KLS Vishwanathrao Deshpande Institute of Technology

(Accredited by NAAC with "A" Grade)

(Approved by AICTE, New Delhi, Affiliated to VTU, Belagavi)

(Recognized Under Section 2(f) by UGC, New Delhi)

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	Dr. Meenal M. Kaliwal
Course Name	:	CALCULUS, LAPLACE TRANSFORM & NUMERICAL TECHNIQUES
Course Code	:	1BMATE201
Year of Question Paper	:	MODEL QP (2025)
Date of Submission	:	11-02-2026

Faculty Member

Head of the Department

Dept. of Electronic & Communication Engg

KLS V.D.I.T., HALIYAL (U.K.)

Dean (Acad.)

12/2/26.

Model Question Paper with effect from 2025 (CBCS Scheme)

USN | | | | | | | | | | |

1BMATE201

Second Semester B.E Degree Examination

Calculus, Laplace Transforms and Numerical Techniques

TIME: 03 Hours **Max. Marks: 100**

Note:
1. *Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**.*
2. *VTU Formula Hand Book is Permitted.*
3. *M: Marks, L: Bloom's level, C: Course outcomes.*

		Module -1	M	L	C
Q.01	a	Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\,dz\,dx\,dy$.	7	L3	CO1
	b	Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ by changing the order of integration.	7	L3	CO1
	c	Derive the relation $\beta(m,n) = \frac{\gamma(m)\,\gamma(n)}{\gamma(m+n)}$	6	L2	CO1
		OR			
Q. 02	a	Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)dxdy$ by changing into polar coordinates.	7	L3	CO1
	b	Using the double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2}+\frac{x^2}{b^2}=1.$	7	L3	CO1
	c	Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{sin\theta}} \times \int_0^{\pi/2} \sqrt{sin\theta}\ \ d\theta = \pi.$	6	L2	CO1
		Module-2			
Q. 03	a	Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at the point (1, -2, 1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$	7	L2	CO1
	b	Evaluate $Curl(Curl\vec{F})$ and $div(curl\vec{F})$, If $\vec{F} = x^2y\,\hat{\imath} + y^2z\,\hat{\jmath} + z^2x\,\hat{k}$	7	L3	CO1
	c	Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\,\hat{\imath} + (3xz + 2xy)\,\hat{\jmath} + (3xy - 2xz + 2z)\,\hat{k}$ is both solenoidal and irrotational.	6	L3	CO1
		OR			
Q.04	a	Find the directional derivative of $\emptyset = 4xz^3 - 3x^2y^2z$ at (2, -1, 2) along 2i-3j+6k.	7	L2	CO1
	b	Using Green's theorem, evaluate $\oint[(3x - 8y^2)dx + (4y - 6xy)\,dy]$ over the boundary of the region $x = 0, y = 0, and\ x + y = 1.$	7	L3	CO1
	c	Find the work done in moving a particle in the force field $\vec{F} = 3x^2\,\hat{\imath} + (2xz - y)\,\hat{\jmath} + z\,\hat{k}$ along the straight line from $(0, 0, 0)$ *to* $(2, 1, 3)$.	6	L3	CO1
		Module-3			
Q. 05	a	Find a real root of $x^3 - 9x + 1 = 0\ in\ (2, 3)$ by the Regula-Falsi method in four iterations.	7	L3	CO2

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<table>
<tr><td></td><td>c</td><td>Using Newton's forward interpolation find y at $x = 5$ from the data
<table>
<tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td></tr>
<tr><td>y</td><td>1</td><td>3</td><td>8</td><td>16</td></tr>
</table></td><td>6</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>b</td><td>Evaluate $\int_0^{\pi/2} \sqrt{sin\theta}d\theta$ by taking seven ordinates using, Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q. 06</td><td>a</td><td>Find the real root of the equation $\cos x = xe^x$, which is nearer to $x = 0.5$ by the Newton-Raphson method, correct to four decimal places.</td><td>7</td><td>L2</td><td>CO2</td></tr>
<tr><td></td><td>b</td><td>Determine $f(x)$ as a polynomial in x for the data given below by using Newton's divided difference formula
<table>
<tr><td>x</td><td>2</td><td>4</td><td>5</td><td>6</td><td>8</td><td>10</td></tr>
<tr><td>$f(x)$</td><td>10</td><td>96</td><td>196</td><td>350</td><td>868</td><td>1746</td></tr>
</table></td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>c</td><td>Evaluate $\int_{0.2}^{1.4}(sinx - logx + e^x)dx$ by taking seven ordinates, using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule.</td><td>6</td><td>L3</td><td>CO2</td></tr>
<tr><td colspan="6">Module-4</td></tr>
<tr><td>Q. 07</td><td>a</td><td>Find an approximate value of y when $x = 0.1$, if $\frac{dy}{dx} = x - y^2$ and $y = 1$ at $x = 0$ using Taylor's series method.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>b</td><td>Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with the initial condition $y = 1$ when $x = 0$. Find approximately y for $x = 0.1$ by Modified Euler's method. Carry out three modifications.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>c</td><td>Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$, compute $y(0.4)$ using Milne's Predictor-Corrector method.</td><td>6</td><td>L3</td><td>CO2</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q. 08</td><td>a</td><td>Using modified Euler's formula, compute y(1.1) correct to three decimal places given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ and $y = 1$ at $x = 1$.</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>b</td><td>Using the Runge-Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2$</td><td>7</td><td>L3</td><td>CO2</td></tr>
<tr><td></td><td>c</td><td>Solve $y' = 3x + y^2$ using Taylor's series method y(0) = 1 and compute y(0.1).</td><td>6</td><td>L3</td><td>CO2</td></tr>
<tr><td colspan="6">Module-5</td></tr>
<tr><td>Q. 09</td><td>a</td><td>Find the Laplace transform of (i) $te^{-t}sin4t$ (ii) $\frac{1-cosat}{t}$</td><td>7</td><td>L3</td><td>CO3</td></tr>
<tr><td></td><td>b</td><td>Find the Laplace transform of square wave function of period $2a$, defined by $f(t) = \begin{cases} k, 0 < t < a \\ -k, a < t < 2a \end{cases}$</td><td>7</td><td>L3</td><td>CO3</td></tr>
</table>

	c	Express $f(t) = \begin{cases} cost, & 0 < t < \pi \\ cos2t, & \pi < t < 2\pi \\ cos3t, & t > 2\pi \end{cases}$ in terms of the Heaviside unit step function and hence find $L\{f(t)\}$.	6	L3	CO3
OR					
Q. 10	a	Find $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$.	7	L3	CO3
	b	Using the convolution theorem, find the inverse Laplace transform of $\frac{1}{s^3(s^2+1)}$.	7	L3	CO3
	c	Solve by Laplace transform method $y'' + 4y' + 3y = e^{-t}$, given $y(0) = y'(0) = 1$.	6	L3	CO3

