^{-st}.dt$$

$$= \int_0^\infty 1.e^{-st} dt$$

$$= \frac{e^{-st}}{-s}\Big|_0^\infty = \frac{-1}{s}\left[e^{-\infty} - e^{0}\right] = \frac{1}{s}$$

$$f(t) = r(t) = t \text{ for } t \geq 0$$

$$L\{f(t)\} = \int_0^\infty f(t).e^{-st} dt$$

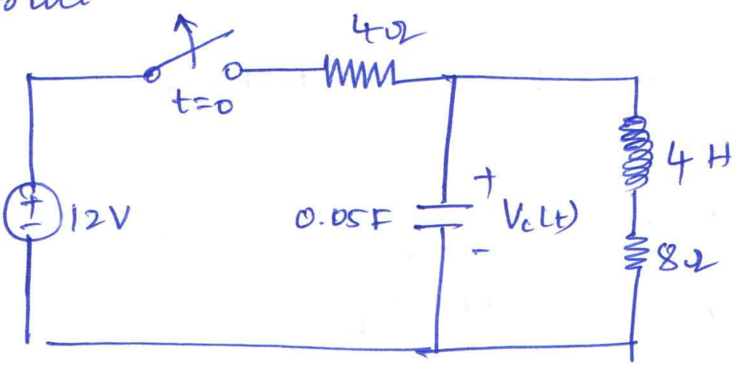
$$= \int_0^\infty t.e^{-st} dt.$$

$$L\{r(t)\} = \left[\frac{t e^{-st}}{-s}\right]_0^\infty - \int_0^\infty \frac{e^{-st}}{-s}.1.dt$$

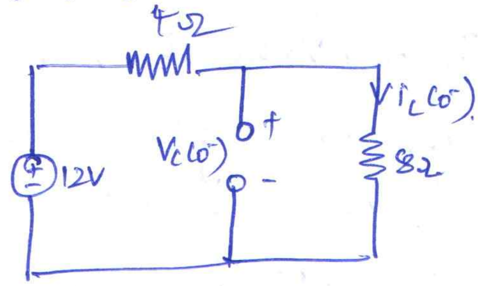
$$= 0 + \frac{1}{s}\int_0^\infty e^{-st} dt = \frac{1}{s}\left[\frac{1}{s}\right]$$

$$= 1/s^2$$

8 a) Determine $i_L(t)$ for $t \geq 0$ using Laplace Transform for the circuit 10



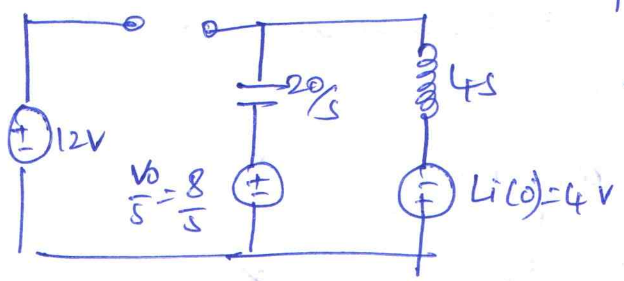
At $t = 0^-$



$i_L(0^-) = \frac{12}{12} = 1\,A = i_L(0^+)$ $\therefore V_c(0^-) = 8 \times i_L(0^-) = 8$ Volts $= V_c(0^+)$

4M

At $t = 0^+$



Apply KVL to loop (2)

$$\frac{8}{s} - \frac{20}{s} I_L(s) - 4s\,I(s) + 4 - 8 I_L(s) = 0$$

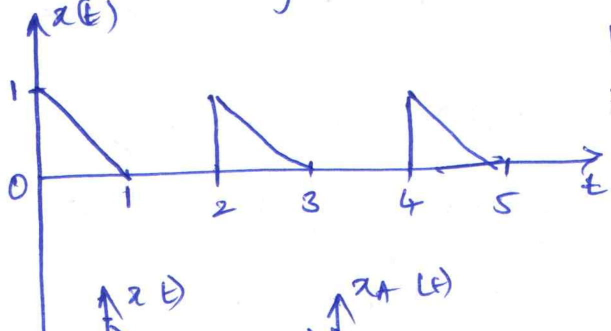
$$\therefore I_L(s) = \frac{8+4s}{20+4s^2+8s} \quad \text{— 4M}$$

$$I_L(s) = \frac{\cancel{4}(s+2)}{\cancel{4}(s^2+2s+5)} = \frac{s+2}{s^2+2s+5} = \frac{(s+1)+1}{(s+1)^2+2^2}$$

$$= \frac{s+1}{(s+1)^2+2^2} + \frac{1}{2} \times \frac{2}{((s+1)^2+2^2}$$

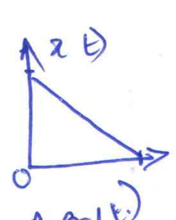
$$i_L(t) = \left[e^{-t}\cos(2t) + \frac{1}{2} e^{-t}\sin(2t)\right]u(t) \quad \text{—— 2M}$$

