USN | | | | | | | | | | |

BCV401

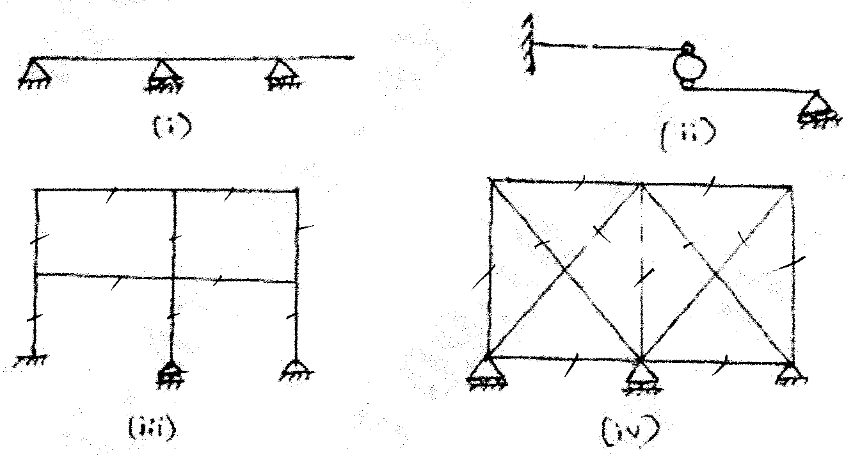
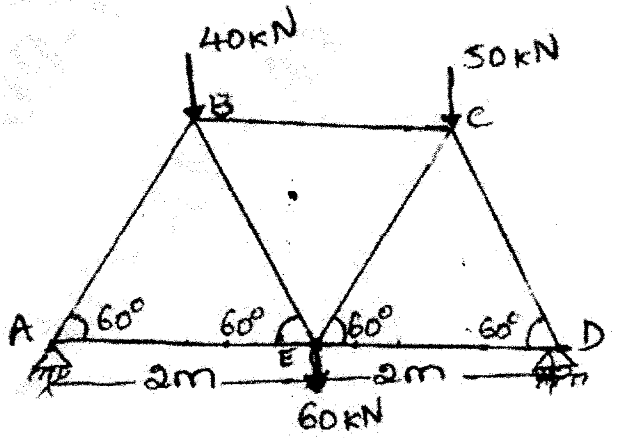
# Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025

## Analysis of Structures



Time: [illegible] hrs.  Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.*
*2. M : Marks , L: Bloom's level , C: Course outcomes.*
*3. Missing data, if any, may be suitably assumed.*
*4. Write legibly.*

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a. | Briefly explain different forms of structures. | 4 | L2 | CO1 |
| | b. | Distinguish between determinate and indeterminate structures. | 6 | L2 | CO1 |
| | c. | Determine degree of static and kinematic indeterminacy for the structures shown in Fig.Q1(c).   Fig.Q1(c) | 10 | L3 | CO1 |
| | | **OR** | | | |
| Q.2 | | Determine the forces in all the members of the truss shown in Fig.Q2 and indicate the magnitude and nature of the forces on the diagram of truss.   Fig.Q2 | 20 | L3 | CO1 |

## Module – 2

**Q.3 a.** Determine maximum slope and deflection for the simply supported beam shown in Fig.Q3(a) by moment area method. | 10 | L3 | CO2

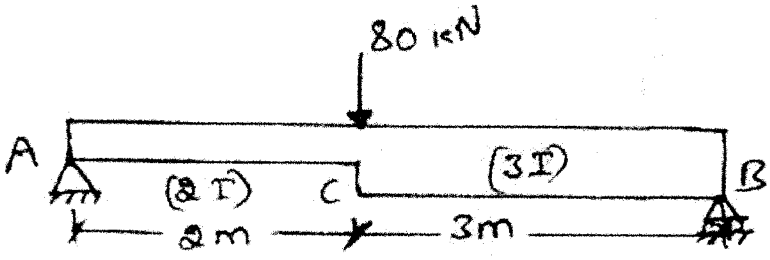



Fig.Q3(a)

**b.** Calculate slope and deflection for the cantilever beam shown in the Fig.Q3(b) by moment area method. | 10 | L3 | CO2

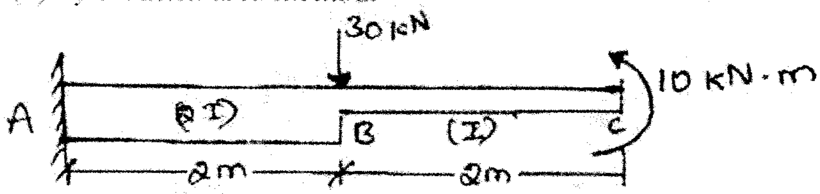



Fig.Q3(b)

**OR**

**Q.4 a.** Determine the vertical deflection at point C for the frame shown in the Fig.Q4(a) by strain energy method. | 10 | L3 | CO2

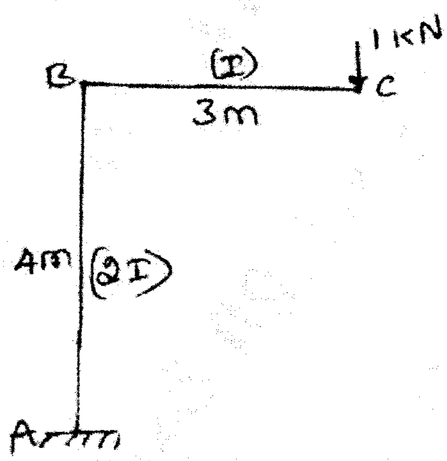



Fig.Q4(a)

**b.** Determine deflection under the load for the simply supported beam shown in Fig.Q4(b) by Castigliano's theorem. | 10 | L3 | CO2

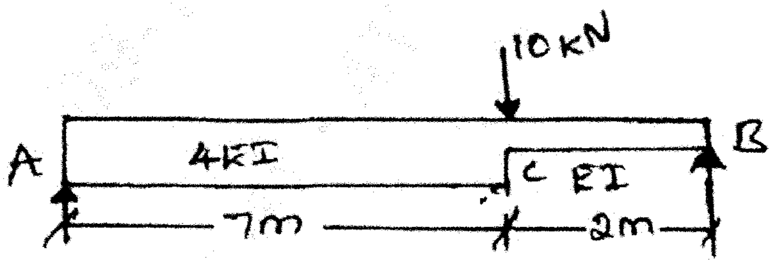



Fig.Q4(b)

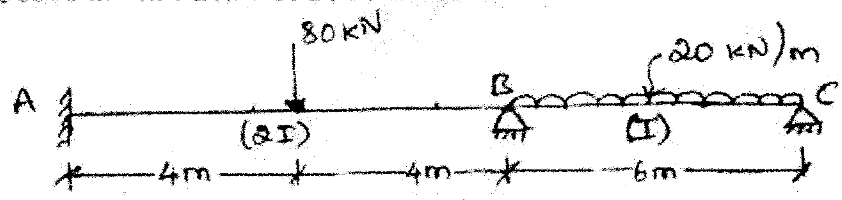
## Module – 3

**Q.5** A three hinged parabolic arch hinged at supports and crown has a span of 24 m and central rise 4 m. It carries a concentrated load of 50 kN at 18 m from left support and udl of 30 kN/m over left half portion. Determine normal thrust, radial shear at 6 m from left support and draw B.M.D. | 20 | L3 | CO3

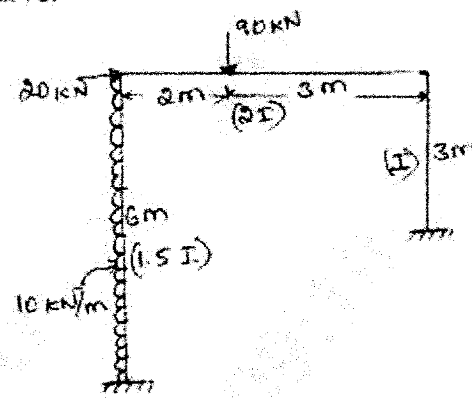
OR

| | | | | | |
|---|---|---|---|---|---|
| Q.6 | | A cable of span 120 m and dip 10 m carries a load of 6 kN/m of horizontal span. Find the maximum and minimum tension in the cable and the inclination of cable at the support. Find the forces transmitted.<br>i) If cables passes over a smooth pulleys<br>ii) If cable passes over a saddle on top of pier.<br>The anchor cable is at $30^\circ$ to the horizontal. Maximum permissible stress is 200 $N/mm^2$ and height of pier is 15 m, determining moment, length of cable and size of cable. | 20 | L3 | CO3 |

