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(Approved by AICTE, New Delhi, Affiliated to VTU, Belagavi)
(Recognized Under Section 2(f) by UGC, New Delhi)
Udyog Vidya Nagar, Haliyal - 581 329, Dist.: Uttara Kannada
www.klsvdit.edu.in | principal@klsvdit.edu.in | hodece@klsvdit.edu.in
DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

University / Model Question Paper Scheme & Solution

Faculty Name	:	Prof. Pooja. C Shindhe
Course Name	:	Electromagnetics Theory
Course Code	:	BEC401
Year of Question Paper	:	June/July - 2025
Date of Submission	:	02/02/2026

Faculty Member   HoD   Dean (Acad.)

CBCS SCHEME

USN 2VD23EC 5

BEC401

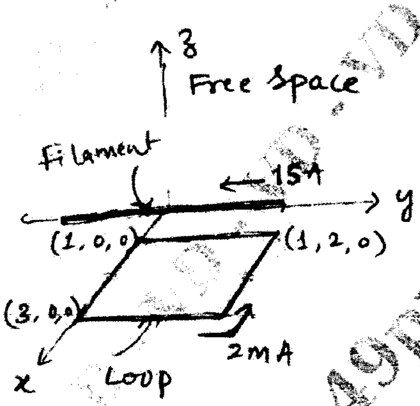
Fourth Semester B.E/B.Tech. Degree Examination, June/July 2025

Electromagnetics Theory

Time: 3 hrs. Max. Marks: 100

Note: ***1. Answer any FIVE full questions, choosing ONE full question from each module.***
2. M : Marks , L: Bloom's level , C: Course outcomes.

			M	L	C
		Module – 1			
1	a.	Derive an expression for electric field intensity due to infinite the charge.	8	L2	CO1
	b.	Define Coulomb's law in the vector from and explain.	5	L1	CO1
	c.	Transform the vector field $\bar{W} = 10\bar{a}_x - 8\bar{a}_y + 6\bar{a}_z$ to cylindrical co-ordinate system at point P(10, –8, 6).	7	L3	CO1
		OR			
2	a.	Define position vector and distance vector with an illustration in Cartesian system.	5	L1	CO1
	b.	A change of $1\mu c$ is at A(2, 0, 0), what charge must be placed at print B(–2, 0, 0), which will make 'y' component of total force per unit charge is zero at point C(1, 2, 2). Assume that the media is free space.	7	L3	CO1
	c.	Electric charge lies in the plane at z = –2m in the form of a square sheet described by $-2 \le x \le +2$m and $-2 \le y \le +2$m with charge density P_s of $2(x^2 + y^2 + 4)^{3/2}\eta$ c/m^2. Determine electric field intensity $\bar{E}$ at the origin.	8	L3	CO1
		Module – 2			
3	a.	If $\bar{E} = -8xy\bar{a}_x - 4x^2\bar{a}_y + \bar{a}_z V/m$, the charge of 6C is to be moved from B(1, 8, 5) to A(2, 18, 6). Find the work done. Selected path is $y = 3x^2 + z$ and $Z = x + 4$.	9	L3	CO2
	b.	State and prove Gauss law.	5	L2	CO2
	c.	Derive the expression for current continuity equation.	6	L2	CO2
		OR			
4	a.	Obtain $\bar{E}$ and $\bar{D}$ for infinite sheet of charge using Gauss law.	8	L2	CO2
	b.	Let $\bar{D} = 5r^2\bar{a}_r$ mC/m^2 for $r < 0.08$m and $\bar{D} = 0.1/r^2\bar{a}_r$ mC/m^2 for $r > 0.1$m, find : i) Volume charge density for r = 0.06m, ii) Volume charge density for r = 0.1m. Assume that $\bar{D}$ is in spherical system.	6	L3	CO2
	c.	The current density vector is given by $\bar{J} = \frac{2}{r}\cos\theta\bar{a}_r + 20e^{-2r}\sin\theta.\bar{a}_\theta$, find : i) $\bar{J}$ at $(r = 3m, \theta = 0^0, \phi = \pi)$ ii) Total current passing through the sphere with r = 3m, $0 \le \theta \le 20°$ and $0 \le \phi \le 2\pi$ in $\bar{a}_r$ direction.	6	L3	CO2
		Module – 3			
5	a.	Find $\bar{E}$ at P(3, 1, 2) for the field of two co-axial conducting cylinders with v = 50V at r = 2m and v = 20V at r = 3 m using Laplace's equation.	9	L3	CO3
	b.	Calculate the value of $\bar{J}$ if $\bar{H} = \frac{1}{\sin}\bar{a}_\theta$ at P(2, 30°, 20°).	5	L3	CO3
	c.	Deduced Poisson's and Laplace's equation using Gauss law in point form. Write Laplacian operation on 'V' for different co-ordinate system.	6	L2	CO3

		OR			
6	a.	Derive the expression for magnetic field $\overline{H}$ due to infinite long straight line using Biot – Savart law.	10	L2	CO3
	b.	A Co-axial cable with radius of inner conductor 'a', inner radius of outer conductor 'b' and its outer radius 'c'. The outer conductor carries current + I and inner conductor carries current – I. Determine and sketch variation of $\overline{H}$ against 'r' for : i) $r < a$ ii) $a < r < b$ iii) $b < r < c$ and iv) $r > c$.	10	L3	CO3
		Module – 4			
7	a.	In a certain region, the magnetic flux density in a magnetic material with $X_m = 6$ is given as $\overline{B} = 0.005y^2\overline{a}_x T$ at $y = 0.4m$, find $\overline{J}, \overline{J}_b$ and $\overline{J}_T$.	8	L3	CO4
	b.	Derive Lorentz force equation and explain.	5	L2	CO4
	c.	Derive an equation for the force between the two differential current elements.	7	L2	CO4
		OR			
8	a.	A square loop of wire in z = 0 plane carrying 2mA in the field of an infinite filament on the y-axis as shown in the Fig.Q8(a). Find the total force on the loop.  Fig.Q8(a)	7	L3	CO4
	b.	Obtain the Tangential component of $\overline{B}$ and $\overline{H}$ is the boundary of two medium having the permeability of μ_1 and μ_2.	8	L2	CO4
	c.	Compare electric and magnetic circuits.	5	L2	CO4
		Module – 5			
9	a.	Explain inconsistency of current continuity equation in detail.	7	L2	CO5
	b.	Derive general wave equation of $\overline{E}$ and $\overline{H}$ for the media with parameters μ, ϵ and σ.	8	L2	CO5
	c.	A circular loop conductor lies in z = 0 plane and has a radius of 0.1 m and resistance of 5Ω. Given $\overline{B} = 0.2 \sin 10^3 t$ Tesla, determine the current in the loop.	5	L3	CO5
		OR			
10	a.	Derive Maxwell's equations in integral and point form for static electric and magnetic fields using Faraday's law, Ampere's circuital law and Coulomb's law.	8	L2	CO5
	b.	A 9375MHz uniform plane wave is propagating in polystyrene. If the amplitude of electric field intensity is 20 V/m and the material is assumed to be lossless, find Attenuation Constant (α), phase constant (β), Wavelength (λ), Velocity of propagation (v), intrinsic impedance (η), propagation constant (γ) and amplitude of the magnetic field. For polystyrene $\mu_r = 1$ and $\epsilon_r = 2.56$.	6	L3	CO5
	c.	State and explain Poynting theorem.	6	L2	CO5

