CBCS SCHEME

USN | | | | | | | | | | |

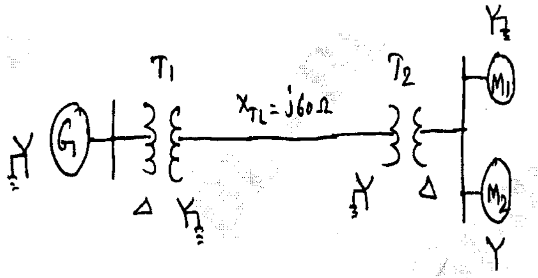
BEE601

Sixth Semester B.E./B.Tech. Degree Examination, June/July 2025

Power System Analysis – I

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define per unit quantity, mention the advantages of per unit system.	6	L1	CO1
	b.	Draw the reactance diagram of the system shown in Fig Q1(b). The ratings of the components are, $X_{TL} = j\,60\Omega$ Gen : 15 MVA, 6.6 KV, X″ = 12% T_1 : 20 MVA, 6.6/66 KV, X = 8% T_2 : 20 MVA, 66/6.6 KV, X = 8% M1 and M2 : 5 MVA, 6.6 KV, X″ = 20%  Fig Q1(b)	8	L3	CO1
	c.	Draw the per phase basis, representation of synchronous machine and Transmission line.	6	L3	CO1
		OR			
Q.2	a.	Show that per unit impedance of two winding transformer will remain as well as secondary.	6	L2	CO1
	b.	Derive an equation for per unit impedance if changes of base occur.	6	L2	CO1
	c.	Two generators rated 10MVA, 13.2 KV and 15 MVA, 13.2 KV are connected in parallel to a bus bar. They feed supply to two motors of input 8 MVA and 12 MVA respectively. The operating voltage of the motors of inputs 12.5 KV. Assuming base quantities as 50 MVA and 13.8 KV, draw the reactance diagram, The percentage reactance of generators is 15% and that for motor is 20%.	8	L1	CO1
		Module – 2			
Q.3	a.	What is doubling effect in the transmission line? Substantiate with equations.	10	L3	CO2
	b.	Explain with the help of oscillogram if short circuit current of synchronous generation operating on no load, distinguish between sub transient, transient and steady state periods. Also show that $x''_d < x'_d < x_d$ with equivalent circuit diagram.	10	L3	CO2

		OR			
Q.4	**a.**	A synchronous generator and motor are rated for 30,000 KVA, 13.2 KV and both have sub transient reactance of 20%. The line connecting them has a reactance of 10% on the base of machine ratings. The motor is drawing 20,000 KW at 0.8 p.f lead. The terminal voltage of motor is 12.8 KV. When a symmetrical 3 Phase Fault occurs art motor terminals, find the sub transient current in generator, motor and the fault point.	**10**	**L4**	**CO2**
	b.	What are the causes for faults in power system and how symmetrical faults are differ from unsymmetrical faults and how to analyze these faults. Explain the procedure to solve the problems using different methods.	**10**	**L2**	**CO2**
		Module – 3			
Q.5	**a.**	Determine the sequence components of the three voltages. $V_a = 200\angle 0^\circ$, $V_b = 200\angle 245^\circ$ and $V_c = 200\angle 105^\circ$.	**8**	**L2**	**CO3**
	b.	Prove that a balanced set of three phase voltages will have only positive sequence components of voltages only.	**6**	**L2**	**CO3**
	c.	Explain the concept of phase shift in star – Delta transformer bank.	**6**	**L1**	**CO3**
		OR			
Q.6	**a.**	Derive relation between sequence components of phase and line currents in delta connected system.	**10**	**L3**	**CO3**
	b.	Draw positive, negative and zero sequence network for the power system shown in Fig 6 (b). Per unit impedance Z_n in neutral of G_1 = j0.02 pu.	**10**	**L4**	**CO3**

Power system Components	Positive sequence Z_1	Negative sequence Z_2	Zero sequence Z_0
G_1, G_2, G_3	J 0.12 Pu	J0.08 Pu	J 0.03 pu
T_1, T_2, T_3	J0.1 Pu	J 0.1 Pu	j 0.1 pu
TL_1, TL_2, TL_3	J 0.084 Pu	J0.08 Pu	J 0.12 Pu

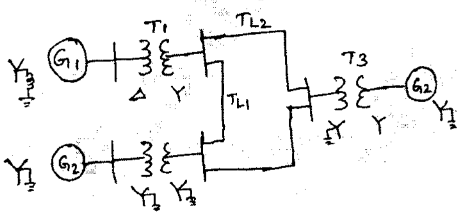


Fig Q6(b)

		Module – 4			
Q.7	**a.**	Derive the expression for fault current if Line – Line (LL) fault occurs through fault impedance Z_f in power system. Show the connection of sequence network to represent the fault.	**10**	**L3**	**CO4**
	b.	A 3-phae generator with an open circuit voltage of 400V is subjected to an LG fault through a fault impedance of j2 Ω. Determine the fault current is Z_1 = j 4Ω, Z_2 = j2 Ω, and Z_0 = j1 Ω. Also calculate the fault current for LL and LLG fault.	**10**	**L3**	**CO4**

		OR			
Q.8	**a.**	What are symmetrical faults? What are the different types of unsymmetrical fault and mention their frequency of occurrence.	**6**	**L1**	**CO4**
	b.	Draw inter connection of sequence network and mention the terminal condition for LG, LL and LLG faults.	**9**	**L2**	**CO4**
	c.	Derive the symmetrical component relation for one conductor open fault.	**5**	**L2**	**CO4**
		Module – 5			
Q.9	**a.**	Derive an expression for the swing equation and explain swing curve.	**8**	**L2**	**CO5**
	b.	A loss free alternator supplies 50 mW to an infinite bus, the SSSL being 100 mW. Determine if the alternator will remain stable if the input to the prime mover of the alternator is abruptly increased by 40 MW.	**8**	**L3**	**CO5**
	c.	Explain methods of improving transient stability.	**4**	**L1**	**CO5**
		OR			
Q.10	**a.**	Explain Equal area criteria concept when a power system is subjected to sudden change in mechanical input?	**10**	**L3**	**CO5**
	b.	Derive an expression for critical clearing angle and critical clearing time.	**10**	**L3**	**CO5**

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Solution of VTU Question Paper

June|July 2025

Power System Analysis-1 [BEE601]

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Module-01

01.a. Define per unit quantity, mention the advantages of per unit system. [06 Marks]

Per unit quantity is a ratio of actual value to the base value.

$$\text{Per unit value} = \frac{\text{actual value in any unit}}{\text{base value in same unit}}$$

Advantages of per unit system are

01. The P.u. value of reactance of a transformer refered to primary or secondary is same. Because of this the P.u. circuit of a power system becomes simple for analysing.
02. The P.u. values of 1-ϕ and 3-ϕ quantity are same, i.e $\sqrt{3}$ factor is not involved in P.U. calculation.
03. The P.U. value of the power system components of different ratings lies in a very narrow range. This makes P.U. calculation with less errors.

01.b. Draw the reactance diagram of the system shown below. The ratings of the components are,

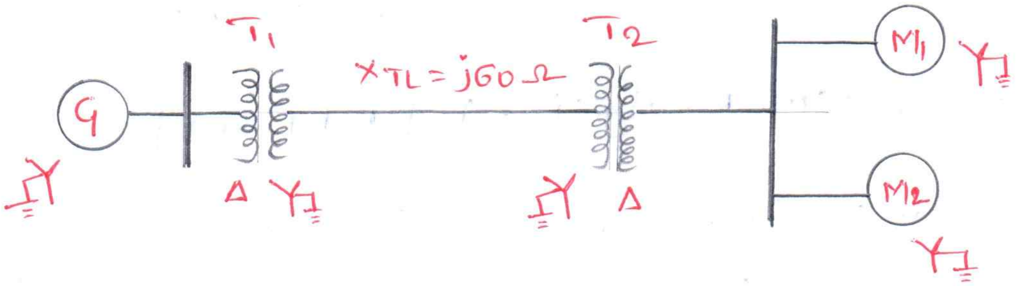
$X_{TL} = j60\,\Omega$

Gen : 15 MVA, 6.6 KV, $X'' = 12\%$.

T_1 : 20 MVA, 6.6|66 KV, $X = 8\%$.

T_2 : 20 MVA, 66|6.6 KV, $X = 8\%$.

M1 and M2 : 5MVA, 6.6kV, x'' = 20%



[08 Marks]

Let us choose a common base MVA of 20MVA
KV base in transmission line = 66KV
in gen and motor circuit = 6.6KV.

P.U. impedance

$$\text{of Gen} = Z_{pu(old)}\,\frac{MVA_{b(new)}}{MVA_{b(old)}}\;\frac{KV_{B(old)}^2}{KV_{B(new)}^2}$$

$$= j0.12 \times \frac{20}{15} \times \frac{6.6^2}{6.6^2} = j0.16\ \text{pu}$$

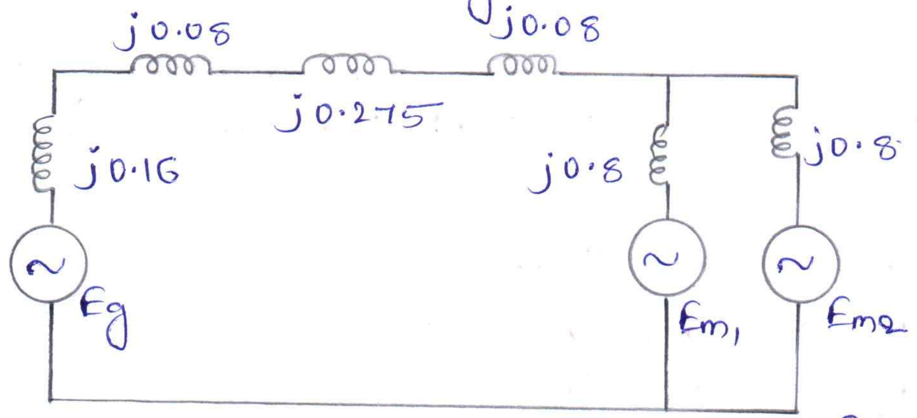
$$\text{of } T_1 \text{ and } T_2 = j0.08 \times \frac{20}{20} \times \frac{6.6^2}{6.6^2} = j0.08\ \text{pu}$$

$$\text{of } M_1 \text{ and } M_2 = j0.2 \times \frac{20}{5} \times \frac{6.6^2}{6.6^2} = j0.8\ \text{pu}$$

$$\text{Transmission line} = \frac{\text{actual value }(MVA_B)}{KV_B^2}$$

$$= \frac{j60 \times 20}{66^2} = j0.275\ \text{pu}.$$

Per unit reactance diagram.

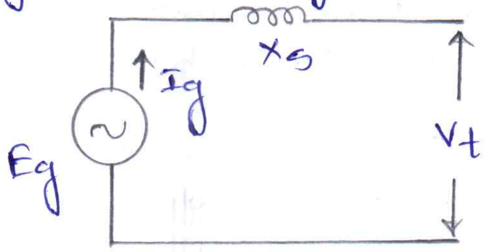


All values are in P.U.

01.c. Draw the per phase basis, representation of synchronous machine and transmission line.

[06 Marks]

Synchronous generator.