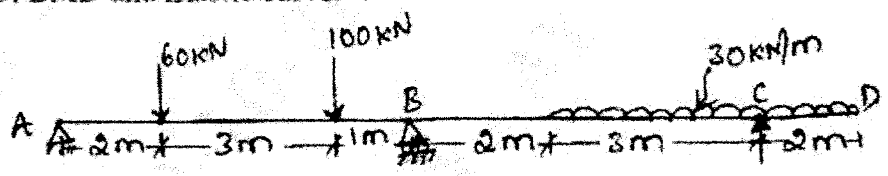
**Module – 4**

| | | | | | |
|---|---|---|---|---|---|
| Q.7 | | Analyze the beam shown in Fig.Q7 by slope deflection method. Relative to support A support 'B' sinks by 1 mm and support C rises by 0.5 mm. Take EI = 30000 $kN\text{-}m^2$. Draw SFD, BMD and elastic curve.<br><br><br>Fig.Q7 | 20 | L3 | CO4 |

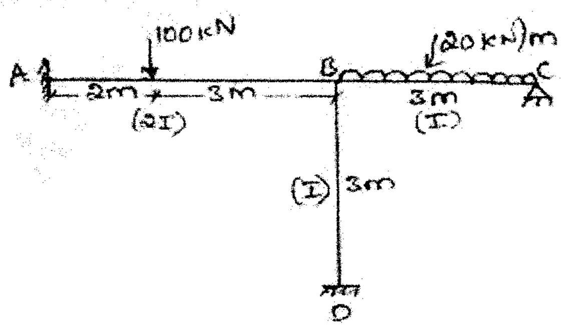
OR

| | | | | | |
|---|---|---|---|---|---|
| Q.8 | | Analyse the frame shown in Fig.Q8 by slope deflection method. Draw BMD and elastic curve.<br><br><br>Fig.Q8 | 20 | L3 | CO4 |

**Module – 5**

| | | | | | |
|---|---|---|---|---|---|
| Q.9 | | Analyse the beam shown in Fig.Q9 by moment distribution method. Draw SFD, BMD and Elastic curve.<br><br><br>Fig.Q9 | 20 | L3 | CO5 |

OR

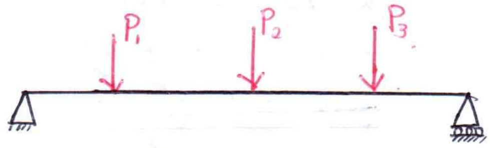
| | | | | | |
|---|---|---|---|---|---|
| Q.10 | | Analyse the frame shown in Fig.Q10 by moment distribution method. Draw BMD and Elastic curve.<br><br><br>Fig.Q10 | 20 | L3 | CO5 |

* * * * *

# KLS, VDIT, HALIYAL.
## DEPARTMENT OF CIVIL ENGINEERING.
## IV SEMESTER B.E DEGREE EXAMINATION JUNE/JULY 2025.
## ANALYSIS OF STRUCTURES. (BCV401).
## QUESTION PAPER SOLUTION.

Q1.a. The different forms of structure are.

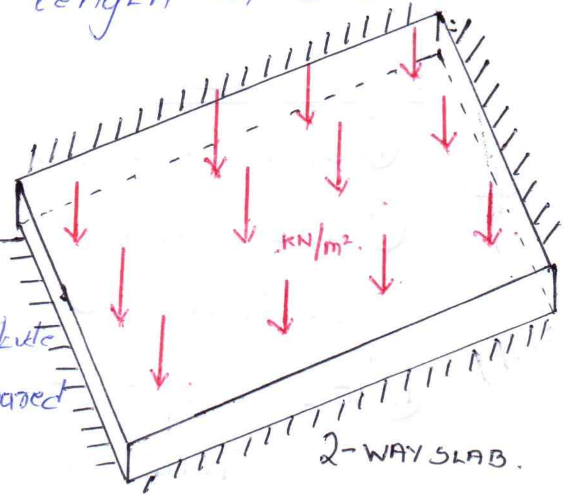
1) ONE-DIMENSIONAL ELEMENT.




In this type of structural element, loads are acting along the length of the member. The length of such members is quite large in comparison with other dimensions. The bending takes place along the length in 1 direction.

ex: Beams, Cables, Axially Loaded Bars.

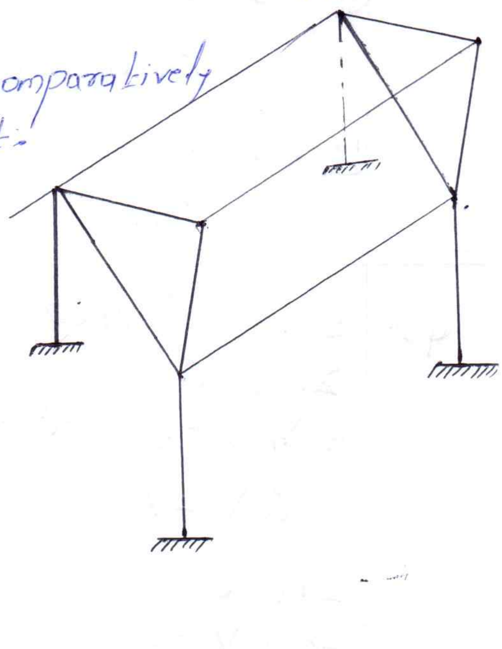
2) TWO-DIMENSIONAL ELEMENT.




These elements are widely used in practice. These elements can bend in both directions length and breadth & the bending moment in each direction is of relative magnitude. The thickness of element is small compared to other dimensions.

ex: 2-way slab, Circular plate, membrane, Shell, Grid beams.

3) THREE-DIMENSIONAL ELEMENT



The radius of curvature, span are comparatively larger than the thickness of the element.

Its deformations are w.r.t 3 axes.

ex: Hyperbolic Paraboloid Shell,
Space frame, Space Structure,
North light Shell

Q1.b.

| Statically Determinate Structure | Statically Indeterminate Structure |
|---|---|
| 1. The determination of internal forces in the structures do not require the cross-sectional areas & moment of inertia of beams & frame membrane. | The c/s areas & moment of inertia are required as the $1^{st}$ step of analysis. These quantities have effect on the values of internal forces & moments. |
| 2. The internal forces and moments are not affected by settlement of supports or lack of it. | The settlement of supports are important factors in determination of internal forces & moments. |
| 3. The Equilibrium equations are adequate to determine the internal forces and moments. | The compatibility conditions of displacements are required over & above the Equilibrium conditions to determine the internal forces & moments. |

Q1.c.

(i) Static Indeterminacy, $(SD) = EI + II$

(i) $EI = (2+1+1) - 3 = 1$. $II = m-(2j-3) = 3-(2\times3-3) = 0$

$\therefore$ Total $SD = 1+0 = 1$.

(ii) $EI = (3+1)-(3+2) = -1$ $II = m-(2j-3) = 2-(2\times2-3) = 1$

Total $SD = -1+1 = 0$

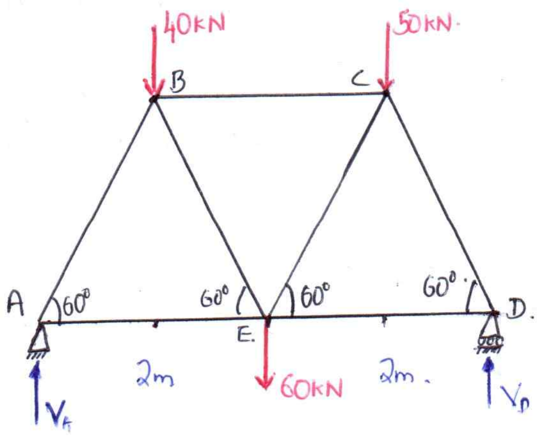
(iii) $EI = (3+1+2)-3 = 3$. $II = m-(2j-3) = 10-(2\times9-3) = -5$.