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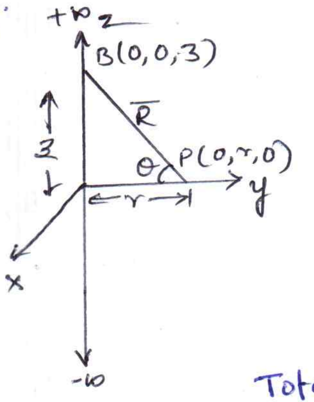
Module - 1

1a. Derive an Expression for electricfield intensity due to infinite line charge.

Ans: Field due to Infinite electricfield intensity

Consider an Infinite long straight line carrying uniform charge having density ρ_L C/m. Let the line lies along z-axis from $-\infty$ to ∞ and hence called "Infinite line charge".

Let point P is on Y axis at which electric field Intensity is to be determined. The distance of point P from the origin is 'r' as shown in fig. —2M



$\bar{P} = r.\bar{a}_y$ $\quad d\bar{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\bar{a}_R$

$\bar{B} = z.\bar{a}_z$

$\bar{R} = r.\bar{a}_y - z\bar{a}_z$ $\quad$ where $dQ = \rho_l\, dz$

$\bar{R} = \sqrt{r^2+z^2}$ $\quad a_R = \frac{\bar{R}}{|R|}$

$\therefore d\bar{E} = \frac{\rho_l r.dz\ \bar{a}_z}{4\pi\epsilon_0[r^2+z^2]^{3/2}}$ —3M

$$\text{Total } \bar{E} = \int d\bar{E} = \int_{-\infty}^{+\infty} \frac{\rho_l\, r.dz.\bar{a}_y}{4\pi\epsilon_0(r^2+z^2)^{3/2}}$$

from diagram $\tan\theta = z/r$.
$z = r\tan\theta$
$dz = r\sec^2\theta . d\theta$

and $\theta = \tan^{-1}(z/r)$
with $z=-\infty$; $\theta = -\pi/2$
$z = +\infty$; $\theta = +\pi/2$

$$\bar{E} = \int_{\theta=-\pi/2}^{+\pi/2} \frac{\rho_l\, r.r.\sec^2\theta\, d\theta\ \bar{a}_y}{4\pi\epsilon_0(r^2+r^2\tan^2\theta)^{3/2}}$$

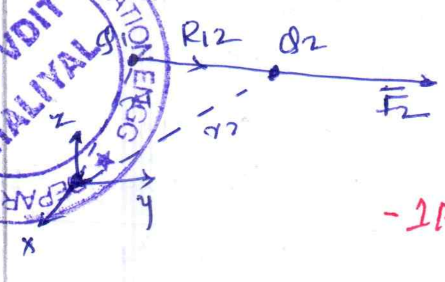
$$\boxed{\bar{E} = \frac{\rho_l}{2\pi\epsilon_0 r}\bar{a}_y}$$ —4M

1b. Define Coulomb's law in the vector from and Explain.

Ans: Coulomb's Law in vector form.

Defination:- The force of attraction or repulsion between any two charges is directly proportional to the product of charges and inversly proportional to the square of distance between them. 2M

$F \propto \frac{Q_1Q_2}{r^2} \Rightarrow F = K\frac{Q_1Q_2}{r^2}$



—1M

K = constant of proportionality

$K = \frac{1}{4\pi\epsilon_0} = $ -m/f -rad.

$\epsilon = \epsilon_0\epsilon_r \rightarrow \epsilon_0$ - permitivity of free space

$\epsilon_0 = \frac{1}{36\pi}\times 10^{-9}$ F/m $= 8.854\times10^{-12}$ F

ϵ_r = relative permitivity —2M

$$\bar{F}_2 = F_2\bar{a}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} \cdot \bar{a}_{12}$$

where $\bar{a}_{12} = \frac{\bar{R}_{12}}{|R_{12}|} = \bar{R}_{12} = \bar{r}_2 - \bar{r}_1$

and $R_{12} = |\bar{R}_{12}| = |\bar{r}_2 - \bar{r}_1|$

Q1C Transform the vector field $W = 10\bar{a}_x - 8\bar{a}_y + 6\bar{a}_z$ to cylindrical ~~in cartesian~~ system. at point P(10, −8, 6).

Ans
$\bar{W} = 10a_x - 8\bar{a}_y + 6\bar{a}_z$

$\bar{W} = W_r\bar{a}_r + W_\phi\bar{a}_\phi + W_z\bar{a}_z$ (in cylindrical system)

where

$W_r = 10\cos\phi - 8\sin\phi + 0$

$= 10\times 0.78 - (8\times -0.624) + 0$

$= 7.8 + 4.996 + 0$

$\boxed{W_r = 12.796}$

$\times$	$\bar{a}_r$	$\bar{a}_\phi$	$\bar{a}_z$
$\bar{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\bar{a}_y$	$\sin\phi$	$\cos\phi$	0
$\bar{a}_z$	0	0	1

− 3M

and at P(10, −8, 6)

$\phi = \tan^{-1}(y/x) = -38.65^\circ$

$\cos\phi = 0.78$

$\sin\phi = -0.624$. − 4M

Similarly

$W_\phi = 10\sin\phi + 8\cos\phi + 0$

$= 10(\times -0.624) + 8(0.78)$

$= -6.24 + 6.24$

$= 0$

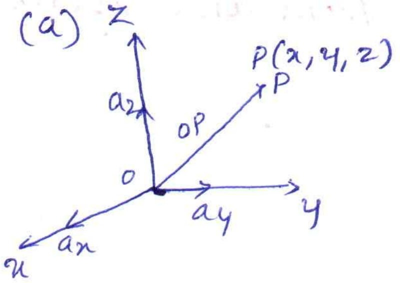
$W_z = 1\times 6 = 6$

$\boxed{\bar{W} = 12.8\bar{a}_r + 6\bar{a}_z}$ in cylindrical co-ordinates system

Q2a Define Position vector and distance vector with an illustration in Cartesian system.