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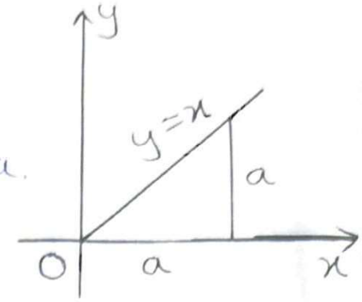
AY: 2025-26

Department: Electronics & Communication Engg
Subject with Sub. Code: Calculus, Laplace Transforms & Numerical Techniques (1BMATE201)
Max. Marks: 100
Semester / Branch / Division: II / ECE / A & B **Name of Faculty: Dr. Meenal Kaliwal**

Q.No.	Solution and Scheme	Marks
1a.	Let, $I = \int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz\, dx\, dy$	
	$= \int_{y=0}^1 \left\{ \int_{x=y^2}^1 x \int_{z=0}^{1-x} 1 \, dz \, dx \right\} dy$	1
	$= \int_{y=0}^1 \int_{x=y^2}^1 x \, [z]_{z=0}^{1-x} \, dx\, dy$	1
	$= \int_{y=0}^1 \int_{x=y^2}^1 x(1-x) \, dx\, dy$	1
	$= \int_{y=0}^1 \left(\frac{x^2}{2} - \frac{x^3}{3} \right)_{x=y^2}^1 dy$	1
	$= \int_{y=0}^1 \left[\frac{1}{2}(1-y^4) - \frac{1}{3}(1-y^6) \right] dy$	1
	$= \left[\frac{1}{2} y - \frac{1}{2} \times \frac{y^5}{5} \right]_{y=0}^1 - \frac{1}{3} \left[y - \frac{y^7}{7} \right]_{y=0}^1$	1
	$= \frac{1}{2} - \frac{1}{10} - \frac{1}{3} + \frac{1}{21} = \frac{4}{35}$	1
		7

Q.No.	Solution and Scheme	Marks

1b. Here y varies from 0 to a, and for each, y, x varies from $x=y$ to $x=a$. 1



Thus, the lower value of x lies on the line $y=x$ and the upper value on the line $x=a$.

$$\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$$

$$= \int_{x=0}^{a} \int_{y=0}^{x} \frac{x}{x^2+y^2}\,dy\,dx$$ 2

$$= \int_0^a \left\{ x . \left[\frac{1}{x}\tan^{-1}\left(\frac{y}{x}\right)\right]_{y=0}^{x} \right\} dx$$ 2

$$= \int_0^a (\tan^{-1} 1 - \tan^{-1} 0)\,dx$$ 1

$$= \int_0^a \frac{\pi}{4}\,dx = \frac{\pi a}{4}$$ 1

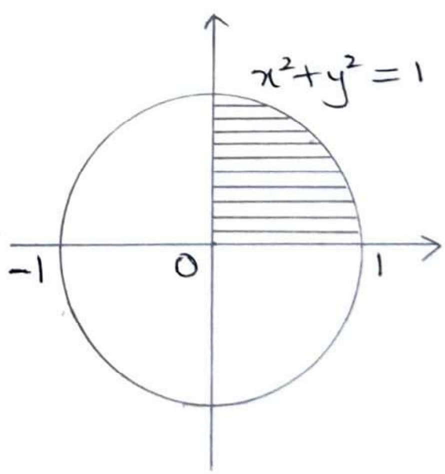
7

1c. $$\Gamma n = 2\int_0^\infty e^{-x^2} x^{2n-1}\,dx$$ 1

Similarly, $$\Gamma m = 2\int_0^\infty e^{-y^2} y^{2m-1}\,dy$$

Therefore,

$$\Gamma m\,\Gamma n = 4\left\{\int_0^\infty e^{-y^2} y^{2m-1}\,dy\right\}\left\{\int_0^\infty e^{-x^2} x^{2n-1}\,dx\right\}$$ 1

Q.No.	Solution and Scheme	Marks
	$= 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)} x^{2n-1} y^{2m-1}\, dx\, dy$	1
	Transforming the repeated integrals to polar coordinates,	
	$\Gamma m\,\Gamma n = 4\int_{r=0}^{\infty}\int_{\theta=0}^{\pi/2} e^{-r^2}(r\cos\theta)^{2n-1}(r\sin\theta)^{2m-1}\, r\, d\theta\, dr$	1
	$= 2\int_0^\infty r^{2(m+n)-1} e^{-r^2}\, dr \times 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$	1
	$= \Gamma m+n\ \beta(m,n)$	1
	$\therefore \beta(m,n) = \dfrac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$	6
	OR	
02. a.	$I = \int_{y=0}^{1}\int_{x=0}^{\sqrt{1-y^2}} (x^2+y^2)\, dx\, dy$ 	
	$x=0$ & $x=\sqrt{1-y^2}$	
	$\Rightarrow x^2 = 1-y^2$	1
	$\therefore x^2+y^2=1$, is a circle with centre at the origin and radius 1.	

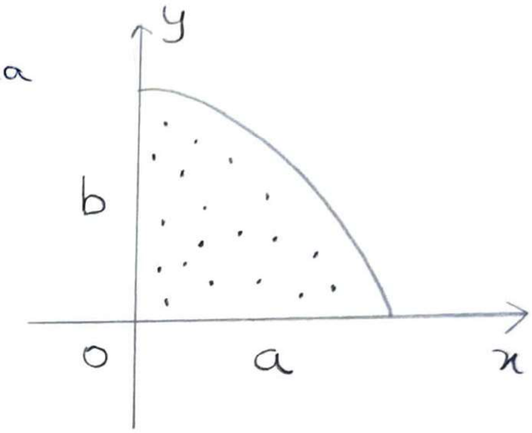
Q.No.	Solution and Scheme	Marks
	Since y varies from 0 to 1, the region of integration is first quadrant of the circle.	
	In polar we have,	1
	$x = r\cos\theta$ & $y = r\sin\theta$	
	$x^2+y^2 = r^2$ i.e. $r^2 = 1^2$ or $r = 1$	
	Also, $x = 0$ & $y = 0 \Rightarrow r = 0$ and hence r varies from 0 to 1.	1
	In the first quadrant θ varies from 0 to $\pi/2$.	1
	Also, $dx\,dy = r\,dr\,d\theta$	
	$I = \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} (r^2\cos^2\theta + r^2\sin^2\theta)\, r\,dr\,d\theta$	1
	$= \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} r^3\,dr\,d\theta = \int_{\theta=0}^{\pi/2}\left(\frac{r^4}{4}\right)_{r=0}^{1} d\theta$	1
	$= \frac{1}{4}\int_{\theta=0}^{\pi/2} 1\,d\theta = \frac{1}{4}\times\frac{\pi}{2}$	
	$\therefore I = \frac{\pi}{8}$	1
		7

