CBCS SCHEME

BME403

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Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025
Fluid Mechanics

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

			M	L	C
		Module – 1			
Q.1	a.	Define the following properties of fluids and mention their SI units: (i) Mass Density (ii) Specific weight (iii) Kinematic viscosity	06	L1	CO1
	b.	Calculate the dynamic viscosity of oil which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with an angle of inclination 30°. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of the oil film is 1.5 mm.	08	L3	CO1
	c.	Calculate the capillary rise in a glass tube of 3.0 mm diameter when immersed vertically in (i) water and (ii) mercury. Take surface tensions for mercury and water as 0.0725 N/m and 0.52 N/m respectively in contact with air. Specific gravity for mercury is given as 13.6.	06	L3	CO1
		OR			
Q.2	a.	Distinguish between (i) Absolute pressure (ii) Gauge pressure (iii) Gauge vacuum (iv) Atmospheric pressure. Indicate their relative position on a chart.	06	L2	CO1
	b.	Derive an expression for the total pressure and the depth of centre of pressure for a inclined surface submerged in water.	08	L3	CO1
	c.	A square plate of 1.5 m side is immersed in water vertically. Find the hydrostatic force on the plate and the depth of centre of pressure from free surface of water. When its upper side is 0.5 m below the free surface of water.	06	L3	CO1
		Module – 2			
Q.3	a.	Define the following : (i) Steady and Unsteady flow (ii) Compressible and Incompressible flow (iii) Laminar and Turbulent flow	06	L2	CO2
	b.	Define the equation of continuity. Obtain an expression for continuity equation for a three-dimensional flow.	08	L3	CO2
	c.	The velocity components in a two-dimensional flow are : $u = 8x^2y - \frac{8}{3}y^3$ and $v = -8xy^2 + \frac{8}{3}x^3$ Show that these velocity component represent a possible case of an irrotational flow.	06	L3	CO2

OR

Q.4	a.	Prove that the maximum velocity in a circular pipe for viscous flow is equal to two times the average velocity of the flow.	08	L3	CO2
	b.	A fluid of viscosity 0.5 poise and specific gravity 1.20 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as 147.15 N/m^2. Find (i) The pressure gradient (ii) The average velocity (iii) The Reynolds number of the flow.	08	L3	CO2
	c.	Define Reynolds number. Explain its significance in fluid flow.	04	L2	CO2

Module – 3

Q.5	a.	Derive Euler's equation of motion along a stream line. Obtain Bernoulli's equation from Euler's equation. Mention the assumptions made.	08	L3	CO3
	b.	Derive the expression for the rate of flow of fluid through a horizontal venturimeter.	06	L3	CO3
	c.	A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 14.715 N/cm^2 and vacuum pressure at the throat is 40 cm of mercury. Find the discharge of water through venturimeter. Take $C_d = 0.98$.	06	L3	CO3

OR

Q.6	a.	Explain the procedure to find the loss of head due to friction in pipes using (i) Darcy formula and (ii) Chezy's formula.	06	L2	CO3
	b.	Obtain expression for head loss in a sudden expansion in the pipe. List all the assumptions made in the derivation.	08	L3	CO3
	c.	Calculate the rate of flow of water through a pipe of diameter 300 mm. When the difference of pressure head between the two ends of a pipe 400 m apart is 5 m of water. Take value of f = 0.009 in the formula. $h_f = \dfrac{4 f l V^2}{d \times 2g}$	06	L3	CO3

Module – 4

Q.7	a.	What do you understand by the terms boundary layer and boundary layer theory?	04	L1	CO4
	b.	Define displacement thickness. Derive an expression for the displacement thickness.	08	L3	CO4
	c.	Oil with a free-stream velocity of 2 m/s flow over a thin plate 2 m wide and 2 m long. Calculate the boundary layer thickness and the shear stress at the trailing end point and determine the total surface resistance of the plate. Take specific gravity as 0.86 and kinematic viscosity as 10^{-5} m^2/s.	08	L3	CO4

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Q.08	a	Define the following dimensionless numbers: (i) Reynolds number (ii) Froude's number (iii) Euler's number (iv) Weber's number (v) Mach number.	10	L2	CO4
	b	The resisting force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft l, velocity V, air viscosity μ, air density ρ and bulk modulus of air K. Express the functional relationship between these variables and resisting force. Use Buckingham's π theorem method.	10	L3	CO4
Module-5					
Q.09	a	Derive an expression for the velocity of a sound wave in terms of change of pressure and change of density.	8	L3	CO5
	b	Differentiate between stagnation and static state.	4	L2	CO5
	c	Compute the velocity of a bullet fired in still air and Mach number when the Mach angle is 30^0. Take R = 0.28714 kJ/kgK and γ = 1.4. Assume air temperature to be 15^0c.	8	L3	CO5
OR					
Q.10	a	Derive the expressions for stagnation temperature and stagnation pressure in terms of Mach number.	8	L3	CO5
	b	Mention the advantages and disadvantages of CFD.	6	L2	CO5
	c	What are the steps involved in solving a CFD problem? Explain.	6	L2	CO5

KLS, Vishwanathrao Deshpande Institute of Technology – Haliyal
Department of Mechanical Engineering
VTU Solved Question paper: June/July 2025

Subject : Fluid Mechanics	Subject Code: BME403	Semester : IV	AY : 2025-26
Staff Name : Chandrakanth M.T			

Q1 (a) (i) Mass Density: Is defined as the ratio of the mass of a fluid to its volume. It is denoted by symbol "ρ" (rho).

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}} \quad \frac{kg}{m^3}.$$

(ii) Specific Weight: is the ratio between the weight of a fluid to its volume.