Ans :

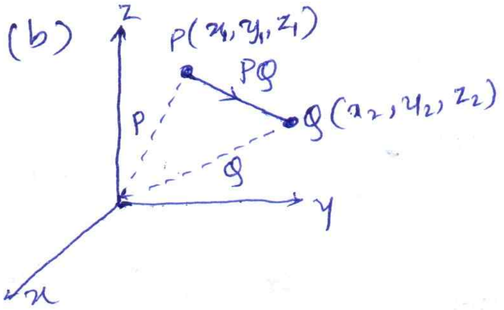
(a)



The position vector Rop of point P is defined as the directed distance from the origin 'O' to 'P' i.e.

$R_{op} = OP = x a_x + y a_y + z a_z$. 2M

(b)



The distance vector is the displacement from one point to another

$PQ = OQ - OP$ = Head − Tail

$= (x_2, y_2, z_2) - (x_1, y_1, z_1)$

$= (x_2 - x_1)a_x + (y_2 - y_1)a_y + (z_2 - z_1)a_z$

$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

3M

(2)

Q2 b. A charge of 1 μC is at A(2,0,0) what charge must be placed at point B(-2,0,0) which will make 'y' component of total force per unit charge is zero at point C(1,2,2). Assume that the media is free space.

Ans:

$\bar{A} = 2\bar{a}_x$

$\bar{B} = -2\bar{a}_x$

$\bar{C} = \bar{a}_x + 2\bar{a}_y + 2\bar{a}_z$

$Q_1 = 1\,\mu C \quad Q_2 = ?$

$q_A = 1\,\mu C$

$q_B = ?$

Condition given.

$E_y = E_{Ay} + E_{By} = 0$

from ① & ②

$$\frac{2\times10^{-6}}{4\pi\varepsilon_0\times27} + \frac{2Q_B}{4\pi\varepsilon_0\, 17\sqrt{17}} = 0$$

$$Q_B = -\frac{\cancel{2}\times10^{-6}}{\cancel{4\pi\varepsilon_0}\times27} \times \frac{\cancel{4\pi\varepsilon_0}\, 17\sqrt{17}}{\cancel{2}}$$

$Q_B = -2.57\,\mu C$

$\boxed{Q_B \simeq 2.6\,\mu C}$ -3M

$\overrightarrow{R_A} = (-1,2,2)\ \{\bar{C}-\bar{A}\}$

$|R_A| = 3$

$$\bar{E}_A = \frac{Q_1}{4\pi\varepsilon_0 R_A^2}\bar{a}_{RA} \quad \text{when } \bar{a}_{RA} = \frac{\overline{R_A}}{|R_A|}$$

$$= \frac{Q_1}{4\pi\varepsilon_0}\frac{\overline{R_A}}{R_A^3}$$

$$\bar{E}_A = \frac{Q_1(-a_x+2a_y+2a_z)}{4\pi\varepsilon_0\times27}$$

$$\bar{E}_{Ay} = \frac{Q_1\times2}{4\pi\varepsilon_0\times27} \quad \{Q_1 = 1\,\mu C\}$$

$$\bar{E}_{Ay} = \frac{2\times10^{-6}}{4\pi\varepsilon_0\times27} \rightarrow ①$$ -2M

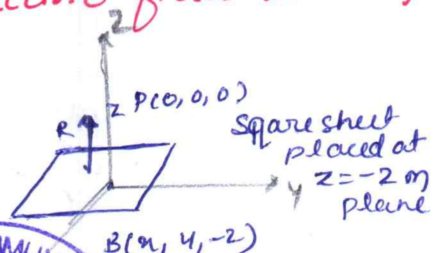
IIIly

$$\bar{E}_B = \frac{Q_B\times\overrightarrow{R_B}}{4\pi\varepsilon_0|R_B|^3}$$

$$\bar{E}_B = \frac{Q_B(-3a_x+2a_y+2a_z)}{4\pi\varepsilon_0\, 17\sqrt{17}}$$

$$\bar{E}_{By} = \frac{Q_B\times2}{4\pi\varepsilon_0\, 17\sqrt{17}} \rightarrow ②$$ -2M

Q2 c. Electric charge lies in the plane at z = -2m in the form of a square sheet described by $-2\le x\le+2$m and $-2\le y\le+2$m with charge density ρ_s of $2(x^2+y^2+4)^{3/2}$ nC/m². Determine electric field intensity $\bar{E}$ at the origin.



$\bar{P} = 0;\ \bar{B} = x\bar{a}_x + y\bar{a}_y + -2\bar{a}_z$

$\bar{R} = \bar{P} - \bar{B} = -x\bar{a}_x - y\bar{a}_y + 2\bar{a}_z$

$$d\bar{E} = \frac{dq}{4\pi\varepsilon_0 R^2}\bar{a}_R$$

where $dq = \rho_s \times ds$.

$\rho_s = 2(x^2+y^2+4)^{3/2}$ nC/m²

$ds = dx\,dy$.

$$d\bar{E} = \frac{\cancel{2}(x^2+y^2+4)^{3/2}(x\bar{a}_x + y\bar{a}_y + z\bar{a}_z)\times dx\,dy \times10^{-9}}{\cancel{2}\,4\pi\varepsilon_0(x^2+y^2+4)^{3/2}}$$

x & y component = 0 ; because of symetry

$$= \frac{x\,dx\,dy}{2\pi\varepsilon_0} \times 10^{-9}\,\bar{a}_z = \frac{dx\,dy}{\pi\varepsilon_0} \times 10^{-9}\,\bar{a}_z$$

$$E = \int d\bar{E} = \int_{y=-2}^{2}\int_{x=-2}^{2} \frac{1}{\pi\varepsilon_0}\,dx\cdot dy \times 10^{-9}\,\bar{a}_z$$

$$\boxed{\bar{E} = 575.21\,\bar{a}_z}$$

Q3 a. If $\bar{E} = -8xy\bar{a}_x - 4x^2\bar{a}_y + a_z$ V/m, the charge of 6C is to be moved from B(1, 8, 5) to A(2, 18, 6). find the work done. selected path is $y = 3x^2 + z$ and $z = x + 4$.

Ans:

$$W = -Q\int_{B(1,8,5)}^{A(2,18,6)} \bar{E}\cdot d\bar{E}$$

where $\bar{E} = -8xy\,a_x - 4x^2\bar{a}_y + \bar{a}_z$ V/m

$d\bar{E} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$

$\bar{E}\cdot d\bar{E} = -8xy\,dx - 4x^2 dy + dz$ — 2M

$$W = -Q\left\{\int_{x=1}^{2} 8xy\,dx - \int_{y=8}^{18} 4x^2 dy + \int_{z=5}^{6} dz\right\}$$

from the given path $y = 3x^2 + z = 3x^2 + x + 4$ — 3M

Differentiate with respect to z.