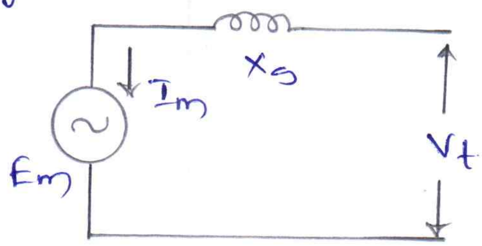
E_g = no load voltage generated

X_s = Synchronous reactance

V_t = terminal voltage

I_g = generator current

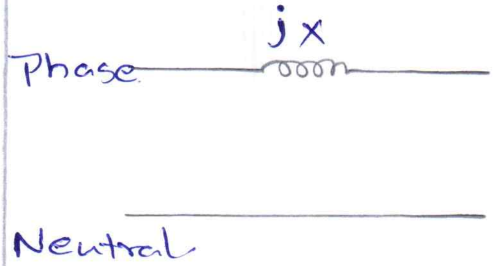
Synchronous motor.



E_m = back emf generated.

I_m = motor current

Transmission line

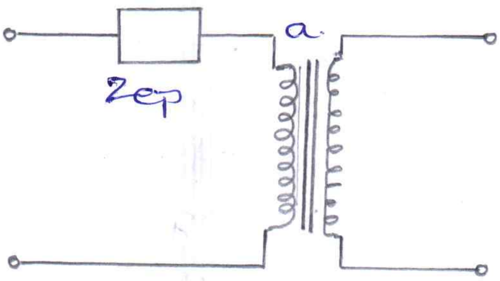


Line represented by π model.

02.a. Show that per unit impedance of two winding transformer will remain same in primary and secondary.

[06 Marks]

Consider a 1-ϕ equivalent of a 3-ϕ transformer as shown below.



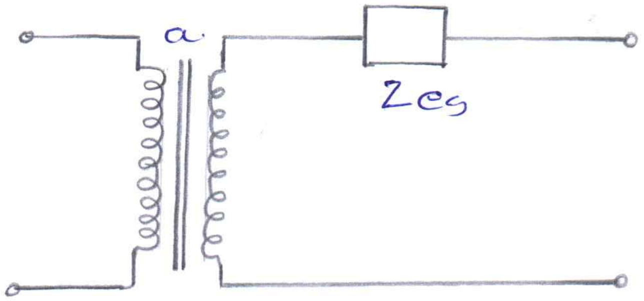
where Zep is the impedance of the transformer refered to Primary side and 'a' is the transformation ratio.

Let $(MVA_B)_P$ and $(kV_B)_P$ be the base voltampere and base voltage on Primary side.

Then P.U. impedance

$$Z_{ep}(pu) = \frac{Z_{ep}}{(KV_B)_P^2}(MVA_B)_P \longrightarrow (1)$$

The single phase equivalent with transformer impedance on secondary side is as shown below.



Where Zes is impedance refered to secondary side of the transformer.
Let $(MVA_B)_S$ and $(KV_B)_S$ be the base voltamper and base voltage on secondary side.

Then P.U. impedance

$$Z_{es}(pu) = \frac{Z_{es}}{(KV_B)_S^2}(MVA_B)_S \longrightarrow (2)$$

But we know that

$$Z_{es} = Z_{ep}/a^2$$

$$(MVA_B)_P = (MVA_B)_S$$

$$(KV_B)_S = (KV_B)_P/a.$$

equation (2) becomes

$$Z_{es}(pu) = \frac{Z_{ep}/a^2}{(KV_B)_P^2/a^2} \times (MVA_B)_P$$

$$= \frac{Z_{ep}}{(KV_B)_P^2}(MVA_B)_P.$$

$$Z_{es}(pu) = Z_{ep}(pu)$$

Thus it shows that P.U. impedance refered to primary and secondary of a transformer are same. Provided base voltampere on both side of the transformer are same, and the base voltages on both side are on ratio of transformation.

Q2.b. Derive an equation for per unit impedance if change of base occur. [06 Marks]

Let Zactual be the actual impedance and $(MVA)_B$ old be the old base MVA and $(KV)_B$ old be the old base KV.

Then $(Z_{pu})_{old} = \frac{Z_{actual}}{(KV_B)^2_{old}} \times (MVA_B)_{old} \longrightarrow (1)$

Now let $(MVA)_B$ new and $(KV)_B$ new be the new base MVA and KV.

Then $(Z_{pu})_{new} = \frac{Z_{actual}}{(KV_B)^2_{new}} \times (MVA_B)_{new} \longrightarrow (2)$

Dividing equation (2) by (1)

$$\frac{(Z_{pu})_{new}}{(Z_{pu})_{old}} = \frac{(MVA_B)_{new}}{(MVA_B)_{old}} \times \frac{(KV_B)^2_{old}}{(KV_B)^2_{new}}$$

$$\therefore Z_{pu\,new} = Z_{pu\,old} \cdot \frac{MVA_{B\,new}}{MVA_{B\,old}} \cdot \frac{KV_B^2\,old}{KV_B^2\,new}$$

Q2.c. Two generators rated 10 MVA, 13.2 KV and 15 MVA, 13.2 KV are connected in parallel to a bus bar. They feed supply to two motors of input 8 MVA and 12 MVA respectively. The operating voltage of the mortors is 12.5 KV. Assuming base quantities as 50 MVA and 13.8 KV, draw the reactance diagram. The percentage reactance of generators are 15% and that of motors are 20%. [08 Marks]

We have change of base formula.

$$Z_{pu\,new} = Z_{pu\,old} \frac{MVA_{B\,new}}{MVA_{B\,old}} \frac{KV_B^2\,old}{KV_B^2\,new}$$

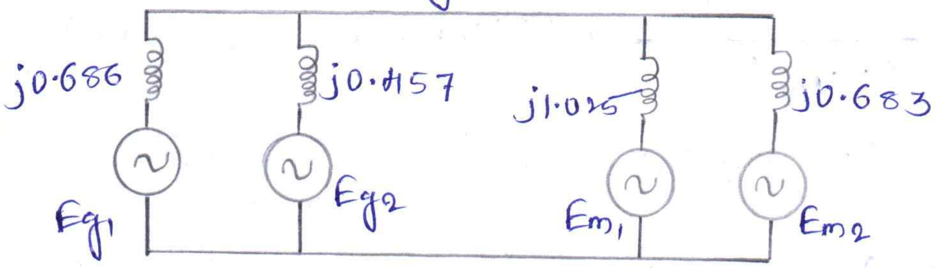
$$Z_{pu\,new}\ gen_1 = 0.15 \times \frac{50}{10} \times \frac{13.2^2}{13.8^2} = j0.686\ pu.$$

$$Z_{pu\,new}\ gen_2 = 0.15 \times \frac{50}{15} \times \frac{13.2^2}{13.8^2} = j0.457\ pu$$

$$Z_{pu\,new}\ motor_1 = 0.2 \times \frac{50}{8} \times \frac{12.5^2}{13.8^2} = j1.025\ pu.$$

$$Z_{pu\,new}\ motor_2 = 0.2 \times \frac{50}{12} \times \frac{12.5^2}{13.8^2} = j0.683\ pu.$$

Pu reactance diagram.

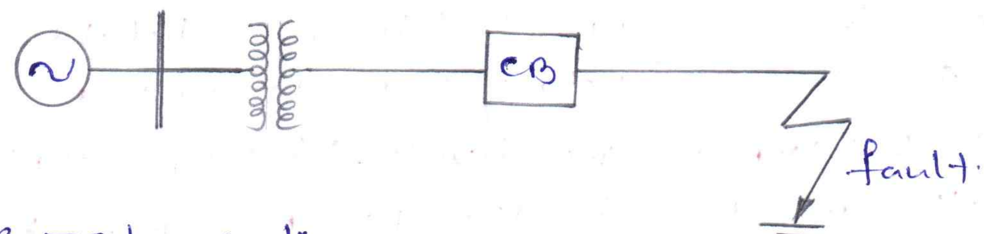


All values are in pu.

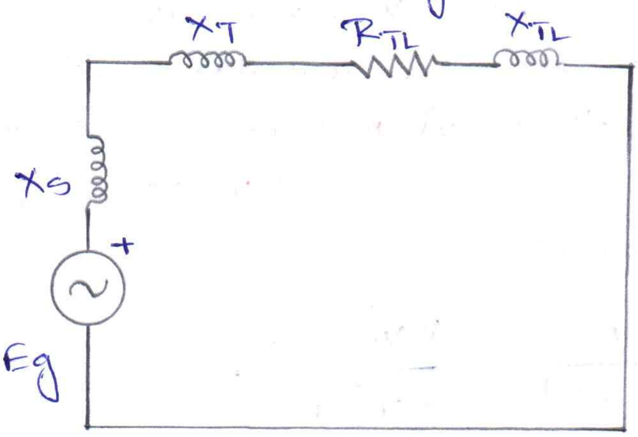
Module-02.

Q3.a. What is doubling effect in the transmission line? Substantiate with equations. [10 Marks]

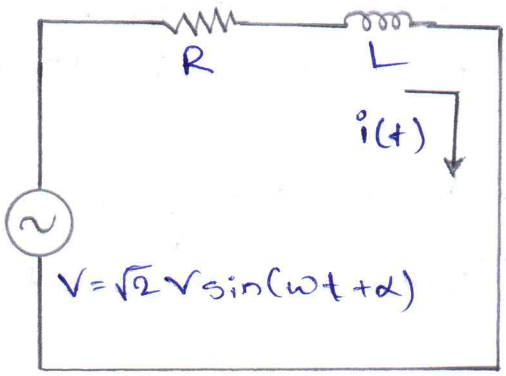
Consider a power system shown below. let 3-ϕ fault occurs at the end of transmission line.



The impedance diagram can be drawn as below.



$\approx$



Short circuit current $i(t)$ can be found by writing KVL

$$Ri(t) + L\frac{di(t)}{dt} = \sqrt{2}\,V\sin(\omega t+\alpha)$$

It is known from circuit theory that the current after short circuit is composed of two parts.

$$i(t) = i_s(t) + i_t(t)$$

where $i_s(t) \rightarrow$ steady state current

$i_t(t) \rightarrow$ transient current

$$i_s(t) = \frac{\sqrt{2}\,V}{|Z|}\sin(\omega t+\alpha-\phi)$$

where $Z = \sqrt{R^2+X^2}$, $X=\omega L$, $\phi = \tan^{-1} X/R$.

$$\therefore i_s(t) = \frac{\sqrt{2}\,V}{\sqrt{R^2+X^2}}\sin(\omega t+\alpha-\phi)$$

and $i_t(t) = A e^{-t/\tau}$

where $\tau = L/R$.

$$\therefore i(t) = \frac{\sqrt{2}\,V}{\sqrt{R^2+X^2}}\sin(\omega t+\alpha-\phi) + A e^{-t/\tau}$$

Constant 'A' can be determined by applying initial condition. i.e at $t=0^-$, $i(t)=0$.

