KLS Vishwanathrao Deshpande Institute of Technology

(Accredited by NAAC with "A" Grade)

(Approved by AICTE, New Delhi, Affiliated to VTU, Belagavi)

(Recognized Under Section 2(f) by UGC, New Delhi)

Udyog Vidya Nagar, Haliyal - 581 329, Dist.: Uttara Kannada

www.klsvdit.edu.in | principal@klsvdit.edu.in | hodece@klsvdit.edu.in

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	Sudheendra Yalagi
Course Name	:	Control Systems
Course Code	:	BEC403
Year of Question Paper	:	June/July - 2025
Date of Submission	:	20/2/2026

Faculty Member HoD Dean (Acad.)

CBCS SCHEME

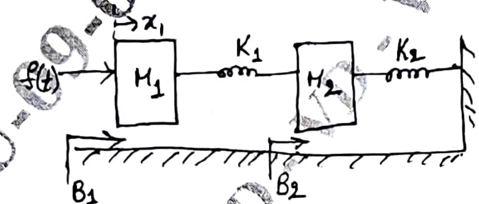
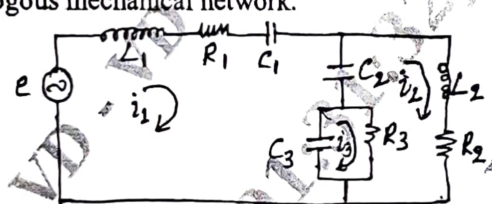
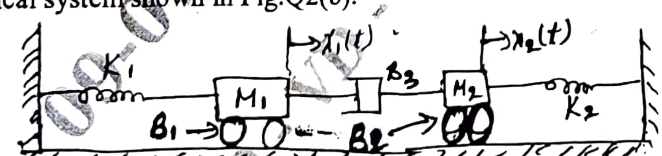
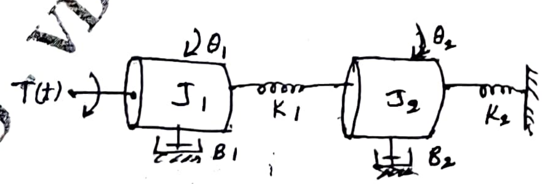
USN | | | | | | | | | | |

BEC403

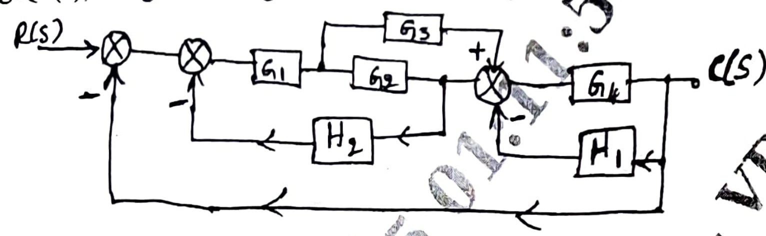
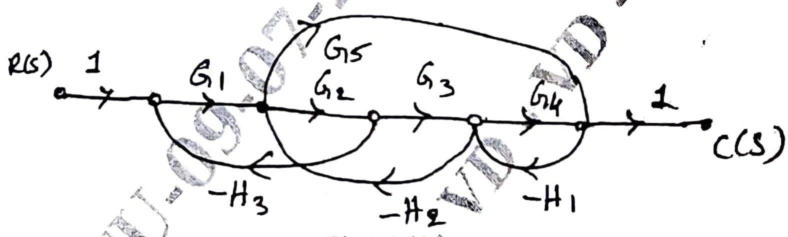
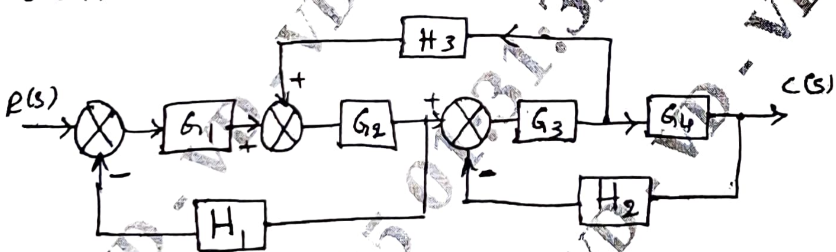
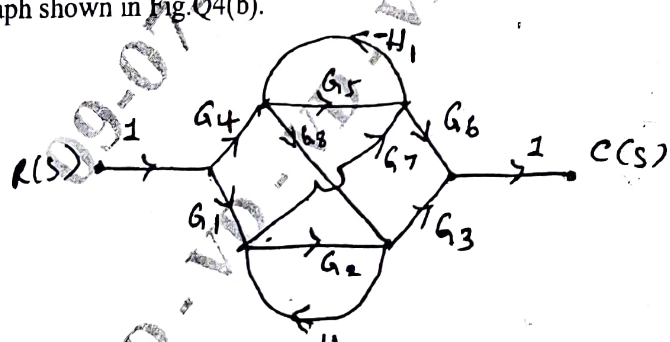
Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025
Control Systems

Time: 3 hrs. Max. Marks: 100

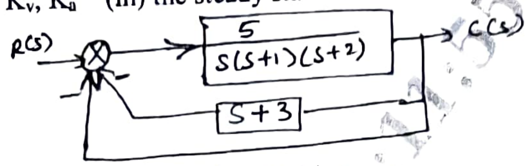
Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define control system with examples. Compare closed loop and open loop control systems.	06	L1 L2 L3	CO1
	b.	For the mechanical system shown in Fig.Q1(b), write the mechanical network, equilibrium equations and obtain the electrical network based on F-V analogy.  Fig.Q1(b)	08	L1 L2 L3	CO1
	c.	The force-voltage analogy of a mechanical system is shown in Fig.Q1(c). Obtain its analogous mechanical network.  Fig.Q1(c)	06	L1 L2 L3	CO1
		OR			
Q.2	a.	Explain the effect of feedback on control systems.	06	L1 L2 L3	CO1
	b.	Find the force-voltage analogous electrical network for the given mechanical system shown in Fig.Q2(b).  Fig.Q2(b)	06	L1 L2 L3	CO1
	c.	Derive the differential equation governing the mechanical rotational system shown in Fig.Q2(c). Draw the equivalent voltage and current analogy circuits.  Fig.Q2(c)	08	L1 L2 L3	CO1

Module – 2

Q.3	a.	Determine the transfer function C(S)/R(S) for the system shown in Fig.Q3(a), using block diagram reduction technique.  Fig.Q3(a)	10	L1 L2 L3	CO2
	b.	Determine the overall transfer function using Mason's gain formula for the signal flow graph shown in Fig.Q3(b).  Fig.Q3(b)	10	L1 L2 L3	CO2
		OR			
Q.4	a.	Find the transfer function by reducing the block diagram shown in Fig.Q4(a).  Fig.Q4(a)	10	L1 L2 L3	CO3
	b.	Find the transfer function by using Mason's gain formula for the signal flow graph shown in Fig.Q4(b).  Fig.Q4(b)	10	L1 L2 L3	CO2

Module – 3

Q.No		Question	Marks	Level	CO
Q.5	a.	For the system shown in Fig.Q5(a), find the (i) System type (ii) Static error constants K_p, K_v, K_a (iii) the steady state error for an input $r(t) = 3 + 2t$.  Fig.Q5(a)	08	L1 L2 L3	CO3
	b.	Find the step response c(t) for the system described by $\frac{C(s)}{R(s)} = \frac{4}{s+4}$ Also find time constant, rise time and settling time.	05	L1 L2 L3	CO3
	c.	Derive the equation steady state error of simple closed loop system.	07	L1 L2 L3	CO3
		OR			
Q.6	a.	Given a unity feedback system with $G(s) = \frac{20(1+s)}{s^2(2+s)(4+s)}$ (i) What is the type of system? (ii) Find static error coefficients. (iii) Find steady error if the input is $r(t) = 40 + 2t + 5t^2$	06	L1 L2 L3	CO3
	b.	Write the general block diagram of the following and explain : (i) PD type of controller (ii) PI type of controller	06	L1 L2 L3	CO3
	c.	Derive the response of an under damped second order system for unit step input.	08	L1 L2 L3	CO3

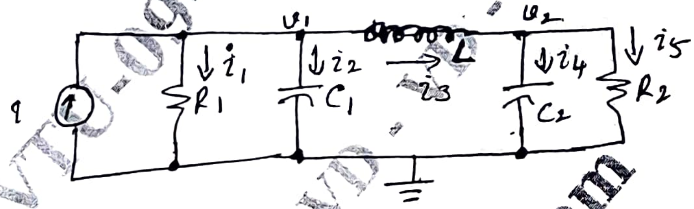
Module – 4

Q.No		Question	Marks	Level	CO
Q.7	a.	Mention limitations of Routh's criterion.	04	L1 L2 L3	CO4
	b.	Determine the range of K for which the system is stable such that a unity feedback system has $G(s) = \frac{K(s+13)}{s(s+3)(s+7)}$ using RH criterion. Also find closed loop, poles more negative than – 1.	08	L1 L2 L3	CO4
	c.	Check the stability of the given characteristic equation using Routh's method. $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$	08	L1 L2 L3	CO4
		OR			
Q.8	a.	Sketch the complete Root locus of system having $G(s)\,H(s) = \frac{K}{s(s+5)(s+10)}$	08	L1 L2 L3	CO4
	b.	Sketch the complete Root locus of system having $G(s)\,H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$	12	L1 L2 L3	CO4

Module – 5

Q.9	a.	Draw the Bode plot for the open loop transfer function of a system is $$G(s) = \frac{K(1+0.2s)(1+0.025s)}{s^3(1+0.001s)(1+0.005s)}$$ Determine that the system is conditionally stable. Find the range of K for which the system is stable.	10	L1 L2 L3	CO5
	b.	The transfer function of a system is $$G(s)\,H(s) = \frac{K}{s(s+2)(s+10)}$$ Sketch the Nyquist plot and hence calculate the range of values of K for stability.	10	L1 L2 L3	CO5

OR

Q.10	a.	Obtain the state model of the network shown in Fig.Q10(a) assuming $R_1 = R_2 = 1\ \Omega$, $C_1 = C_2 = 1F$, and $L = 1H$.  Fig.Q10(a)	10	L1 L2 L3	CO5
	b.	Obtain the state transition matrix for the state model whose A matrix is given by $$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$	10	L1 L2 L3	CO5

* * * * *

Q1 ⓐ Define Control System With examples. Compare Closed loop and Open loop Control systems. [Definition & Comparision] 6M

Ans:- A Control system is a systematically arranged interconnection of components which together act to provide a desired response when excited by an input.

Comparision between openloop & Closed loop Control systems

Open loop	Closed loop
① Construction & Design are simple, hence less expensive	Complicated design, more expensive.
② Generally stability is not a problem	Designer should be carefull as there is less stability.
③ Less accurate & less reliable.	Highly accurate & more reliable.
④ No feedback, o/p has no effect on input	Feedback is present, o/p affects input.
⑤ Sensitive to disturbances & environment changes	Almost insensitive to both.
⑥ Small Bandwidth	Larger Bandwidth

Q1 ⓑ For the mechanical system shown in Fig Q1ⓑ, Write the mechanical network, equilibrium equations and obtain the electrical network based on FV analogy.