$dy = (6x+1)\,dx$.

$$W = -Q\left\{-\int_{x=1}^{2}(24x^3 + 8x^2 + 32x)\,dx - \int_{x=1}^{2}(24x^3 + 4x^2)\,dx + \int_{x=5}^{6} dz\right\}$$ (solving eqn) — 3M

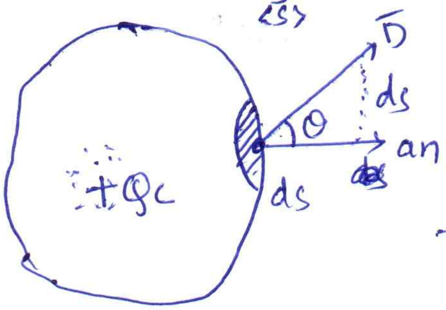
with $Q = 6C$

$$\boxed{W = 1530 \text{ Jouls}}$$ — 2M

Q3 b. State and Prove Gauss Law

Defination: "The electric flux passing through any closed surface is equal to the total charge enclosed by that surface."

$$\Psi_{total} = \oint_{\langle S\rangle} \bar{D}\cdot d\bar{s} = Q_{enclosed} \text{ — Coulombs.}$$



At a point P consider an differential element of structure 'ds' & let D makes an angle 'θ' with ds as shown in fig-

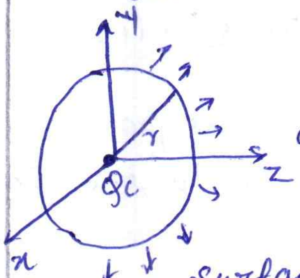
$d\bar{s} = ds\,a_n$.

The flux comming at ds is

$d\Psi$ = flux coming ds = D normal ds

$d\psi = D_s \cdot ds.$

The total flux $\psi = \oint_{\langle s \rangle} D_s \cdot ds.$



considering the charge Q_c situated at the center of an imaginary sphere of radius "r".

The electric field D is every where, normal to the surface of spherical due to point charge.

$$\bar{D} = \frac{Q}{4\pi r^2}\,\bar{a}_r \; - \; C/m^2$$ 2M

The differential element of area on spherical surface in spherical coordinates

$$ds = r^2 \sin\theta \, d\theta \, d\phi$$

$$d\bar{s} = r^2 \sin\theta \, d\theta \, d\phi \; a_r$$

$$\psi_{total} = \oint_{\langle s \rangle} D \cdot ds.$$

$$= \oint_{\langle s \rangle} \frac{Q}{4\pi r^2} a_r \cdot r^2 \sin\theta \, d\theta \, d\phi \; a_r$$

$\because a_r \cdot a_r = 1$

$$= \frac{Q}{4\pi} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{0}^{2\pi} d\phi$$

$$= \frac{Q}{4\pi} (-\cos\theta)_0^{\pi} \, (\phi)_0^{2\pi} = \frac{Q}{4\pi} \cdot [-1-(1)]\, 2\pi$$

$$= \frac{Q}{\cancel{4\pi}} \cancel{(2)} \cancel{(2\pi)} = Q.$$ 3M

$$\boxed{\psi_{total} = Q_c}$$

Q3c. Derive the expression for current continuity equation.

$I = \int_s \bar{I} \cdot d\bar{s} = \frac{-dQ_i}{dt} \rightarrow (1)$; $Q_i \rightarrow$ charge inside the closed surface 2M

Integral form of a continuity eqn of current, if the current flow out of the volume, which is enclosed by its closed system's using divergence theorem. 3M

$$Q_s = d\bar{s} = \int_{vol.} (\nabla \cdot \bar{s})\, dV$$

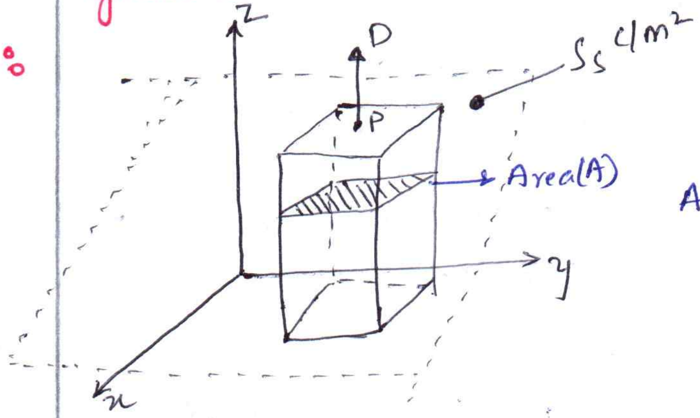
from (1)

$$\frac{-dQ_i}{dt} = \int_{vol} (\nabla \cdot \bar{J})\, \cancel{dv} = \int \frac{\partial \rho_v}{\partial t} \cancel{dv}$$

$$\boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{dt}}$$ 2M

Q4 a. Obtain $\bar{E}$ and $\bar{D}$ for infinite sheet of charge using Gauss law.

Ans:



Infinite sheet of charge of uniform charge density ρ_s C/m², lying the z=0.

To determine D at point 'P'

As D is normal to the sheet

$$D = D_z a_z$$

$$d\bar{s} = ds\, a_z = dx\,dy\, a_z$$

$$\rho_s \int_{\langle s \rangle} ds = Q = \oint_s D \cdot ds = D_z \left[\int_{top} ds + \int_{bottom} ds \right]$$

D has no component in x & y direction

$$Q = D_z (2A)$$

$$= \rho_s A \quad \text{Let } \rho_s = 2D_z$$

$$D_z = \left(\frac{\rho_s}{2}\right)$$

$$D = D_z a_z = \frac{\rho_s}{2} a_z$$

$$E = \frac{D}{\varepsilon_0} = \frac{\rho_s}{2\varepsilon_0} a_z \quad V/m$$

Q4 b. Let $\bar{D} = 5r^2 \bar{a}_r$ m C/m² for $r < 0.08$m and $\bar{D} = 0.1/r^2 \bar{a}_r$ m C/m² for $r > 0.1$m. Find i) Volume charge density for $r = 0.06$m. ii) Volume charge density for $r = 0.1$m. Assume that $\bar{D}$ is in spherical system.