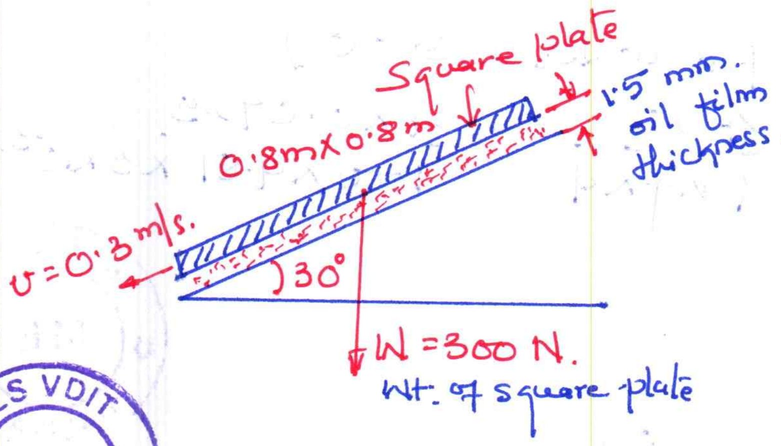
$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{\text{mass} \times g}{\text{volume}}.$$

$$w = \frac{\rho \times g}{\text{volume}} \quad N/m^3.$$

(iii) Kinematic Viscosity: It is defined as the ratio of dynamic viscosity and density of fluid.

$$\vartheta = \frac{\mu}{\rho} \quad \frac{m^2}{s}$$

Q1 (b)



$t = dy = 1.5mm \times 10^{-3}$ m

component of Weight W along the plane

$= W\cos 60°$

$= 300 \times \cos 60$

$= 150$ N.

and shear stress, $\tau = \frac{F}{Area} = \frac{150}{0.64}$ N/m²

Now, We have $\tau = \mu \dfrac{du}{dy}$

$du = u - 0 \Rightarrow u = 0.3 \text{ m/s}.$

$dy = 1.5 \times 10^{-3} \text{ m}.$

$$\therefore \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ Ns/m}^2$$

$$\mu = 1.17 \times 10 = 11.7 \text{ poise.}$$

Q1 (c) Given data :

Dia. of glass tube $d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}.$

σ for mercury $= 0.0725 \text{ N/m}$

σ for Water $= 0.52 \text{ N/m}.$

Sp. gr. of mercury $= 13.6.$

$\therefore$ density $= 13.6 \times 1000 = 13600 \text{ kg/m}^3.$

(i) Capillary rise for water ($\theta = 0$)

$$\therefore h = \frac{4\sigma}{\rho \times g \times d} = \frac{4 \times 0.0725}{1000 \times 9.81 \times 3 \times 10^{-3}}.$$

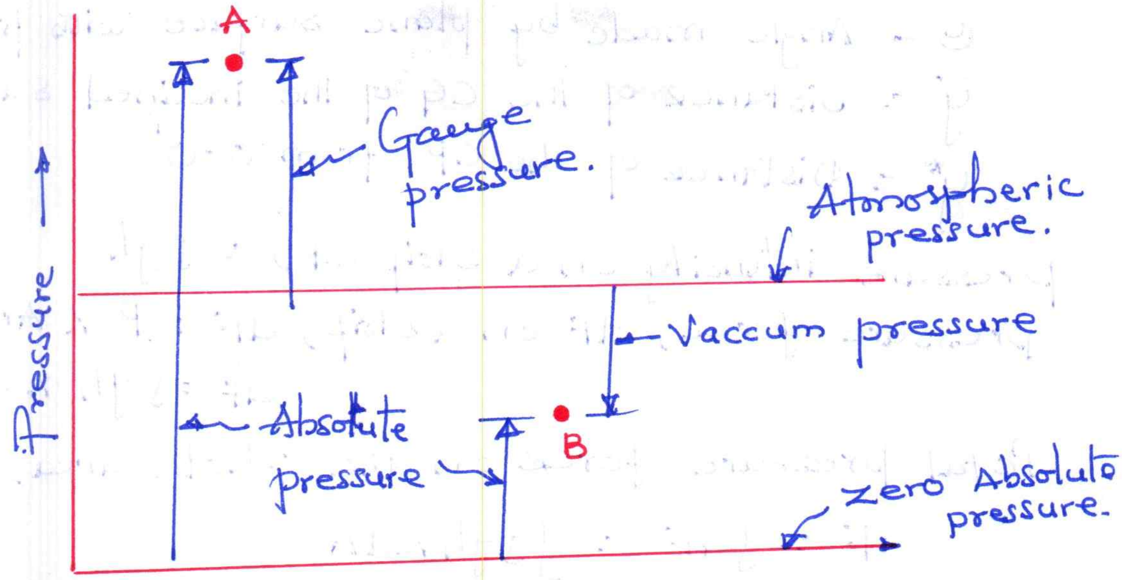
$=$

(ii) for mercury ($\theta = 130^\circ$)

$$\therefore h = \frac{4\sigma \cos\theta}{\rho \times g \times d} = \frac{4 \times 0.52 \times \cos 130^\circ}{13600 \times 9.81 \times 3 \times 10^{-3}}$$

$=$

Q2 (a)