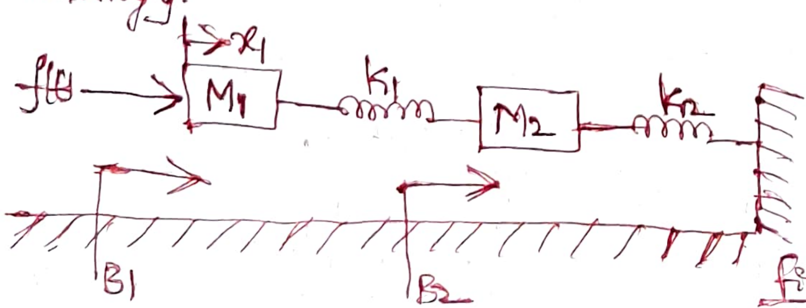
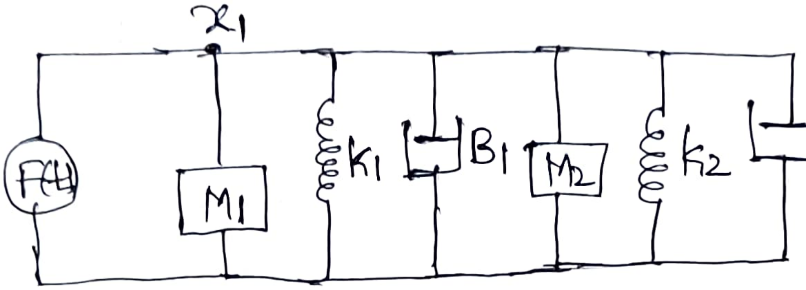


Fig Q1ⓑ

Ans— The mechanical network for the given Q.1(b) is given as,



2M

Equation for the above n/w is,

$$F(t) = M_1 \frac{d^2x_1(t)}{dt^2} + B_1 \frac{dx_1(t)}{dt} + K_1 x_1(t) + M_2 \frac{d^2x_1(t)}{dt^2} + B_2 \frac{dx_1(t)}{dt} + K_2 x_1(t)$$

Now for F-V Analogy $M \rightarrow L$, $B \rightarrow R$ & $K \rightarrow \frac{1}{C}$, $F \rightarrow V$ $x_1 \rightarrow q_1$

$$\therefore V(t) = L_1 \frac{d^2q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{1}{C_1} q_1 + L_2 \frac{d^2q_1}{dt^2} + R_2 \frac{dq_1}{dt} + \frac{1}{C_2} q_1$$

$$= L_1 \frac{d}{dt}\left(\frac{dq_1}{dt}\right) + R_1 \frac{dq_1}{dt} + \frac{1}{C_1} q_1 + L_2 \frac{d}{dt}\left(\frac{dq_1}{dt}\right) + R_2 \frac{dq_1}{dt} + \frac{1}{C_2} q_1$$

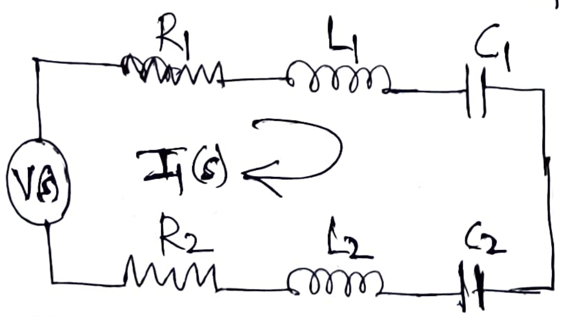
2M

Replace $\frac{dq}{dt} \rightarrow i$, $q \rightarrow \int i$, $\frac{d}{dt} \rightarrow s$ & $\int \rightarrow \frac{1}{s}$

$$\therefore V(s) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 + L_2 \frac{di_1}{dt} + R_2 i_1 + \frac{1}{C_2}\int i_1$$

$$= L_1 s I_1(s) + R_1 I_1(s) + \frac{1}{C_1 s} I_1(s) + L_2 s I_1(s) + R_2 I_1(s) + \frac{1}{C_2 s} I_1(s)$$

2M



2M

Q1 © The Force-Voltage analogy of a mechanical system is shown in Fig Q1©. Obtain its analogous mechanical network.

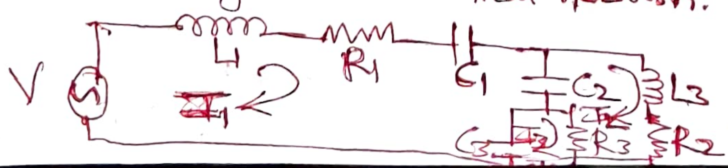
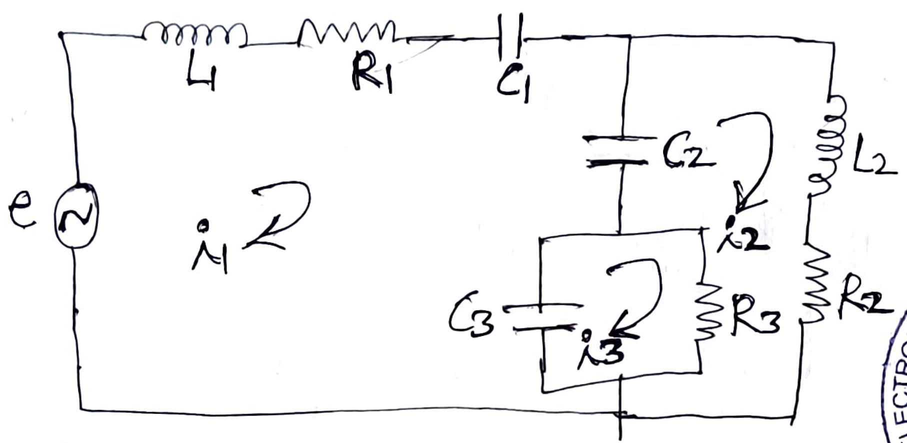


Fig Q1©

Ans – Mechanical System in F-V Analogy is given below,



Apply KVL for loop1;

$$V(t) = L_1 \frac{d}{dt} i_1(t) + R_1 i_1(t) + \frac{1}{C_1}\int i_1(t)\,dt + \frac{1}{C_2}\int (i_1(t) - i_2(t))\,dt + \frac{1}{C_3}\int (i_1(t) - i_3(t))\,dt \quad \text{——} \ (1)$$

lly KVL for loop2

$$0 = R_3(i_3(t) - i_2(t)) + \frac{1}{C_3}\int (i_3(t) - i_2(t))\,dt$$

ie $\frac{1}{C_3}\int (i_2(t) - i_3(t))\,dt = R_3(i_3(t) - i_2(t))$ —— (2)

lly KVL for loop3

2M

$$0 = L_2 \frac{d}{dt} i_2(t) + R_2 i_2(t) + R_3(i_2(t) - i_3(t)) + \frac{1}{C_2}\int (i_2(t) - i_1(t))\,dt$$

ie $R_3(i_3(t) - i_2(t)) = L_2 \frac{d}{dt} i_2(t) + R_2 i_2(t) + \frac{1}{C_2}\int (i_2(t) - i_1(t))\,dt$ —— (3)

For F-V Analogy $\rightarrow$ $F \rightarrow V$, $L \rightarrow M$, $R \rightarrow B$, $\frac{1}{C} \rightarrow K$, $x(t) \rightarrow i(t)$

Equations (1), (2) & (3) Becomes,

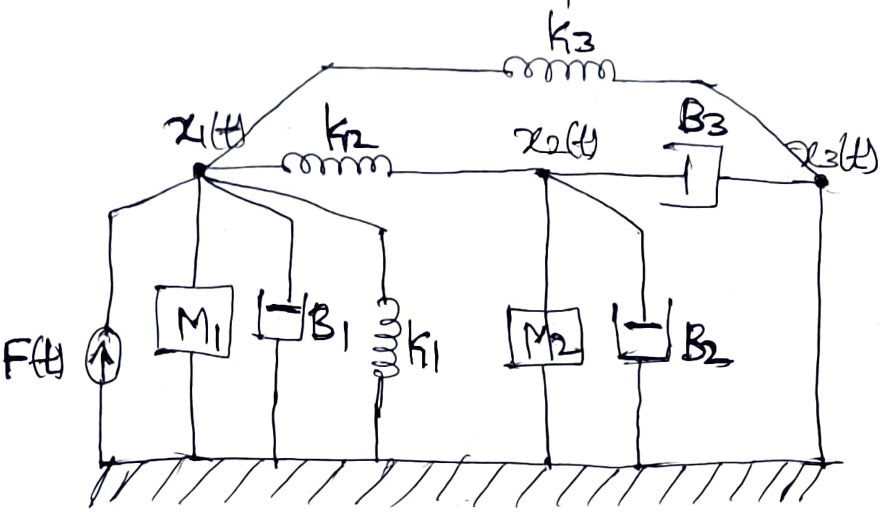
$$F(t) = M_1 \frac{d^2}{dt^2} x_1(t) + B_1 x_1(t) + K_1 x_1(t) + K_2(x_1(t) - x_2(t)) + K_3(x_1(t) - x_3(t)) \quad \text{——} \ (4)$$

$$K_3(x_2(t) - x_3(t)) = B_3 \frac{d}{dt}[x_3(t) - x_2(t)] \quad \text{——} \ (5)$$

2M

$$B_3 \frac{d}{dt}(x_3(t) - x_2(t)) = M_2 \frac{d^2}{dt^2} x_2(t) + B_2 \frac{d}{dt} x_2(t) + K_2(x_2(t) - x_1(t)) \quad (6)$$

From eq's no's ④,⑤ & ⑥ Draw the equivalent mechanical n/w as below



2M

Q2 ⓐ Explain the effect of feedback on Control Systems.

Ans:- Effect of feedback (Explanation — 6M)

① Overall gain

Let the transfer function be $T = \frac{G}{1+GH}$ — ①

2M

if $1+GH$ increases then Gain decreases
& $1+GH$ decreases then Gain Increases

② Sensitivity

$S_G^T = \frac{\%\text{ change in } T}{\%\text{ change in } G}$ where T — Variable, G — parameter

ie $S_G^T = \frac{\delta T/T}{\delta G/G} = \frac{\delta T}{\delta G} \cdot \frac{G}{T}$

$\frac{\delta T}{\delta G} = \frac{\delta}{\delta G}\left[\frac{G}{1+GH}\right] = \frac{1+GH \cdot 1 - GH}{(1+GH)^2}$ 2M

$\frac{\delta T}{\delta G} = \frac{1}{(1+GH)^2}$ $\left(\frac{G}{T} = 1+GH\right)$

$S_G^T = \frac{1}{(1+GH)^2}$ High $\Rightarrow S_G^T = \frac{1}{\text{High}}$ ie $1+GH\uparrow, S\downarrow$
& $1+GH\downarrow, S\uparrow$

③ Stability

$T = \frac{G}{1+GH}$ for -Ve Feedback, $GH = -1, T = \infty$ (Unstable)

if $GH > 0$ (Stable) this is for +ve feedback 2M

Q2(b) Find the force voltage analogous electrical network for the given mechanical system shown in fig Q2(b).