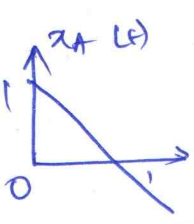
8b) Find the Laplace Transform of the periodic Signal 10

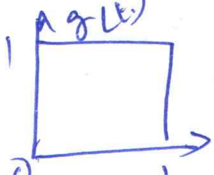


Laplace Transform of a periodic signal is given by $L[f(t)] = \frac{1}{1-e^{-s}} \int_0^1 e^{-st} f(t)\,dt$

1 M







—2M

$x_1(t) = x_A(t).\,g(t)$

$= [-t+1]\,[u(t) - u(t-1)]$

$= -t\,u(t) + t\,u(t-1) + u(t) - u(t-1)$

$= -t\,u(t) + (t-1)\,u(t-1) + u(t-1)$

$+ u(t) - u(t-1)$

$= u(t) - r(t) + r(t-1)$ — 4m

~~Apply~~ L.T $X_1(s) = 1/s - 1/s^2 + e^{-s}/s^2 = \frac{s-1+e^{-s}}{s^2}$

$$\therefore X(s) = \frac{X_1(s)}{1-e^{-sT}} = \frac{s-1+e^{-s}}{s^2(1-e^{-2s})} \quad \text{——— 3M}$$

9a Define Z-Parameters. Determine Y parameters interms of Z-parameters 6

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$, $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ $Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$, $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

$\Delta = Z_{11}Z_{22} - Z_{12}Z_{21} = \Delta Z$ $\Delta_1 = Z_{22}V_1 - Z_{12}V_2$ $\Delta_2 = Z_{11}V_2 - Z_{21}V_1$

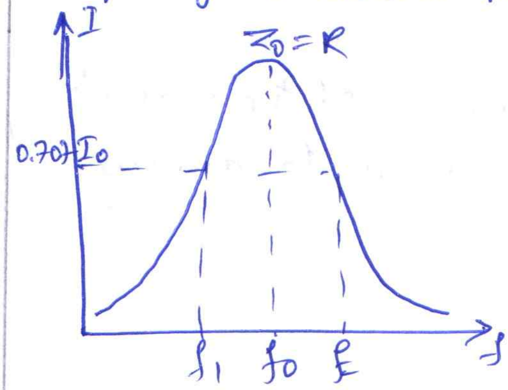
2M

$I_1 = \frac{\Delta_1}{\Delta} = \frac{Z_{22}}{\Delta Z}V_1 - \frac{Z_{12}}{\Delta Z}V_2$

$I_2 = \frac{\Delta_2}{\Delta} = \frac{-Z_{21}}{\Delta Z}V_1 + \frac{Z_{11}}{\Delta Z}V_2$ —— 2M

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{22}/\Delta Z & -Z_{12}/\Delta Z \\ -Z_{21}/\Delta Z & Z_{11}/\Delta Z \end{bmatrix}$$ —— 2M

9b Show that Resonant frequency is geometric mean of cut off frequency in Series R-L-C circuit 7



$Z_1 = \sqrt{R^2 + (X_{C1} - X_{L1})^2}$

$Z_2 = \sqrt{R_2^2 + (X_{L2} - X_{C2})^2}$

$Z_1 = Z_2$

$R^2 + (X_{C1} - X_{L1})^2 + R^2 + (X_{L2} - X_{C2})^2$

$X_{C1} - X_{L1} = X_{L2} - X_{C2}$ — 2M

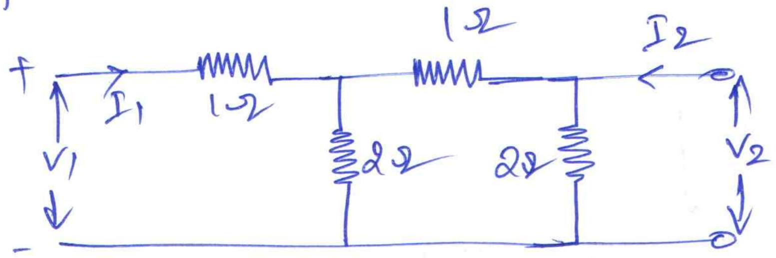
$\frac{1}{\omega C_1} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$

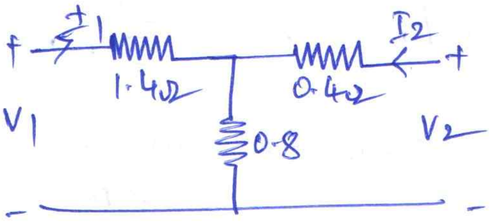
$\omega_1\omega_2 = \frac{1}{LC}$ (1) 2M WKT $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 = \frac{1}{LC}$ — (2)

From (1) & (2) $f_0 = \sqrt{f_1 f_2}$ —— 2M

9c Apply two-Port network Analysis to determine ABCD parameters of the network 7





$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$ 2 M

Apply KVL

$V_1 = 2.2I_1 + 0.8I_2$ —— (1)