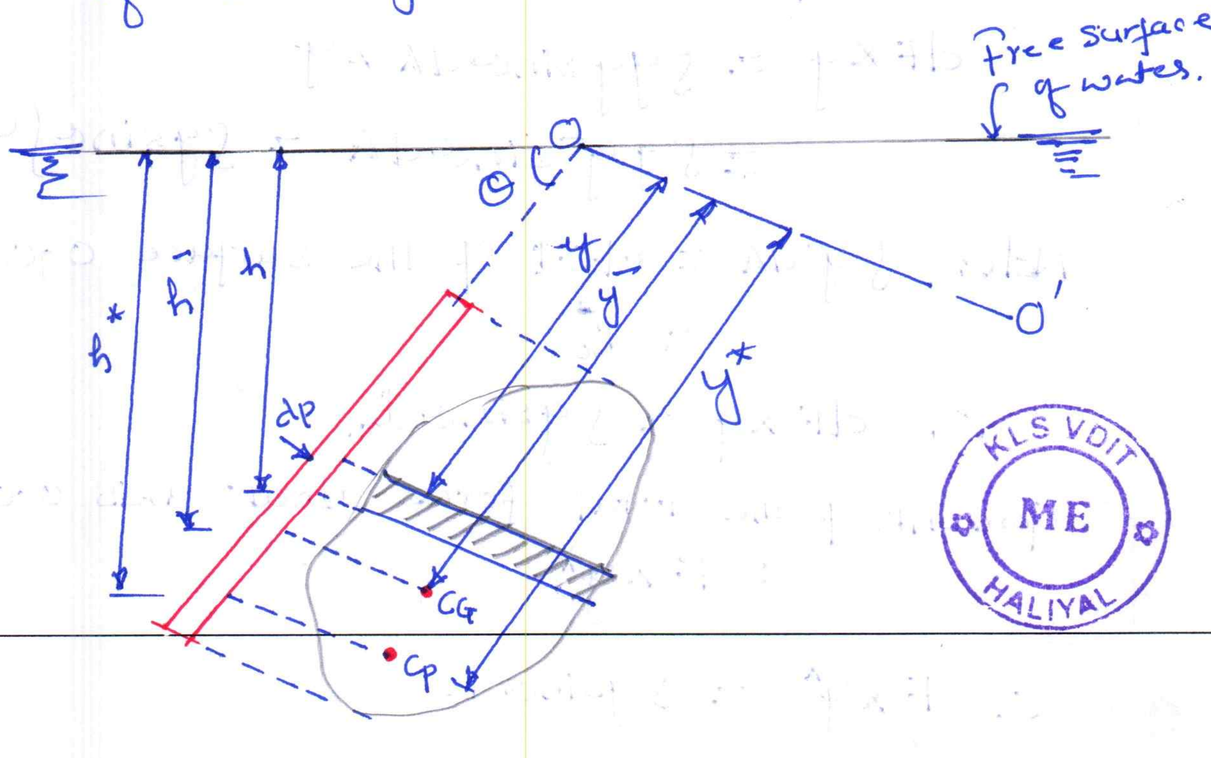
(i) Absolute pressure : is defined as the pressure which is measured with reference to absolute vaccum pressure.

(ii) Gauge pressure: is defined as the pressure which is measured with the help of pressure measuring instrument, in which the atm. pressure is taken as datum.

(iii) Gauge Vaccume : is defined as the pressure below the atmospheric pressure.

(iv) Atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Q2 (b)



θ - Angle made by plane surface with free surface

$\bar{y}$ = Distance of the CG of the inclined surface.

y^* = Distance of the CP from 0-0'

pressure intensity on a strip (P) = $\rho g h$

pressure force, dF on a strip, $dF = P \times Area$

$$dF = \rho g h \times dA$$

Total pressure force on the whole area, A

$$F = \int dF = \int \rho g h \times dA$$

But from fig: $\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin\theta.$

$$\therefore \quad F = \rho g \sin\theta \, \bar{y} \, dA \qquad | \bar{y}\sin\theta = \bar{h}$$

$$\boxed{F = \rho g \bar{h} \, dA} \quad \text{—— (1)}$$

Centre of pressure (h^*)

$$dF = \rho g h \, dA = \rho g y \sin\theta \, dA \qquad | h = y\sin\theta$$

Moment of force, dF about axis 0-0'

$$dF \times y = \rho g y \sin\theta \, dA \times y$$

$$= \rho g y^2 \sin\theta \, dA = \rho g \sin\theta \int y^2 dA$$

Note: $\int y^2 dA$ = M.O.I of the surface 0-0'

$$= I_0$$

$$\therefore \quad dF \times y = \rho g \sin\theta \, I_0.$$

Moment of the total force about axis 0-0' is

$$= F \times y^*$$

$$\therefore \quad F \times y^* = \rho g \sin\theta \, I_0$$

$$\frac{h^*}{\sin\theta} = \frac{\sin\theta\left[I_G + A\bar{y}^2\right]}{A\bar{h}}$$

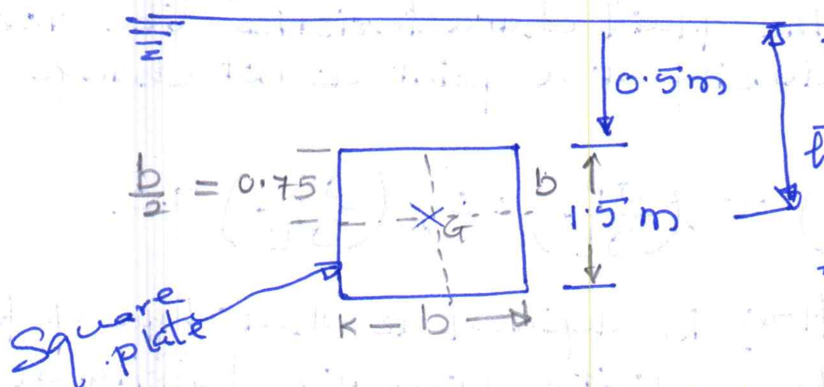
$$h^* = \frac{\sin^2\theta}{A\bar{h}}\left[I_G + A \times \frac{\bar{h}^2}{\sin^2\theta}\right]$$

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \cancel{A} \times \frac{\cancel{\sin^2\theta}}{\cancel{A}\bar{h}} \times \frac{\bar{h}^2}{\cancel{\sin^2\theta}}$$

$$\therefore \boxed{h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}}$$

Q2 (c)

Square plate of 1.5 m



$\bar{h} = 1.25$ m.