Ans:

i) For $r < 0.08$m, $\bar{D} = 5r^2 \bar{a}_r$ m C/m² ; $D_r = 5r$ $D_\theta = D_\phi = 0$

$$\rho_v = \nabla \cdot \bar{D} = \frac{1}{r^2} \cdot \frac{\partial [r^2 D_r]}{\partial r} \quad \text{[D is only function of r only]}$$

$= 20r$. where $r = 0.06$m

$$\boxed{\rho_v = 1.2\text{m C/m}^3}$$

ii) ρ $r = 0.1$m $\bar{D} = \frac{0.1}{r^2} \bar{a}_r$ m C/m² $D_r = \frac{0.1}{r^2}$ $D_\theta = D_\phi = 0$.

$$\rho_v = \nabla \cdot \bar{D} = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{0.1}{r^2} \right] = 0 \quad (\because \text{derivative of constant is } 0)$$

$$\boxed{\rho_v = 0}$$

Q4 c. The current density vector is given by $J = \frac{2}{r} \cos\theta\, \bar{a}_r + 20 e^{-2r} \sin\theta\, \bar{a}_\theta$

find: i) J at ($r = 3$m, $\theta = 0°$, $\phi = \pi$)

ii) Total current passing through the sphere with $r = 3$m, $0 \le \theta \le 20°$, $0 \le \phi \le 2\pi$ in $\bar{a}_r$ direction.

i) $\vec{J} = \frac{2}{r} \cos\theta\, \hat{a}_r + 20 e^{-2r} \sin\theta\, a_\theta$ ($r = 3$m, $\theta = 0°$, $\phi = \pi$)

$\vec{J} = \frac{2}{3} \cos(0)\, \hat{a}_r + 20\, e^{-2(3)} \sin(0)\, a_\theta$

$\vec{J} = \frac{2}{3} \hat{a}_r = 0.666\, \hat{a}_r$ A/m² —2M

ii) $I = \oint J \cdot ds = \oint J r \sin\theta\, d\theta\, d\phi$

$$I = \int_{\theta=0}^{20°} \int_{\phi=0}^{2\pi} r \sin\theta\, d\theta\, d\phi \quad @ r = 3m$$

$$\boxed{I = 2.204\,A}$$

→4M

Q5 a. Find $\bar{E}$ at P(3,1,2) for the field of two co-axial conducting cylinders with V=50V at $r=2m$ and V=20V at $r=3m$ using Laplace's equation.

Ans: E=? at P(3,1,2) in cylindrical system.

$\nabla^2 V = 0 \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left[r.\frac{\partial V}{dr}\right] + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$ — 3M

Since V in function of 'r' only

$\frac{1}{r}\frac{\partial}{\partial r}(r.\frac{\partial V}{\partial r}) = 0$; By double integration, we get

$V = C_1 \ln(r) + C_2 \longrightarrow (1)$

using Boundary condition:

V=50 at $r=2m$ & V=20m at $r=3m$

from (1) we get,

$C_1 = 30/\ln(2/3) = -75$

$C_2 = 50 - \frac{30}{\ln(2/3)} \times \ln(2) = 102.$ — 4M

$\boxed{V = -75\ln(r) + 102}$

$\therefore \quad \bar{E} = -\nabla V = -\frac{\partial V}{\partial r}\bar{a}_r \Rightarrow \boxed{\bar{E} = \frac{75}{r}\bar{a}_r}$ — 2M.

Q5 b. Calculate the value of $\bar{J}$ of $\bar{H} = \frac{1}{\sin\theta}\bar{a}_\theta$ at P(2,30°,20°).

Ans: $\bar{J} = \text{curl}\,\bar{H} = \frac{1}{r^2}\begin{vmatrix} \bar{a}_r & r a_\theta & r\sin\theta\,\bar{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ H_r & rH_\theta & r\sin\theta H_\phi \end{vmatrix}$ — 2M

with $\bar{H} = \frac{1}{\sin\theta}\bar{a}_\theta$; $H_\theta = \frac{1}{\sin\theta}$; $H_r = H_\phi = 0$

$J = \sin\theta\,\bar{a}_\phi$ At P(2,30°,20°)

$\boxed{\bar{J} = 0.25\,\bar{a}_\phi \text{ A/m}}$ — 3M

Q5 c. Deduced Poisson's and Laplace's equation using Gauss law in point form, write Laplacian operation on 'V' for diffe co-ordinate system.

Ans: Gauss law in point form,

$\nabla.\bar{D} = \rho_v$; $\nabla.(\epsilon\bar{E}) = \rho_v$ since $\bar{E} = -\nabla V$

$\nabla.[\epsilon(-\nabla V)] = \rho_v$

$\boxed{\nabla^2 V = -\rho_v/\epsilon} \rightarrow$ poissons eqn.

for certain region $\rho_v = 0$; $\boxed{\nabla^2 V = 0} \rightarrow$ Laplaces fn — 3M

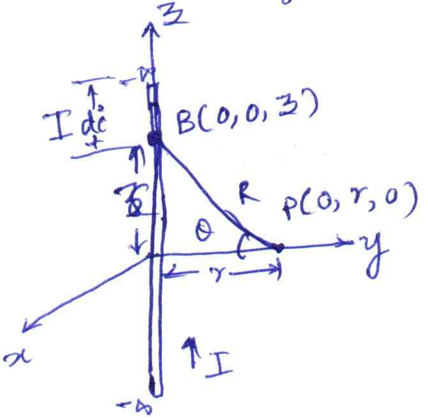
i) Cartessan system: $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$ — 1M

ii) Cylindrical system: $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}(r.\frac{\partial V}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 r}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$ — 1M

iii) Spherical system: $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial V}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial V}{\partial\theta}) + \frac{1}{\ldots}\frac{\partial^2 V}{\partial\phi^2}$

Q6 a. Derive the expression for magnetic field $\bar{H}$ due to infinite long straight line using Biot-Savart law.

Ans: $\bar{H}$ due to infinite long straight line :-



$$d\bar{H} = \frac{I \cdot d\bar{l} \times \bar{a}_R}{4\pi R^2}$$

$$I \cdot d\bar{l} = I\, dz\, \bar{a}_z$$

$$\bar{a}_R = \frac{(r \cdot \bar{a}_r - z \cdot \bar{a}_z)}{\sqrt{r^2+z^2}}$$

$\{ \bar{P} = r \cdot \bar{a}_r,\ \bar{B} = z \cdot \bar{a}_z,\ \bar{R} = \bar{P} - \bar{B} = r\bar{a}_r - z\,\bar{a}_z \}$ -2M

$$d\bar{l} \times \bar{a}_R = \begin{vmatrix} a_r & a_\phi & \bar{a}_z \\ 0 & 0 & dz \\ r & 0 & -z \end{vmatrix}$$

$$= r \cdot dz \cdot \bar{a}_\phi$$

$$d\bar{H} = \frac{I \cdot r \cdot dz}{4\pi (r^2+z^2)^{3/2}} \bar{a}_\phi$$

$$\text{Total } \bar{H} = \int_{z=-\infty}^{\infty} d\bar{H} = \int_{-\infty}^{\infty} \frac{I r\, dz}{4\pi (r^2+z^2)^{3/2}} \bar{a}_\phi$$

with $z = r\tan\theta$ $\quad z^2 = r^2\tan^2\theta$; $\theta = \tan^{-1}(z/r)$ -2M