Total $SD = 3-5 = -2$.

(iv) $EI = (1+1+2)-3 = 1$ $II = m-(2j-3) = 11-(2\times6-3) = 2$.

Total $SD = 1+2 = 3$

Q2. STEP 1: SUPPORT REACTION




$\Sigma M_D = 0$

$\Rightarrow V_A \times 4 - 40\times3 - 60\times2 - 50\times1 = 0$

$\therefore V_A = 72.5\,kN$.

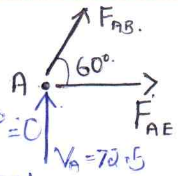
$\Sigma F_V = 0$.

$\Rightarrow V_A - 40 - 60 - 50 + V_B = 0$.

$\Rightarrow V_B = 77.5\,kN$.

STEP 2:

① At Joint A.




$\Sigma F_y = 72.5 + F_{AB}\sin 60° = 0$

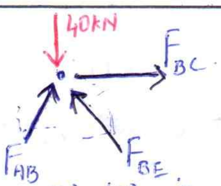
$F_{AB} = \frac{-72.5}{\sin 60} = -83.71\text{ kN}$ (C).

$\Sigma F_x = 0$

$\Rightarrow F_{AB}\cos 60° + F_{AE} = 0$

$\Rightarrow F_{AE} = -F_{AB}\cos 60° = 41.85\text{ kN}$ (T).

② At Joint B




$\Sigma F_y = 0$

$\Rightarrow -40 + F_{AB}\sin 60° + F_{BE}\sin 60° = 0$.

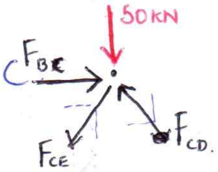
$\Rightarrow F_{BE} = -37.52\text{ kN}$ (T).

$\Sigma F_x = 0$

$\Rightarrow F_{AB}\cos 60 - F_{BE}\cos 60° + F_{BC} = 0$.

$\therefore F_{BC} = -60.615\text{ kN}$ (C).

④ At Joint C.




$\Sigma F_y = 0$

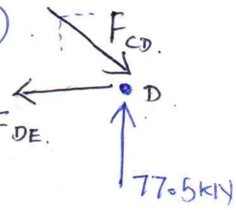
$\Rightarrow -50 - F_{CE}\sin 60 + F_{CD}\sin 60 = 0$.

$\therefore F_{CE} = 31.75\text{ kN}$. (T).

$\Sigma F_x = 0 \cdot \Rightarrow F_{BE} - F_{CE}\cos 60 - F_{CD}\cos 60 = 0$

$\Rightarrow F_{BC} = 60.62\text{ kN}$ (C) (CHECK).

③ At Joint D.




$\Sigma F_y = 0$

$\Rightarrow 77.5 - F_{CD}\sin 60 = 0$.

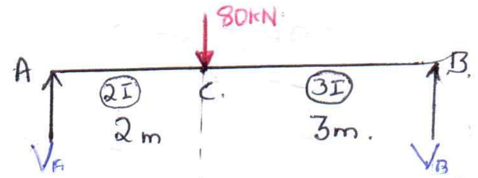
$\therefore F_{CD} = 89.49\text{ kN}$. (C)

$\Sigma F_x = 0 \quad \Rightarrow F_{CD}\cos 60 - F_{DE} = 0$.

$\therefore F_{DE} = 44.745\text{ kN}$ (T).

| MEMBER | FORCE | NATURE. |
|---|---|---|
| AB. | 83.71 kN | COMPRESSIVE. |
| AE | 41.85 kN | TENSILE. |
| BE. | 37.52 kN | TENSILE. |
| BC. | 60.615 kN | COMPRESSIVE |
| CD. | 89.49 kN | COMPRESIVE |
| DE. | 44.75 kN. | TENSILE. |
| CE. | 31.75 kN | TENSILE. |

Q3.a. STEP 1: SUPPORT REACTION.

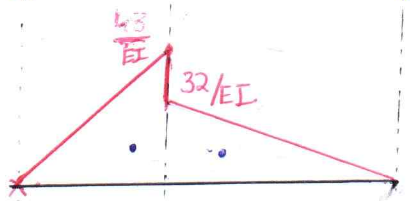



$\Sigma M_B = 0 \Rightarrow V_A \times 5 - 80 \times 3 = 0 \quad \therefore V_A = 48\,kN.$

$\Sigma F_v = 0 \Rightarrow V_A + V_B - 80 = 0 \quad \therefore V_B = 32\,kN.$

$d_{AB}$ = Moment of Area of $\frac{M}{EI}$ diagram b/w A and B about B

$$d_{AB} = \left(\frac{1}{2} \times 2 \times \frac{48}{EI}\right) \times \left(\frac{2}{3} + 3\right) + \left(\frac{1}{2} \times 3 \times \frac{32}{EI}\right) \times 2 = \frac{272}{EI}.$$

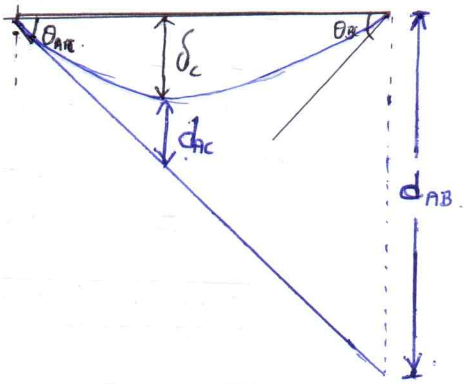



M/EI Diagram.

$$\theta_A = \frac{d_{AB}}{5} = \frac{272}{5EI} = \frac{54.4}{EI}.$$

$d_{AC}$ = Moment of Area of $\frac{M}{EI}$ diagram b/w A and C about C.

$$= \left(\frac{1}{2} \times 2 \times \frac{48}{EI}\right) \times \left(\frac{2}{3}\right) = \frac{32}{EI}.$$




$$\delta_c = \theta_A \times 2m - d_{AC} = \frac{54.4}{EI} \times 2 - \frac{32}{EI} = \frac{76.8}{EI}.$$

Maximum slope occurs at 'A', $\theta_A = \frac{54.4}{EI}.$

Maximum deflection occurs at 'C' $\delta_c = \frac{76.8}{EI}.$

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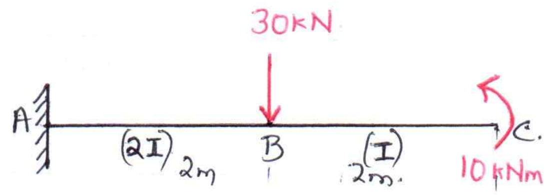
Q3.b. Using Moment Area theorem I.

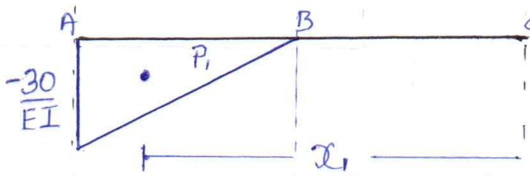



$\theta_c$ = Area of $\frac{M}{EI}$ diagram between A and C

$= P_1 + P_2 + P_3.$

$$= \frac{1}{2} \times 2 \times \left(\frac{-30}{EI}\right) + 2 \times \left(\frac{-5}{EI}\right) + 2 \times \left(\frac{-10}{EI}\right)$$

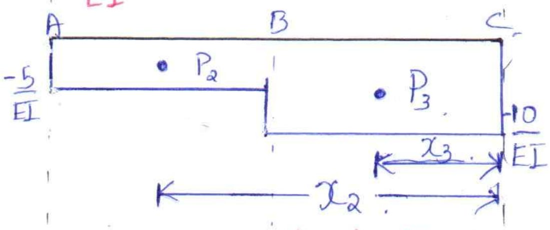
$$\theta_c = \frac{-30}{EI} - \frac{10}{EI} - \frac{20}{EI} = \quad \therefore \theta_c = \frac{-60}{EI}.$$




$\frac{M}{EI}$ diagram due to 30 kN.

Using Moment Area theorem II.