C.G $\frac{b}{2}$

M.I $b^4/12$

$A = 1.5 \times 1.5$
$= 2.25\ m^2$

Hydrostatic force on the plate

$$F = \rho g A \bar{h}$$

|| $\rho_{water} = 1000\ \frac{kg}{m^3}$

$$\therefore F = 1000 \times 9.81 \times 2.25 \times 1.25$$

$$F = 27{,}590.62\ N.$$

Centre of pressure is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$I_G = \frac{b^4}{12}$$

$$I_G = 0.421$$

$$= \frac{0.421}{2.25 \times 1.25} + 1.25$$

$$\therefore \boxed{h^* = 1.39\ m}$$

Module - 2

Q3 (a)

(i) steady flow and unsteady flow.

Steady flow is defined as that type of flow in which the fluid characteristics like velocity pressure, density at a point do not change with time.

$$\left(\frac{\partial V}{\partial t}\right)=0, \left(\frac{\partial p}{\partial t}\right)=0, \left(\frac{\partial \rho}{\partial t}\right)=0.$$

Unsteady flow is defined as that type of flow in which the velocity, pressure, density at a point changes w.r.t time.

$$\left(\frac{\partial V}{\partial t}\right)\neq 0, \left(\frac{\partial p}{\partial t}\right)\neq 0, \left(\frac{\partial \rho}{\partial t}\right)\neq 0.$$

(ii) ~~an~~ compressible and incompressible flow.

Compressible : is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ).

$$\rho \neq \text{constant}.$$

Incompressible : Is that type of flow in which the density is constant.

$$\rho = \text{constant}.$$

(iii) Laminar and Turbulent flow.

Laminar flow : The fluid elements move in well defined path and they remain the same relative position at cross-section of the flow passage.

Turbulent flow : The fluid element move in erratic and unpredictable flow.

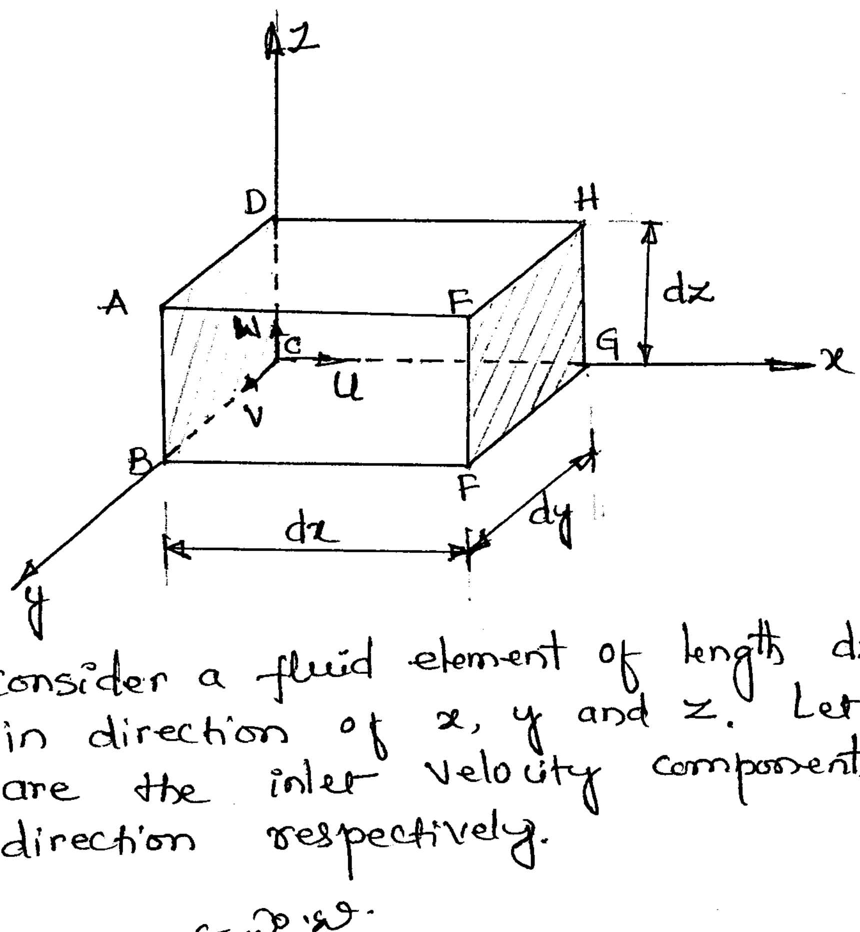
Q.No	Solution and Scheme	Marks

(iii) Laminar and Turbulent flow.

Laminar flow : When the various fluid particles moves in layers (Laminae) with one layer of fluid sliding smoothly over another layer, the flow is known as Laminar flow.

Turbulent flow: When the fluid particles move in an entirely indisciplined manner which results in rapid and continuous mixing of the fluid leading to formation of eddies. The flow is known as turbulent flow.

3b



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Consider a fluid element of length dx, dy and dz in direction of x, y and z. Let u, v and w are the inlet velocity components in x, y & z direction respectively.