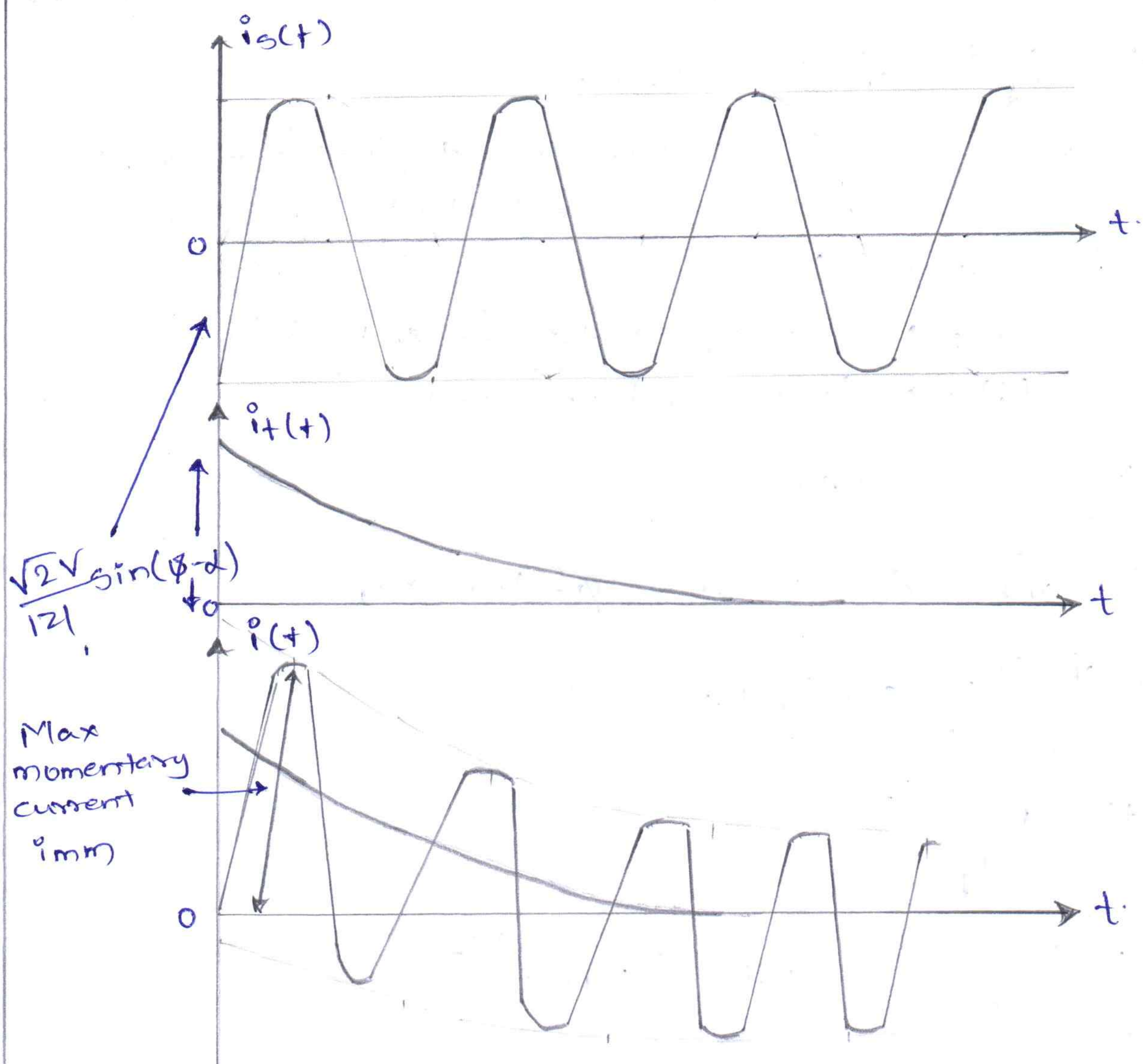
$$A = -\frac{\sqrt{2}\,V}{|Z|}\sin(\alpha-\phi)$$

$$A = \frac{\sqrt{2}\,V}{|Z|}\sin(\phi-\alpha)$$

$$\therefore i(t) = \frac{\sqrt{2}\,V}{|Z|}\sin(\omega t+\alpha-\phi) + \frac{\sqrt{2}\,V}{|Z|}\sin(\phi-\alpha)\,e^{-t/\tau}$$

The first term is called symmetrical short circuit current and the second term is DC off-set current.

The waveforms are as shown below.



Maximum momentary current is the first peak value reached in the first cycle of short circuit current, if delay in transient current is neglected.

$$i_{mm} = \frac{\sqrt{2}V}{|Z|}\sin(\phi-\alpha) + \frac{\sqrt{2}V}{|Z|}$$

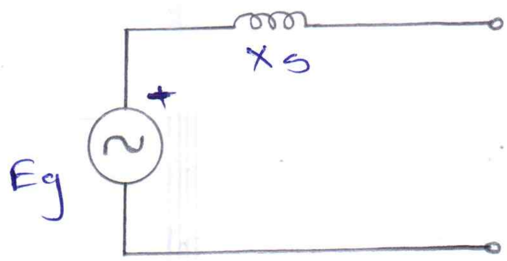
Maximum possible momentary current will result if $\phi = 90^\circ$ and $\alpha = 0^\circ$. i.e resistance of transmission line is zero and short circuit occurs when the voltage wave goes through zero.

$$i_{mm(possible)} = \frac{2\sqrt{2}V}{|Z|}$$

03.b. Explain with the help of oscillogram if short circuit current of synchronous generator operating on noload. Distinguish between sub transient, transient and steady state periods. Also show that $X_d'' < X_d' < X_d$ with equivalent circuit diagram.

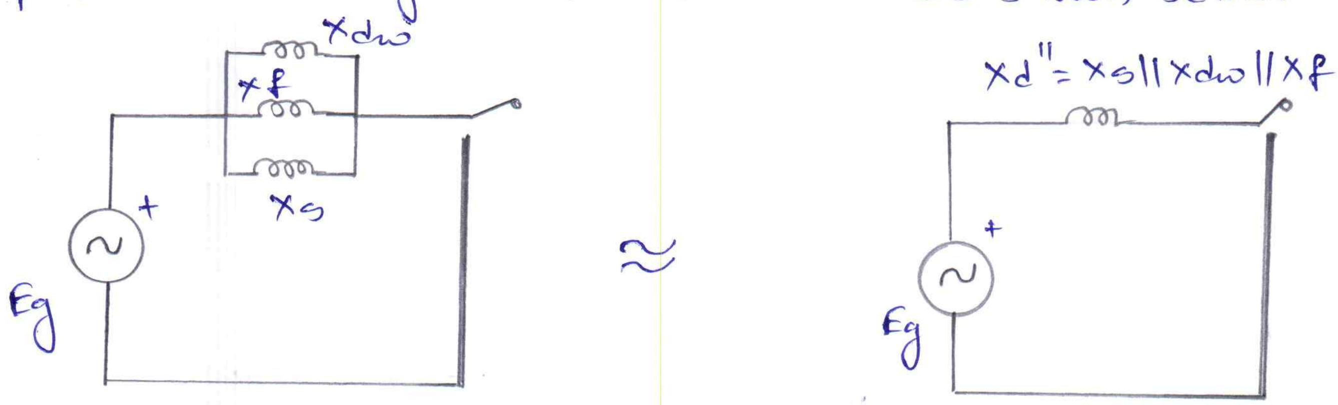
[10 Marks]

Consider a synchronous generator operating at no-load. The current in the stator winding is zero. So, there is only main flux in air gap. The circuit model is as shown below.



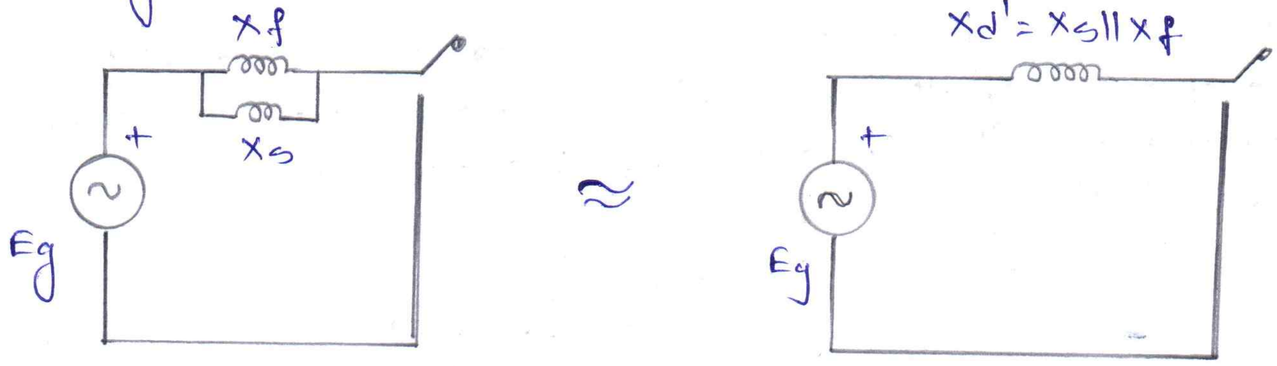
where
$X_s \rightarrow$ Synchronous reactance.

Now sudden 3-ϕ fault occurs at terminals of synchronous generator. Suddenly very high current flows through stator winding. This current will produce stator flux which demagnetises the rotor flux and resultant flux in the air gap is reduced. But according to theory of constant flux linkages, change in flux in infadecimally small time is not permitted. Because of this current is induced in damper and field windings in a direction to help the main flux. So to take this effect the reactance of damper and field winding refered to stator winding is included in parallel with synchronous reactance as shown below.



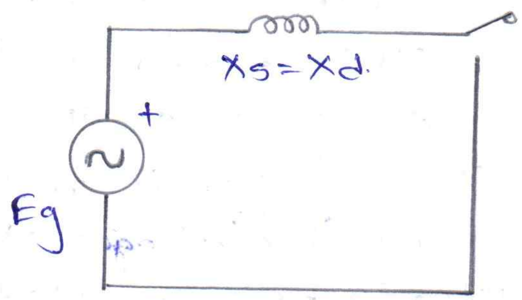
where X_d'' = subtransient reactance

currents in the damper winding and field winding decay in accordance with the winding time constant. The time constant of the damper winding which has low leakage reactance is much less than that of field winding.
So current induced becomes zero after some instant of time. So its reactance is not needed to be included.
So we get reduced circuit model.



X_d' = transient reactance.

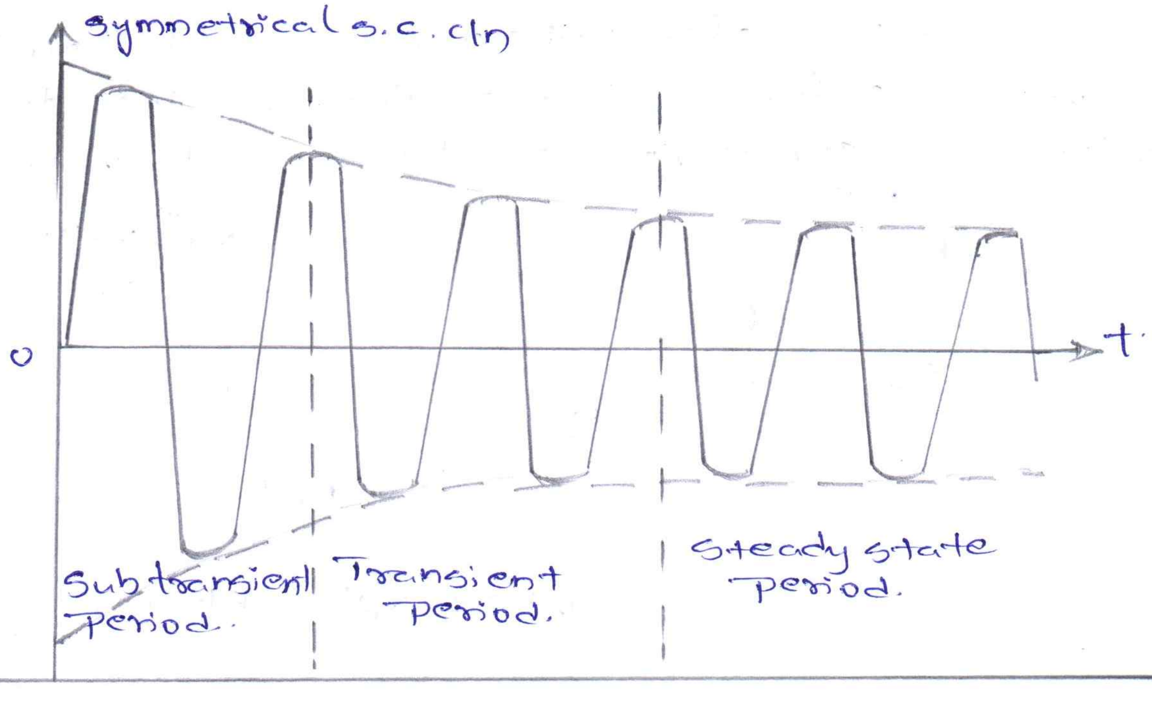
After some instant current in field winding also becomes zero. So its reactance is also removed from the circuit model. The reduced circuit model is as shown below.