$\Delta_c$ = Moment of Area of $\frac{M}{EI}$ diagram between A and C.




$\frac{M}{EI}$ diagram due to 10 kNm.

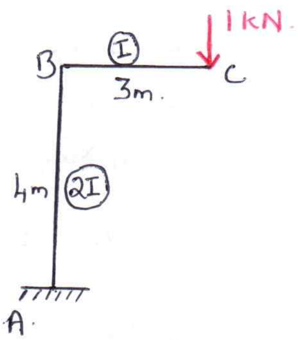
$= P_1 x_1 + P_2 x_2 + P_3 x_3.$

$$= \frac{-30}{EI} \times \left(\frac{4}{3} + 2\right) + \frac{-10}{EI} \times 3 + \frac{-20}{EI} \times 1$$

$$\therefore \Delta_c = \frac{-150}{EI}.$$

Q4.a.

| PORTION | CB | BA |
|---|---|---|
| ORIGIN | C | B |
| LIMIT | 0 to 3m | 0 to 4m |
| M | $-x$ | $-3$ |
| $m_v$ | $-x$ | $-3$ |
| FLEXURAL RIGIDITY | EI | 2EI |



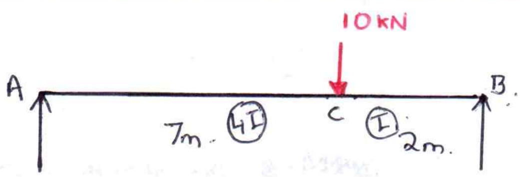

$$EI\Delta_{cv} = \int M m_v \, dx = \int_0^3 (-x)(-x)\,dx + \frac{1}{2}\int_0^4 (-3)(-3)\,dx = \int_0^3 x^2\,dx + \frac{9}{2}\int_0^4 dx$$

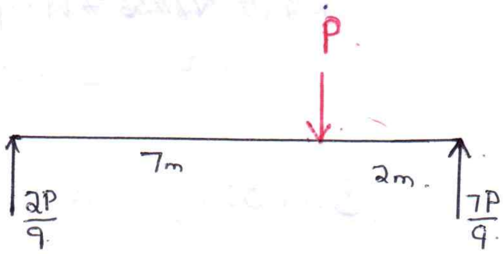
$$= \left[\frac{x^3}{3}\right]_0^3 + \frac{9}{2}\left[x\right]_0^4 = \frac{3^3}{3} + \frac{9}{2}[4] = 27$$

$$\therefore \Delta_{cv} = \frac{27}{EI}$$

Q4.b.

| PORTION | AC | CB |
|---|---|---|
| ORIGIN | A | C |
| LIMIT | 0 to 7m | 0 to 2m |
| M | $+\frac{2P}{9}x = 0.22Px$ | $+\frac{7P}{9}x = 0.78Px$ |
| FLEXURAL RIGIDITY | 4EI | EI |




$$U = \int_0^L \frac{M^2}{2EI}\,dx = \int_0^7 \frac{M^2}{2\times 4EI}\,dx + \int_0^2 \frac{M^2}{2EI}\,dx$$

$$= \int_0^7 \frac{(0.22Px)^2}{8EI}\,dx + \int_0^2 \frac{(0.78Px)^2}{2EI}\,dx$$

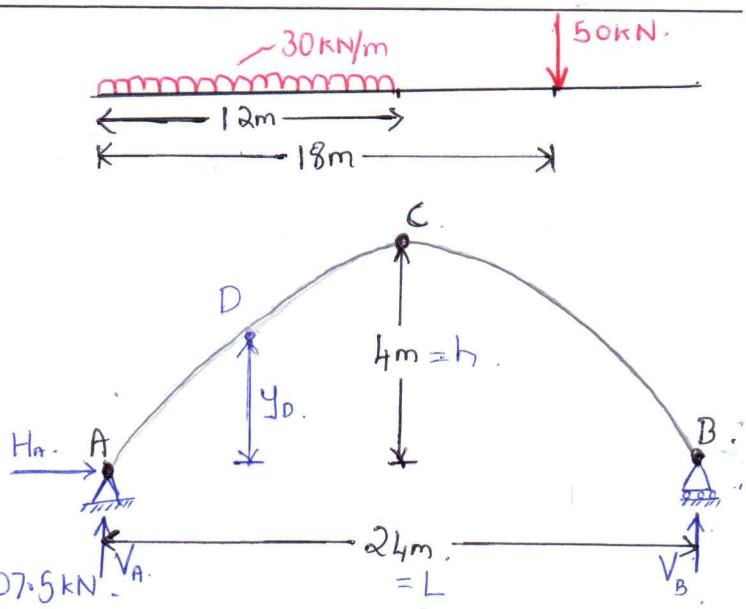
$$U = \frac{0.22^2P^2}{8EI}\int_0^7 x^2\,dx + \frac{0.78^2P^2}{2EI}\int_0^2 x^2\,dx = \frac{6.05\times10^{-3}P^2}{EI}\left[\frac{x^3}{3}\right]_0^7 + \frac{0.3}{EI}P^2\left[\frac{x^3}{3}\right]_0^2$$

$$U = \frac{2.017\times10^{-3}P^2}{EI}\left[7^3 - 0^3\right] + \frac{0.1P^2}{EI}\left[2^3-0^3\right] = \frac{0.69P^2}{EI} + \frac{0.8P^2}{EI} = \frac{1.49P^2}{EI}$$

$$\Delta_c = \frac{dU}{dP} = \frac{d}{dP}\left[\frac{1.49P^2}{EI}\right] = \frac{2.92P}{EI}$$

Substituting $P = 10kN$ we get, $\Delta_c = \frac{2.92\times10}{EI} = \frac{29.2}{EI}$

Q5. STEP 1: SUPPORT REACTION.




$\Sigma M_B = 0$.

$\Rightarrow V_A \times 24 - (30\frac{kN}{m} \times 12m) \times 18 - 50 \times 6 = 0$.

$\Rightarrow V_A = \frac{6780}{24} = 282.5 kN$.

$\Sigma F_V = 0$,

$\Rightarrow V_A - (30\frac{kN}{m} \times 12m) - 50 + V_B = 0$.

$\Rightarrow V_B = 127.5 kN$.

$\Sigma M_C = 0$.

$\Rightarrow V_B \times 12 - H \times 4 - 50 \times 6 = 0 \Rightarrow H = 307.5 kN$.

STEP 2: Y CO-ORDINATE.

For a Parabolic Arch, $y = \frac{4hx(L-x)}{L^2}$.

At 6m from Left Support i.e point D, $x = 6m$

$\therefore y_D = \frac{4 \times 4 \times 6(24-6)}{24^2} = 3m$,

$\frac{dy}{dx} = \frac{4h(L-2x)}{L^2} = \tan\theta$

$\therefore \tan\theta = \frac{4 \times 4(24 - 2 \times 6)}{24^2} = \frac{1}{3}$ $\Rightarrow \theta = \tan^{-1}\left[\frac{1}{3}\right] = 18.43°$.

STEP 3: BENDING MOMENT, NORMAL THRUST & RADIAL SHEAR.

Bending Moment, $M = V_A \times 6 - H \times y_D - (30\frac{kN}{m} \times 6m) \times 3$

$\Rightarrow M = 282.5 \times 6 - 307.5 \times 3 - 30 \times 6 \times 3 = 232.5 kNm$.

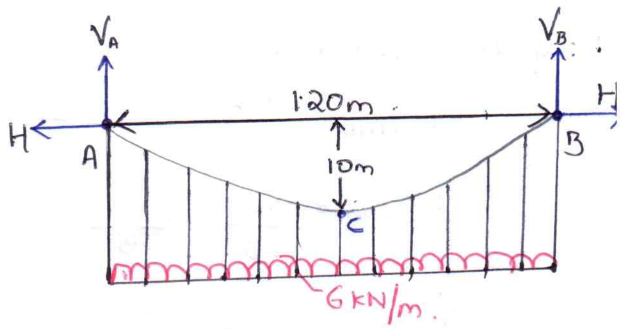
Normal Thrust, $N = V\sin\theta + H\cos\theta = 102.5\sin 18.43 + 307.5\cos 18.43$

$\therefore N = 324.13 KN$.