8

Q.No	Solution and Scheme	Marks
	Mass of fluid entering the face ABCD per second $= \rho \times$ Velocity in x-direction $\times$ Area of ABCD $= \rho \times u \times (dy \times dz)$ then, Mass of fluid leaving the face EFGH per second $= \rho u\, dy\, dz + \frac{\partial}{\partial x}(\rho u\, dy\, dz)dx.$ $\therefore$ Gain of mass in x-directions = Mass through ABCD − Mass through EFGH per sec. $= \rho u \cancel{dy} dz - \rho u \cancel{dy} dz - \frac{\partial}{\partial x}(\rho u\, dy\, dz)dx.$ $= -\frac{\partial}{\partial x}(\rho u)\, dx\, dy\, dz$ || $\because dy\, dz$ is constant. Similarly the net gain of mass in y-directions $= -\frac{\partial}{\partial y}(\rho v)\, dx\, dy\, dz$ and in z-direction $= -\frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz$ $\therefore$ Net gain of mass $= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\, dy\, dz$ — (1) Mass of fluid in the element is $\rho\, dx\, dy\, dz$ and its rate of increase with time is. $\frac{\partial}{\partial t}(\rho \cdot dx \cdot dy \cdot dz)$ or $\frac{\partial \rho}{\partial t} dx \cdot dy \cdot dz$ — (2)	

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Equating the two expressions,

$$\text{or} \quad -\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]dx\,\cancel{dy}\,dz = \frac{\partial \rho}{\partial t}\,dx\,\cancel{dy}\,dz$$

$$\text{or} \quad \boxed{\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0} \quad (3)$$

Equation (3) is the continuity equation in Cartesian co-ordinates in its general form.

This equation is applicable to:

i) steady and unsteady flow.

ii) Uniform and non-Uniform flow,

iii) compressible and incompressible fluid.

Q3 (c) Given.

$$u = 8x^2y - \frac{8}{3}y^3 \quad \text{and} \quad V = -8xy^2 + \frac{8}{3}x^3$$

$$u = 8x^2y - \frac{8}{3}y^3$$

$$\frac{\partial u}{\partial x} = 8 \times 2xy = 16xy$$

Also, $\frac{\partial u}{\partial y} = 8x^2 - \frac{8}{\cancel{3}} \times \cancel{3} y^2 = 8x^2 - 8y^2$

$$\frac{\partial v}{\partial x} = -8y^2 + 8x^2$$

$$\frac{\partial v}{\partial y} = -16xy$$

(i) for a two-dimensional flow continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$16xy + (-16xy) = 16xy - 16xy = 0$$

$\therefore$ It is a possible case of flow.

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(ii) Rotation, ω_z is given by.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left[(-8y^2 + 8x^2) - (8x^2 - 8y^2)\right]$$

$$= \frac{1}{2}\left[\cancel{-8y^2} + \cancel{8x^2} - \cancel{8x^2} + \cancel{8y^2}\right]$$

$$= 0$$

$\therefore$ Rotation is zero, which means it is case of irrotational flow.

Q4 (a)

prove that $U_{max} = 2\bar{u}$

The velocity is maximum, when $r=0$ in equation $u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right]$

$$\therefore U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2 \quad \text{—} (1)$$

The fluid flowing per second through elementary ring

dQ = velocity at a radius 'r' x area of ring.

$= u \times 2\pi r\,dr$

$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] \times 2\pi r\,dr.$

$$Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \times 2\pi r\,dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2)\,r \cdot dr.$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 r - r^3)\,dr.$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$Q = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4} = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4$$

$$\therefore \bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4}{\pi R^2} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2 \quad \text{—} (2)$$

Dividing equation (1) by eq^n(2)

$$\therefore \frac{U_{max}}{\bar{u}} = 2 \Rightarrow \boxed{U_{max} = 2\bar{u}}$$

Q4 (b)

Given data :

Fluid Viscosity $= 0.5$ poise $= \frac{0.5}{10} \frac{Ns}{m^2}$

specific gravity 1.20

$\therefore$ density $(\rho) = 1.20 \times 1000 = 1200\ kg/m^3$.

Dia. of pipe $(D) = 100 mm = 0.1 m$.

max. shear stress, $\tau_{max} = 147.15\ N/m^2$.

(i) Pressure gradient, $\frac{dp}{dx}$

$$\tau_{max} = -\frac{\partial p}{\partial x} \frac{R}{2}$$

$$147.15 = -\frac{\partial p}{\partial x} \times \frac{D}{4} = -\frac{\partial p}{\partial x} \times \frac{0.1}{4}$$

$$\therefore \frac{\partial p}{\partial x} = -\frac{147.15 \times 4}{0.1} =$$

$\therefore$ Pressure Gradient =

(ii) Average Velocity, $\bar{u}$

$$\bar{u} = \frac{1}{2} U_{max} = \frac{1}{2}\left[-\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2\right]$$

$$= \frac{1}{8\mu} - \left(-\frac{\partial p}{\partial x}\right) R^2$$

$$\bar{u} =$$

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(iii) Reynold number $(R_e) = \frac{\bar{u} \times D}{\nu} = \frac{\bar{u} \times D}{\mu/\rho}$

$$R_e = \frac{\rho \times \bar{u} \times D}{\mu} = 1200 \times \quad \times 0.1$$

Q4 (c)

Defenation :