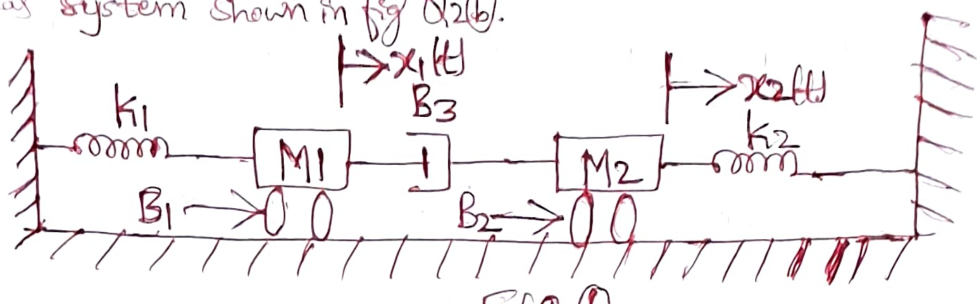
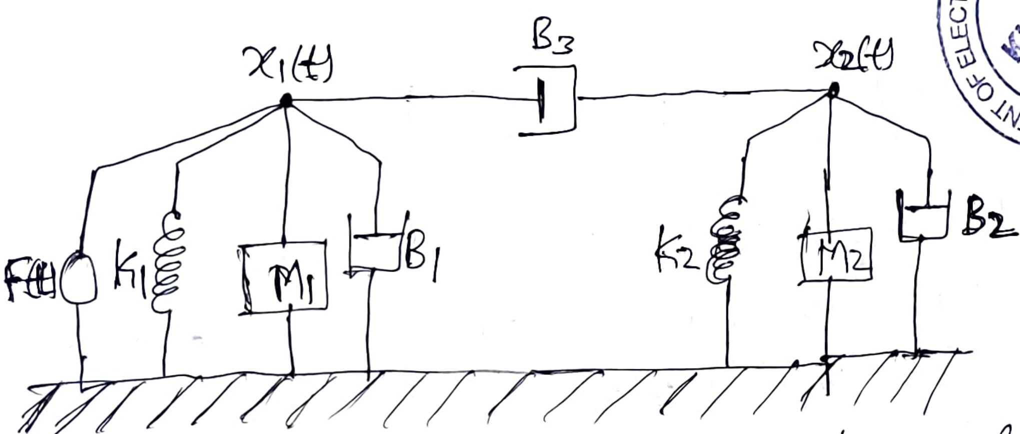


Fig Q2(b)

Ans — Equivalent Mechanical Network



Now writing the equilibrium equations at node I & node 2 we have

for node I $(x_1(t))$

$$F(t) = M_1 \frac{d^2}{dt^2} x_1(t) + B_1 \frac{d}{dt} x_1(t) + B_3 \left[\frac{d}{dt} x_1(t) - \frac{d}{dt} x_2(t)\right] + K_1 x_1(t) \quad \text{—— (1)}$$

Illy at node 2 $(x_2(t))$

$$0 = M_2 \frac{d^2}{dt^2} x_2(t) + B_2 \frac{d}{dt} x_2(t) + B_3 \left[\frac{d}{dt} x_2(t) - \frac{d}{dt} x_1(t)\right] + K_2 x_2(t) \quad \text{—— (2)}$$

FV Analogy

eqn (1) becomes

2M

$$V(t) = L_1 \frac{d^2}{dt^2} q_1 + R_1 \frac{d}{dt} q_1 + R_3 \left[\frac{d}{dt} q_1 - \frac{d}{dt} q_2\right] + \frac{1}{C_1} q_1 \quad \text{—— (3)}$$

Illy eqn (2) becomes,

$$0 = L_2 \frac{d^2}{dt^2} q_2 + R_2 \frac{d}{dt} q_2 + R_3 \left[\frac{d}{dt} q_2 - \frac{d}{dt} q_1\right] + \frac{1}{C_2} q_2 \quad \text{—— (4)}$$

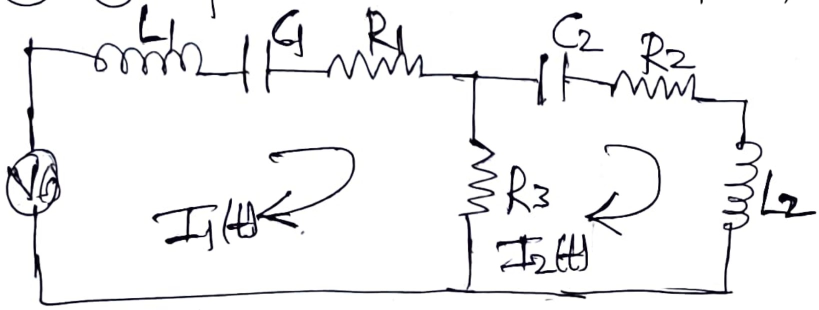
Taking Laplace transform of both eqs ③ & ④ we get

2M

$$V(s) = L_1 s I_1(s) + R_1 I_1(s) + R_3[I_1(s) - I_2(s)] + \frac{1}{C_1 s}[I_1(s)] \quad \text{——— ⑤}$$

$$\& \quad 0 = L_2 s I_2(s) + R_2 I_2(s) + R_3[I_2(s) - I_1(s)] + \frac{1}{C_2 s}[I_2(s)] \quad \text{——— ⑥}$$

from ⑤ & ⑥ Equivalent F-V Electric n/w is,



2M

Q2(c) Derive the differential equation governing the mechanical rotational system shown in fig Q2(c) Draw the equivalent Voltage & current analogy circuits.

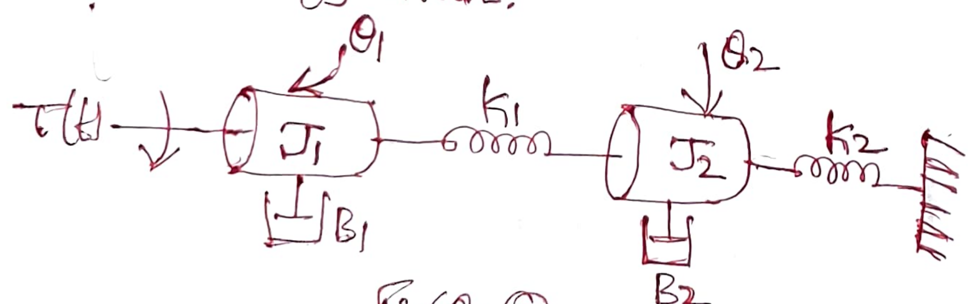


Fig Q2(c)

Ans — To draw equivalent voltage analogy circuit writing the equations

$$J_1 \frac{d^2\theta_1}{dt^2} + K_1(\theta_1 - \theta_2) + B_1 \frac{d}{dt}\theta_1 = T(t) \quad \text{——— ①}$$

Ily for J_2,

2M

$$J_2 \frac{d}{dt^2}\theta_2 + K_2(\theta_2) + K_1(\theta_2 - \theta_1) + B_2 \frac{d}{dt}\theta_2 = 0 \quad \text{——— ②}$$

Now, $\frac{d\theta}{dt} \to \omega$, $\theta = \int \omega\, dt$, $\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$

Eqn ① becomes,

$$J_1 \frac{d\omega_1}{dt} + K_1 \int (\omega_1 - \omega_2)\, dt + B_1 \omega_1 = T \quad \text{——— ③}$$

2M

Eqn ② becomes,

$$J_2 \frac{d\omega_2}{dt} + K_2 \int \omega_2\, dt + B_2 \omega_2 + K_1 \int (\omega_2 - \omega_1)\, dt = 0 \quad \text{——— ④}$$

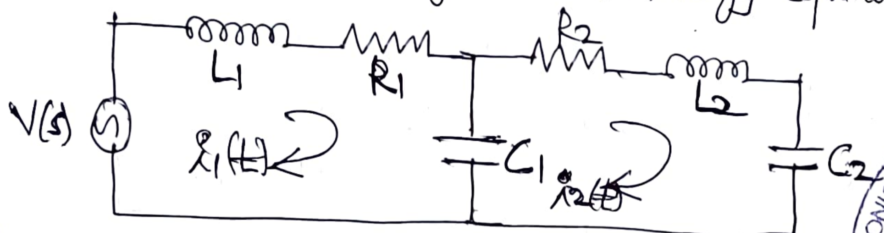
~~for~~ force-Voltage analogy F-V, T → Voltage, J → L, B → R & K = $\frac{1}{C}$
Eqⁿ ③ becomes, ω → i

$$L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int (i_1 - i_2)\,dt + R_1 i_1 = V(t) \quad \text{—— (5)}$$

Eqⁿ ④ becomes,

$$R_2 i_2 + \frac{1}{C_2}\int (i_2)\,dt + L_2 \frac{di_2}{dt} + \frac{1}{C_1}\int (i_2 - i_1)\,dt = 0 \quad \text{—— (6)}$$

From eqⁿs (5) & (6) Drawing the F-V Analogy equivalent circuit,



2M

Now for ~~Voltage~~ Current Analogy circuit,

J → C, B → $\frac{1}{R}$, K = $\frac{1}{L}$, ω → V

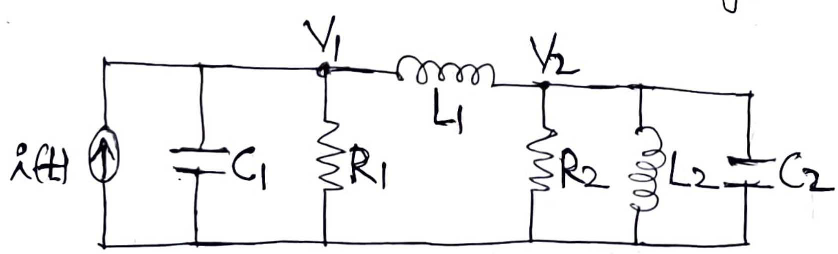
From eqⁿ ③ we have,

$$C_1 \frac{dV_1}{dt} + \frac{1}{L_1}\int (V_1 - V_2)\,dt + \frac{1}{R_1} V_1 = i(t) \quad \text{—— (7)}$$

& Eqⁿ ④ Becomes,

$$\frac{1}{R_2} V_2 + \frac{1}{L_2}\int V_2\,dt + C_2 \frac{dV_2}{dt} + \frac{1}{L_1}\int (V_2 - V_1)\,dt = 0 \quad \text{—— (8)}$$

From eqⁿ (7) & (8) Drawing the Current Analogy circuit,



2M

Module-②

Q3 ⓐ Determine the transfer function C(s)/R(s) for the system shown in fig Q3ⓐ using block diagram reduction techniques.

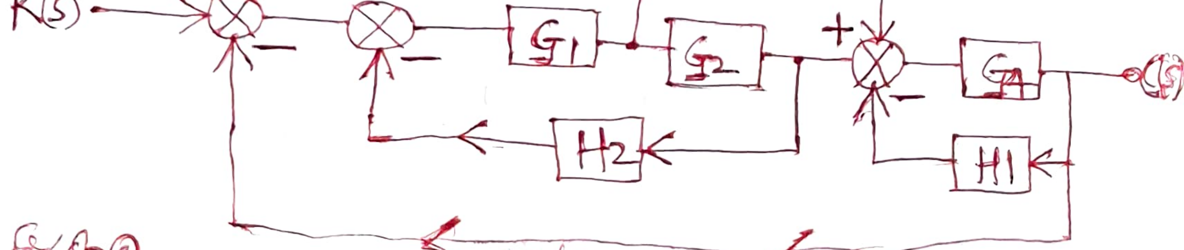
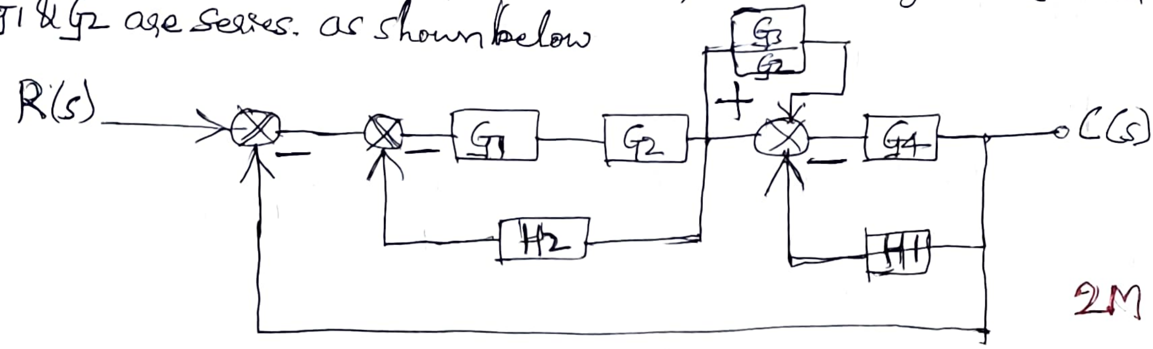
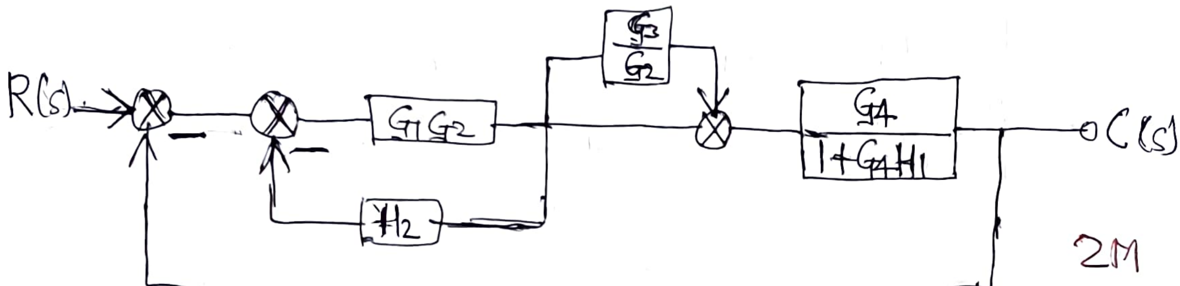