Q. No.	Solution	Marks

2b. In the given region, x varies from 0 to a, and for each x, y varies from 0 to a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ i.e. to the point for which

$$y = b\left(1 - \frac{x^2}{a^2}\right)^{1/2}$$

Hence, the required area is



$$A = \int_{x=0}^{a} \int_{y=0}^{b(1-x^2/a^2)^{1/2}} 1 \, dy\, dx$$ 2

$$= \int_{0}^{a} b\left(1 - \frac{x^2}{a^2}\right)^{1/2} dx$$ 1

$$= \frac{b}{a} \int_{0}^{a} (a^2 - x^2)^{1/2} dx$$ 1

$$= \frac{b}{a} \left\{ \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1}(x/a) \right\}_{x=0}^{a}$$ 2

$$= \frac{b}{a} \cdot \frac{a^2}{2} \cdot \frac{\pi}{2}$$

$$A = \frac{\pi}{4} ab \text{ sq. units}$$ 1

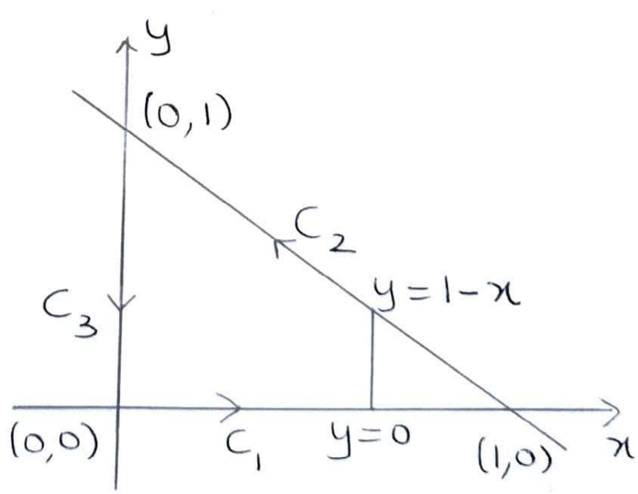
Total: 7

Q. No.	Solution	Marks
2c.	Let, $I_1 = \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \int_0^{\pi/2} (\sin\theta)^{-1/2}\, d\theta$	
	$p-1 = -\frac{1}{2} \Rightarrow p = \frac{1}{2}$	
	Using, $\int_0^{\pi/2} (\sin\theta)^{p-1}\, d\theta = \frac{1}{2}\beta\left(\frac{p}{2}, \frac{1}{2}\right)$	
	$= \frac{1}{2} \frac{\Gamma(p/2)\cdot\Gamma(1/2)}{\Gamma\left(\frac{p+1}{2}\right)}$	1
	$I_1 = \frac{1}{2}\frac{\Gamma(1/4)\,\Gamma(1/2)}{\Gamma(3/4)} = \frac{1}{2}\frac{\Gamma(1/4)\sqrt{\pi}}{\Gamma(3/4)} \longrightarrow (i)$	1
	Consider, $I_2 = \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \int_0^{\pi/2} (\sin\theta)^{1/2}\, d\theta$	
	$p-1 = \frac{1}{2} \Rightarrow p = 3/2$	
	$I_2 = \frac{1}{2}\frac{\Gamma(3/4)\,\Gamma(1/2)}{\Gamma(5/4)} = \frac{1}{2}\cdot\frac{\Gamma(3/4)\sqrt{\pi}}{\Gamma(5/4)} \longrightarrow (ii)$	1
	$I_1 I_2 = \frac{1}{2}\cdot\frac{\Gamma(1/4)\sqrt{\pi}}{\Gamma(3/4)} \times \frac{1}{2}\cdot\frac{\Gamma(3/4)\sqrt{\pi}}{\Gamma(5/4)}$	1
	$= \frac{1}{4}\frac{\Gamma(1/4)\,\Gamma(3/4)\,\pi}{\Gamma(3/4)\,\Gamma(5/4)} = \frac{1}{4}\cdot\frac{\Gamma(1/4)}{\frac{1}{4}\Gamma(1/4)} \times \pi$	1
	$I_1 I_2 = \frac{1}{4} \times 4\pi = \pi$	
	$\therefore \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$	1
		6

Q.No.	Solution and Scheme	Marks
	MODULE-2	
03 a.	$\phi = x^2yz + 4xz^2 \; ; \; P \equiv (1, -2, 1)$	
	$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$	1
	$\nabla\phi = (2xyz + 4z^2) i + (x^2z) j + (x^2y + 8xz) k$	1
	$[\nabla\phi]_{(1,-2,1)} = (-4+4) i + j + (-2+8) k$	2
	$= j + 6k$	
	The unit vector in the direction of $2i - j - 2k$,	
	$\hat{n} = \frac{2i - j - 2k}{\sqrt{4+1+4}} = \frac{2i - j - 2k}{3}$	1
	$\therefore$ The required directional derivative is	
	$\nabla\phi \cdot \hat{n} = (j + 6k) \cdot \frac{(2i - j - 2k)}{3}$	1
	$= \frac{(-1) + (6)(-2)}{3} = \frac{-13}{3}$	1
	Thus, $\boxed{\nabla\phi \cdot \hat{n} = \frac{-13}{3}}$	
		7
b.	$\vec{F} = x^2y\hat{i} + y^2z\hat{j} + z^2x\hat{k}$	
	$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & y^2z & z^2x \end{vmatrix}$	1
	$= i\left\{\frac{\partial}{\partial y}(z^2x) - \frac{\partial}{\partial z}(y^2z)\right\} - j\left\{\frac{\partial}{\partial x}(z^2x) - \frac{\partial}{\partial z}(x^2y)\right\}$	