Radial Shear, $Q = V\cos\theta - H\sin\theta = 102.5\cos 18.43 - 307.5\sin 18.43$

$\therefore Q = 0$.

Q6. STEP 1: SUPPORT REACTIONS.




1) Due to Symmetry, $V_A + V_B$ = TOTAL LOAD.

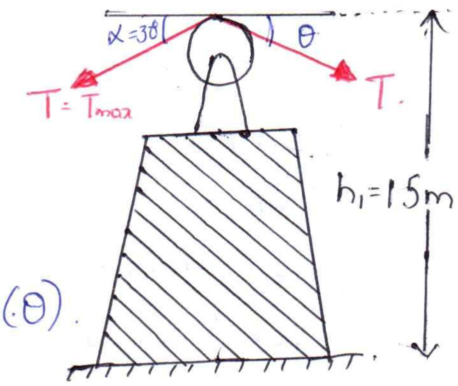
$V_A = V_B = \frac{\text{TOTAL LOAD}}{2} = \frac{6\,kN/m \times 120m}{2} = 360\,kN$.

$\Sigma M_c = 0$

$\Rightarrow -H\times 10 + V_A \times 60 - (6\,kN/m \times 60m)\times 30m = 0$.

$\Rightarrow 360\times 60 - 6\times 60\times 30 = H\times 10$. $\Rightarrow 10800 = H\times 10$.

$\therefore H = 1080\,kN$.




Tension in the main cable, $T_{max} = \sqrt{V^2 + H^2}$.

$\Rightarrow T_{max} = \sqrt{360^2 + 1080^2} = 1138.42\,kN$.

Inclination of main cable to horizontal ($\theta$).

$\cos\theta = \frac{H}{T_{max}} = \frac{1080}{1138.42} = 0.948$.

$\theta = \cos^{-1}(0.948) = 18.435°$.

CASE (i): IF CABLE PASSES OVER SMOOTH PULLEY.

VERTICAL FORCE ON TOWER $= T_{max}(\sin\alpha + \sin\theta)$.

$= 1138.42(\sin 30 + \sin 18.43) =$

HORIZONTAL FORCE ON TOWER $= T_{max}(\cos\theta - \cos\alpha)$.

$= 1138.42\,(\cos 18.435° - \cos 30)$.

CASE (ii): IF CABLE PASSES OVER THE SADDLE.

The anchor cable tension $T_1$ is given by

$T_1 \cos\alpha = T_{max}\cos\theta$ $\Rightarrow T_1 = \frac{1138.42 \times \cos 18.435°}{\cos 30°} = 1247.07\,kN$.

VERTICAL FORCE ON TOWER $= T_1 \sin\alpha + T_{max}\sin\theta$.

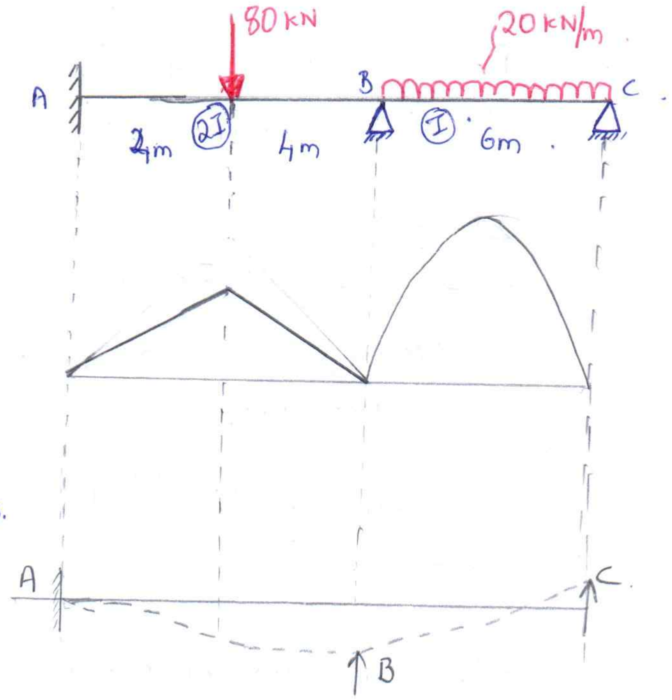
$= 1247.07 \sin 30 + 1138.42 \sin 18.435$

$= 983.53\,kN$.

HORIZONTAL FORCE ON TOWER = 0.

Q7.

STEP 1: FIXED END MOMENTS.




$$M_{FAB} = -\frac{WL}{8} = -\frac{80\times 8}{8} = -80\,kNm.$$

$$M_{FBA} = +\frac{WL}{8} = \frac{80\times 8}{8} = 80\,kNm$$

$$M_{FBC} = -\frac{WL^2}{12} = -\frac{20\times 6^2}{12} = -60\,kNm.$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{20\times 6^2}{12} = 60\,kNm.$$

STEP 2: SLOPE DEFLECTION EQUATIONS.

$\theta_A = 0$ ($\because$ End A is Fixed).

$\delta_B = 1mm$. $\delta_c = 0.5mm$.

For span AB. $\delta_B = +0.001m$

For span BC = $\delta_B = -0.001m$ and $\delta_c = 0.0005$. $\therefore \delta_{BC} = 0.001 + 0.0005 = 0.0015$

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right] = -80 + \frac{2E(2I)}{8}\left[0 + \theta_B - \frac{3\times 0.001}{4}\right]$$

$$M_{AB} = -80 + 0.5EI\theta_B - 0.5(EI)\times 0.00075 = -80 + 0.5EI\theta_B - 0.5\times 30000\times 0.00075$$

$$M_{AB} = -91.25 + 0.5EI\theta_B.$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[\theta_A + 2\theta_B - \frac{3\delta}{L}\right] = 80 + \frac{2E(2I)}{8}\left[0 + 2\theta_B - \frac{3\delta}{L}\right].$$

$$M_{BA} = 80 + 2EI\theta_B - \frac{0.5\times 30000\times 3\times 0.001}{8} = 74.375 + 2EI\theta_B.$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left[2\theta_B + \theta_c - \frac{3\delta}{L}\right] = -60 + \frac{2EI}{6}\left[2\theta_B + \theta_c - \frac{3(-0.0015)}{6.}\right]$$

$$M_{BC} = -60 + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_c + 22.5 = -37.5 + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_c.$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left[\theta_B + 2\theta_c - \frac{3\delta}{L}\right] = 60 + \frac{2EI}{6}\left[\theta_B + 2\theta_c - \frac{3\times(-0.0015)}{6}\right].$$

$$= 60 + \frac{1}{3}EI\theta_B + \frac{2}{3}EI\theta_c + 22.5 = 82.5 + \frac{1}{3}EI\theta_B + \frac{2}{3}EI\theta_c$$

STEP 3: EQUILIBRIUM CONDITION.

$\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0.$

$\Rightarrow 74.375 + 2EI\theta_B + (-37.5) + \frac{2}{3}EI\theta_B + \frac{1}{3}EI\theta_c = 0$

$\Rightarrow 36.875 + 2.667EI\theta_B + 0.333EI\theta_c = 0 \longrightarrow (1).$

$\Sigma M_c = 0 \quad \Rightarrow \quad M_{CB} = 0$

$\Rightarrow 82.5 + \frac{1}{3} EI\theta_B + \frac{2}{3} EI\theta_C = 0 \longrightarrow (2)$

Solving Equation (1) & (2) we get

$EI\theta_B = 1.75$ and $EI\theta_C = -124.625$.

$\therefore M_{AB} = 91.25 + 0.5(1.75) = 92.125 \text{ kNm}$.

$M_{BA} = 74.375 + 2(1.75) = 77.875 \text{ kNm}$.

$M_{BC} = -37.5 + \frac{2}{3}(1.75) + \frac{1}{3}(-124.625) = -77.875 \text{ kNm}$.

$M_C = 82.5 + \frac{1}{3}(1.75) + \frac{2}{3}(-124.625) = 0$.