$dz = r\sec^2\theta\, d\theta$ and where $z = -\infty$; $\theta = -\pi/2$

$z = +\infty$; $\theta = +\pi/2$

$$\bar{H} = \int_{\theta=-\pi/2}^{\pi/2} \frac{I \cdot r \cdot \sec^2\theta\, d\theta}{4\pi (r^2 + r^2\tan^2\theta)^{3/2}} \bar{a}_\phi$$

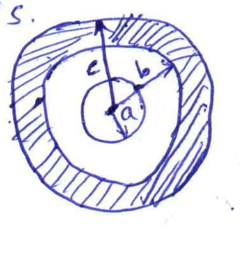
-3M

$$\boxed{\bar{H} = \frac{1}{2\pi r} \bar{a}_\phi}$$

Q6 b. A co-axial cable with radius of inner conductor 'a' inner radius of outer conductor 'b' and its outer radius 'c'. The outer conductor carries current +I and inner conductor carries current -I. Determine and sketch variation of $\bar{H}$ against 'r' for; i) $r < a$ ii) $a < r < b$ iii) $b < r < c$ & iv) $r > c$.

Ans: Assume the ~~the~~ co-axial cable is along z-axis.

$\bar{H}$ is calculated as follows,



i) within the inner conductor.

$(r < a)$

current enclosed by a closed path.

$$I' = \frac{\pi r^2}{\pi a^2} I = \frac{r^2}{a^2} I$$

also $I' = \oint \bar{H}\, d\bar{l} \rightarrow$ using Ampere's circuital law.

$$\oint \bar{H} \cdot d\bar{l} = \frac{r^2}{a^2} I$$

$$\bar{H} = H_\phi \cdot \bar{a}_\phi \ \& \ d\bar{l} = r \cdot d\phi\ \bar{a}_\phi$$

$$\bar{H} = d\bar{l} = r H_\phi\, d\phi$$

$$\int_{\phi=0}^{2\pi} r H_\phi \cdot d\phi = \frac{r^2}{a^2} I \Rightarrow \boxed{\bar{H} = H_\phi \cdot \bar{a}_\phi = \frac{Ir}{2\pi a}\bar{a}_\phi} \text{ A/M}$$

-3M

ii) Between a & b ; $\bar{H} = \frac{I}{2\pi r}\bar{a}_r$ A/m

-2M

iii) Within outer ~~boundary~~ conductor $b < r < c$; current Enclosed by a closed path as $-I$

$$I' = \frac{\pi(r^2-b^2)}{\pi(c^2-b^2)}(-I) = -\frac{(r^2-b^2)}{(c^2-b^2)} I$$

Total Current = $I_{enc} = I' + I''$; where I'' - current inner cond.

$$I_{enc} = -I\left(\frac{r^2-b^2}{c^2-b^2}\right) + I = I\left(\frac{c^2-r^2}{c^2-b^2}\right)$$

by Ampere circuital law.

$$I_{enc} = \oint H\, d\bar{l} = r H_\phi 2\pi ;\ \therefore r H_\phi 2\pi = I\left(\frac{c^2-r^2}{c^2-b^2}\right)$$

-3M

(iv) Outside the cable $(r > c)$ Here closed.

$$I_{enc} = 0$$

$$\boxed{\oint H\, dL = 0}$$

-2M

Q7 a. In a certain region, the magnetic flux density in a magnetic material with $\chi_m = 6$ is given as $\bar{B} = 0.005 y^2 \bar{a}_x$ T at $y = 0.4$ m find $\bar{J}, \bar{J}_b$ and $\bar{J}_T$.

Ans: Given $\bar{B} = 0.005 y^2 \bar{a}_x$ at $y = 0.4$ m ; $\chi_m = 6$, $\mu_0 = 4\pi \times 10^{-4}$

i) $\bar{J} = \nabla \times \bar{H}$, $\bar{H} = \frac{\bar{B}}{\mu} = \frac{\bar{B}}{\mu_0(1+\chi_m)}$, $\bar{J} = \frac{1}{\mu_0(1+\chi_m)} \nabla \times \bar{B}$

where $\nabla \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0.005y^2 & 0 & 0 \end{vmatrix} = -0.01y\,\bar{a}_z$ At $y = 0.4$m

$$\boxed{\bar{J} = -45.72\, a_z}$$

3M

ii) $\bar{J}_b = \frac{\nabla \times \chi_m \bar{B}}{\mu_0(1+\chi_m)} = \frac{\chi_m}{\mu_0(1+\chi_m)} \nabla \times \bar{B}$ where $\nabla \times B = -0.01 y\, a_z$

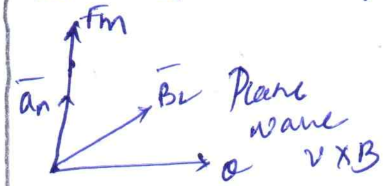
At $y = 0.4$m

$|\bar{J}_b| = -2728.37\ \bar{a}_z$ 3M

iii) $\bar{J}_T = \frac{\nabla \times \bar{B}}{\mu_0} = \frac{-0.014}{\mu_0} \bar{a}_z = -3183.09\, \bar{a}_z$ A/m — 2M

Q7 b. Derive Lorentz force equation and explain

Ans: Lorentz force Equation.



Using coulombs law = $\bar{F}_e = Q\bar{E} \rightarrow (1)$ — 1M

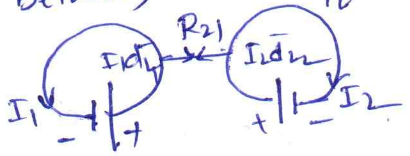
for magnetic field : $\bar{F}_m = Q\bar{v} \times \bar{B}$ Newton $\rightarrow (2)$ — 1M

$\rightarrow$ 2M from (1) & (2) $F = \bar{F}_e + \bar{F}_m$

$\boxed{F = -Q[\bar{E} + \bar{v} \times \bar{B}]}$ Newton. — 2M

Q7 c. Derive an equation for the force between the two differential current elements.

Ans: Force Between two differential Current Element.