Q.No.	Solution and Scheme	Marks
	$+k\left\{\frac{\partial}{\partial x}(y^2 z) - \frac{\partial}{\partial y}(x^2 y)\right\}$	1
	$= i\{0 - y^2\} - j\{z^2 - 0\} + k\{0 - x^2\}$	
	$\text{curl } \vec{F} = -y^2\hat{i} - z^2\hat{j} - x^2\hat{k}$	1
	$\text{curl}(\text{curl } \vec{F}) = \nabla \times (\nabla \times \vec{F})$	
	$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y^2 & -z^2 & -x^2 \end{vmatrix}$	1
	$= i\left\{\frac{\partial}{\partial y}(-x^2) - \frac{\partial}{\partial z}(-z^2)\right\} - j\left\{\frac{\partial}{\partial x}(-x^2) - \frac{\partial}{\partial z}(-y^2)\right\}$ $+ k\left\{\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y^2)\right\}$	
	$= i\{0 + 2z\} - j\{-2x + 0\} + k\{0 + 2y\}$	
	$= 2z\hat{i} + 2x\hat{j} + 2y\hat{k}$	1
	$\text{div}(\text{curl } \vec{F}) = \nabla \cdot (\nabla \times \vec{F})$	
	$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (2z\hat{i} + 2x\hat{j} + 2y\hat{k})$	1
	$= \frac{\partial}{\partial x}(2z) + \frac{\partial}{\partial y}(2x) + \frac{\partial}{\partial z}(2y)$	
	$\text{div}(\text{curl } \vec{F}) = 0$	1
		7
3c.	$\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j}$ $+ (3xy - 2xz + 2z)\hat{k}$	
	We have, $\text{div } \vec{F} = \nabla \cdot \vec{F}$	1

Q.No.	Solution and Scheme	Marks
	$= \frac{\partial}{\partial x}(y^2-z^2+3yz-2x)+\frac{\partial}{\partial y}(3xz+2xy)$	1
	$+\frac{\partial}{\partial z}(3xy-2xz+2z)$	
	$= -2+2x-2x+2=0$	1
	$\therefore$ div $\vec{F}=0 \Rightarrow \vec{F}$ is solenoidal.	
	curl $\vec{F} = \nabla \times \vec{F}$	
	$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (y^2-z^2+3yz-2x) & (3xz+2xy) & (3xy-2xz+2z) \end{vmatrix}$	1
	$= \hat{i}[3x-3x]-\hat{j}[3y-2z+2z-3y]$	1
	$+\hat{k}[3z+2y-2y-3z]$	
	$= \vec{0}$	1
	$\Rightarrow \vec{F}$ is irrotational.	6
	OR	
4a.	Given, $\vec{F}=3x^2\hat{i}+(2xz-y)\hat{j}+z\hat{k}$	
	Let, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then	
	$d\vec{r}=dx\hat{i}+dy\hat{j}+dz\hat{k}$	1
	$\therefore \vec{F}\cdot d\vec{r}=(3x^2)dx+(2xz-y)dy+z\,dz$	
	$\int_C \vec{F}\cdot d\vec{r}=\int_C 3x^2\,dx+\int_C(2xz-y)\,dy+\int_C z\,dz$	1
	The equations of the straight line from $(0,0,0)$ to $(2,1,3)$ are $x=2t$, $y=t$ & $z=3t$	1

Q.No.	Solution and Scheme	Marks
	$\Rightarrow dx = 2dt,\ dy = dt$ and $dz = 3dt$	1
	and t varies from $t=0$ to $t=1$	
	$\therefore \int_C \vec{F}\cdot d\vec{r} = \int_0^1 [3(2t)^2\, 2dt + \{(4t)(3t)-t\}dt + (3t)3\,dt]$	1
	$= \int_0^1 [36t^2 + 8t]\,dt$	
	$= 36\left[\frac{t^3}{3}\right]_{t=0}^{1} + 8\left[\frac{t^2}{2}\right]_{t=0}^{1}$	1
	$= 12 + 4 = 16$	
	$\therefore$ Workdone $= \underline{\underline{16}}$	1
		7
4b.	Here, $M = 3x - 8y^2$, $N = 4y - 6xy$	1
	$\therefore \frac{\partial M}{\partial y} = -16y$, $\frac{\partial N}{\partial x} = -6y$	
		
	Equation to C_1 : $y=0$ ($\because dy=0$) and x varies from $x=0$ to $x=1$	1
	Equation to C_2 : $y=1-x$ ($\because dy=-dx$) and x varies from $x=1$ to $x=0$.	

Q. No. | Solution | Marks

Equations to C_3: $x = 0 \Rightarrow dx = 0$ and y varies from $y = 1$ to $y = 0$.

Now,

$$\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = -6y + 16y = 10y$$ [1]

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy = \int_0^1 \int_0^{1-x} 10y\,dy\,dx$$ [1]

$$= \int_{x=0}^{1} \left(10 \times \frac{y^2}{2}\right)_{y=0}^{1-x} dx$$ [1]

$$= \int_{x=0}^{1} 5y^2 \Big|_{y=0}^{1-x} dx = \int_{x=0}^{1} 5(1-x)^2\,dx$$ [1]

$$= \left[-\frac{5}{3}(1-x)^3\right]_{x=0}^{1} = -5/3$$

$$\therefore \int_C (3x - 8y^2)\,dx + (4y - 6xy)\,dy = -5/3$$ [1]

[7]

4c. The directional derivative of ϕ in the direction of a vector $\vec{a} = \nabla\phi \cdot \frac{\vec{a}}{|\vec{a}|}$

Given, $\phi = 4xz^3 - 3x^2y^2z$

$\vec{a} = 2i - 3j + 6k$

$$\therefore \nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$$

$$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}(4xz^3 - 3x^2y^2z) = 4z^3 - 6xy^2z$$