fig Q3ⓐ

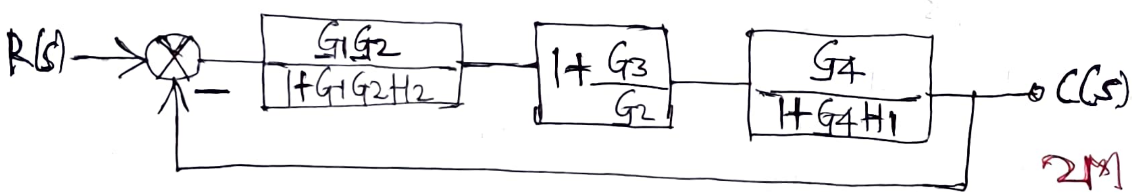
Ans — Check for series & parallel combination, we are not seeing here hence shift the takeoff point (remove) towards right side then G_1 & G_2 are series. as shown below



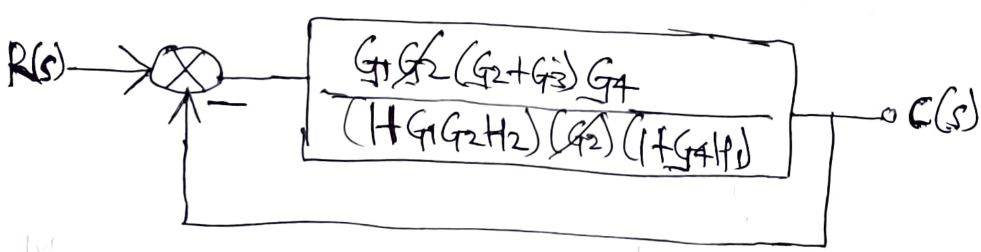
2M



2M



2M



2M

Now $\frac{C(s)}{R(s)} = \dfrac{\dfrac{G_1(G_2+G_3)G_4}{(1+G_1G_2H_2)(1+G_4H_1)}}{1+\dfrac{G_1(G_2+G_3)G_4}{(1+G_1G_2H_2)(1+G_4H_1)}} = \dfrac{\dfrac{G_1(G_2+G_3)G_4}{\cancel{(1+G_1G_2H_2)(1+G_4H_1)}}}{\dfrac{(1+G_1G_2H_2)(1+G_4H_1)+G_1(G_2+G_3)G_4}{\cancel{(1+G_1G_2H_2)(1+G_4H_1)}}}$

$$\therefore \boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_4+G_1G_3G_4}{(1+G_4H_1)+G_1G_2H_2+G_1G_2G_4H_1H_2+G_1G_4G_2+G_1G_4G_3}}$$ 2M

Q3 (b) Determine the overall transfer function using Mason's gain formula for the signal flow graph shown in fig Q3(b).

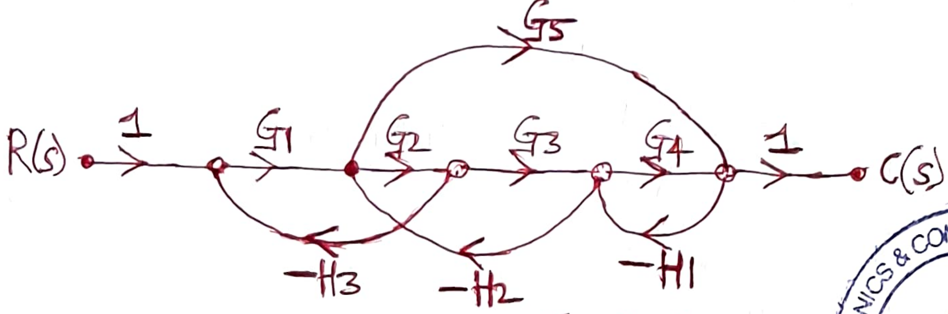
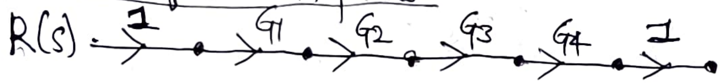


Fig Q3(b)

Ans— We need to find $TF = \frac{C(s)}{R(s)}$

Step ① No of Forward paths

2M



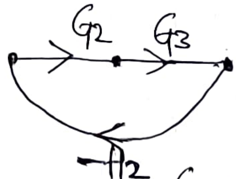
; $T_1 = G_1G_2G_3G_4$

; $T_2 = G_1G_5$

Step ② Individual loops

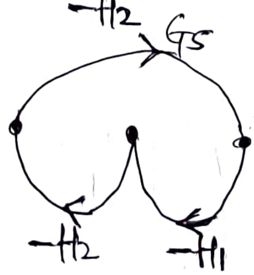
2M

$L_1 = -G_1G_2H_3$



$L_2 = -G_2G_3H_2$

$L_3 = -G_4H_1$



$L_4 = +G_5H_1H_2$

Step ③ To find 'Δ'

2M

$\Delta = 1 -$ [Sum of individual loop gains] + [Σ gain product of all possible combination of two non touching loops] $-$ [Σ gain product of all possible combination of three non touching loops] + - - - - - - - -

$$\therefore \Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1L_3]$$

$$= 1 - [-G_1G_2H_3 - G_2G_3H_2 - G_4H_1 + G_5H_1H_2] + [G_1G_2G_4H_1H_3]$$

$$\therefore \Delta = 1 + G_1G_2H_3 + G_2G_3H_2 + G_4H_1 - G_5H_1H_2 + G_1G_2G_4H_1H_3$$

Step ④ To find Δ_1 & Δ_2 2M

$\Delta_1 = 1$ & $\Delta_2 = 1$

Step ⑤ Mason's gain formula

$$TF = \frac{C(s)}{R(s)} = \frac{\sum T_k \Delta_k}{\Delta} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$$

2M

$$\therefore \boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4\cdot 1 + G_1G_5\cdot 1}{1+G_1G_2H_3+G_2G_3H_2+G_4H_1-G_5H_1H_2+G_1G_2G_4H_1H_3}}$$

Q4 ⓐ Find the transfer function by reducing the block diagram shown in fig Q4ⓐ

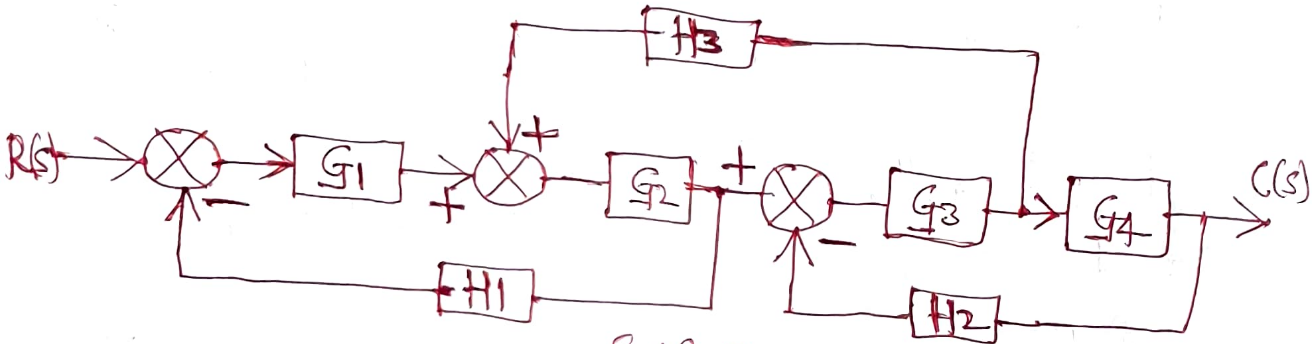
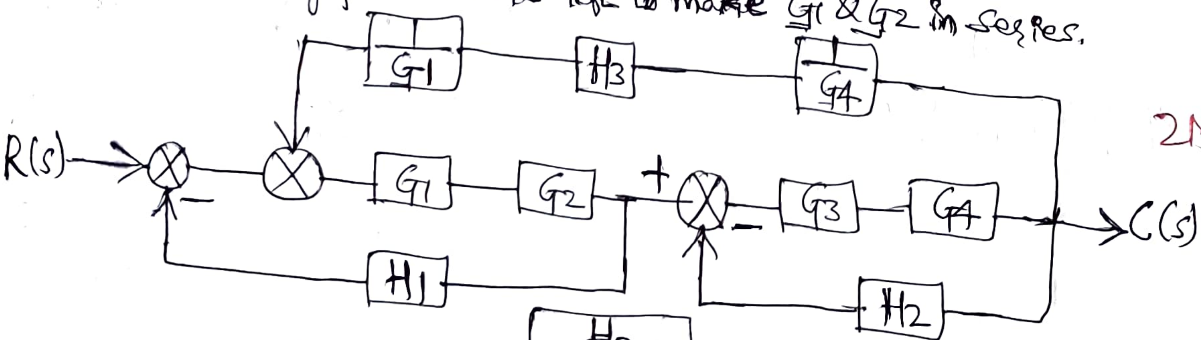
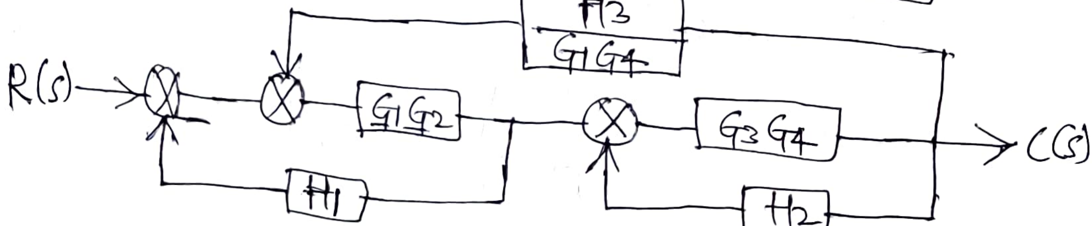


Fig Q4ⓐ

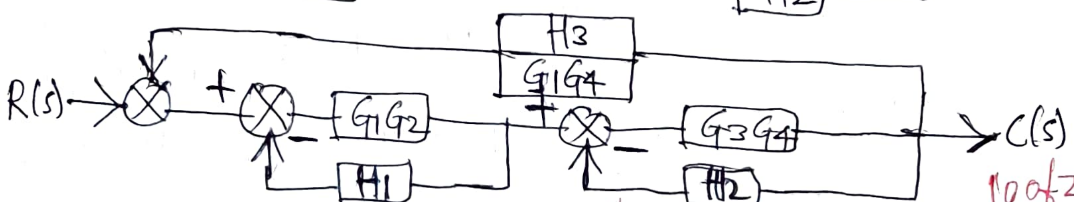
Ans— Always check for blocks are series/parallel if not shift summing point towards left & take off points towards right.
Shift summing point towards left to make G_1 & G_2 in series.