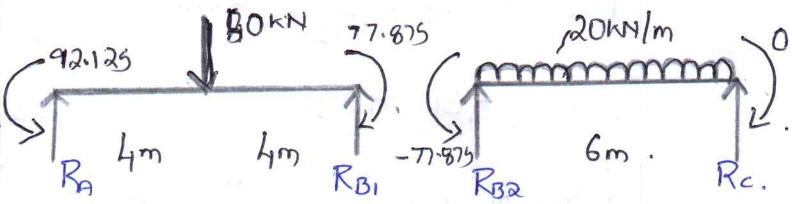
STEP 4: BMD & SFD.

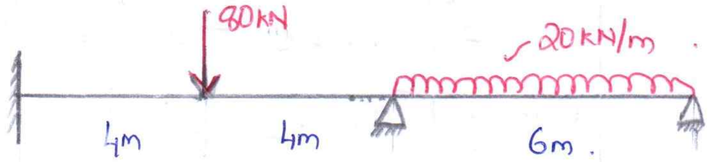
$R_A = \dfrac{92.125 + 80\times4 - 77.875}{8} = 41.78 \text{ kN}$

$R_{B1} = 80 - 41.78 = 38.22 \text{ kN}$.

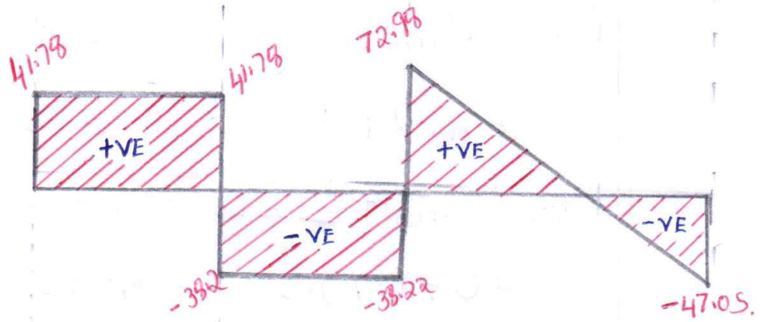
$R_{B2} = \dfrac{77.875 + 20\times6\times3}{6} = 72.98 \text{ kN}$

$R_C = 20\times6 - 72.95 = 47.05 \text{ kN}$.

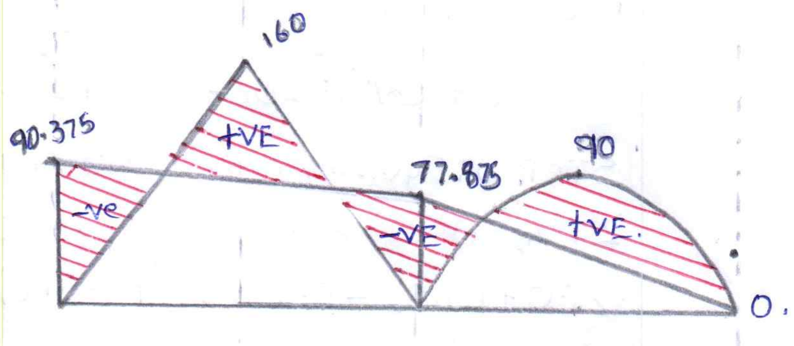



SFD




BMD



Q8.

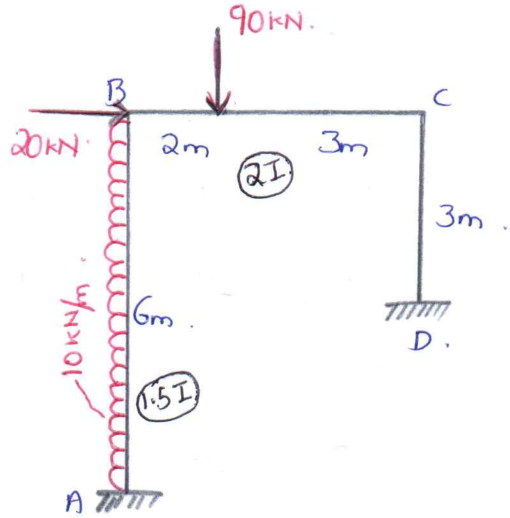
STEP 1: FIXED END MOMENT.

$$M_{FAB} = \frac{-wL^2}{12} = \frac{-20 \times 6^2}{12} = -60\text{kNm}.$$

$$M_{FBA} = \frac{wL^2}{12} = 60\text{kNm}.$$

$$M_{FBC} = -\frac{Pab^2}{L^2} = -\frac{90 \times 2 \times 3^2}{5^2} = -129.6\text{kNm}$$

$$M_{FCB} = \frac{Pa^2b}{L^2} = \frac{90 \times 2^2 \times 3}{5^2} = 43.2\text{kNm}.$$




STEP 2: SLOPE DEFLELTION EQUATION.

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right] = -60 + \frac{2E(1.5I)}{6}\left[0 + \theta_B - \frac{3\delta}{6}\right].$$

$$M_{AB} = -60 + 0.5EI\theta_B - 0.25EI\delta.$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[\theta_A + 2\theta_B - \frac{3\delta}{L}\right] = 60 + \frac{2E(1.5I)}{6}\left[0 + 2\theta_B - \frac{3\delta}{6}\right].$$

$$= 60 + EI\theta_B - 0.25EI\delta.$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left[2\theta_B + \theta_C - \frac{3\delta}{L}\right] = -129.6 + \frac{2E(2I)}{5}\left[2\theta_B + \theta_C - 0\right]$$

$$= -129.6 + (8/5)EI\theta_B + (4/5)EI\theta_C.$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(\theta_B + 2\theta_C - \frac{3\delta}{L}\right] = 43.2 + \frac{2E(2I)}{5}\left[\theta_B + 2\theta_C - 0\right].$$

$$= 43.2 + (4/5)EI\theta_B + (8/5)EI\theta_C.$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left[2\theta_C + \theta_D - \frac{3\delta}{L}\right] = 0 + \frac{2EI}{3}\left[2\theta_C + 0 - \frac{3\delta}{3}\right].$$

$$= 0 + (4/3)EI\theta_C - (2/3)EI\delta.$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left[\theta_C + 2\theta_D - \frac{3\delta}{L}\right] = 0 + \frac{2EI}{3}\left[\theta_C + 0 - \frac{3\delta}{3}\right].$$

$$= 0 + (2/3)EI\theta_C - (2/3)EI\delta.$$

STEP 3: EQUILIBRIUM CONDITION.

$\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0.$

$\Rightarrow 60 + EI\theta_B - 0.25EI\delta + (-129.6) + (8/5)EI\theta_B + (4/5)EI\theta_C = 0.$

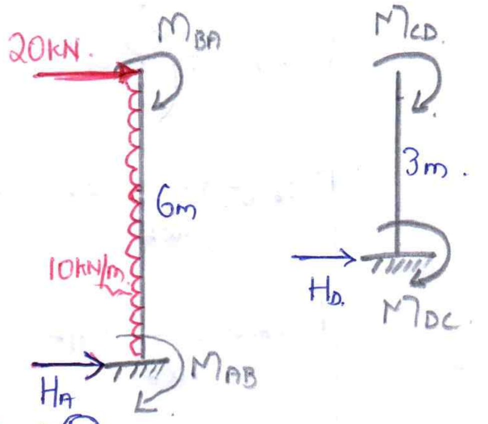
$\Rightarrow -69.6 + (13/5)EI\theta_B + (4/5)EI\theta_C - 0.25EI\delta = 0 \longrightarrow (1)$

$\Sigma M_c = 0 \Rightarrow M_{CB} + M_{CD} = 0$

$\Rightarrow 43.2 + (4/5)EI\theta_B + (8/5)EI\theta_C + 0 + (4/3)EI\theta_C - (2/3)EI\delta = 0$

$\Rightarrow 43.2 + (4/5)EI\theta_B + (44/15)EI\theta_C - (2/3)EI\delta = 0 \longrightarrow (2)$

STEP 4:- SHEAR CONDITION.




Taking $\Sigma M_B = 0$.

$\Rightarrow M_{AB} + M_{BA} - H_A \times 6 - 10 \times 6 \times 3 = 0$

$\Rightarrow H_A = \dfrac{M_{AB} + M_{BA} - 180}{6} \longrightarrow (3)$

lll'y $\Sigma M_c = 0$.