Q. No.	Solution	Marks
	$\frac{\partial}{\partial y}(4xz^3 - 3x^2y^2z) = -6x^2yz$	
	$\frac{\partial}{\partial z}(4xz^3 - 3x^2y^2z) = 12xz^2 - 3x^2y^2$	
	$\nabla\phi = (4z^3 - 6xy^2z)\hat{i} + (-6x^2yz)\hat{j} + (12xz^2 - 3x^2y^2)\hat{k}$	
	At $(2,-1,2)$, $\nabla\phi = [4(2)^3 - 6(2)(-1)^2(2)]\hat{i} - 6(2)^2(-1)(2)\hat{j} + [12(2)(2)^2 - 3(2)^2 \times (-1)^2]\hat{k}$	3
	$= 8\hat{i} + 48\hat{j} + 84\hat{k}$	
	The directional derivative,	
	$\nabla\phi \cdot \frac{\vec{a}}{\lvert\vec{a}\rvert} = (8\hat{i} + 48\hat{j} + 84\hat{k}) \cdot \frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{2^2 + (-3)^2 + 6^2}}$	
	$= [8\hat{i} + 48\hat{j} + 84\hat{k}) \cdot \frac{(2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{49}}$	3
	$= \frac{8.2 - 48(3) + 84.6]}{7} = \frac{376}{7}$	
	$= \frac{376}{7}$	6
	Module - 3	
5a.	Let $f(x) = x^3 - 9x + 1$, $f(2) = -9 < 0$, $f(3) = +1 > 0$	1
	$\therefore$ The root lies in $(2,3)$	
	1^{st} approximation : $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$	1
	$x_1 = 2.9000$	1

Q.No.	Solution and Scheme	Marks

<u>2nd approximation:</u>

$f(2.9) = (2.9)^3 - 9(2.9) + 1 = -0.7110$

$a = 2.9, \; b = 3$ — 1

$\therefore x_2 = 2.9416$

<u>3rd approximation:</u>

$f(2.9416) = -0.0207.$ — 1

$a = 2.9416, \; b = 3$

$x_3 = 2.9428$

<u>4th approximation:</u> $f(2.9428) = -0.0207$

$a = 2.9428, \; b = 3$ — 1

$x_4 = 2.9428$

$\therefore$ The real roots for the given equation — 1

is $x = 2.9428$

Total: 7

b. $h = 6 - 4 = 2$ — 1

$r = \dfrac{x - x_0}{h} = \dfrac{5-4}{2} = 0.5$

Forward difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
$x_0 = 4$	$y_0 = 1$			
		2		
$x_1 = 6$	$y_1 = 3$		3	
		5		0
$x_2 = 8$	$y_2 = 8$		3	
		8		
$x_3 = 10$	$y_3 = 16$			

— 3

Q.No.	Solution and Scheme	Marks

Newton's forward interpolation formula is,

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \cdots$$ 1

$$= 1 + (0.5)2 + \frac{(0.5)(0.5-1)}{2}(3) + \frac{(0.5)(0.5-1)(0.5-2)}{6} \times 0$$ 1

$$= 1 + 1 - 0.375$$ 1

$$y_{(5)} = \underline{\underline{1.6250}}$$ 7

5c. Let, $f(\theta) = \sqrt{\sin\theta}$. $a = 0$ & $b = \pi/2$

7 ordinates = 6 equal parts $\Rightarrow n = 6$

$$h = \frac{b-a}{n} = \frac{\pi/2 - 0}{6} = \pi/12$$ 1

θ	$\theta_0 = 0$	$\theta_1 = \pi/12$	$\theta_2 = \pi/6$	$\theta_3 = \pi/4$
$f(\theta)$	$y_0 = 0$	$y_1 = 0.5087$	$y_2 = 0.7071$	$y_3 = 0.8409$

$\theta_4 = \pi/3$	$\theta_5 = 5\pi/12$	$\theta_6 = \pi/2$
$y_4 = 0.9306$	$y_5 = 0.9828$	$y_6 = 1$

2

Simpson's $(1/3)^{rd}$ Rule is,

$$\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \frac{h}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right]$$ 1

$$= \frac{\pi}{36}\left[(0+1) + 2(0.7071 + 0.9306) + 4(0.5087 + 0.8409 + 0.9828)\right]$$ 1

$$= \pi/36\,[1 + 3.2754 + 9.6176]$$

Q.No.	Solution and Scheme	Marks

$$\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \frac{\pi}{36}[13.893]$$ 1

$$\simeq 1.2124$$ 6

OR

6a. Let, $f(x) = \cos x - xe^x$ 1

$$f'(x) = -\sin x - xe^x - e^x$$

Newton Raphson formula is, 2

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1st approximation:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$ 1

$$x_1 \simeq 0.51803$$ 1

2nd approximation: $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$x_2 = 0.51776$$ 1

3rd approximation: $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$$x_3 = 0.5177$$

Thus, the root correct to four decimal places is $x = 0.5177$. 1

7

b. Newton's general interpolation formula is,

$$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3) + \cdots$$ 1

Q.No.	Solution and Scheme	Marks

The divided difference table is

x	$f(x)$	1st Divided Difference
$x_0=2$	$f(x_0)=10$	
		$f(x_0,x_1)=\frac{96-10}{4-2}=43$
$x_1=4$	$f(x_1)=96$	
		$f(x_1,x_2)=\frac{196-96}{5-4}=100$
$x_2=5$	$f(x_2)=196$	
		$f(x_2,x_3)=\frac{350-196}{6-5}=154$
$x_3=6$	$f(x_3)=350$	
		$f(x_3,x_4)=\frac{868-350}{8-6}=259$
$x_4=8$	$f(x_4)=868$	
		$f(x_4,x_5)=\frac{1746-868}{10-8}=439$
$x_5=10$	$f(x_5)=1746$	

2

2nd Divided Differences	3rd Divided Differences
$f(x_0,x_1,x_2)=\frac{100-43}{5-2}=19$	
	$f(x_0,x_1,x_2,x_3)=\frac{27-19}{6-2}=2$
$f(x_1,x_2,x_3)=\frac{154-100}{6-4}=27$	
	$f(x_1,x_2,x_3,x_4)=\frac{35-27}{8-4}=2$
$f(x_2,x_3,x_4)=\frac{259-154}{8-5}=35$	
	$f(x_2,x_3,x_4,x_5)=\frac{45-35}{10-5}=2$
$f(x_3,x_4,x_5)=\frac{439-259}{10-6}=45$	

2

Q.No.	Solution and Scheme	Marks

$f(x) = 10 + (x-2)43 + (x-2)(x-4)19$ 1

$\quad + (x-2)(x-4)(x-5)2$

$= 10 + (x-2)\left\{43 + (19x-76) + (x^2-9x+20)2\right\}$

$= 10 + (x-2)(2x^2+x+7)$ 1

$\therefore f(x) = 2x^3 - 3x^2 + 5x - 4$

7

c. Let, $f(x) = \sin x - \log x + e^x$

$a = 0.2$ and $b = 1.4$, $n = 6$, $h = \dfrac{b-a}{n}$

$\therefore h = 0.2$ 1

x	$x_0 = 0.2$	$x_1 = 0.4$	$x_2 = 0.6$
$f(x)$	$y_0 = 3.0295$	$y_1 = 2.7975$	$y_2 = 2.8976$