X_d = direct axis reactance.

So we can conclude that $X_d'' < X_d' < X_d$

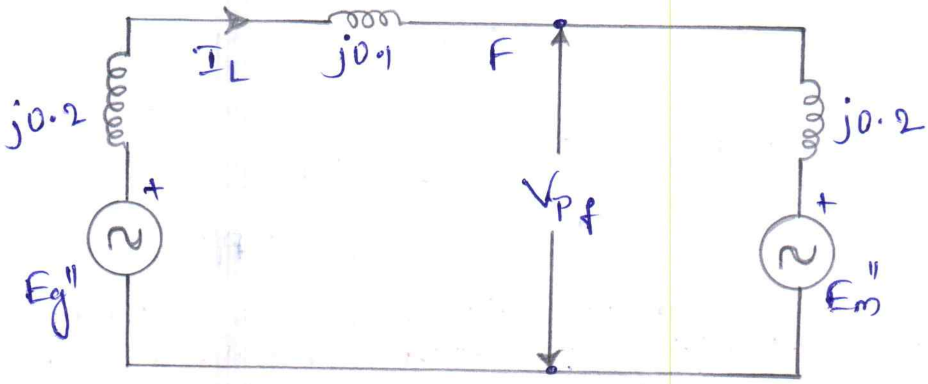
The current wave form of stator current is as shown below.



04. a. A synchronous generator and motor are rated for: 30,000 KVA, 13.2 kV and both have sub transient reactance of 20%. The line connecting them has a reactance of 10% on the base of machine ratings. The motor is drawing 20,000 kW at 0.8 pf lead. The terminal voltage of motor is 12.8 kV. When a symmetrical 3 phase fault occurs at motor terminals, find the subtransient current in generator, motor, and at fault point.

[10 Marks]

Equivalent circuit before fault.



Let base MVA = 30 MVA

base kV = 13.2 kV

Prefault voltage at fault point $V_{pf} = 12.8\,kV$

$$V_{pf}\ pu = \frac{12.8}{13.2} = 0.97\angle 0^\circ\ pu.$$

$$\text{Base current } I_B = \frac{MVA_B}{\sqrt{3}\,kV_B} = \frac{30\times10^6}{\sqrt{3}\times13.2\times10^3} = 1312\,A$$

$$\text{load current } I_L = \frac{20000\times10^3}{\sqrt{3}\times12.8\times10^3\times0.8}\ \angle\cos^{-1}0.8$$

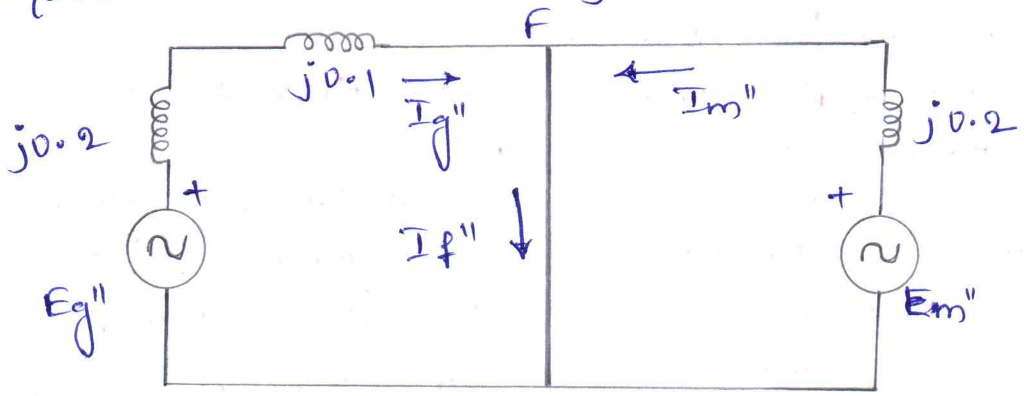
$$= 1128\angle 36.86^\circ\ A$$

$$I_L\ pu = \frac{1128}{1312} = 0.8597\angle 36.86^\circ\ pu.$$

$$E_g'' = [j0.2 + j0.1]\, I_L + V_{pf} = 0.84\angle 14.2^\circ\ pu.$$

$$E_m'' = V_{pf} - j0.2 I_L = 1.0819\angle -7.3^\circ\ pu.$$

Equivalent circuit during fault.



$I_g'' = E_g''/j0.3 = 2.8\angle{-75.8^\circ}\,pu = 3673.6\angle{-75.8^\circ}\,A$

$I_m'' = E_m''/j0.2 = 5.409\angle{-97.3^\circ}\,pu = 7096.61\angle{-97.3^\circ}\,A$

fault current $I_f'' = I_g'' + I_m'' = 8.065\angle{-90^\circ}\,pu$

$= 10581.28\angle{-90^\circ}\,A$

04.b. What are the causes for faults in power system and how symmetrical faults are different from unsymmetrical faults and how to analyze these faults. Explain the procedure to solve the problems using different methods. [10 Marks]

A fault in a circuit is any failure which interferes with the normal flow of current. The faults occur in power system due to insulation failure of equipments, flash over of lines initiated by a lightning stroke, due to permanent damage to conductors and towers or due to accidental faulty operations.

In symmetrical faults the fault current remains the same in all the phases. Only there will be a phase difference of 120° electrical. Hence the system remains balanced even after fault occurance. So it can be analysed on a single phase basis. The unsymmetrical faults are analyzed using symmetrical components.

Symmetrical faults are analysed by.

a. Using Kirchoff's law.

b. Using Thevenin's theorem.

c. By forming bus impedance matrix.

a. Symmetrical fault current estimation using Kirchoff's law

01. Choose appropriate base values and determine the prefault condition reactance diagram of the given power system.

02. Calculate the internal emfs of synchronous machines and the prefault voltage at the fault point using prefault current.

03. Draw the fault condition reactance diagram. The currents in this reactance diagram are fault condition currents.

04. Calculate the P.U. values of fault currents in the various parts of the system and in fault.

05. The actual values of fault currents are obtained by multiplying the P.U values by the respective base currents.

b. Symmetrical fault current estimation using Thevenin's theorem.

01. Choose appropriate base values and determine the prefault condition reactance diagram of the given power system.

02. Calculate the prefault voltage at the fault point using the prefault current. If the system is unloaded then the prefault voltage is 1 P.U. The prefault voltage at the fault point is the Thevenin's voltage

03. Determine the Thevenin's impedance of the system at the fault point.

04. Draw the Thevenin's equivalent at the fault point

Fault current $I_f = V_{th} / Z_{th}$. PU

The actual value of fault current is obtained by multiplying the P.U. value with base current.

Module - 03

05.a. Determine the sequence components of the three voltages. $V_a = 200\angle 0^\circ$, $V_b = 200\angle 245^\circ$, and $V_c = 200\angle 105^\circ$.

[08 Marks]

We have

$$V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right)$$

$$= \frac{1}{3}\left[200\angle 0^\circ + 200\angle 245^\circ + 200\angle 105^\circ\right]$$

$$= 21.60\angle 10.60^\circ$$

$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2V_c\right]$$

$$= \frac{1}{3}\left[200\angle 0^\circ + 1\angle 120^\circ \times 200\angle 245^\circ + 1\angle 240^\circ \times 200\angle 105^\circ\right]$$

$$= 197.80\angle -3.31^\circ$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2V_b + aV_c\right]$$

$$= \frac{1}{3}\left(200\angle 0^\circ + 1\angle 240^\circ \times 200\angle 245^\circ + 1\angle 120^\circ \times 200\angle 105^\circ\right]$$

$$= 20.14\angle 158.23^\circ$$

Also

$$V_{b1} = a^2 V_{a1} = 1\angle 240^\circ \times 197.80\angle -3.31^\circ$$

$$= 197.8\angle -123.31^\circ$$

$$V_{b2} = a V_{a2} = 1\angle 120^\circ \times 20.14\angle 158.23^\circ$$

$$= 20.14\angle -81.77^\circ$$

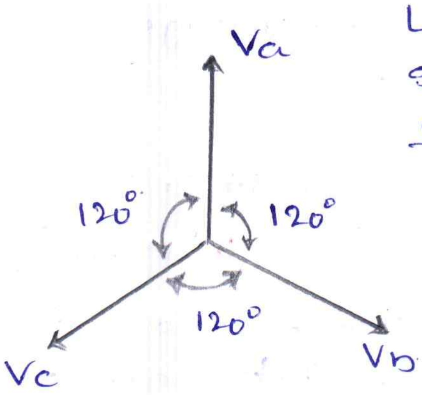
$$V_{c1} = aV_{a1} = 197.8\angle 116.69^\circ$$

$$V_{c2} = a^2 V_{a2} = 20.14\angle 38.23^\circ$$

$$V_{b0} = V_{c0} = V_{a0} = 21.60\angle 10.60^\circ$$

05. b. Prove that a balanced set of three phase voltages will have only positive sequence components of voltages only. [06 Marks]

Consider a balanced 3-ϕ system of voltages where in all the phase voltages are of equal magnitude and symmetrically displaced by 120° as shown.



Let Va, Vb and Vc be the balanced system of 3-ϕ voltages.

from fig.

$$V_a = V_a.$$
$$V_b = a^2 V_a. \longrightarrow (1)$$
$$V_c = a V_a.$$

We have

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = 1/3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a. \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c. \end{bmatrix}$$

Using equation 1

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a. \end{bmatrix} \begin{bmatrix} V_a \\ a^2 V_a \\ a V_a \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} V_a + a^2 V_a + a V_a \\ V_a + a^3 V_a + a^3 V_a. \\ V_a + a^4 V_a + a^2 V_a \end{bmatrix}$$

but $a^3 = 1$ and $a^4 = a.$

$$= \frac{1}{3} \begin{bmatrix} V_a + a^2 V_a + a V_a. \\ V_a + V_a + V_a. \\ V_a + a V_a + a^2 V_a \end{bmatrix} = \frac{1}{3} \begin{bmatrix} V_a(1+a+a^2) \\ 3V_a. \\ V_a(1+a+a^2) \end{bmatrix}$$

but $1 + a + a^2 = 0.$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 0 \\ 3aV \\ 0 \end{bmatrix}$$

Thus. $\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$

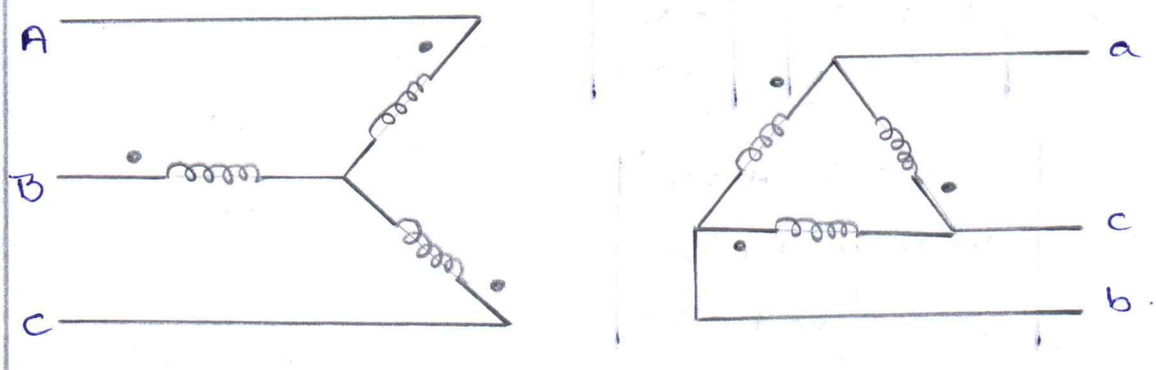
So $V_{a0} = 0$, $V_{a1} = V_a$ and $V_{a2} = 0$.