$d(F_1)$ — Force exerted on element $I_1 d\bar{L}_1$ due to magnetic field $d\bar{B}$ by $I_2 d\bar{L}_2$ — 3M

$d(\bar{F}_1) = I_1 d\bar{L}_1 \times d\bar{B}_2$

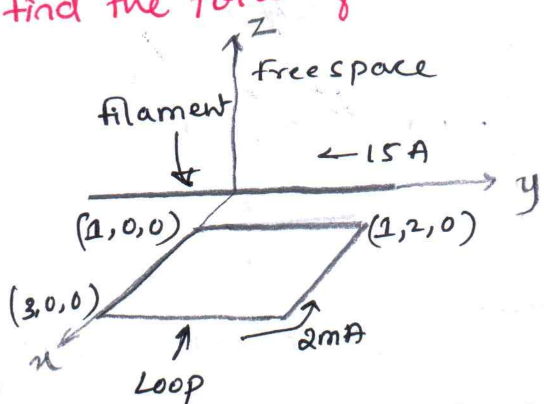
where $d\bar{B}_2 = \mu_0 d\bar{H}_2 = \mu_0 \frac{I_2 d\bar{L}_2 \times \bar{a}_{21}}{4\pi R_{21}^2}$

To be free $\bar{F}_1 = \oint d(\bar{F}_1) = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1}\oint_{L_2} \frac{d\bar{L}_1 \times (d\bar{L}_2 \times \bar{a}_{12})}{R_{21}}$ — 3M

If force exerted on $I_2 d\bar{L}_2$ due to $I_1 dL_1$ is given by

$I_2 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1}\oint_{L_2} \frac{d\bar{L}_2 \times (d\bar{L}_1 \times a_{12})}{R_{12}^2}$ — 1M.

8a. A square loop of wire in z=0 plane carrying 2mA in the field of an infinite filament on the y-axis as shown in the fig Q8 (a) find the total force on the loop.



Ans: Field produced by filament is given by $\bar{H} = \frac{I}{2\pi x} \bar{a}_z = \frac{15}{2\pi x} \bar{a}_z$

$\bar{B} = \mu_0 \bar{H} = 4\pi \times 10^{-7} \times \frac{15}{2\pi x} \bar{a}_z$

$= \frac{3 \times 10^{-6}}{x} a_z$ — 3M

$\bar{F} = -I \oint \bar{B} \cdot d\bar{L}$ where $I = 2 \times 10^{-3}$

Total Force → sum of the forces on the four sides,

$$\bar{F} = -2\times10^{3}\times3\times10^{6}\left\{\int_{x=1}^{3}\frac{dx}{2}\,\bar{a}_z\times a_x + \int_{z=0}^{1}\frac{dy}{3}\,a_z\times\bar{a}_y + \int_{x=3}^{1}\frac{dx}{x}\,\bar{a}_z\times\bar{a}_x + \int_{2}^{0} dy\,\bar{a}_z\times\bar{a}_y\right\}$$

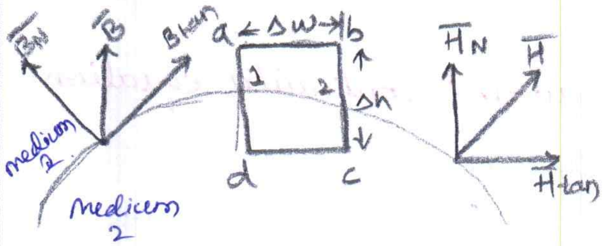
$$= -6\times10^{-9}\left\{\ln(x_1)\Big|_1^3 a_y + \frac{1}{3}(y)\Big|_0^2(-\bar{a}_x) + \ln(x)\Big|_3^1\bar{a}_y + \left(y\Big|_2^0(-\bar{a}_z)\right)\right\}$$

$$\boxed{\bar{F} = -8\,\bar{a}_x}\ \eta N$$

– 4M

Q8 b. Obtain the Tangential component of $\bar{B}$ and $\bar{H}$ is the boundary of two medium having the permeability of μ_1 & μ_2.

Ans: Tangential component of $\bar{B}$ & $\bar{H}$,



By Amperes circuital law

$$\oint \bar{H}\, d\bar{L} = I$$

– 2M

LHS

$$\oint H\,d\bar{L} = \left\{\int_a^b + \int_b^1 + \int_1^2 + \int_c^d + \int_d^1 + \int_1^a\right\}\times dL$$

$$\int_a^b \bar{H}\,d\bar{L} = H_{tan_1}(\Delta n), \quad \int_c^d \bar{H}\,dL = -H_{tan_2}(\Delta n)$$

$$\int_b^2 \bar{H}\cdot d\bar{L} = H_{N_1}\left(\frac{\Delta n}{2}\right), \quad \int_d^1 \bar{H}\cdot dL = -H_{N_2}\left(\frac{\Delta n}{2}\right)$$

$$\int_2^c \bar{H}\cdot d\bar{L} = H_{N_2}\left(\frac{\Delta n}{2}\right), \quad \int_1^d H\cdot d\bar{L} = -H_{N_1}(\Delta n/2)$$

– 2M

$$\oint H\,d\bar{L} = (H_{tan_1} - H_{tan_2})\,\Delta w \quad \text{let } \oint \bar{H}\,d\bar{L} = K\cdot dw$$

$$K\cdot dw = (H_{tan_1} - H_{tan_2})\,\Delta w$$

$$\bar{H}_{tan_1} - \bar{H}_{tan_2} = K\,\bar{a}_{N_1}$$

– 2M

$\bar{a}_{N_1} \rightarrow$ unit vector normal to boundary from medium

$B_{tan_1} = \mu_1 H_{tan_1}$ & $B_{tan_2} = \mu_2 H_{tan_2}$ from ① $\boxed{\dfrac{B_{tan_1}}{\mu_1} - \dfrac{B_{tan_2}}{\mu_2} = K}$

If Boundary n free of conductors, $K=0$

from ① $H_{tan_1} = H_{tan_2}$ & from ② $\dfrac{B_{tan_1}}{B_{tan_2}} = \dfrac{\mu_1}{\mu_2}$ // – 2M

Q8 c compare electric and Magnetic circuits.

Ans: comparision of Electric and Magnetic circuits.

Electric Circuits	Magnetic Circuits
* Path Traced by current	* Path traced by Magnetic flux.
* Driving force = emf	* Driving force = mmf
* $\sigma, \bar{E}, \bar{J} = \frac{I}{S} = \sigma \cdot E$ A/m²	* $\mu, \bar{H}, B = \frac{\phi}{S} = \mu \bar{H}$
* Conductance $= \frac{1}{R}$ $R = \frac{emf}{current}$	* Permeance $= \frac{1}{R}$ $R = \frac{mmf}{flux}$
* $R = IR$	* $em = \phi R$
* $\Sigma I = 0$ $\Sigma emf = 0$	* $\Sigma \phi = 0$ $\Sigma mmf = \Sigma IR$.

Q9 a. Explain inconsistency of current continuity equation in detail.

Ans: Inconsistany of current continuity equation.