$x_3 = 0.85$	$x_4 = 1.0$	$x_5 = 1.2$	$x_6 = 1.4$
$y_3 = 3.1660$	$y_4 = 3.5597$	$y_5 = 4.0698$	$y_6 = 4.4042$

2

Simpson's $(3/8)^{th}$ Rule is,

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 2(y_3+y_6+\cdots+y_{n-3}) + 3(y_1+y_2+\cdots+y_{n-1})\right]$$ 1

$$\int_{0.2}^{1.4} (\sin x - \log x + e^x)\,dx = \frac{3h}{8}\left[(y_0+y_6) + 2(y_3) + 3(y_1+y_2+y_4+y_5)\right]$$ 1

$$= \frac{3(0.1)}{8}\left[(3.0295+4.4042) + 2(3.166) + 3(2.7975+2.8976+3.5597+4.4042)\right]$$

$$\int_a^b f(x)\,dx = 4.053$$ 1

6

Q.No.	Solution and Scheme	Marks

Module - 4

7a. $y' = x - y^2, \; x_0 = 0, \; y_0 = 1$

$y'(0) = -1$

$y'' = 1 - 2yy' \Rightarrow y''(0) = 3$

$y''' = -2yy'' - 2y'y' \Rightarrow y'''(0) = -8$ — **3**

$y'''' = -2[yy''' + y'y'' + 2y'y'']$

$= -2[yy''' + 3y'y'']$

$y''''(0) = 34$

Taylor's series is given by,

$$y(x) = y_0 + (x - x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y''''(x_0) + \cdots$$ — **1**

$$y(x) = y_0 + x\,y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y''''(0) + \cdots$$ — **1**

$$y(0.1) = 1 + x(-1) + \frac{x^2}{2} \times 3 + \frac{x^3}{6} \times (-8) + \frac{x^4}{24}(34)$$

$$y(0.1) = 1 - x + \frac{3}{2}x^2 - \frac{4}{3}x^3 + \frac{17}{12}x^4$$ — **1**

$$= 1 - (0.1) + \frac{3}{2}(0.1)^2 - \frac{4}{3}(0.1)^3 + \frac{17}{12}(0.1)^4$$

$$= 1 - 0.1 + 0.015 - 0.001333 + 0.000141667$$

$y(0.1) = 0.91381$ — **1**

Total: **7**

Q.No.	Solution and Scheme	Marks

b. $y_0 = 1,\ x_0 = 0,\ h = 0.1$

$f(x_0, y_0) = 1$

Euler's formula ; $y_1^{(0)} = y_0 + hf(x_0, y_0)$

$$y_1^{(0)} = 1.1$$

Modified Euler's formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(0)})\right]$$ 1

$$y_1^{(1)} = 1.0917$$ 1

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(1)})\right]$$ 1

$$= 1 + \frac{0.1}{2}\left[1 + \frac{1.0917 - 0.1}{1.0917 + 0.1}\right]$$

$$= 1 + 0.05\left[1.832172527\right]$$ 2

$$= 1.0916$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(2)})\right]$$ 2

$$\simeq 1.0916$$

$\therefore\ y(0.1) = 1.0916$ 7

9c.

x	y	$y' = xy + y^2$
$x_0 = 0$	$y_0 = 1$	$y'_0 = 1$
$x_1 = 0.1$	$y_1 = 1.1169$	$y'_1 = 1.3592$
$x_2 = 0.2$	$y_2 = 1.2773$	$y'_2 = 1.8869$
$x_3 = 0.3$	$y_3 = 1.5049$	$y'_3 = 2.7162$

1

Q.No.	Solution and Scheme	Marks
	Milne's Predictor formula is,	
	$y_4^{(P)} = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3')$	
	$= 1 + \frac{4(0.1)}{3}[2(1.3592) - 1.8869 + 2(2.7162)]$	
	$y_4^{(P)} = 1.835186667$	1
	$\simeq 1.8352$	
	$y_4' = x_4 y_4 + y_4^2 = (0.4)(1.8352) + (1.8352)^2$	
	$y_4' = 4.10203904 \simeq 4.1020$	
	Milne's corrector formula is,	
	$y_4^{(c)} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$	
	$= 1.2773 + \frac{(0.1)}{3}[1.8869 + 4(2.7162) + 4.1020]$	1
	$y_4^{(c)} = 1.83909$	
	$y_4' = x_4 y_4 + (y_4)^2 = 4.11789$	1
	$y_4^{(c)} = 1.83962$	
	$y_4' = 4.1200$	1
	$y_4^{(c)} = 1.83969$	1
	$\therefore y(0.4) \simeq 1.8396$	
		6

Q.No.	Solution and Scheme	Marks
	OR	
8a.	$\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x} = \frac{1-xy}{x^2}$; $x_0 = 1$, $y_0 = 1$	
	$x_1 = x_0 + h \Rightarrow 1.1 = 1 + h \Rightarrow h = 0.1$	
	Euler's formula is,	
	$y_1^{(0)} = y_0 + hf(x_0, y_0)$	
	$= 1 + (0.1) f(1, 1)$	
	$= 1 + (0.1)\left(\frac{1-1}{1}\right) = 1 + 0 = 1$	1
	$\therefore y_1^{(0)} = 1$	
	Modified Euler's formula is,	1
	$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(0)})]$	
	$= 1 + \frac{0.1}{2}[f(1, 1) + f(1.1, 1)]$	
	$= 1 + (0.05)\left[0 + \frac{1-(1.1)(1)}{(1.1)^2}\right]$	2
	$y_1^{(1)} = 0.9959$	
	$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$	1
	$y_1^{(2)} \simeq 0.99605$	
	$y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$	1
	$\simeq 0.99605$	
	Thus, $y(1.1) \simeq 0.99605$	1
		7