$\Rightarrow M_{CD} + M_{DC} - H_D \times 3 = 0 \quad \Rightarrow H_D = \dfrac{M_{CD} + M_{DC}}{3} \longrightarrow (4)$

$\Sigma H = 0$

$\Rightarrow H_A + H_D + 10 \times 6 + 20 = 0 \quad \Rightarrow \dfrac{M_{AB} + M_{BA} - 180}{6} + \dfrac{M_{CD} + M_{DC}}{3} + 80 = 0$

$\Rightarrow -10 + (1/12)EI\theta_B - (1/24)EI\delta + 10 + (1/6)EI\theta_B - (1/24)EI\delta - 30$
$\quad + 0 + (4/9)EI\theta_C - (2/9)EI\delta + (2/9)EI\theta_C - (2/9)EI\delta = 0$

$\Rightarrow -30 + 0.25EI\theta_B + (6/9)EI\theta_C - 0.5278EI\delta = 0 \longrightarrow (5)$

Solving equation (1), (2) and (5) we got.

$EI\theta_B = 37.26$ , $EI\theta_C = 1.4$ $EI\delta = 113.6$

$\therefore M_{AB} = -60 + 0.5 \times 37.26 - 0.25 \times 113.6 = -69.77$ kNm

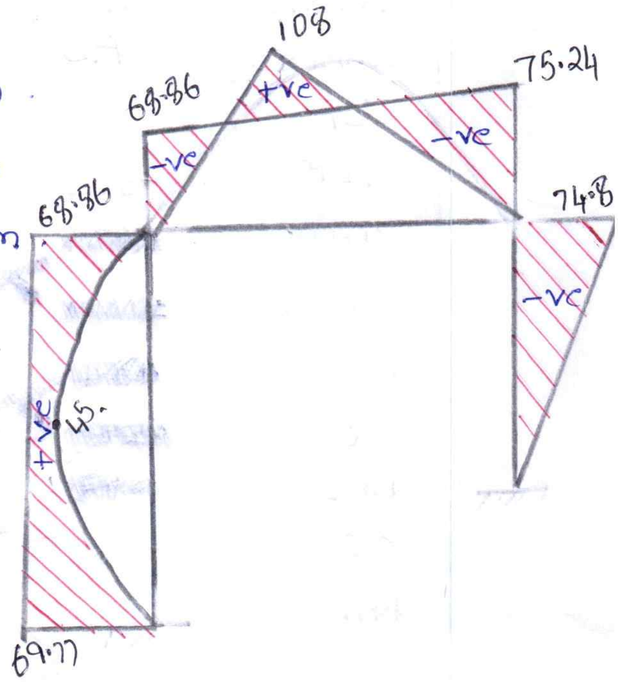
$M_{BA} = 60 + 37.26 - 0.25 \times 113.6 = 68.86$ kNm

$M_{BC} = -129.6 + (8/5)37.26 + (4/5)1.4 = -68.86$ kNm

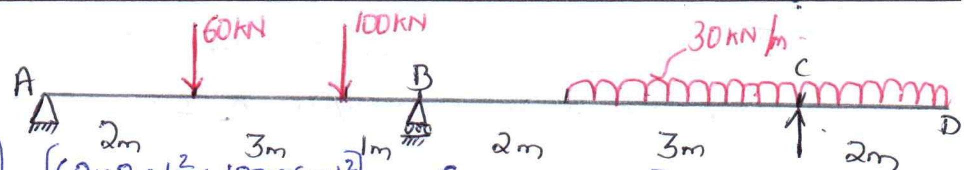
$M_{CB} = 43.2 + (4/5)37.26 + (8/5)1.4 = 75.248$ kNm

$M_{CD} = (4/3)EI\theta_C - (2/3)EI\delta = -75.248$ kNm

$M_{DC} = (2/3)EI\theta_C - (2/3)EI\delta = -74.8$ kNm



Q7. STEP 1 :- FIXED END MOMENTS




$$M_{FAB} = -\left(\frac{P_1 a_1 b_1^2}{L^2} + \frac{P_2 a_2 b_2^2}{L^2}\right) = -\left[\frac{60\times2\times4^2}{6^2} + \frac{100\times5\times1^2}{6^2}\right] = -[53.33 + 13.89] = -67.219 \text{ kNm}$$

$$M_{FBA} = +\left(\frac{P_1 a_1^2 b_1}{L^2} + \frac{P_2 a_2^2 b_2}{L^2}\right) = \left[\frac{60\times2^2\times4}{6^2} + \frac{100\times5^2\times1}{6^2}\right] = 26.67 + 69.44 = 96.114 \text{ kNm.}$$

$$M_{FBCB} = +\frac{wa^2}{12L^2}(6L^2 - 8La + 3a^2) = +\frac{30\times3^2}{12\times5^2}\left[6\times5^2 - 8\times5\times3 + 3\times3^2\right] = +51.3 \text{ kNm.}$$

$$M_{FBC} = -\frac{wa^3}{12L^2}(4L - 3a) = \frac{-30\times3^3}{12\times5^2}\left[4\times5 - 3\times3\right] = -29.7 \text{ kNm.}$$

STEP 2: DISTRIBUTION FACTORS

| JOINTS | MEMBERS. | K. | ΣK. | K/ΣK = DISTRIBUTION FACTOR |
|---|---|---|---|---|
| B | BA | $\frac{4EI}{6} = \frac{2}{3}EI$. | 1.467EI. | 0.454 |
| | BC. | $\frac{4EI}{5} = 0.8EI$. | | 0.546 |
| C | CB | $\frac{4EI}{5.} = 0.8EI$. | 0.8EI | 1 |
| | CD. | 0 | | 0. |

STEP 3: MOMENT DISTRIBUTION TABLE.

| JOINTS | A | B | | C | |
|---|---|---|---|---|---|
| MEMBERS | AB | BA | BC | CB | CD |
| DF. | 1 | 0.454 | 0.546 | 1 | 0 |
| FEM | -67.2 | 96.11 | -29.7 | 51.3 | -60. |
| BALANCE | | | | 8.7 | |
| CARRY OVER | | | 4.35 | | |
| BAL | 67.2 | -32.13 | -38.63 | | |
| CO | -16.07 | 33.6. | | | |
| BAL | 16.07 | -15.25 | -18.35 | | |
| CO | -7.63 | 8.04. | | | |
| BAL. | 7.64 | -3.65 | -4.39 | | |
| CO | 1.82 | 3.82. | | | |
| BAL | -1.82 | -1.73 | -2.09 | | |
| FINAL | 0 | 88.81 | -88.81 | 60 | -60 |

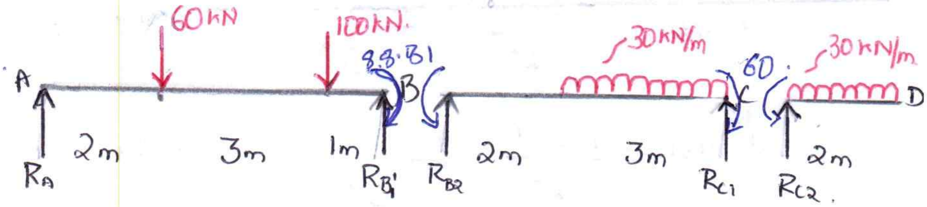
STEP 4 :- SFD and BMD.

For Span AB, $\Sigma M_B = 0$.

$R_A \times 6 - 60 \times 4 - 100 \times 1 + 88.81 = 0$.

$\Rightarrow R_A \times 6 - 251.19 = 0$.

$\therefore R_A = 41.865\, kN$.




$R_{B_1} = 60 + 100 - 41.865 = 118.13\, kN$.

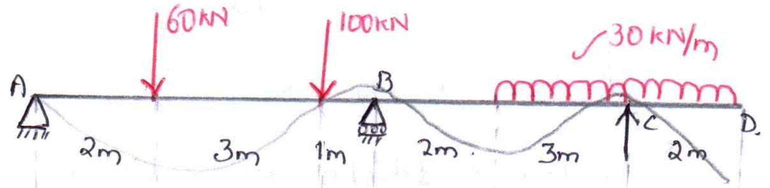
FOR SPAN BC, $\Sigma M_C = 0$.