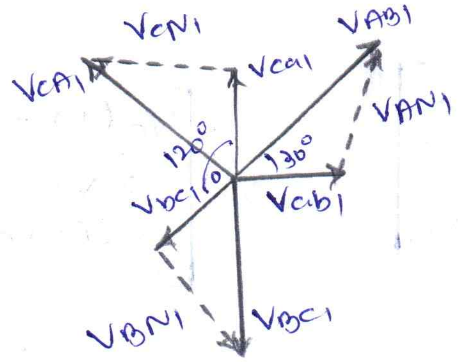
It shows that a balanced set of 3-ϕ voltages will have only positive sequence voltage.

05.c. Explain the concept of phase shift in star-delta transformer bank. [06 Marks]

Positive and negative sequence voltages and currents undergo a phase shift in passing through a star-delta transformer, which depends on the labelling of terminals.

Consider a star-delta transformer with terminal labelling as indicated in fig. Windings shown parallel to each other are magnetically coupled. Assume that the transformer is excited with positive sequence voltages and carries positive sequence currents. Phasor diagram is as shown below.





+ve sequence.

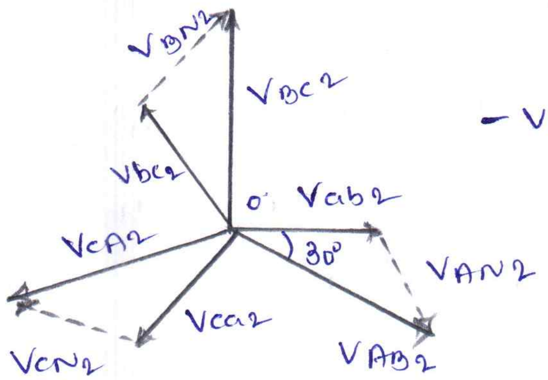
From phasor diagram

$V_{AB1} = x\, V_{ab1} \angle 30^\circ$

where x = phase transformation ratio.

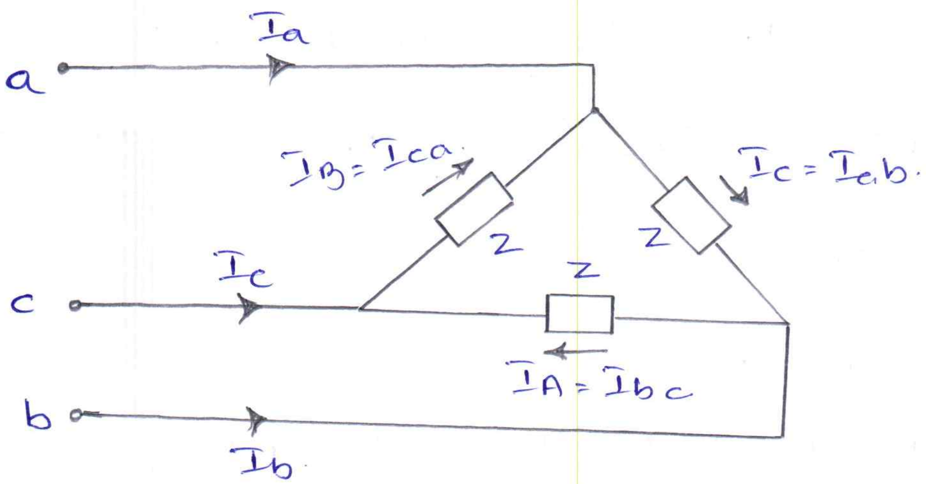
So the positive sequence line voltages on star side lead the corresponding voltages on the delta side by 30°. The same also applies for line currents.

Now assume that the transformer is excited by negative sequence voltages and currents the voltage phasor diagram will be as shown below. The phase shift in comparison to the positive sequence case now reverses. i.e the stator side quantities lag the delta side quantities by 90°.



06. a. Derive relation between sequence components of phase and line currents in delta connected system. [10 Marks]

Consider a delta connected three-phase system where in the line currents I_a, I_b and I_c are entering the delta connected system shown below.



The phase currents are I_{ab}, I_{bc} and I_{ca}. Also.
$I_{ab} = I_C$, $I_{bc} = I_A$ and $I_{ca} = I_B$.
Applying KCL we get

$$\left.\begin{aligned} I_a &= I_C - I_B \\ I_b &= I_A - I_C \\ I_c &= I_B - I_A \end{aligned}\right\} \longrightarrow (1)$$

The sequence components of line currents are.

$$I_{a_1} = \frac{1}{3}\left(I_a + a \cdot I_b + a^2 I_c\right]$$

$$= \frac{1}{3}\left[(I_C - I_B) + a(I_A - I_C) + a^2(I_B - I_A)\right]$$

$$= \frac{1}{3}\left[a(I_A + aI_B + a^2 I_C) - a^2(I_A + a \cdot I_B + a^2 I_C)\right]$$

$$= \frac{1}{3}(a - a^2)(I_A + a \cdot I_B + a^2 \cdot I_C)$$

$$= \frac{1}{3}\left(j\sqrt{3}\right](3 I_{A_1}) \qquad \because (I_A + aI_B + a^2 I_C) = 3 \cdot I_{A1}$$

$$\therefore I_{a_1} = j\sqrt{3}\, I_{A_1}$$

$$I_{a_2} = \frac{1}{3}\left(I_a + a^2 I_b + aI_c\right)$$

$$= \frac{1}{3}\left[(I_C - I_B) + a^2(I_A - I_C) + a(I_B - I_A)\right]$$

$$= \frac{1}{3}\left[a^2(I_A + a^2 \cdot I_B + a \cdot I_C) - a(I_A + a^2 I_B + aI_C)\right]$$

$$= \frac{1}{3}\left((a^2 - a)(I_A + a^2 \cdot I_B + a \cdot I_C)\right]$$

$$= \frac{1}{3}\left(-j\sqrt{3}\right](3 \cdot I_{A2}) \qquad \because (I_A + a^2 I_B + aI_C) = 3I_{A2}$$

$$\therefore I_{a_2} = -j\sqrt{3}\, I_{A2}.$$

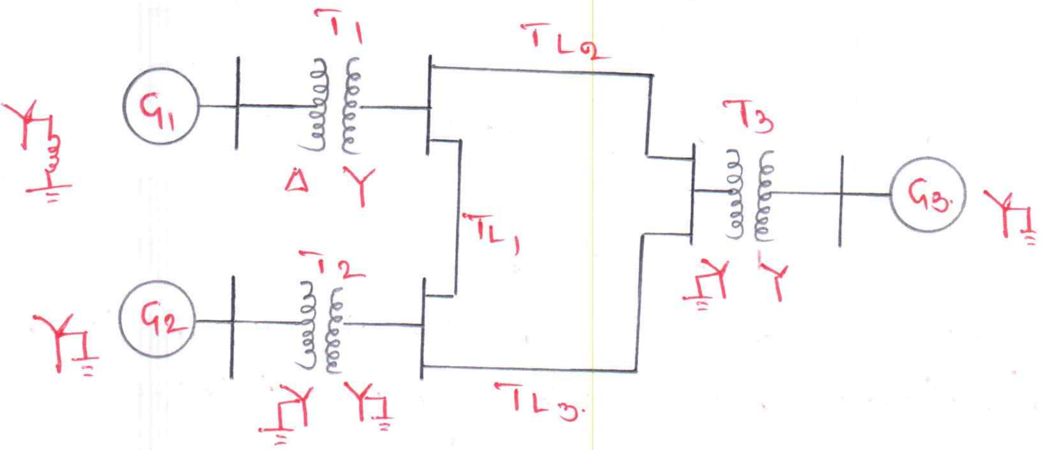
$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$= \frac{1}{3}\left((I_C - I_B) + (I_A - I_C) + (I_B - I_A)\right]$$

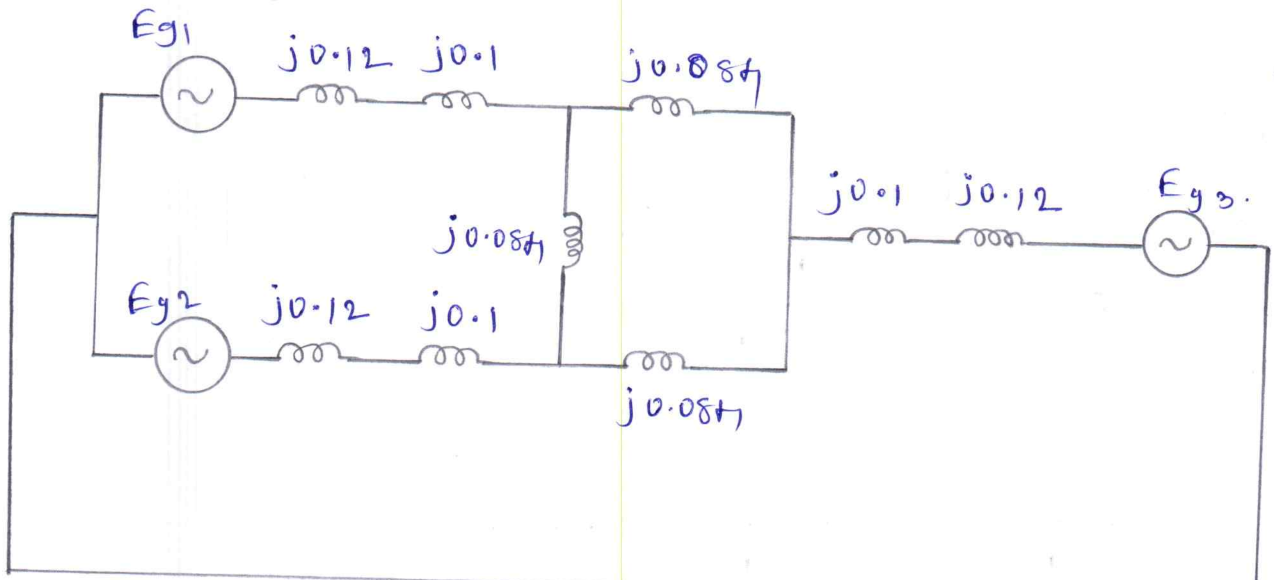
$$I_{a0} = 0.$$

06.b. Draw positive, negative and zero sequence networks for the power system shown below. Per unit impedance Z_n in neutral of G_1 = j0.02 pu.