Q.No.	Solution and Scheme	Marks
b.	Given, $y' = \frac{y^2-x^2}{y^2+x^2}$, $x_0 = 0$, $y_0 = 1$	
	$x_1 = x_0 + h \Rightarrow 0.2 = 0 + h \Rightarrow h = 0.2$	
	$k_1 = hf(x_0, y_0) = (0.2) f(0,1)$	
	$= 0.2\left[\frac{1-0}{1+0}\right] = 0.2$	1
	$k_1 = 0.2$	
	$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$	
	$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$	
	$= (0.2) f(0.1, 1.1) = (0.2)\left[\frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}\right]$	
	$= (0.2)(0.9836065574)$	1
	$k_2 = 0.1967$	
	$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$	
	$= (0.2) f(0.1, 1.09835)$	
	$= 0.2\left[\frac{(1.09835)^2 - (0.1)^2}{(1.09835)^2 + (0.1)^2}\right]$	1
	$k_3 = 0.1967$	
	$k_4 = hf(x_0 + h, y_0 + k_3) = (0.2) f(0.2, 1.1967)$	1
	$k_4 = 0.1891$	
	$y(x_0 + h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + k_4 + 2k_3)$	1
	$y(x_1) = y(0.2) = 1 + \frac{1}{6}(0.2 + 0.3934 + 0.3934 + 0.1891)$	
	$y(0.2) = 1.1959$	2
		7

Q. No.	Solution	Marks
8c.	$y' = 3x + y^2$, $x_0 = 0$, $h = 0.1$, $y_0 = 1$ $y(x) = y(x_0) + (x - x_0) y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \frac{(x-x_0)^3}{3!} y'''(x_0) + \cdots \longrightarrow (I)$	1
	$y'(0) = 1$ $y'' = 3 + 2yy'$ $\therefore y''(0) = 3 + 2(1)(1) = 5$	2
	$y''' = 2yy'' + 2(y')^2$ $y'''(0) = 2(1)(5) + 2(1)^2 = 12$	2
	Substituting these values in eqn (I), $y(x) = 1 + \frac{x}{1!} \times 1 + \frac{x^2}{2!} \times 5 + \frac{x^3}{3!} \times 12$ $y(0.1) = 1 + x + \frac{5}{2} x^2 + 2x^3 + \cdots\cdots$	1
		6

Q. No.	Solution	Marks
9a.	Consider, $L[\sin 4t] = \frac{4}{s^2+16}$	
	$L[e^{-t}\sin 4t] = \frac{4}{(s+1)^2+16}$ [Using shifting property]	1
	$\therefore L[t e^{-t}\sin 4t] = (-1)\frac{d}{ds}\left\{\frac{4}{s^2+2s+17}\right\}$	
	$= (-1)\left\{\frac{-4(2s+2)}{(s^2+2s+17)^2}\right\}$	1
	$= \frac{8(s+1)}{(s^2+2s+17)^2}$	1
	(ii) Let, $f(t) = 1-\cos at$	
	$\therefore L[f(t)] = L[1] - L[\cos at] = \frac{1}{s} - \frac{s}{s^2+a^2}$	1
	$= \bar{f}(s)$	
	Applying Division property,	
	$L\left[\frac{1-\cos at}{t}\right] = \int_s^\infty \bar{f}(s)\,ds = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+a^2}\right) ds$	1
	$= \log s \Big\vert_s^\infty - \frac{1}{2}\log(s^2+a^2)\Big\vert_{s=s}^\infty$	
	$= \log s - \log\sqrt{s^2+a^2}\Big\vert_{s=s}^\infty = \log\frac{s}{\sqrt{s^2+a^2}}\Big\vert_{s=s}^\infty$	1
	$= \log\frac{s}{s\sqrt{1+\frac{a^2}{s^2}}}\Bigg\vert_{s=s}^\infty$	
	$= \log 1 - \log\frac{s}{\sqrt{s^2+a^2}} = \log\frac{\sqrt{s^2+a^2}}{s}$	1
	$[\because \log 1 = 0]$	7

Q. No.	Solution	Marks
b.	Laplaca transform of periodic function is, $L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$	1
	f(t) is a periodic function with period 2a. $L[f(t)] = \frac{1}{1-e^{-s2a}} \int_0^{2a} e^{-st} f(t)\, dt$	
	$= \frac{1}{1-e^{-2as}} \left[\int_0^a f(t)\, e^{-st} dt + \int_a^{2a} e^{-st} f(t) dt \right]$	1
	$= \frac{1}{1-e^{-2as}} \left[\int_0^a k e^{-st} dt + \int_a^{2a} -k e^{-st} dt \right]$	1
	$= \frac{k}{1-e^{-2as}} \left[\int_0^a e^{-st} dt - \int_a^{2a} e^{-st} dt \right]$	
	$= \frac{k}{1-e^{-2as}} \left\{ \frac{e^{-st}}{-s} \Big\vert_{t=0}^{a} - \left(\frac{e^{-st}}{-s} \right)_{t=a}^{2a} \right\}$	1
	$= \frac{k}{1-e^{-2as}} \left\{ -\frac{1}{s} \left(e^{-as} - e^{0} \right) + \frac{1}{s} \left(e^{-2as} - e^{-as} \right) \right\}$	
	$= \frac{k}{s(1-e^{-2as})} \left[-e^{-as} + 1 + e^{-2as} - e^{-as} \right]$	
	$= \frac{k}{s(1-e^{-2as})} \left[-2e^{-as} + 1 + e^{-2as} \right]$	1
	$= \frac{k}{s(1-e^{-2as})} \left[(e^{-as})^2 + 2(1)(e^{-as}) + 1^2 \right]$	

Q. No.		Marks
	$= \dfrac{k(1-e^{-as})^2}{s(1-e^{-as})(1+e^{-as})} = \dfrac{k(1-e^{-as})}{s(1+e^{-as})}$	1
	$=$	7
9c.	$f(t) = \begin{cases} \cos t, & 0<t<\pi \\ \cos 2t, & \pi<t<2\pi \\ \cos 3t, & 2\pi<t \end{cases}$	
	$f(t) = \cos t + (\cos 2t - \cos t)u(t-\pi) + (\cos(3t - \cos 2t)\, u(t-2\pi)$	1
	using, $f(t) = f_1(t) + [f_2(t) - f_1(t)]u(t-\pi) + [f_3(t) - f_2(t)]u(t-2\pi)$	
	$L[f(t)] = L[\cos t] + L[(\cos 2t - \cos t)u(t-\pi)] + L[(\cos 3t - \cos 2t)u(t-2\pi)]$	1
	$L[\cos t] = \dfrac{s}{s^2+1} \longrightarrow \text{(i)}$	
	Next consider, $L[(\cos 2t - \cos t)u(t-\pi)]$	
	Let, $F(t) = \cos 2t - \cos t$	
	$L[F(t)u(t-\pi)] = e^{-\pi s} L[F(t+\pi)]$	1
	$F(t+\pi) = \cos(2t+2\pi) - \cos(t+\pi) = \cos 2t - (-\cos t) = \cos 2t + \cos t$	
	$L[F(t+\pi)] = \dfrac{s}{s^2+4} + \dfrac{s}{s^2+1}$	1
	Thus, $L\{(\cos 2t - \cos t)u(t-\pi)\} = e^{-\pi s}\left(\dfrac{s}{s^2+4} + \dfrac{s}{s+1}\right) \longrightarrow \text{(ii)}$	