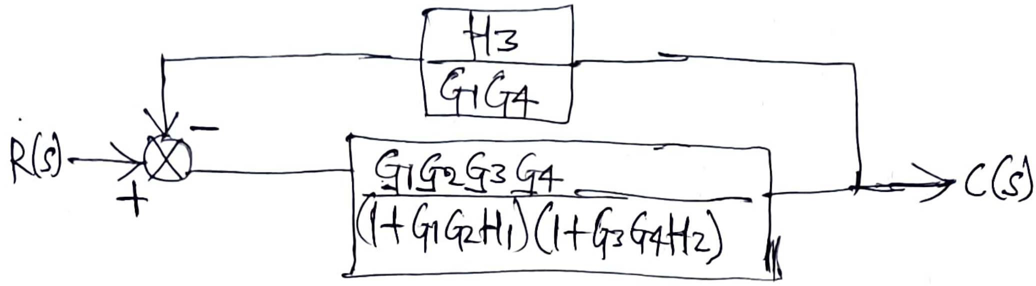
2M



2M



2M



2M

$$\therefore \boxed{\frac{C(s)}{R(s)} = \frac{\frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)}}{1+\frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+G_3G_4H_2)} \times \frac{H_3}{G_1G_4}}}$$ 2M

Q4(b) Find the transfer function by using Mason's gain formula for the signal flow graph shown in fig Q4(b).

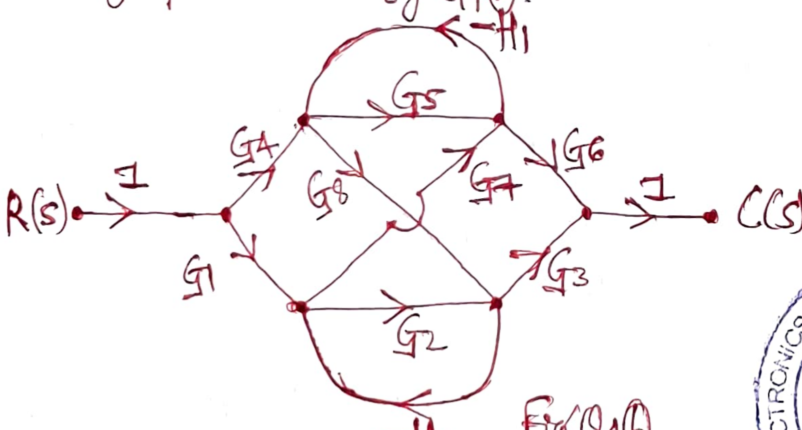


Fig Q4(b)

Ans — According to Mason's gain formula,

$$TF = \frac{\sum T_k \Delta_k}{\Delta}$$

Step① No of forward paths

$T_1 = G_1G_2G_3$ $T_4 = G_4G_8G_3$
$T_2 = G_4G_5G_6$ $T_5 = G_4G_8(-H_2)G_7G_6$
$T_3 = G_1G_7G_6$ $T_6 = G_1G_7(-H_1)G_8G_3$

$\} \; K = 6$ 2M

Step② Individual Loops

$L_1 = -G_5H_1$, $L_2 = -G_2H_2$, $L_3 = G_7G_8H_1H_2$ 2M

Step ③ Non-touching loop pairs

two non touching loop pairs

$L_1L_2 = G_2G_5H_1H_2$

Step ④ To find Δ

$$\Delta = 1 - [-G_5H_1 - G_2H_2 + G_7G_5H_1H_2] + [G_2G_5H_1H_2]$$

$$\therefore \boxed{\Delta = 1 + G_5H_1 + G_2H_2 - G_7G_5H_1H_2 + G_2G_5H_1H_2}$$ 2M

To find Δ_1 $T_1 \rightarrow L_1$ is non-touching

$\Delta_1 = 1 - [-G_5H_1] \Rightarrow \boxed{\Delta_1 = 1 + G_5H_1}$ 2M

To find Δ_2 $T_2 \rightarrow L_2$ is non-touching

$\Delta_2 = 1 - [-G_2H_2] \Rightarrow \boxed{\Delta_2 = 1 + G_2H_2}$

Since T_3, T_4, T_5 & T_6 are loops are all touching each other

$$\therefore \boxed{\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1}$$

2M

$$\therefore \boxed{TF = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3[1+G_5H_1] + G_4G_5G_6[1+G_2H_2] + G_1G_7G_8 + G_4G_8G_3 + G_4G_2(-H_2)G_7G_8 + G_1G_7(-H_1)G_8G_3}{1 + G_5H_1 + G_2H_2 - G_7G_5H_1H_2 + G_2G_5H_1H_2}}$$

Module-③

Q5 ⓐ For the system shown in Fig Q5ⓐ, find the (i) System type, (ii) Static error constants K_p, K_v, K_a (iii) the study state error for an input $r(t) = 3 + 2t$.

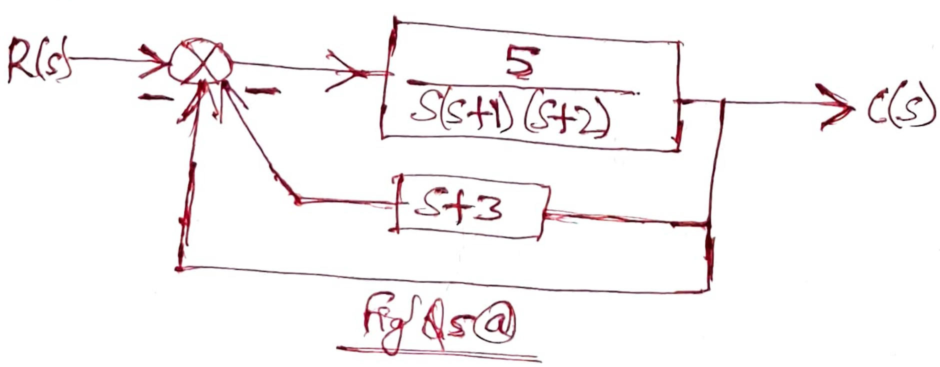
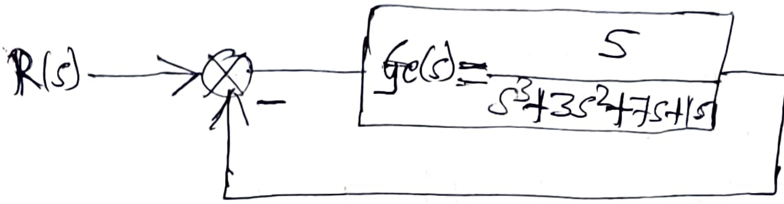


Fig Q5ⓐ

Ans — Reducing the inner loop using the formula $G_e(s) = \frac{G(s)}{1+G(s)H(s)}$

with $G(s) = \frac{5}{s(s+1)(s+2)}$ & $H(s) = (s+3)$, we get,



2M

① Since no poles of $G_e(s)$ lie at the origin of the s-plane, the system under investigation is type 0. 1M

② For type 0 system, the static error constants are,

$$K_p = \lim_{s\to 0} G_e(s) = \frac{5}{15} = \frac{1}{3}$$ 3M

$$K_v = \lim_{s\to 0} s\,G_e(s) = 0$$

$$\& \; K_a = \lim_{s\to 0} s^2 G_e(s) = 0$$

③ Now steady state error for ramp input $r(t) = 3+2t$ is, 2M

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

Q5 (b) Find the step response c(t) for the system described by $\frac{C(s)}{R(s)} = \frac{4}{s+4}$, Also find time constant, rise time & settling time.

Ans — WKT $G(s) = \frac{C(s)}{R(s)} \Rightarrow C(s) = G(s) \times R(s) = \frac{4}{s+4} \times \frac{1}{s}$ 1M

$= \frac{1}{s} - \frac{1}{s+4}$ Taking Laplace transform, we get,

$$c(t) = 1 - e^{-4t} ; t \geq 0$$ 1M

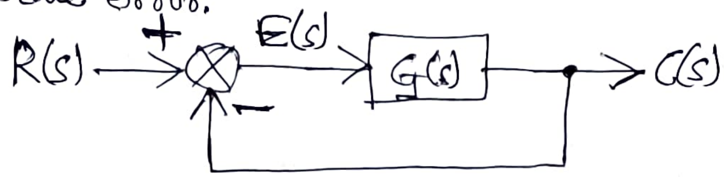
Time constant $T = \frac{1}{a} = \frac{1}{4} = 0.25$ secs 1M

Rise Time $= t_r = \frac{2.2}{a} = \frac{2.2}{4} = 0.55$ Secs 1M

Settling time $= t_s = \frac{4}{a} = \frac{4}{4} = 1$ Secs 1M

Q5(c) Derive the steady state error equation for simple closed loop system. [Derivation — 7M]

Ans — Many steady state errors in control systems arise from non linear sources. Consider a unit -ve feedback system for finding steady state error.



The steady state error of a unity feedback system is defined as,

$$e_{ss} = \lim_{t \to \infty} [r(t) - c(t)]$$

From above fig, $E(s) = R(s) - C(s)$

The o/p of the system is, $C(s) = E(s) \cdot G(s)$ ——— ①

The closed loop transfer function is, $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$\Rightarrow C(s) = \frac{G(s) \cdot R(s)}{1+G(s)} \quad \text{——— ②}$$

Equating ① & ② we get,

$$E(s) \cdot G(s) = \frac{G(s) \cdot R(s)}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$$

The steady state error is given by,

$$e_{ss} = \lim_{t \to \infty} e(t)$$

Using final value theorem we can write,

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s E(s)$$

$$\therefore \boxed{e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1+G(s)}}$$

Q6 (a) Given a unity feedback system with

$$G(s) = \frac{20(1+s)}{s^2(2+s)(4+s)}$$

(i) What is the type of system?
(ii) Find Static error Coefficients
(iii) Find Steady state error if input is $r(t) = 40 + 2t + 5t^2$

Ans— Given $H(s) = 1$ (Unity Feedback)

$$\therefore G(s)H(s) = \frac{20(1+s)}{s^2(2+s)(4+s)} = \frac{20(1+s)}{s^2\left[2\left(1+\frac{s}{2}\right)4\left(1+\frac{s}{4}\right)\right]}$$

$$= \frac{\overset{2.5}{\cancel{20}}(1+s)}{s^2[8(1+0.5s)(1+0.25s)]} = \frac{2.5(1+s)}{s^2[(1+0.5s)(1+0.25s)]} \quad 1M$$

Which is Analogous to $\frac{K(1+T_1 s)}{s^2((1+T_2 s)(1+T_3 s))}$

$\therefore$ It is type 2 system

Now $K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{2.5(1+s)}{s^2((1+0.5s)(1+0.25s))} = \infty$ 1M

$K_v = \lim_{s\to 0} s[G(s)H(s)] = \lim_{s\to 0} \cancel{s}\frac{2.5(1+s)}{s^{\cancel{2}}(1+0.5s)(1+0.25s)} = \infty$ 1M

& $K_a = \lim_{s\to 0} s^2[G(s).H(s)] = \lim_{s\to 0} \cancel{s^2}\frac{2.5(1+s)}{\cancel{s^2}(1+0.5s)(1+0.25s)}$

$= 2.5$ 1M

Input $r(t) = 40 + 2t + 5t^2$

$r(t) = A_1 + A_2 t + \frac{A_3}{2}t^2$ 1M

Step $\to A$, ramp $\to At$, Parabola $\to \frac{A}{2}t^2$

40 steps, 2 ramp & 10 parabolic

$\therefore e_{ss} = e_{ss1} + e_{ss2} + e_{ss3}$

$$= \frac{A_1}{1+K_p} + \frac{A_2}{K_v} + \frac{A_3}{K_a} = \frac{40}{1+\infty} + \frac{2}{\infty} + \frac{10}{2.5}$$

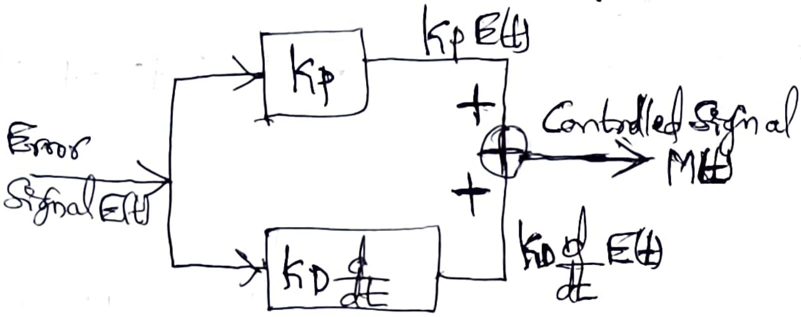
$= 0 + 0 + 4$

$\therefore \boxed{e_{ss} = 4}$ 1M

Q6 (b) Write the block diagram of following & explain.
(i) PD-type of Controller (ii) PI-type of Controller.