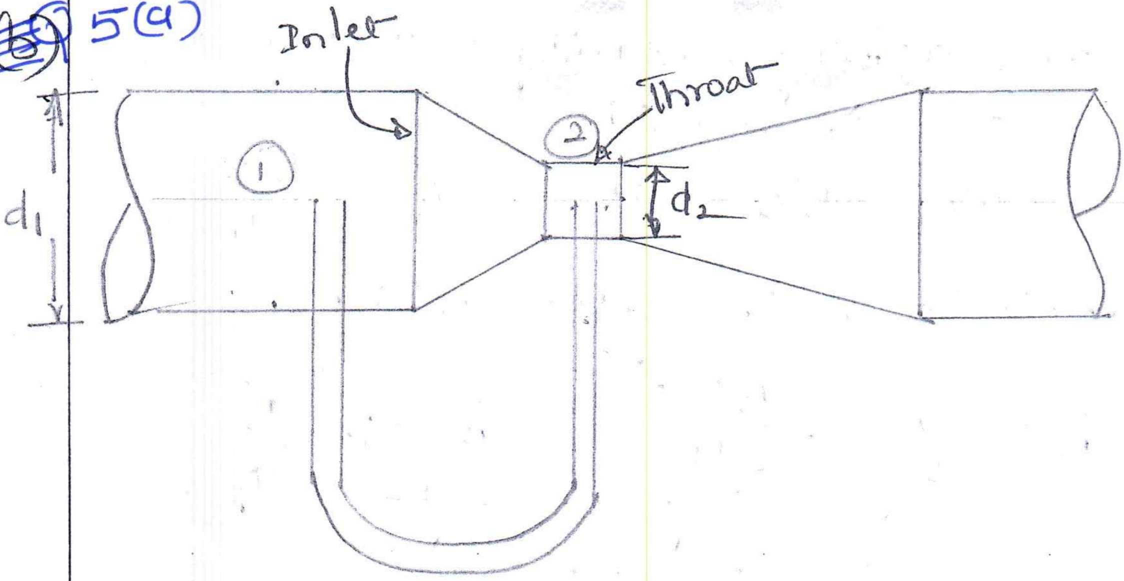
Reynolds number is a dimensionaless quanity used in fluid mechanics to predict the type of flow pattern in a fluid.

$$R_e = \frac{\rho V L}{\mu}$$

Significance : The Reynolds number is crucial because it helps to predict wheather a fluid flow will be laminar or Turbulent flow.

If $R_e < 2000$ then flow is laminar

If $R_e > 4000$ then flow is turbulent.

Q.No.	Solution and Scheme	Marks
~~3(b)~~ 5(a)		
	let, d_1 = diamter at inlet (1) d_2 = diameter at throat (2) v_1 = velocity at inlet v_2 = Velocity at throat, a_1 = area at (1) = $\frac{\pi}{4}(d_1)^2$ a_2 = area at (2) = $\frac{\pi}{4}(d_2)^2$	10
	Applying Bernoulli's equation at section 1 and 2, we get. $\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{v_2^2}{2g}+Z_2$ for Horizontal position of Venturimeter $Z_1=Z_2$ $\therefore \frac{P_1}{\rho g}+\frac{v_1^2}{2g} = \frac{P_2}{\rho g}+\frac{v_2^2}{2g}$. or $\frac{P_1-P_2}{\rho g} = \frac{V_2^2}{2g}-\frac{V_1^2}{2g}$ | $\frac{P_1-P_2}{\rho g}=h$ $h = \frac{V_2^2}{2g}-\frac{V_1^2}{2g}$	

1

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Q.No.	Solution and Scheme	Marks
	Now applying continuity equation at section (1) and (2).	

$a_1 V_1 = a_2 V_2$ or $V_1 = \frac{a_2 V_2}{a_1}$

Substitute V_1 in equation (1)

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$h = \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or $V_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$

$$\therefore V_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$\therefore$ Discharge $Q = a_2 V_2$

$$Q = a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$\boxed{Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$

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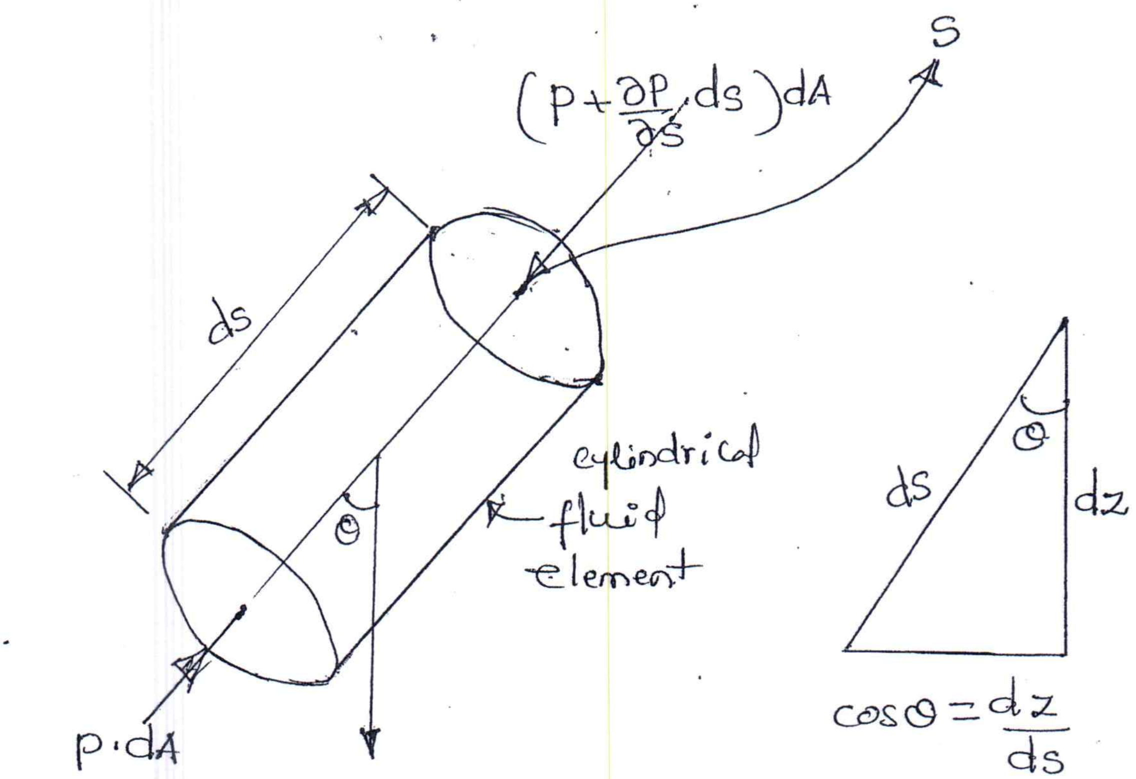
OR