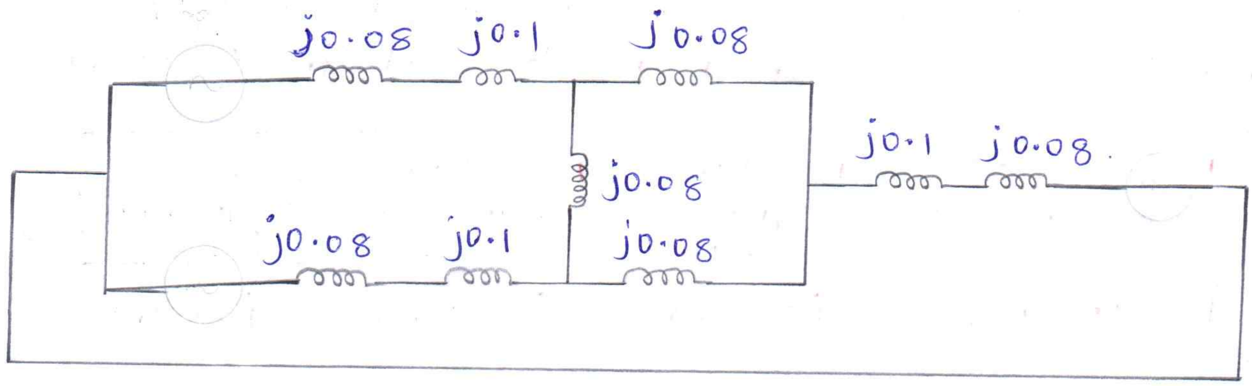
Power system components.	Positive sequence Z_1	Negative sequence Z_2	zero sequence Z_0.
G_1, G_2, G_3.	j0.12pu	j0.08pu	j0.03pu
T_1, T_2, T_3	j0.1pu	j0.1pu	j0.1pu.
T_{L1}, T_{L2}, T_{L3}	j0.084pu	j0.08pu	j0.12pu.



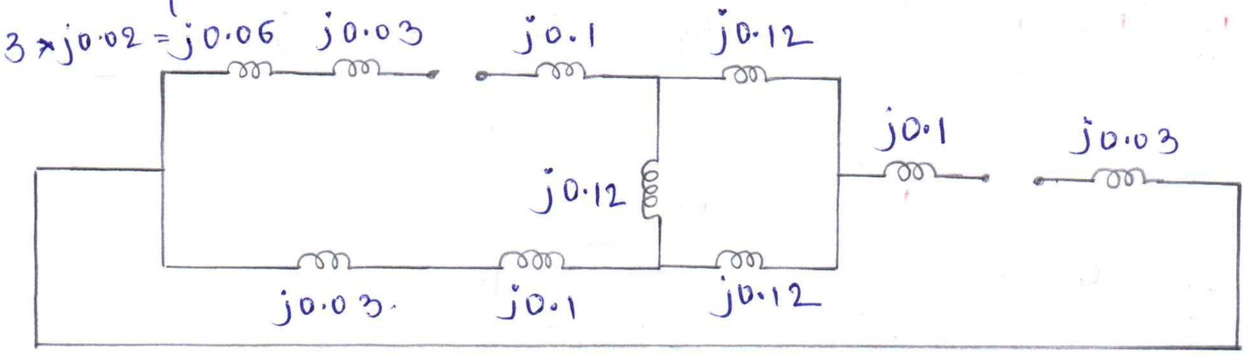
The positive sequence network.



Negative sequence networks.



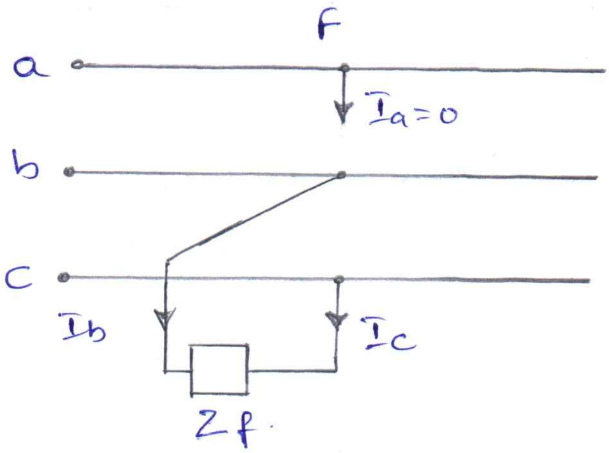
Zero sequence networks.



Module-04

07.a. Derive the expression for fault current if line - line (LL) fault occurs through fault impedance Z_f in power system. Show the connection of sequence network to represent the fault. [10 Marks]

Figure shows a Line to line fault at location F in a power system on phases b and c through a fault impedance Z_f.



Terminal condition.

$I_a = 0$

$I_b + I_c = 0$ or $I_b = -I_c$

$V_b = V_c + I_b Z_f$

Symmetrical component of current given by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a = 0 \\ I_b \\ I_c = -I_b \end{bmatrix}$$

$I_{a0} = 0$

$I_{a1} = \frac{1}{3}(a I_b - a^2 I_b) = 1/3 (a - a^2) I_b.$

$I_{a2} = \frac{1}{3}(a^2 I_b - a I_b) = \frac{1}{3}(a^2 - a) I_b$

So $I_{a0} = 0$.

$I_{a1} = -I_{a2}$

consider

$$V_{a1} - V_{a2} = 1/3 \left[V_a + a V_b + a^2 V_c - (V_a + a^2 V_b + a V_c) \right]$$

$$= \frac{1}{3}\left[(a - a^2) V_b + (a^2 - a) V_c \right]$$

$$= \frac{1}{3}(a - a^2)(V_b - V_c)$$

$$= \frac{1}{3}(a - a^2)(I_b Z_f) \qquad \because V_b = V_c + I_b Z_f.$$

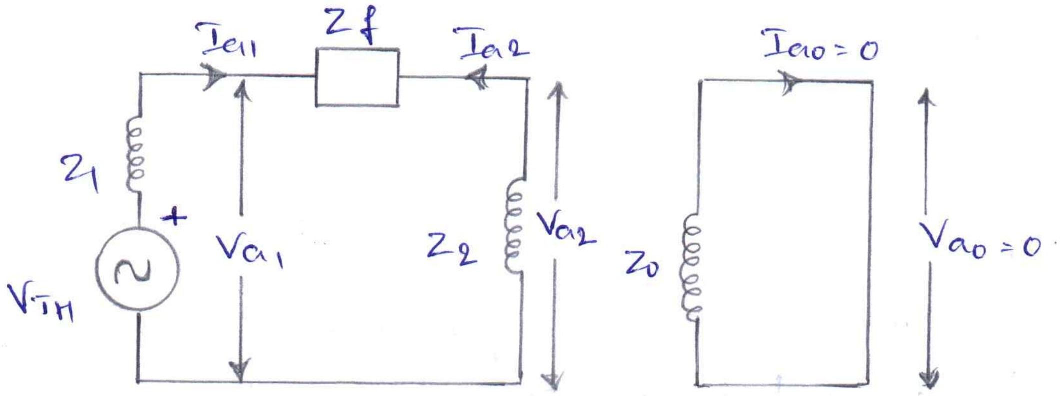
$$= I_{a1} Z_f \qquad \because 1/3 (a - a^2) I_b = I_{a1}$$

So $V_{a1} = V_{a2} + I_{a1} Z_f$

as $I_{a0} = 0$

$V_{a0} = -I_{a0} Z_0 = 0.$

Equations suggest parallel connection of positive and negative network through a series impedance Z_f. Since $I_{a0} = V_{a0} = 0$ Zero sequence network is connected separately and short circuited on itself.



Fault current

$$I_f = I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$= 0 + a^2 I_{a1} + a(-I_{a1})$$

$$= (a^2 - a) I_{a1} = -j\sqrt{3}\, I_{a1}$$

$$|I_f| = \sqrt{3}\, I_{a1}$$

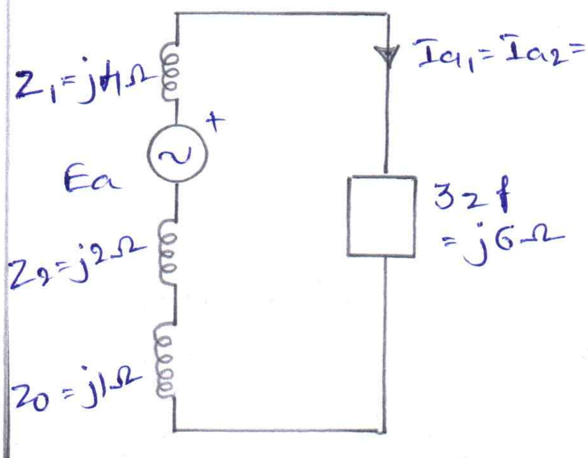
$$= \sqrt{3}\, \frac{V_{TH}}{(Z_1 + Z_2 + Z_f)}$$

07. b. A 3-phase generator with an open circuit voltage of 400V is subjected to an LG fault through a fault impedance of $j2\,\Omega$. Determine the fault current if $Z_1 = j4\,\Omega$, $Z_2 = j2\,\Omega$, and $Z_0 = j1\,\Omega$. Also calculate the fault current for LL and LLG fault.

[10 Marks]

*LG fault.

Interconnection of sequence network for LG fault is.



$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_1 + Z_2 + Z_0 + 3Z_f}$$

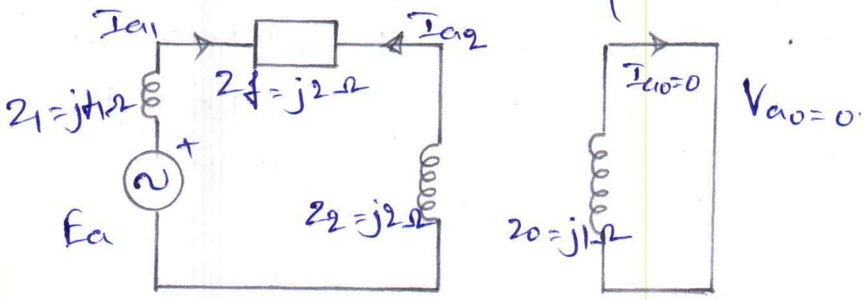
$$= \frac{400/\sqrt{3}}{j(4+2+1+6)}$$

$$= -j\,17.76\ A$$

Fault current $I_f = 3|I_{a0}| = 3 \times 17.76 = 53.28\,A$.

* LL fault

Interconnection of sequence networks for LL fault is.



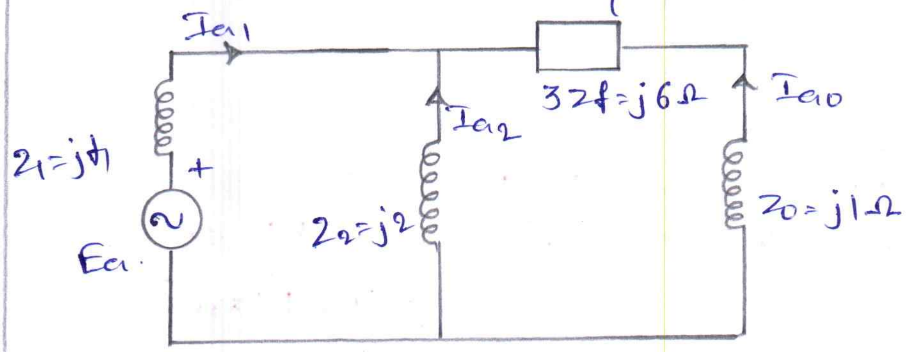
$$I_{a1} = \frac{E_a}{(Z_1 + Z_2 + Z_f)} = \frac{400/\sqrt{3}}{j(4+2+2)} = -j28.86\,A$$

Fault current $I_f = \sqrt{3}\,|I_{a1}| = \sqrt{3}\,(28.86)$

$= 49.98\,A$.