Ans— (i) PD type of Controller

It is the combination of proportional & derivative Controller.



3M

Output of PD Controller is given by,

$$m(t) = K_P E(t) + K_D \frac{d}{dt} E(t)$$

Applying Laplace Transform,

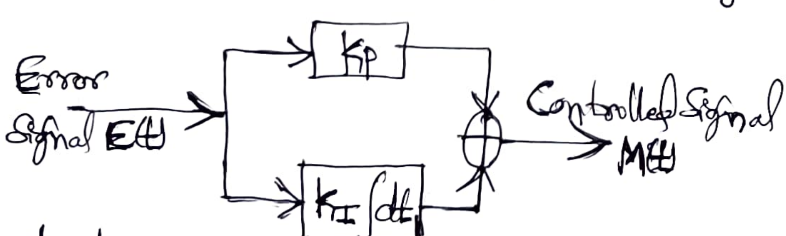
$$M(s) = K_P E(s) + K_D\, sE(s)$$

$$m(s) = E(s)\left[K_P + sK_D\right]$$

Now Transfer Function is $\boxed{\frac{m(s)}{E(s)} = K_P + sK_D}$

(ii) PI type of Controller

It is the combination of proportional & Integral Controller



3M

O/p of PI Controller is given by,

$$m(t) = K_P E(t) + K_I \int E(t)\, dt$$

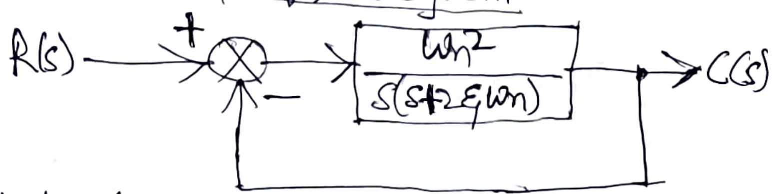
Applying Laplace Transform,

$$m(s) = K_P E(s) + K_I \frac{E(s)}{s}$$

Now transfer Function is $\boxed{\frac{m(s)}{E(s)} = K_P + \frac{K_I}{s}}$

Q6(c) Derive the response of an under damped second order system for Unit step input. [Derivation — 8M]

Ans— Second order reference system



Under damped case — if $0<\xi<1$, Closed loop poles are complex conjugates & lie in the left half of s-plane. The transfer function is,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}\times 1}$$

$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \times R(s) \quad \text{—— (1)}$$

Since input $r(t) = u(t)$, we have, $R(s) = \frac{1}{s}$

Equation (1) becomes,
$$C(s) = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$
$$= \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}$$
$$= \frac{1}{s} - \frac{(s+\xi\omega_n)+\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2+\xi^2\omega_n^2-\xi^2\omega_n^2}$$
$$= \frac{1}{s} - \frac{(s+\xi\omega_n)+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)}$$
$$= \frac{1}{s} - \frac{(s+\xi\omega_n)+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$ is frequency of damped oscillations in rad/sec

$$C(s) = \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$
$$= \frac{1}{s} - \frac{(s+\xi\omega_n)}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi}{\sqrt{1-\xi^2}}\left[\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}\right]$$

WKT, $L^{-1}\left\{\frac{s+b}{(s+b)^2+a^2}\right\} = e^{-bt}\cos at \,; t\geq 0$, $L^{-1}\left\{\frac{a}{(s+b)^2+a^2}\right\} = e^{-bt}\sin at \,; t\geq 0$

& $L^{-1}\left\{\frac{1}{s}\right\} = 1 \,; t\geq 0$

Hence, $\boxed{c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin\omega_d t}$

Module-4

Q7 (a) Mention limitations of Routh's criterion.

[Limitations — 4M]

Ans— ① The Routh's criterion does not give the location of the roots that lies on right half of the S-plane.

② The Routh's criterion does not tell whether roots which lies on the right of the S-plane are real or complex.

③ The Routh's criterion cannot be applied to any other stability boundaries in a complex plane. Such as the unit circle in the Z plane.

Q7 (b) Determine the range of K for which the system is stable such that a unity feedback system has $G(s) = \frac{K(s+13)}{s(s+3)(s+7)}$ using RH criterion, Also find closed loop poles more negative than −1. [8M]

Ans— Given, $G(s) = \frac{K(s+3)}{s(s+3)(s+7)}$ & $H(s) = 1$ (Unity feedback system)

The characteristic equation is $1+G(s)H(s) = 0$

ie $1 + \frac{K(s+3)}{s(s+3)(s+7)} \cdot 1 = 0$

$$1 + \frac{Ks+3K}{s[s^2+10s+21]} = 0$$

$$\frac{s^3+10s^2+21s+Ks+3K}{s[s^2+10s+21]} = 0$$

$\therefore s^3+10s^2+21s+Ks+3K = 0$

$a_0 = 1,\ a_1 = 10,\ a_2 = 21+\cancel{k}\ ,\ a_3 = 13k$

s^3	1	$(21+k)$
s^2	10	$13k$
s^1	$\frac{210+10k-13k}{10}$	0
s^0	$13k$	

$\frac{210-3k}{10}$

For the system to be stable $13k > 0$ & $\frac{210-3k}{10} > 0$

ie $210 - 3k > 0$

$210 > 3k \Rightarrow k < \frac{210}{3}$

ie $k < 70$

∴ The range of k is

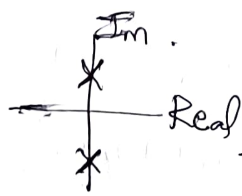
$\boxed{13k < k < 70}$

Q7 ⓒ Check the stability of the given characteristic equation using Routh's method.

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0 \qquad (8M)$$

Ans Let us construct the Routh's table as below,

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	$16 \Rightarrow A$	
$\boxed{s^3}$	0(4)	0(12)		
s^2	6	16		
s^1	1.33			
s^0	16			



$\frac{2\times 8 - 1\times 12}{2} = \frac{16-12}{2} = 2$

$\frac{2\times 20 - 1\times 16}{2} = \frac{40-16}{2} = 12$

Marginally Stable $\frac{2\times 16 - 1\times 0}{2} = \frac{32}{2} = 16$

Auxiliary equation $A = 2s^4 + 12s^2 + 16 = 0$

$= 2(s^4 + 6s^2 + 8) = 0$

$\therefore A = s^4 + 6s^2 + 8$

Now $\frac{dA}{ds} = 4s^3 + 12s$

There are totally 6 roots & Auxiliary eqn has highest power of 4 hence 4 roots are on imaginary axis

Hence $6-4 = 2$ Now we need to find 2 roots are on RHS or LHS of s-plane by observing under Auxiliary eqn no sign changes ∴ RHS = 0 roots LHS = 2 root & Img axis = 4 roots ∴ System is marginally stable.

Q8@ Sketch the complete Root-locus of system having

$$G(s)\,H(s) = \frac{K}{s(s+5)(s+10)}$$ [8M]

Ans — Step ① No of poles by given TF denominator, $= 3$ poles $s = 0, -5, -10$

& No of Zero's $= 0$

Total no of root loci $= \text{Max}(\phi, z) = \text{Max}(3, 0) = 3$

Step ② Total no of asymptotes are $=$ No of poles $-$ No of Zeros $= 3 - 0 = 3$

Step ③ Angle of Asymptotes

$$\theta = \frac{(2q+1)\,180^\circ}{\phi - z}$$

$q = 0, 1 \dots \phi - z - 1$

$= 0, 1, \dots 2 = 0, 1, 2$

Now $\theta_1 = \frac{(2(0)+1)\,180^\circ}{3-0} = \frac{180^\circ}{3} = 60^\circ$

$\theta_2 = \frac{(2(1)+1)\,180^\circ}{3} = \frac{540^\circ}{3} = 180^\circ$

& $\theta_3 = \frac{(2(2)+1)\,180^\circ}{3} = \frac{900^\circ}{3} = 300^\circ$

Step ④ Centroid (σ)

$$\sigma = \frac{\sum \text{real part of poles} - \sum \text{real part of Zero's}}{\phi - z}$$

ie $\sigma = \frac{-15}{3} = -5$

Step ⑤ Breakaway point

$1 + G(s)H(s) = 0$

$1 + \frac{K}{s(s+5)(s+10)} = 0$

$1 + \frac{K}{s(s^2+15s+50)} = 0$

ie $s^3 + 15s^2 + 50s + K = 0$

$K = -s^3 - 15s^2 - 50s$

$\frac{dK}{ds} = -3s^2 - 30s - 50 = 0$

$\therefore 3s^2 + 30s + 50 = 0$

$\boxed{s = -2.11, -7.88}$

Q8 (b) Sketch the complete Root locus of system having,

$$G(s)H(s) = \frac{k}{s(s+1)(s+2)(s+3)}$$ [12M]

Ans — Step ①

No of poles (ϕ) = 4 ie $s = 0, -1, -2, -3$

& No of Zero's (Z) = 0 $\therefore$ Total no of root loci = Max(ϕ, Z) = Max(4, 0) = 4

Step ② Total no of Asymptotes = No of poles − No of Zero's = $\phi - Z = 4 - 0 = 4$

Step ③ Angle of Asymptotes

$$\theta = \frac{(2q+1)180^\circ}{\phi - Z}$$

$q = 0, 1 \ldots \phi - Z - 1$
$= 0, 1, 2, 3$

Now $\theta_1 = \frac{(2(0)+1)180^\circ}{4} = \frac{180^\circ}{4} = 45^\circ$

$\theta_2 = \frac{(2(1)+1)180^\circ}{4} = \frac{540}{4} = 135^\circ$

$\theta_3 = \frac{(2(2)+1)180^\circ}{4} = \frac{900}{4} = 225^\circ$

& $\theta_4 = \frac{(2(3)+1)180^\circ}{4} = \frac{1260}{4} = 315^\circ$

Step ④ Centroid (σ) = $\frac{\sum \text{Real part of poles} - \sum \text{Real part of Zero's}}{\phi - Z}$

ie $\sigma = \frac{-6}{4} = \frac{-3}{2} = -1.5$

Step ⑤ Breakaway point

$1 + G(s)H(s) = 0$

$1 + \frac{k}{s(s+1)(s+2)(s+3)} = 0$

$1 + \frac{k}{s[s^2+3s+2](s+3)}$

$1 + \frac{k}{s(s^3+6s^2+11s+6)}$

ie $1 + \frac{k}{s^4+6s^3+11s^2+6s} = 0$

ie $s^4+6s^3+11s^2+6s+k = 0$

Now $k = -s^4 - 6s^3 - 11s^2 - 6s$

$\frac{dk}{ds} = -4s^3 - 18s^2 - 22s - 6 = 0$

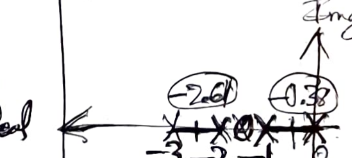
$\therefore 4s^3+18s^2+22s+6 = 0$

Three roots are

$s_1 = -2.61$ ✓

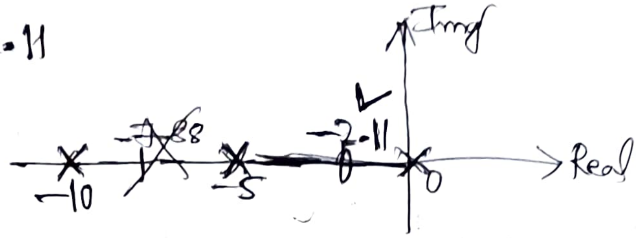
$s_2 = -1.5$ ✗

& $s_3 = -0.38$ ✓



Valid Breakaway points are -2.61 & -0.38

Valid Breakaway point $= -2.11$



Step ⑥ Since poles are real one not required to calculate angle of departure.