Maxwell's equation derived from Amperes circutal law for static Magnetic field is given by

$$\nabla \times \bar{H} = \bar{J} \longrightarrow (1)$$

take divergance on both sides

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} \longrightarrow (2)$$

from vector identity $\nabla \cdot \nabla \times \bar{H} = 0$

$$\therefore \boxed{\nabla \cdot \bar{J} = 0} \longrightarrow (3)$$

-3M

Eqn (3) is Contervversary to current continuity Eqn

$$\nabla \cdot J = \frac{-\partial \rho_v}{\partial t} \quad \text{i.e} \quad \nabla \cdot \bar{J} \neq 0.$$

-so need modification in Eqn (1)

$$\nabla \times \bar{H} = \bar{J} + \bar{q} \rightarrow (4)$$

$$\nabla \cdot (\nabla \times \bar{H}) = (\nabla \cdot \bar{J} + \nabla \cdot \bar{q})$$

when $\nabla \cdot (\nabla \times \bar{H}) = 0$

$$\nabla \cdot \bar{J} + \nabla \cdot \bar{q} = 0$$

$$\nabla \cdot \bar{q} = -\nabla \cdot \bar{J} = \frac{\partial \rho_v}{\partial t}$$

-4M.

$$\boxed{\nabla \times H = \bar{J} + \frac{\partial v}{\partial t}}$$ Displacement current Density.

Q9 b. Derive general wave equation of $\bar{E}$ and $\bar{H}$ for the media with parameters μ, ϵ and σ.

Ans: General Maxwell's Eqn

$$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t} \longrightarrow (1)$$

$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{d} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t} \longrightarrow (2)$

$\nabla \cdot \bar{B} = 0$; $\nabla \cdot \bar{H} = 0$; $\nabla \cdot \bar{D} = 0$; $\nabla \cdot \bar{E} = 0$

take curl on both side of Eqn (1)

$\nabla \times (\nabla \times \bar{E}) = -\mu \nabla \times \frac{\partial \bar{H}}{\partial t} = -\mu \frac{\partial}{\partial t}(\nabla \times \bar{H}) \rightarrow (5)$

from (2) $\nabla \times (\nabla \times \bar{E}) = -\mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2}$ — 4M

from vector identity $\nabla \times \nabla \times E = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$

$= -\nabla^2 \bar{E}$

Eqn (5) becomes

$$\boxed{\nabla^2 E = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}}$$

and with $\bar{D} = \epsilon \bar{E}$

$$\boxed{\nabla^2 D = \mu\sigma \frac{\partial \bar{D}}{\partial t} + \mu\epsilon \frac{\partial^2 D}{\partial t^2}}$$

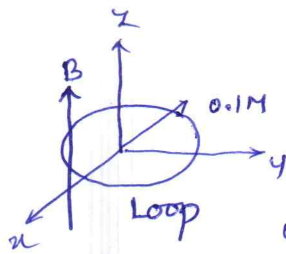
Similarly by taking curl on both sides of Eqn (2) we get

$\nabla^2 H = \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}$ and.

hence $\nabla^2 \bar{B} = \mu\sigma \frac{\partial \bar{B}}{\partial t} + \mu\epsilon \frac{\partial^2 B}{\partial t^2}$ — 4M

Q9c. A circular loop conductor lies in z=0 plane. & has a radius of 0.1m and resistance of 5Ω. Given $\bar{B} = 0.2 \sin 10^3 t \hat{a}_z$ Tesla. determine the current in the loop.

Ans:



$\bar{B} = 0.2 \sin 10^3 t \, \bar{a}_z$

$d\bar{s} = r \cdot dr \cdot d\phi \, \bar{a}_z$

$\bar{B} ds = 0.2 \sin 10^3 t \cdot r \, dr \, d\phi$

where $\phi = \int_S \bar{B} \cdot d\bar{s} = \int_{\phi=0}^{2\pi} \int_{r=0}^{0.1} 0.2 r \sin 10^3 t \, dr \cdot d\phi$

$= 6.28 \sin 10^3 t$

$e = \frac{-d\phi}{\partial t} = -6.28 \cos 10^3 t$

$i = \frac{e}{R} = (-6.283 \cos 10^3 t)/5 = -1.256 \cos 10^3 t$ A

Q10 a. Derive Maxwell's EQn in integral and point form for static electric and Magnetic field.

Ans: From (a) Faraday's law

$$\oint \bar{E} d\bar{L} = 0 \;\&\; \nabla \times \bar{E} = 0$$ 2M

(b) Ampere's circuital law

$$\oint \bar{H} \cdot d\bar{L} = \oint_s \bar{J} \cdot ds \text{ and } \nabla \times \bar{H} = \bar{J}$$ 2M

(C) Gauss law

(i) for static electric field $\oint \bar{D} \cdot ds = \oint_v \rho_v dv \;\&\; \nabla \cdot D = \rho_v$

(ii) for magnetic field $\oint_s \bar{B} \cdot d\bar{s} = 0 \;\&\; \nabla \cdot B = 0$ 4M

Q10 b. A 9375MHz uniform plane wave is propagating in polystyrene. If the amplitude of electric field intensity is 20 V/m and the material is assumed to be lossless, find Attenuation constant (α), Phase constant (β), wave length (λ), Velocity of propagation (v), intrinsic impedance (η), propagation constant (γ) and amplitude of magnetic field. $\mu_r = 1$ and $\epsilon_r = 2.56$.

Ans:

$f = 9375\,MHz$

$E_x = 20\,V/m$

$\mu_r = 1 \quad \epsilon_r = 2.56$

$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \mu_r$

$\epsilon = \epsilon_0 \epsilon_r = 8.85 \times 10^{-12} \epsilon_r$

$\alpha = 0$

$\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 2.56} = 314.37\,r$

$\lambda = 2\pi/\beta = 2\pi/314.37 = 0.0199$

$\eta = E_x / H_y$

$H_y = E_x/\eta \quad \therefore \eta = \sqrt{\mu/\epsilon} = 235.45$

$H_y = 20/235.45 = 0.08495\ A/m$

-6M

$v = f\lambda = 1.873 \times 10^8\,m/s$

$\gamma = \alpha + j\beta = j\,314.37$

Q10c State and Explain Poynting theorem.

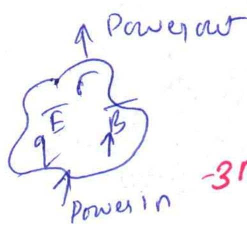
Ans: States - that the rate at which electromagnetic energy decreases within a given volume is equal to the net electromagnetic power flowing out through the surface of the volume plus the power dissipated inside the volume.

$$-\frac{d}{dt}\int_v u\,dv = \oint_s \vec{S} \cdot d\vec{A} + \int_v \vec{J} \cdot \vec{E}\, dv$$ 2M

$\bar{P} = \bar{E} \times \bar{H}$ if $E = E_m \cos(\omega t - \beta l)\, a_x$, $H = H_m(\cos(\omega t - \beta l))\, a_y$ with $H_m = E_m/\eta_0$

$\bar{P} = E_x \cdot E_y$ as

$(\bar{P} = \bar{P}_z \bar{a}_z)$



-3M