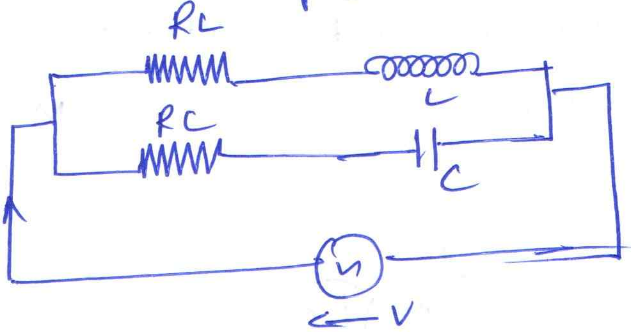
$V_2 = 0.8I_1 + 1.2I_2$ —— (2) 2 M

$$\therefore \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2.2 & 0.8 \\ 0.8 & 1.2 \end{bmatrix}$$

NKT

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} Z_{11}/Z_{21} & -\Delta Z/Z_{21} \\ 1/Z_{2} & Z_{22}/Z_{21} \end{bmatrix} = \begin{bmatrix} 2.75 & 2.5 \\ 1.25 & 1.5 \end{bmatrix}$$ 3 M

10a) Derive Expression for Resonant frequency of the circuit Also show that the circuit resonate at all frequency if $R_L = R_c = \sqrt{\frac{L}{C}}$ 10



$$Y_L = \frac{1}{Z_L} = \frac{1}{R_L + jX_L} = \frac{R_L - jX_L}{R_L^2 + X_L^2}$$

$$Y_c = \frac{1}{Z_c} = \frac{1}{R_c - jX_c} = \frac{R_c + jX_c}{R_c^2 + X_c^2}$$

$$Y_T = \left[\frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2}\right] + j\left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

$$\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} = 0$$

$$R_L^2 C + \omega^2 L^2 C = L\omega^2 C^2 R_C^2 + L$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{R_L^2 - L/C}{LC(R_C^2 - L/C)}}$$

$$Y = \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_L^2 + X_C^2}$$

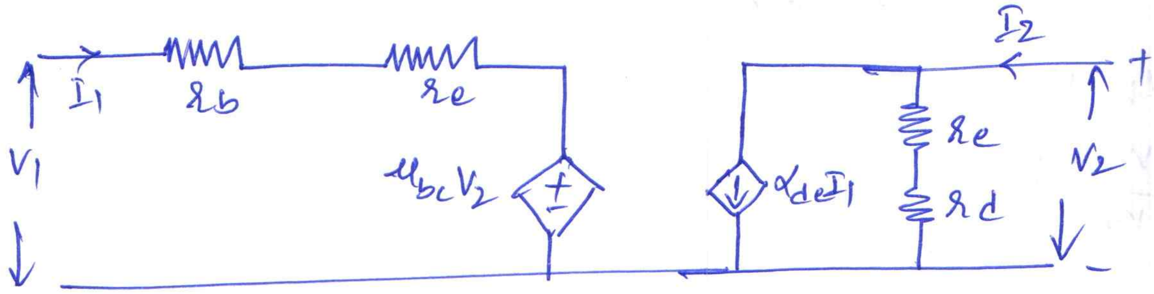
Assuming $R_L = R_C = R$

$$Y = \frac{R}{R^2}\left[\frac{\cancel{2R^2 + X_C^2 + X_L^2}}{\cancel{2R^2 + X_C^2 + X_L^2}}\right]$$

$$Y = \frac{1}{R}$$

$$\therefore R_L = R_C = R = \sqrt{\frac{L}{C}}$$

10 b) Determine h- Parameters



10

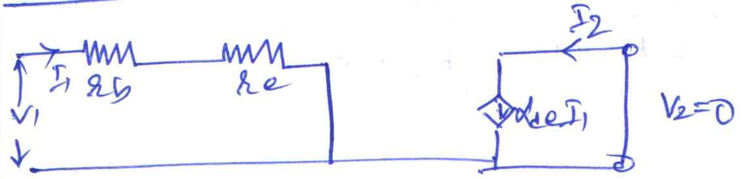
$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\therefore h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$ 2M

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Case(i) : $V_2 = 0$



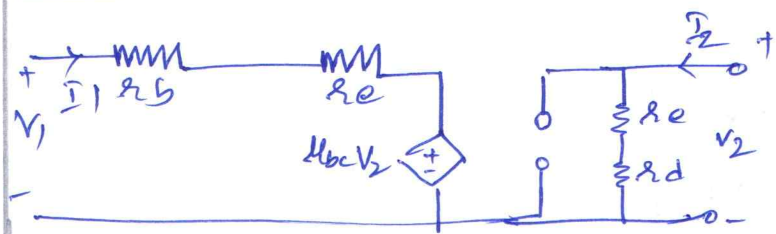
$$V_1 = I_1 (r_b + r_e)$$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = r_b + r_e$$

$$I_2 = \alpha_{de} I_1$$

$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0} = \alpha_{de}.$$

Case 2 : $I_1 = 0$



$$V_2 = I_2 (r_e + r_b)$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} = \frac{1}{r_e + r_b}$$

$$V_1 = \mu_{bc} . V_2$$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$

$$= \mu_{bc}$$