Q. No.		Marks

Next consider,

$$H(t) = \cos 3t - \cos 2t.$$

$$L\{H(t)u(t-2\pi)\} = e^{-2\pi s} L[H(t+2\pi)]$$

$$H(t+2\pi) = \cos(3t+6\pi) - \cos(2t+4\pi)$$

$$= \cos 3t - \cos 2t$$

$$L[H(t+2\pi)] = L[\cos 3t] - L[\cos 2t]$$

$$= \frac{s}{s^2+9} - \frac{s}{s^2+4}$$ 1

$$L\{(\cos 3t - \cos 2t)u(t-2\pi)\}$$

$$= e^{-2\pi s}\left(\frac{s}{s^2+9} - \frac{s}{s^2+4}\right) \longrightarrow \text{(iii)}$$

Combining (i), (ii), (iii)

$$L\{f(t)\} = \frac{s}{s^2+1} + e^{-\pi s}\left[\left(\frac{s}{s^2+4}\right) + \frac{s}{s^2+1}\right]$$ 1

$$+ e^{-2\pi s}\left(\frac{s}{s^2+9} - \frac{s}{s^2+4}\right)$$

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OR

10a. $s^3 - 6s^2 + 11s - 6 = (s-1)(s-2)(s-3)$

So, $$\frac{2s^2-6s+5}{s^3-6s^2+11s-6} = \frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$$ 1

Using partial fractions,

$$\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)} = \frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-3}$$ 1

$$2s^2 - 6s + 5 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$$ 1

Q. No.	Solution	Marks
	put, $s=1 \Rightarrow A = 1/2$	
	$s=2 \Rightarrow B=-1$	1
	$s=3 \Rightarrow C = 5/2$	
	$$\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)} = \frac{1/2}{s-1} - \frac{1}{s-2} + \frac{5/2}{s-3}$$	2
	Using, $L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$	
	$$L^{-1}\left\{\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}\right\} = \frac{1}{2}L^{-1}\left\{\frac{1}{s-1}\right\} - L^{-1}\left\{\frac{1}{s-2}\right\} + 5/2\, L^{-1}\left\{\frac{1}{s-3}\right\}$$	
	$$= \frac{1}{2}e^{t} - e^{2t} + \frac{5}{2}e^{3t}$$	1
		7
b.	$$\frac{1}{s^3(s^2+1)} = \frac{1}{s^3}\cdot\frac{1}{s^2+1}$$	
	Let, $\bar{f}(s) = \frac{1}{s^3}$ and $\bar{g}(s) = \frac{1}{s^2+1}$	
	$$L^{-1}[\bar{f}(s)] = L^{-1}\left[\frac{1}{s^3}\right] = \frac{t^2}{2} = f(t)$$	2
	$$L^{-1}[\bar{g}(s)] = L^{-1}\left[\frac{1}{s^2+1}\right] = \sin t = g(t)$$	
	$$L^{-1}\{\bar{f}(s)\cdot\bar{g}(s)\} = \int_0^t f(u)\,g(t-u)\,du$$	
	$$= \int_0^t \frac{u^2}{2}\sin(t-u)\,du$$	1

Q.No.	Solution and Scheme	Marks
	Applying integration by parts,	
	$= \frac{1}{2}\left\{u^2\left[-\frac{\cos(t-u)}{-1}\right] - 2u\left[\frac{\sin(t-u)}{-1}\right] + 2\left[-\frac{\cos(t-u)}{(-1)(-1)}\right]\right\}_{u=0}^{t}$	2
	$= \frac{1}{2}\left[u^2\cos(t-u) + 2u\sin(t-u) - 2\cos(t-u)\right]_0^t$	1
	$= \frac{1}{2}\left\{\left[t^2\cos(0) + 2t\sin t - 2\cos(0)\right] - \left[0+0-2\cos t\right]\right\}$	
	$= \frac{1}{2}\left[t^2+0-2+2\cos t\right] = \frac{1}{2}(t^2+2\cos t-2)$	1
		7
c.	$y''(t) + 4y'(t) + 3y(t) = e^{-t}$	
	$L[y''(t)] + 4L[y'(t)] + 3L[y(t)] = L[e^{-t}]$	
	$\{s^2L[y(t)] - sy(0) - y'(0)\} + 4\{sL[y(t)] - y(0)\} + 3L[y(t)] = \frac{1}{s+1}$	1
	$(s^2+4s+3)L[y(t)] - s - 1 - 4 = 1/s+1$	
	$(s+1)(s+3)L[y(t)] = (s+5) + \frac{1}{s+1}$	
	$L[y(t)] = \frac{s^2+6s+1}{(s+1)^2(s+3)}$	1
	$\therefore y(t) = L^{-1}\left[\frac{s^2+6s+1}{(s+1)^2(s+3)}\right]$	

Q.No.	Solution and Scheme	Marks
	Using partial fractions,	
	$$\frac{s^2+6s+6}{(s+1)^2(s+3)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+3}$$	1
	$$s^2+6s+6 = A(s+1)(s+3) + B(s+3) + C(s+1)^2 \quad \hookrightarrow (i)$$	
	put, $s=-1 \Rightarrow 1 = B(2) \Rightarrow B = 1/2$	
	$s=-3 \Rightarrow -3 = C(4) \Rightarrow C = -3/4$	2
	Equating the coefficient of s^2 of eqn (i),	
	$1 = A + C \quad \therefore A = 7/4$	
	Hence,	
	$$L^{-1}\left[\frac{s^2+6s+6}{(s+1)^2(s+3)}\right] = \frac{7}{4}L^{-1}\left[\frac{1}{s+1}\right] + \frac{1}{2}L^{-1}\left[\frac{1}{(s+1)^2}\right] - \frac{3}{4}L^{-1}\left[\frac{1}{s+3}\right]$$	
	Thus,	
	$$y(t) = \frac{7}{4}e^{-t} + \frac{1}{2}t.e^{-t} - \frac{3}{4}e^{-3t}$$	1
		6
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