hence, $s^3 + 15s^2 + 50s + K = 0$

According to Routh's array,

s^3	1	50	0
s^2	15	K	0
s^1	$\frac{750-K}{15}$	0	
s^0	K		

$K > 0$

$\frac{750 - K_{mar}}{15} = 0$

$\boxed{K_{mar} = 750}$

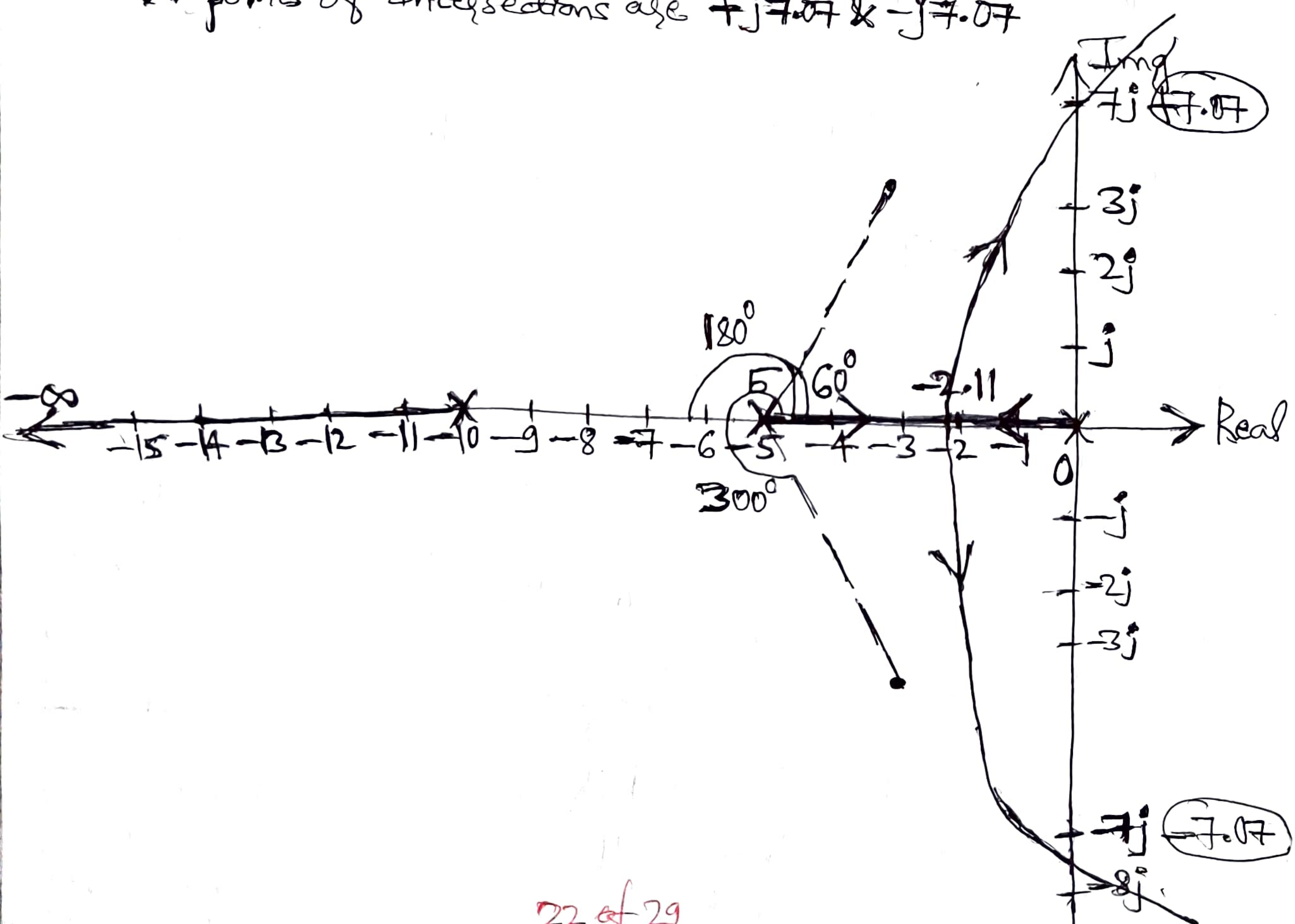
$A(s) = 15s^2 + K_{mar} = 0$

$15s^2 + 750 = 0$

$15s^2 = -750$

$s^2 = \frac{-750}{15} = -50 \therefore s = \pm\sqrt{-50} = \pm j7.07$

$\therefore$ points of Intersections are $+j7.07$ & $-j7.07$



Step 6 $s^4+6s^3+11s^2+6s+K=0$

Routh's Array.

s^4	1	11	K
s^3	6	6	0
s^2	10	K	
s^1	$\frac{60-6K}{10}$		
s^0	K		

$\frac{66-6}{6}=\frac{60}{6}=10$

$\frac{6K-0}{6}$; $\frac{60-6K}{10}$

$K>0$

$\frac{60-6K_{mar}}{10}=0$

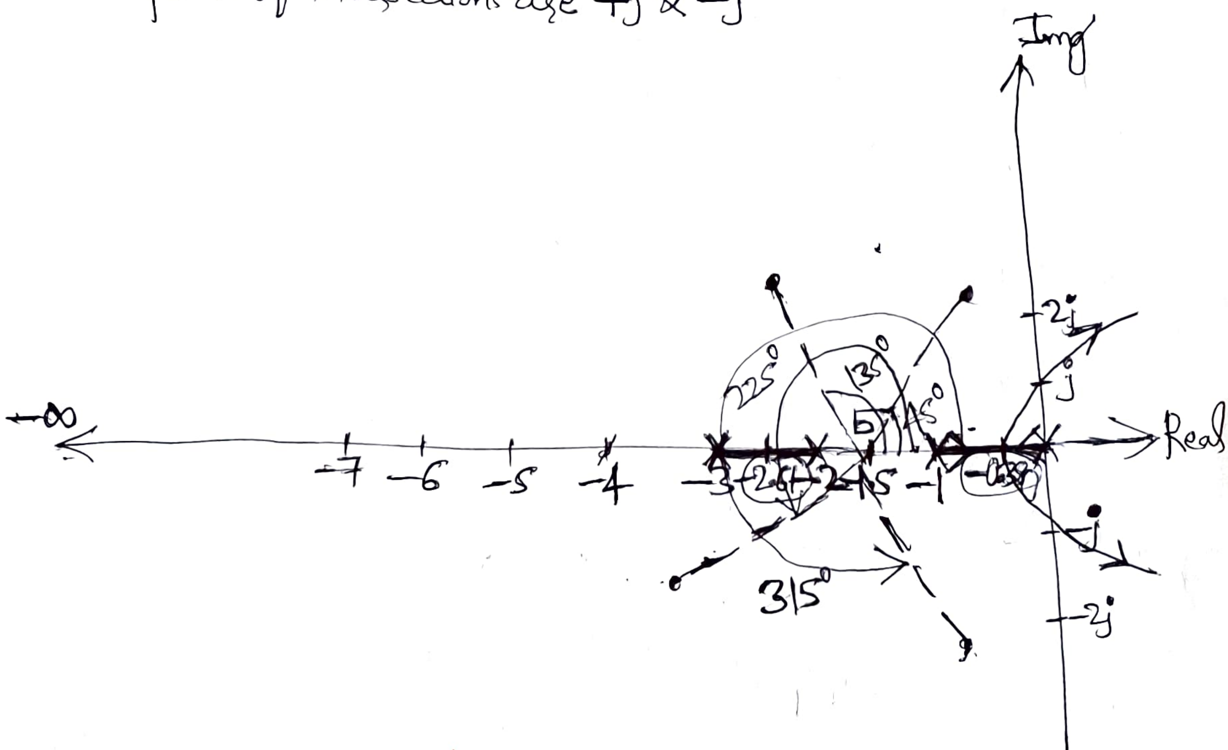
$6K_{mar}=60$

$K_{mar}=\frac{60}{6}=10$

$A(s)=10s^2+K_{mar}=0$

$10s^2+10=0 \Rightarrow s^2=-1 \therefore s=\pm\sqrt{-1}=\pm j1$

$\therefore$ points of intersections are $+j$ & $-j$



Module-5

Q9 a) Draw the Bode plot for the open loop transfer function of a system $G(s)=\frac{K(1+0.2s)(1+0.025s)}{s^3(1+0.001s)(1+0.005s)}$ [10M]

Determine that the system is Conditionally stable, find the range of K for which the system is stable.

Ans— Let $K=1$ & put $s=j\omega$ we get,

Then, $G(s)H(s) = \frac{(1+0.2j\omega)(1+0.025j\omega)}{j^3\omega^3(1+0.001j\omega)(1+0.005j\omega)}$

Let $Y(j\omega) = \frac{1}{j^3\omega^3}$

$\Rightarrow 20\log|Y(j\omega)| = -60\log\omega$ — ①

Put $\omega=0.1$ in above eqn ①, we get, $20\log|Y(j\omega)| = 60\,dB$

→ Table for Constructing of Bode plot.

Factor	Corner frequency	Magnitude & Slope characteristics
$\frac{1}{j^3\omega^3}$	—	Mag $=60\,dB$ at $\omega=0.1$ & Slope $=-60\,dB$/decade upto ω_1
$1+0.2j\omega$	$\omega_1 = \frac{1}{0.2} = 5$	No slope between ω_1 & ω_2 = Slope $= +20-60 = -40\,dB$/decade.
$1+0.025j\omega$	$\omega_2 = \frac{1}{0.025} = 40$	No slope between ω_2 & ω_3 = Slope $= +20-40 = -20\,dB$/decade
$\frac{1}{1+0.005j\omega}$	$\omega_3 = \frac{1}{0.005} = 200$	Slope $= -20-20 = -40\,dB$/decade
$\frac{1}{1+0.001j\omega}$	$\omega_4 = \frac{1}{0.001} = 1000$	Slope $= -20-40 = -60\,dB$/decade

→ Table for plotting phase plot

$\phi(\omega) = \angle GH(\omega) = -270^\circ - \tan^{-1}0.001\omega - \tan^{-1}0.005\omega + \tan^{-1}0.2\omega + \tan^{-1}0.025\omega$

ω (rad/sec)	$\phi(\omega)$ (deg)	ω (rad/sec)	$\phi(\omega)$ (deg)
0.1	−268.74	200	−159.05
0.5	−263.75	382	−180°
1	−257.60	500	−189.91
2	−246.02	700	−202.72
5	−219.59	1000	−216.21
7	−208.02	2000	−239.01
10	−195.96	5000	−256.91
16.556	−180	10000	−263.40
20	−174.32		
50	−151.27		
70	−147.12		
100	−146.94		

It is found that $\phi(\omega)$ is greater than -180 only between $\omega_{p1} = 16.556$ rad/sec & $\omega_{p2} = 382$ rad/sec