Q.No	Question, solution and scheme.	Marks
5b	Derive an expression for Euler's equation of motion along a stream line and deduce Bernoulli's equation	10

Ans



$\cos\theta = \frac{dz}{ds}$

consider a stream-line in which flow is taking place in S-direction. consider a cylindrical element of cross-section dA and length ds.

The forces acting on the cylindrical elements are

1. pressure force in the direction of flow $= p.dA$
2. pressure force in the opposite to the direction of flow
$$= \left(p+\frac{\partial p}{\partial s}.ds\right)dA$$
3. Weight of element $= \rho g\, dA\, ds$

let, 'θ' is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element X acceleration in the direction S.

Q.No	Solution and scheme.	Marks

$$pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s \quad \text{--- (1)}$$

Where a_s is the acceleration in the direction of s.

Now, $a_s = \frac{dv}{dt}$, where v is a function of s & t

$$= \frac{\partial v}{\partial s} \cdot \frac{\partial s}{\partial t} + \frac{\partial v}{\partial t} = \frac{v \partial v}{\partial s} + \frac{\partial v}{\partial t} \qquad \because \frac{ds}{dt} = v$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore a_s = \frac{v \partial v}{\partial s}.$$

Substituting the value of a_s in equation (1)

$$-\frac{\partial p}{\partial s} ds dA - \rho g dA ds \cos\theta = \rho dA ds \times v \frac{\partial v}{\partial s} \quad \text{--- (2)}$$

Dividing by $\rho ds dA$ to equation (2) — $\rho \partial s dA$ is constant.

$$-\frac{\partial p}{\rho \partial s} - g\cos\theta = v \frac{\partial v}{\partial s}$$

or $\frac{\partial p}{\rho \partial s} + g\cos\theta + v \frac{\partial v}{\partial s} = 0.$ — $\cos\theta = \frac{dz}{ds}$

$$\therefore \frac{1}{\rho} \frac{\partial p}{\partial s} + g \frac{dz}{ds} + v \frac{\partial v}{\partial s} = 0$$

or $\frac{\partial p}{\rho} + g dz + v dv = 0 \quad \text{--- (3)}$

Equation (3) is known as Euler's equation of motion.

Bernoulli's equation is obtained by integrating the Euler's equation of motion.

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}.$$

If flow is incompressible, ρ is constant and

$$\therefore \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}.$$

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$$\frac{p}{\rho g} + \frac{V^2}{2g} + Z = \text{constant.}$$

or $$\boxed{\frac{p}{\rho g} + \frac{V^2}{2g} + Z = \text{constant}} \quad \text{--- (4)}$$

Equation (4) is a Benoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid

$\frac{V^2}{2g}$ = Kinetic head.

Z = potential head.

Q5 (c)

Given data

$d_1 = 20\,cm \Rightarrow a_1 = \frac{\pi}{4}(20)^2 = 314.16\,cm^2$

$d_2 = 10\,cm \Rightarrow a_2 = \frac{\pi}{4}(10)^2 = 78.74\,cm^2$

$P_1 = 14.715\,N/cm^2 = 14.715 \times 10^4\,N/m^2$

ρ for water $= 1000\,\frac{kg}{m^3}$

$$\frac{P_1}{\rho g} = \frac{14.715 \times 10^4}{9.81 \times 1000} = 15\,m \text{ of water}$$

$$\frac{P_2}{\rho g} = -40\,cm \text{ of mercury}$$

$= -0.40\,m$ of mercury

$= -0.40 \times 13.6 = -5.08\,m$ of water

$\therefore$ Differential head $= h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$

$= 15 - (-5.08)$

$= 20.08\,m$ of water

$h = 2044\,cm$ of water.

Discharge $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.98 \times \frac{314.16 \times 78.74}{\sqrt{(314.16)^2 - (78.74)^2}} \sqrt{2 \times 9.81 \times 2044}$$

$$= \frac{5032883721}{304} \times 200.25 =$$

Q6
(a)

(I) Using Darcy formula.

Formula $h_f = \frac{4fL}{D} \frac{v^2}{2g}$

where: h_f = loss of head due to friction (m)

f = coefficient of friction (dimensionless)

L = length of pipe (m)

D = Diameter of pipe (m)

v = mean velocity of flow (m/s)

g = acceleration due to gravity (9.81 m/s²)

Procedure:

(a) Find Velocity of flow: $v = \frac{Q}{A}$

where Q is discharge and A is cross sectional area.

(b) Determine the friction factor 'f'

* From Moody's chart or
* Given directly in the problem.

(c) Substitute values of L, D, f, v and g into Darcy formula.

(d) Calculate h_f, which gives the loss of head due to friction.