* LLG fault

Interconnection of sequence networks for LLG fault is.



$$I_{a1} = E_a \Big/ Z_1 + \frac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f} = \frac{400/\sqrt{3}}{j\left[4 + \frac{2(1+6)}{(2+1+6)}\right]}$$

$= -j41.57\,A$

$$I_{a0} = -I_{a1}\left[Z_2 / (Z_2 + Z_0 + 3Z_f)\right]$$

$= j41.57\left[2/(2+1+6)\right]$

$= j9.23\,A$

Fault current $I_f = 3|I_{a0}| = 3 \times 9.23$

$= 27.71\,A$.

08.a. What are symmetrical faults? what are the different types of unsymmetrical faults and mention their frequency of occurrence. [06 Marks]

Symmetrical faults are type of faults where the fault current remains the same in all the phases. Only there will be a phase difference of 120° electrical. Hence system remains balanced even after fault occurance. So it can be analysed on a single phase basics.

Different types of unsymmetrical faults are

* Shunt type faults.
  01. Single line to ground (LG) faults. - 70%
  02. Line to line (LL) faults - 15%
  03. Double line to ground (LLG) faults. 10%

* Series type faults.
  01. One conductor open fault
  02. Two conductor open fault

08.b. Draw inter connection of sequence network and mention the terminal condition for LG, LL and LLG faults. [09 Marks]

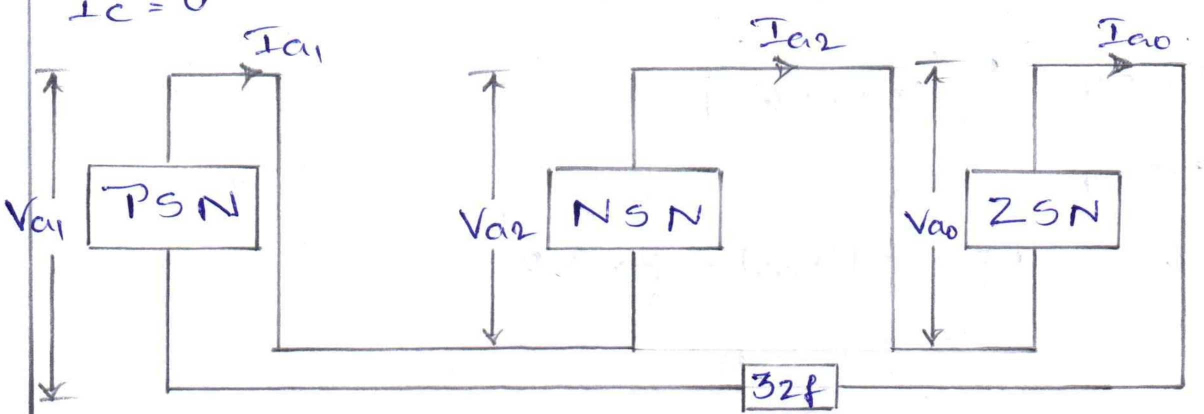
* LG fault

Terminal condition

$V_a = I_a Z_f$

$I_b = 0$

$I_c = 0$



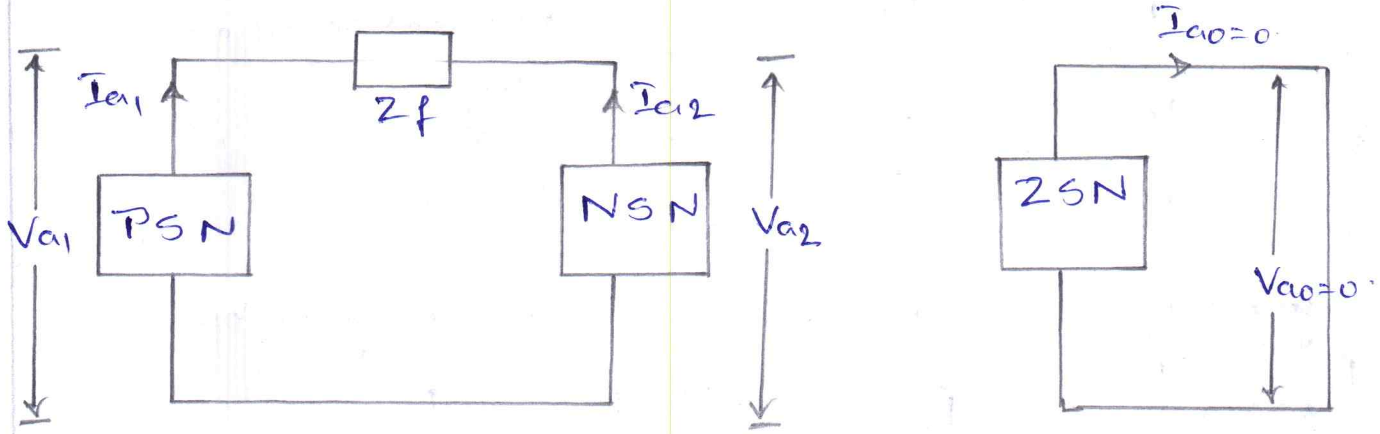
* LL fault

Terminal condition.

$I_a = 0$

$I_b = -I_c$

$V_b = V_c + I_b Z_f$

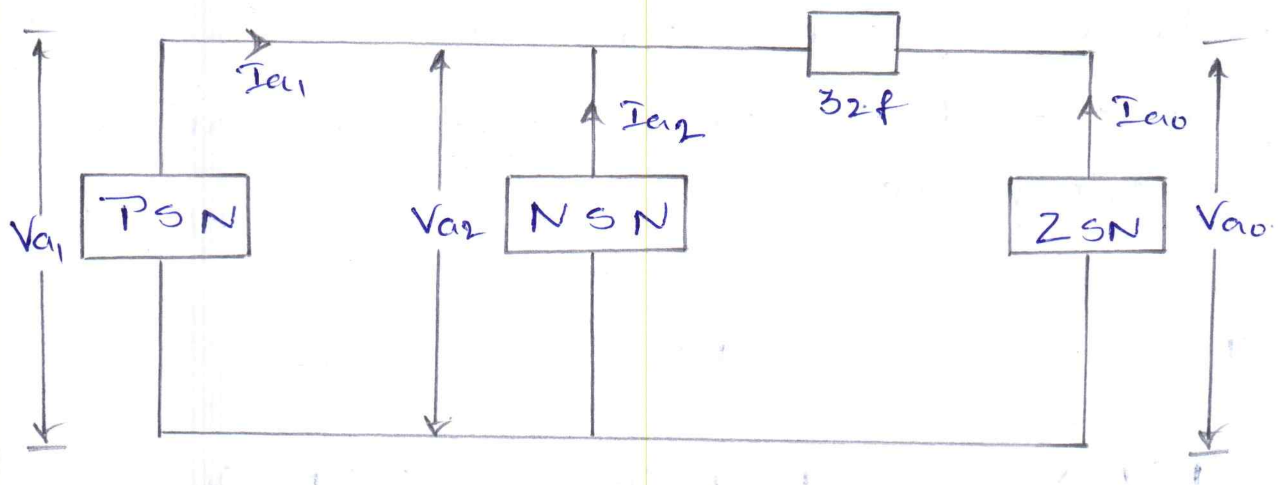


* LLG fault

Terminal condition

$I_a = 0$

$V_b = V_c = (I_b + I_c) Z_f$



where PSN = Positive sequence network.

NSN = Negative sequence network.

ZSN = Zero sequence network.

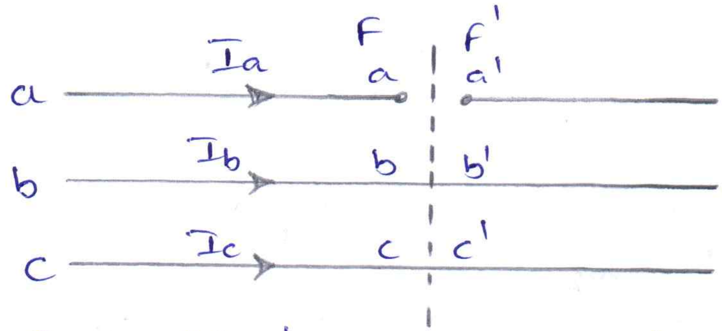
Z_f = fault impedance

V_{a1}, V_{a2}, V_{a0} = Sequence voltages.

I_{a1}, I_{a2}, I_{a0} = sequence currents.

08.c. Derive the symmetrical component relation for one conductor open fault. [05 Marks]

Assume that the conductor 'a' of a system gets opened as shown below.



Terminal conditions

$I_a = 0$

$V_{bb'} = 0$

$V_{cc'} = 0.$

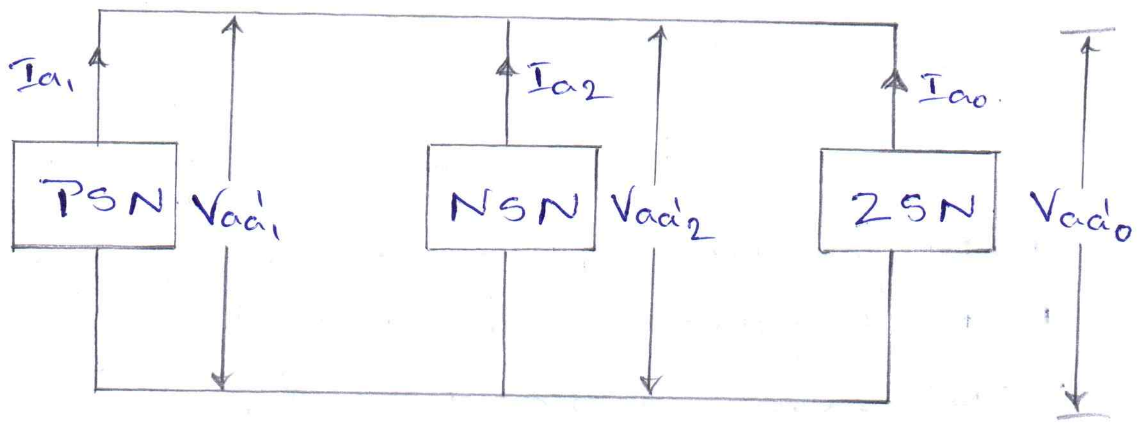
Symmetrical components of voltages are given by

$$\begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{aa'} \\ V_{bb'} = 0 \\ V_{cc'} = 0 \end{bmatrix}$$

$\therefore V_{aa'0} = V_{aa'1} = V_{aa'2} = \frac{1}{3} V_{aa'}$

also $I_a = 0 \Rightarrow I_{a0} + I_{a1} + I_{a2} = 0.$

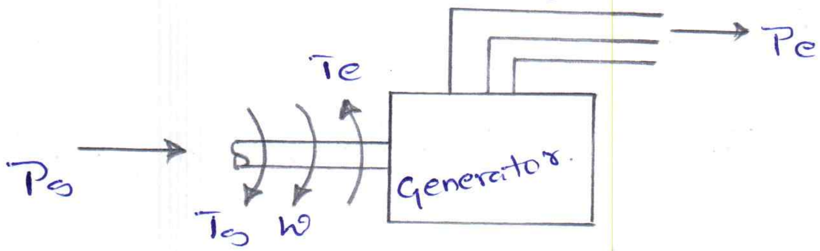
Equation suggests that three sequence networks are connected in parallel.



Module-05.