For gain crossover to be at 16.556 rad/sec, the magnitude plot becomes $20\log K = 60.89$

$\therefore K = 10^{60.89/20} = 1108$

For gain crossover to be at 382 rad/sec, the magnitude plot is $20\log K = 103.76$

Q.9@

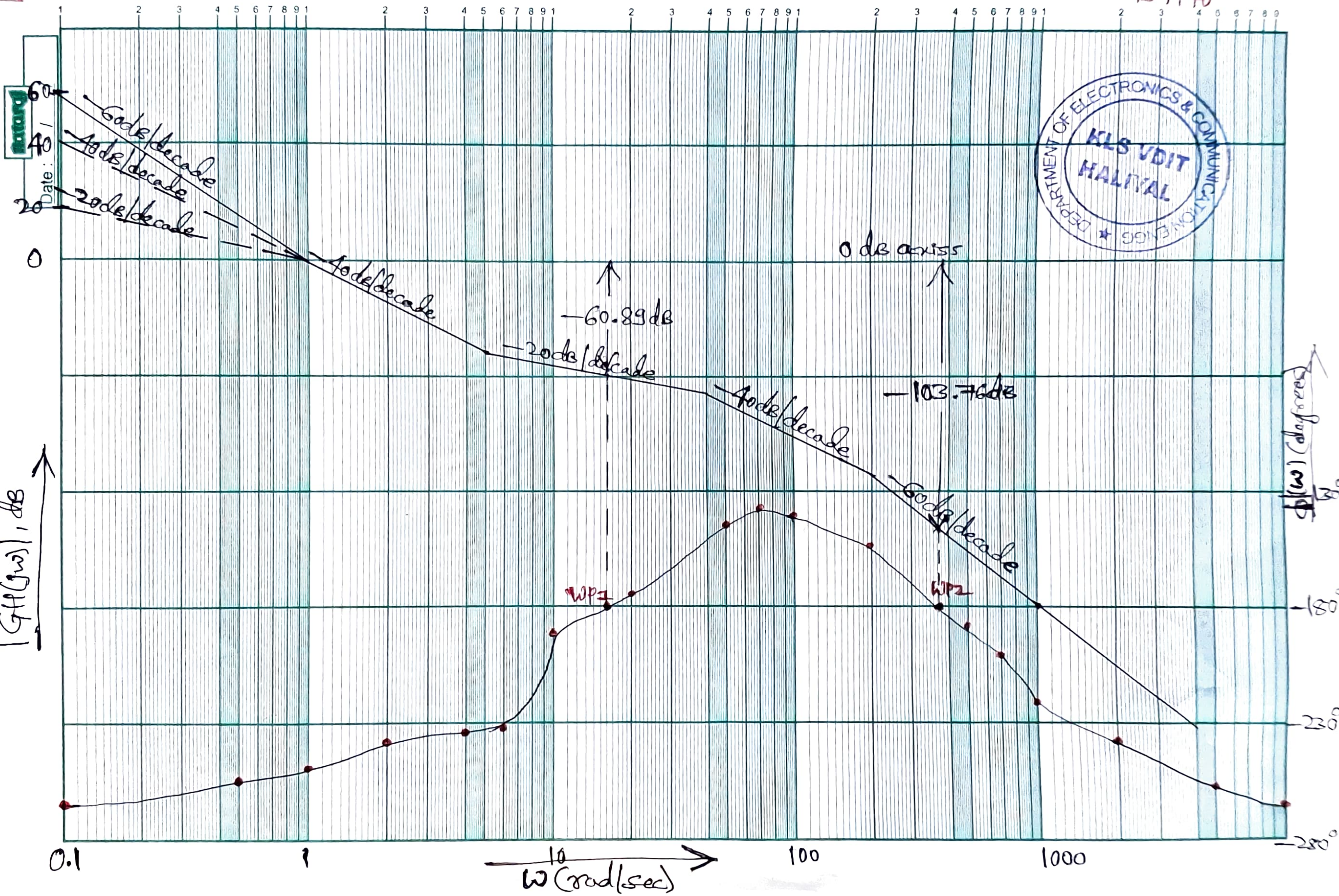
$\omega_{P1} = 16.55\ rad/sec$ & $\omega_{P2} = 382\ rad/sec$

$20\log K = 60.89\ dB$

$\therefore K = 10^{60.89/20} = 1108$

lly $20\log K = 103.76\ dB$

$\therefore K = 10^{103.76/20} = 154170$



$20\log K = 103.76$

$\therefore K = 154170$

Hence range of K is $1108 < K < 154170$

Since range is +ve the closed loop system is conditionally stable.

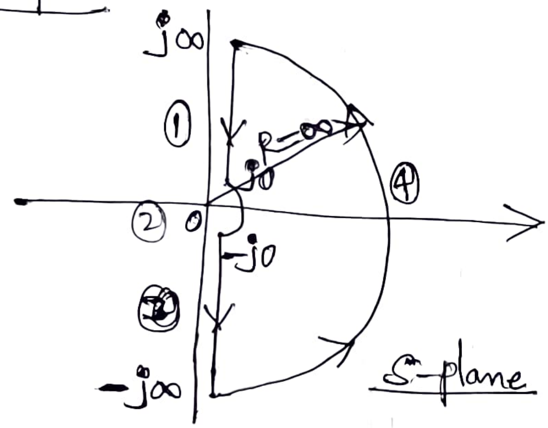
Q9(b) The transfer function of a system is

$$G(s)H(s) = \frac{K}{s(s+2)(s+10)}$$ Sketch the Nyquist plot and hence calculate the range of values of K for stability. [10M]

Ans — Step ① — Poles on right hand side of s plane $P=0$

Step ② — No of encirclements of critical point in F-plane $= N = -P \Rightarrow N=0$ (System is stable)

Step ③ — Nyquist path



Step ④

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(2+j\omega)(10+j\omega)}$$

$$Mag = \frac{K}{\omega\sqrt{4+\omega^2}\sqrt{100+\omega^2}}$$

$$\& \ \phi = \frac{\tan^{-1}(0/K)}{\tan^{-1}\left(\frac{\omega}{0}\right)\tan^{-1}\left(\frac{\omega}{2}\right)\tan^{-1}\left(\frac{\omega}{10}\right)}$$

$$= -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

For section ① $s = \pm j\infty$ to $s = \pm j0$ $\quad s = j\omega$ (Anticlk rotation) $+180^\circ$

$\omega \to \infty$ to 0

Starting point $\omega = \infty$ $0\angle -270^\circ$, end point $\omega \to 0 \Rightarrow \infty\angle -90^\circ$

② 2nd section : $s = +j0$ to $s = -j0 \Rightarrow \omega \to 0$ to -0

Starting point $\omega \to 0$ · $\infty \angle -90°$

end point $\omega \to -0$ $\infty \angle +90°$ } $+180°$

3rd section — Mirror image of section ①

4th section — is at origin.

Step ⑤ $G(j\omega)H(j\omega) = \dfrac{K(-j\omega)(10-j\omega)(2-j\omega)}{(j\omega)(-j\omega)(10+j\omega)(10-j\omega)(2+j\omega)(2-j\omega)}$

$$= \frac{-Kj\omega\,[20 - 12j\omega - \omega^2]}{\omega^2(4+\omega^2)(100+\omega^2)} = \frac{\omega(20-\omega^2)}{\omega^2(4+\omega^2)(100+\omega^2)} = 0$$

$\Rightarrow \omega(20-\omega^2) = 0$

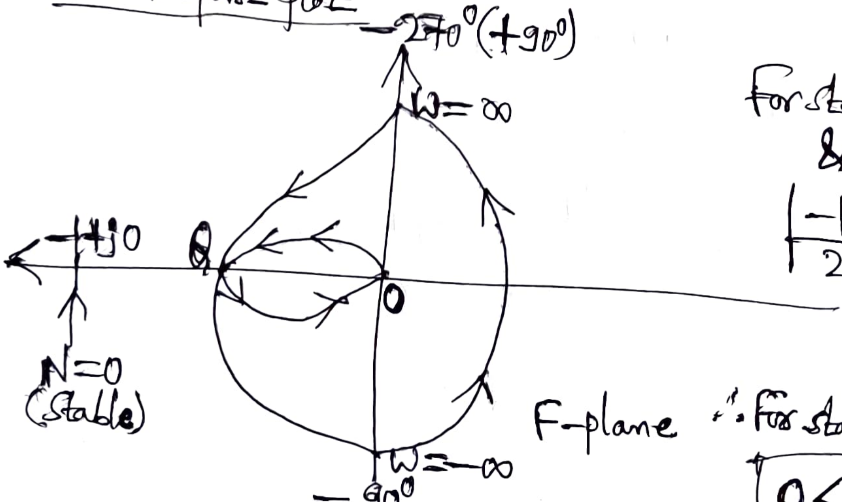
$\omega^2 = 20 \Rightarrow \omega = \sqrt{20} \Rightarrow \omega_{pc}$

Realpart

$$\frac{-12K\omega^2}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$\text{point } Q = \frac{-12K \times 20}{20(4+20)(100+20)} \Rightarrow \frac{-K}{240}$$

Step ⑥ The Nyquist plot



For stability $N = 0$

& $|OQ| < 1$

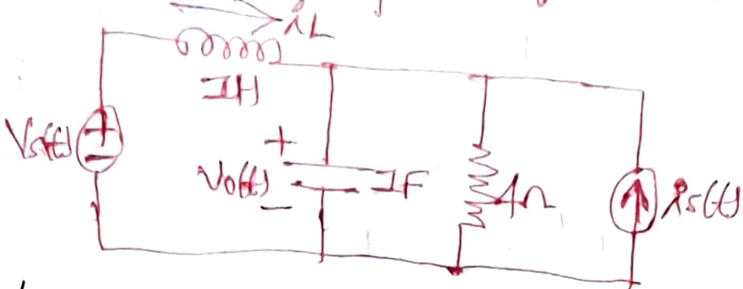
$$\left|\frac{-K}{240}\right| < 1$$

$\therefore K < 240$

$\therefore$ For stability

$$\boxed{0 < K < 240}$$

Q10 (a) Develope the state equations for the n/w shown in below fig Q10(a). [10M]



Ans — Let $i_L = x_1$ & $V_0 = x_2$ be the state variables. Also denote $V_s(t)$ as u_1 & $i_s(t)$ by u_2

Applying KCL, we get

$$-i_L + \frac{V_0}{4} + \frac{dV_0}{dt} - i_s = 0$$

$$\Rightarrow \frac{dV_0}{dt} = -0.25V_0 + i_L + i_s$$

$$\Rightarrow x_2 = x_1 - 0.25x_2 + u_2 \quad \text{—— (1)}$$

$$\frac{di_L}{dt} = V_s(t) - V_0(t)$$

$$\Rightarrow x_1 = -x_2 + u_1 \quad \text{—— (2)}$$

putting eqn (1) in eqn (2) in matrix form we get,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -0.25 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Denoting V_0 by y_1 we get,

$$y_1 = x_2$$

Hence, the o/p equation is,

$$\boxed{y_1 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

Q10(b) Obtain the state transition matrix for the state model whose A matrix is given by $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ [10M]

Ans— The state transition matrix $\phi(t)$ is given by,

$$\phi(t) = e^{At} = L^{-1}[(sI-A)^{-1}]$$

Consider, $sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

Now, $\text{Adj}[sI-A] = \begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix}^T$

$$= \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$\text{Det}[sI-A] = s(s+3)+2 = (s+1)(s+2)$

Hence, $(sI-A)^{-1} = \dfrac{\text{Adj}[sI-A]}{\text{Det}(sI-A)}$

$$= \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

Hence, $\phi(t) = e^{At} = L^{-1}[(sI-A)^{-1}]$

$$\therefore \phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Prepared by – Prof. Sudheendra Yalagur