$R_{B2} \times 5 - 30 \times 3 \times 1.5 - 88.81 + 60 = 0 \Rightarrow R_{B2} \times 5 - 163.81 = 0 \therefore R_{B2} = +32.76\, kN$

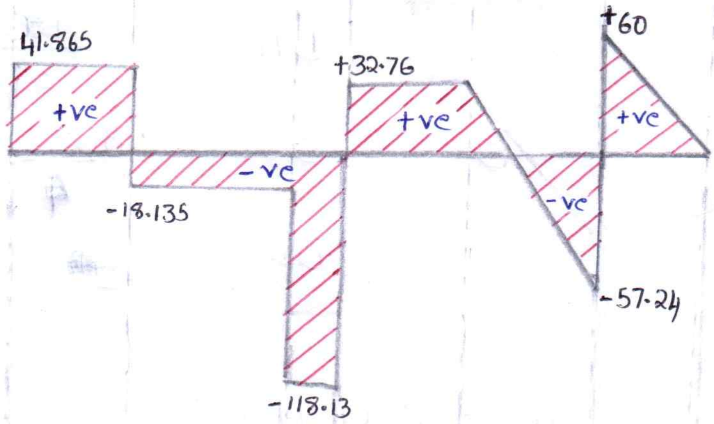
$R_{C1} = 30 \times 3 - 32.67 = 57.24\, kN$.

FOR SPAN CD

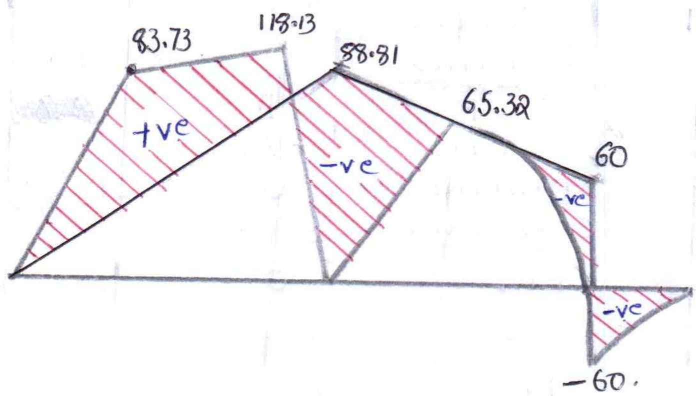
$R_{C2} = 30 \times 2 = 60\, kN$.




SFD.

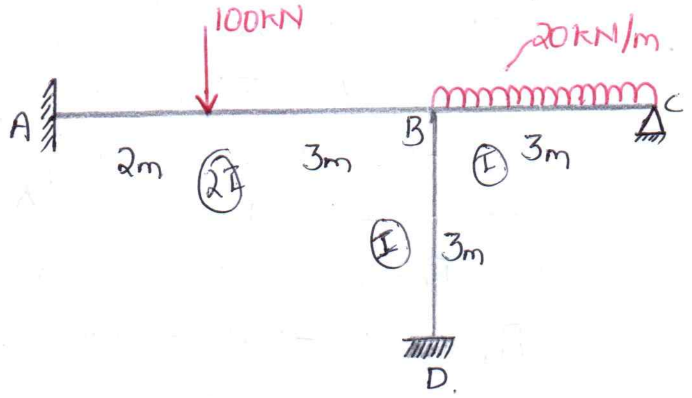



BMD.



Q 10. STEP 1: FIXED END MOMENT.

$M_{FAB} = -\frac{Pab^2}{L^2} = \frac{-100 \times 2 \times 3^2}{5^2} = -72\,kNm$




$M_{FBA} = \frac{Pa^2b}{L^2} = \frac{100 \times 2^2 \times 3}{5^2} = 48\,kNm$

$M_{FBC} = -\frac{WL^2}{12} = \frac{-20 \times 3^2}{12} = -15\,kNm$

$M_{FCB} = \frac{WL^2}{12} = \frac{20 \times 3^2}{12} = 15\,kNm.$

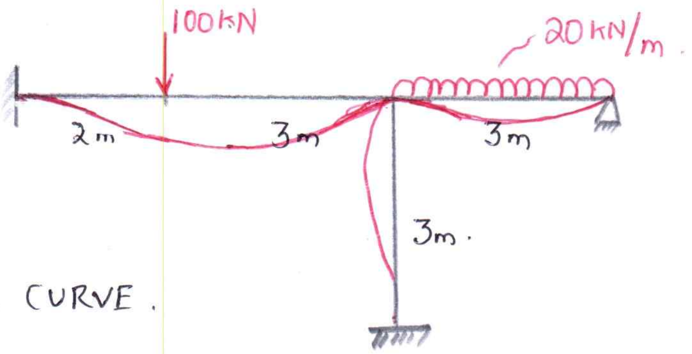
$M_{FBD} = M_{FDB} = 0.$

STEP 2: DISTRIBUTION FACTORS.

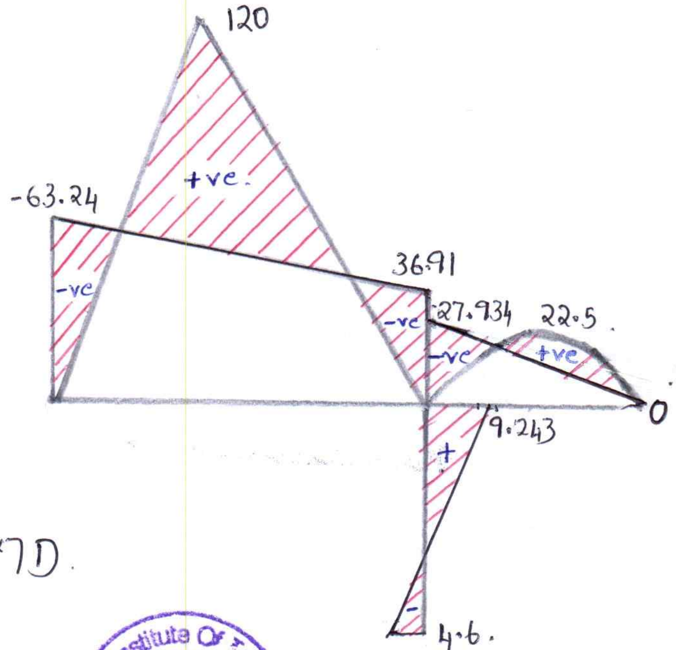
| JOINTS | MEMBER | K | $\Sigma K$ | DISTRIBUTION FACTOR |
|---|---|---|---|---|
| B | BA | $\frac{4E(2I)}{5} = 1.6EI.$ | 3.93EI. | $\frac{1.6EI}{3.93EI} = 0.407$ |
| | BC | $\frac{3EI}{3} = 1\,EI.$ | | $\frac{1\,EI}{3.93EI} = 0.254$ |
| | BD. | $\frac{4EI}{3} = 1.33EI$ | | $\frac{1.33EI}{3.93EI} = 0.339$ |

STEP 3: MOMENT DISTRIBUTION TABLE.

| JOINTS | A | B | | | C. | D |
|---|---|---|---|---|---|---|
| MEMBERS. | AB | BA | BD | BC | CB. | DB |
| DF | 0 | 0.407 | 0.339 | 0.254. | 01 | 0 |
| FEM. | -72 | 48 | 0 | -15 | 15. | |
| BALANCE. | 0 | -13.43 | -11.19 | -8.38 | -15. | |
| CARRYOVER. | 6.715. | | | -7.5 | -4.19 | -5.595. |
| BAL | 0 | 3.05 | 2.54 | 1.91 | +4.19 | |
| CO | 1.525 | | | +2.09 | 0.955. | 1.27. |
| BAL | | -0.85 | -0.71 | -0.53 | -0.955. | |
| CO | 0.425 | | | -0.478 | -0.265 | -0.355 |
| BAL. | | 0.195 | 0.162 | 0.121 | +0265. | |
| CO | 0.098 | | | +0.133 | 0.06. | 0.081 |
| BAL | | -0.054. | -0.045 | -0.034. | -0.06. | |
| FINAL | -63.24 | 36.911 | -9.243 | -27.934 | 0 | -4.6. |




ELASTIC CURVE.




BMD.

(HARSHAVARDHAN.V.S)