09.a. Derive an expression for the swing equation and explain swing curve. (08 Marks)

Consider a generator shown below. It receives mechanical power P_s at torque T_s and rotor speed ω via shaft from prime mover. It delivers electrical power P_e to the power system network via a busbar. The generator developes electromechanical torque T_e in opposition to T_s.



Assuming that winding and friction losses to be negligible, the accelerating torque on the rotor is given by.

$T_a = T_s - T_e$

multiplying by 'ω' on both sides

$$\omega . T_a = \omega T_s - \omega T_e$$

but $\omega T_a = P_a$ = accelerating power.

$\omega T_s = P_s$ = mechanical power input

$\omega T_e = P_e$ = electrical power output.

$\therefore P_a = P_s - P_e$

Under steady state condition $P_s = P_e$ so, $P_a = 0$.

When the balance between P_s and P_e is disturbed the machine dynamics is governed by

$$P_a = T_a \omega = I \alpha \omega = M \frac{d^2\theta}{dt^2}$$

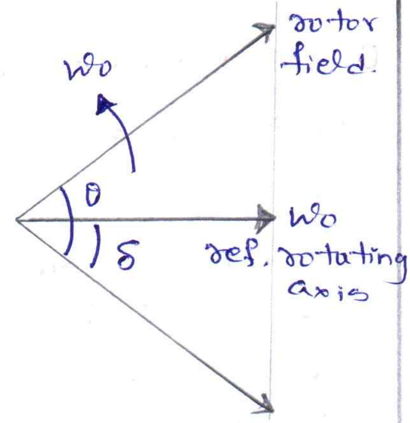
where $\alpha = \frac{d^2\theta}{dt^2}$ = angular acceleration of the rotor.

$\delta = \theta - \omega_0 t$.

taking time derivatives

$$\frac{d\delta}{dt} = \frac{d\theta}{dt} - \omega_0$$

and $$\frac{d^2\delta}{dt^2} = \frac{d^2\theta}{dt^2}$$



We have $P_a = P_s - P_e = M \frac{d^2\delta}{dt^2}$

and $M = GH/180f$

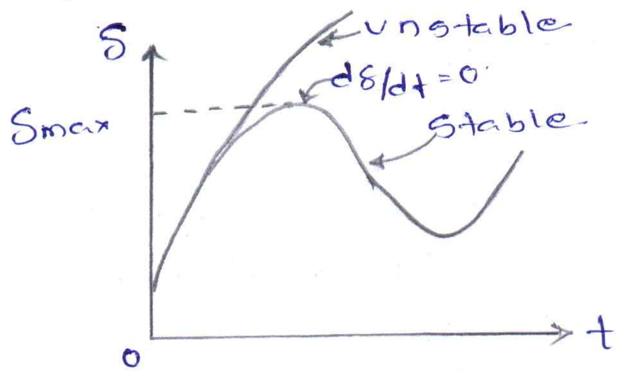
$$\therefore \frac{GH}{180f} \frac{d^2\delta}{dt^2} = P_a = P_s - P_e$$

G is in MVA rating, divide by G will get P in per unit

$$\frac{H}{180f} \frac{d^2\delta}{dt^2} = P_a = P_s - P_e \quad \text{pu.}$$

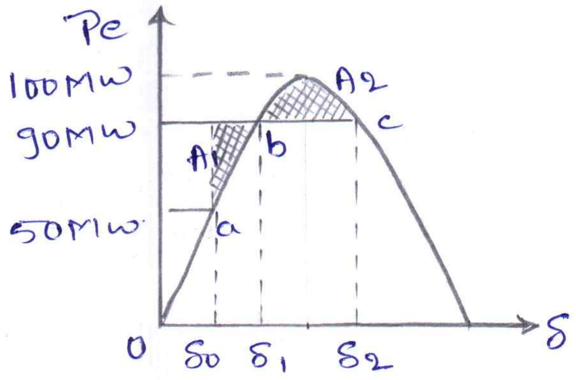
It is called swing equation.

Swing equation is a second-order differential equation Solution can be determined by using numerical solution techniques like Euler's or Runge-Kutta's method. The plot of δ versus t is called as swing curve. Swing curve provides information regarding stability of the system.



Q9.b. A loss free alternator supplies 50MW to an infinite bus, the SSSL being 100MW. Determine if the alternator will remain stable if the input to the prime mover of the alternator is abruptly increased by 40MW. (08 Marks)

Power angle curve is shown below.



$\delta_0 = 30^\circ$

$\delta_1 = 64^\circ$

$\delta_2 = 116^\circ$

$50 = 100 \sin \delta_0 \qquad \delta_0 = 30^\circ$

at b.

$50 + 40 = 90 MW. \quad \delta_1 = \sin^{-1}(90^\circ/100^\circ) = 64^\circ$

at c

$\delta_2 = 180^\circ - 64^\circ = 116^\circ$

$$A_1 = \int_{30^\circ}^{64^\circ} (90 - 100\sin\delta)\, d\delta$$

$$= 90(64^\circ - 30^\circ)\frac{\pi}{180^\circ} + 100(\cos 64^\circ - \cos 30^\circ) = 10.6$$

$$A_2 = \int_{64^\circ}^{116^\circ} (100 \sin\delta - 90)\, d\delta$$

$$= -100(\cos 116^\circ - \cos 64^\circ) - 90(116^\circ - 64^\circ)\frac{\pi}{180^\circ} = 5.99$$

Area A_1 is greater than the area A_2. Hence the machine will fall out of synchronisation and lose stability when the input is suddenly increased by 40 MW.

09.c. Explain methods of improving transient stability. (04 Marks)

Methods used to improve transient stability are

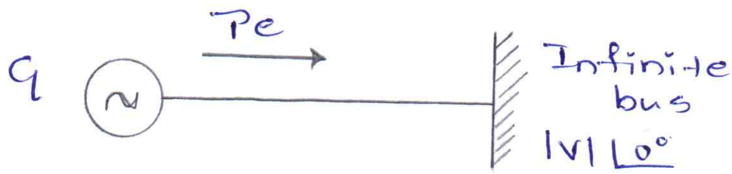
01. Increase the system voltage.
02. Reduceing transfer reactance
03. Use of high speed circuit breakers and auto

System voltage can be increased by using AVR's.
Reactance of the line can be reduced by.

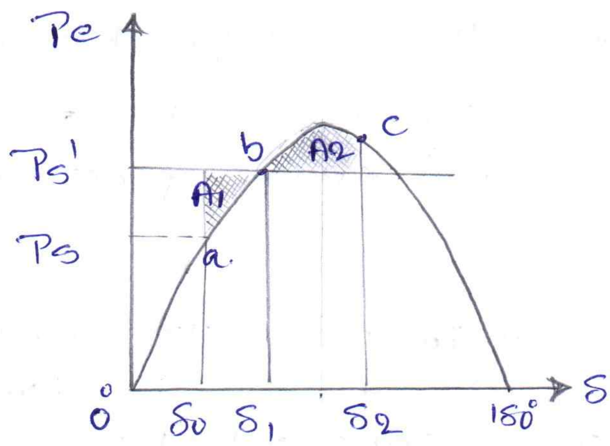
a. Reducing conductor spacing.
b. Using bundled conductors.
c. increasing number of parallel lines.
d. Using series capacitor in lines.

10.a. Explain equal area criteria concept when a power system is subjeted to sudden change in mechanical input. (10 Marks)

Fig. below shows the single line diagram of a synchronous generator connected to an infinite bus.



consider sudden increase in mechanical input. Fig below shows the plot of $(P_e - \delta)$, the power angle curve with the system operating at point 'a' corresponding to input P_s. Let the mechanical input be suddenly increased to P_s' as shown



The accelerating power $P_a = P_s' - P_e$ causes the rotor to accelerate. Hence the rotor angle δ increases, the electrical power transfer increases reducing P_a, till a point 'b' at which $P_a = 0$.

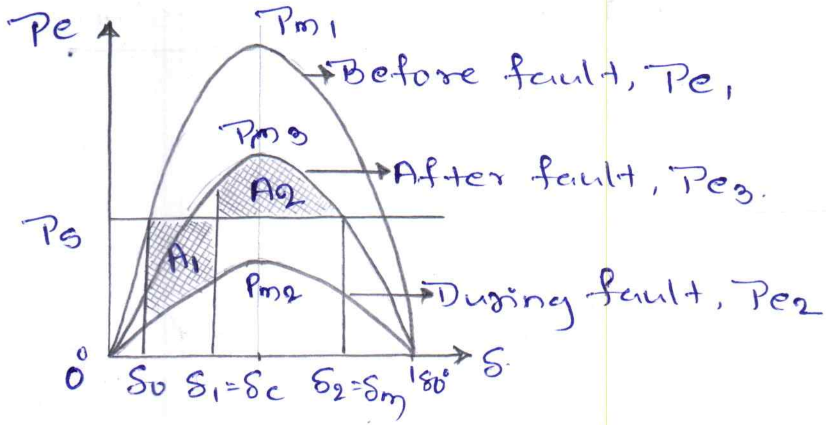
The rotor angle δ_1 however continues to increase because of the inertia of the rotor and Pa becomes negative causing the rotor to decelerate. At some point 'C' where area A_1 = area A_2 the rotor velocity becomes zero and then starts to become negative owing to continued negative Pa. the rotor angle thus reaches the maximum value δ_2 and then starts to decrease.

$$A_1 = \int_{\delta_0}^{\delta_1} (P_s' - P_e)\, d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_s')\, d\delta$$

For system to be stable, find angle δ_2 such that $A_1 = A_2$

10.b. Derive an expression for critical clearing angle and critical clearing time. (10 Marks)



If Ps is increased then δ_1 increases and area A_1 increases, for finding the condition $A_2 = A_1$. δ_2 is increased till it reaches the maximum value δ_m the maximum allowable limit for stable operation. The system is then critically stable. The load angle δ_1 is called as the critical clearing angle δ_{cc}. If the disturbance is cleared before this angle is reached, the generator can continue its operation. If the angle is exceeded, then it will result in instability. The corresponding time is called as the critical clearing time (t_{cc}). This denotes the maximum value of time allowed for the protective device to operate without rendering the system stability.

The critical clearing angle can be determined from EAC and t_{cc} by solving swing equation.

Applying EAC

$$A_1 = A_2$$

$$\int_{\delta_0}^{\delta_{cc}} (P_s - P_{m2}\sin\delta)\, d\delta = \int_{\delta_{cc}}^{\delta_m} (P_{m3}\sin\delta - P_s)\, d\delta$$

where $\delta_0 = \sin^{-1}(P_s/P_{m1})$, $\delta_m = \pi - \sin^{-1}(P_s/P_{m3})$

Integrating we get

$$(P_s\delta + P_{m2}\cos\delta)\Big|_{\delta_0}^{\delta_{cc}} = (-P_{m3}\cos\delta - P_s\delta)\Big|_{\delta_{cc}}^{\delta_m}$$

$$P_s(\delta_{cc} - \delta_0) + P_{m2}(\cos\delta_{cc} - \cos\delta_0) + P_s(\delta_m - \delta_{cc}) + P_{m3}(\cos\delta_m - \cos\delta_{cc}) = 0$$

or $$\cos\delta_{cc} = \frac{P_s(\delta_m - \delta_0) - P_{m2}\cos\delta_0 + P_{m3}\cos\delta_m}{(P_{m3} - P_{m2})}$$

δ_{cc} will be in radians.

$$t_{cc} = \sqrt{\frac{2H(\delta_{cc} - \delta_0)}{\pi \cdot f \cdot P_s}}$$

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