(II) Using Chezy's formula:

Formula for velocity: $V = C\sqrt{mi}$

where V = velocity of flow (m/s)

C = Chezy's constant.

m = hydraulic mean depth

i = hydraulic gradient

for circular pipe flowing full: $m = \frac{D}{4}$

$h_f = iL$

Procedure :

1. Calculate hydraulic mean depth

$$m = \frac{D}{4}$$

2. Find velocity of flow
   * From discharge: $v = \frac{Q}{A}$ or
   * Given directly.

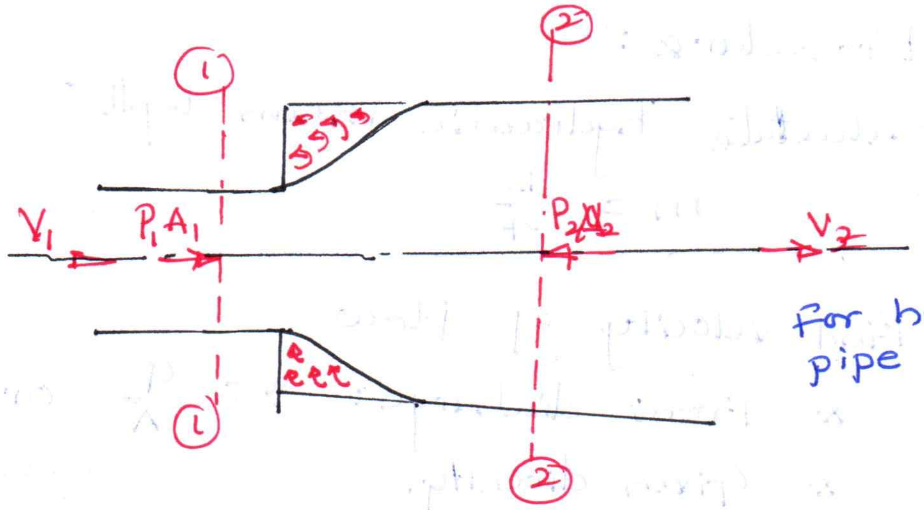
3. Rearrange Chezy's formula to find hydraulic gradient

$$i = \frac{v^2}{C^2 m}.$$

4. Calculate loss of head

$$h_f = iL = \frac{v^2 L}{C^2 m}$$

Q6
(b)



for horizontal pipe $Z_1 = Z_2$

let, P_1 = pressure intensity at section 1-1
V_1 = Velocity of flow at " 1-1
A_1 = Area of pipe at " 1-1

P_2, V_2, A_2 = corresponding values at section 2-2.

Applying Bernoulli's equations to section 1-1 & 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e. \quad \Big\| \; Z_1 = Z_2$$

$$\therefore \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e.$$

$$\text{or} \quad h_e = \left(\frac{P_1 - P_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad \text{—(i)}$$

momentum of liquid/sec at section 1-1 = mass x Velocity
$$= \rho A_1 V_1 \times V_1 = \rho A_1 V_1^2$$

Momentum of liquid/sec at section 2-2
$$= \rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$$

$$\therefore \text{change of Momentum/sec} = \rho A_2 V_2^2 - \rho A_1 V_1^2$$

But from continuity equation, we have.
$$A_1 V_1 = A_2 V_2 \quad \underline{\underline{\text{or}}} \quad A_1 = \frac{A_2 V_2}{V_1}$$

$\therefore$ change of momentum/sec $= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1^2$

$$= \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2 [V_2^2 - V_1 V_2]$$

or $\frac{P_1 - P_2}{\rho} = V_2^2 - V_1 V_2$

$\div$ by g to both sides, we have.

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g} \quad \underline{\text{or}} \quad \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}.$$

Substituting the values of $\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ in eq^n (i)

We get,

$$h_e = \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{g}.$$

$$= \frac{V_2^2 + V_1^2 - 2V_1V_2}{2g} = \frac{(V_1 - V_2)^2}{2g}$$

$$\therefore \boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

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Assumption made in the derivation.

i) Steady flow incompressible fluid.

ii) Uniform velocity distribution at section 1 & 2

iii) One dimensional flow

iv) No energy loss except due to expansion.

v) Viscous effects neglected.

Q6 (c)

Given data :

pipe of diameter $d = 300 \text{ mm} = 0.3 \text{ m}$

length of pipe $L = 400 \text{ m}$

Difference of pressure head, $h_f = 5 \text{ m}$ of water.

$f = 0.009$ in the formula $h_f = \frac{4fLV^2}{d \times 2g}$

$$h_f = \frac{4fLV^2}{d \times 2g}$$

$$5 = \frac{4 \times 0.009 \times 400 \times V^2}{0.3 \times 2 \times 9.81}$$

or $$V^2 = \frac{4 \times 0.3 \times 2 \times 9.81}{5 \times 0.009 \times 400} = 1.308$$

$V = 1.143 \text{ m/s}.$

$\therefore$ Discharge, Q = Velocity × Area.

$$= 1.143 \times \frac{\pi}{4} d^2$$

$$= 1.143 \times \frac{\pi}{4} (0.3)^2$$

$$= 0.1028 \text{ m}^3/\text{s}.$$

$$\boxed{Q = 102.28 \text{ litres/s.}}$$

Q7 (a)

Boundary Layer and Boundary layer Theory

When a fluid (like water or air) flows over a solid surface (like a pipe wall or flat plate), the fluid particales in direct contact with the surface stick to it due to viscosity. This is called the NO-SLIP condition (velocity at the surface = 0).

— As we move away from the surface, the fluid velocity gradually increases until it reaches the free stream velocity.

The thin layer near the solid surface where the velocity changes from zero at the surface to nearly the free stream velocity is called the boundary layer.

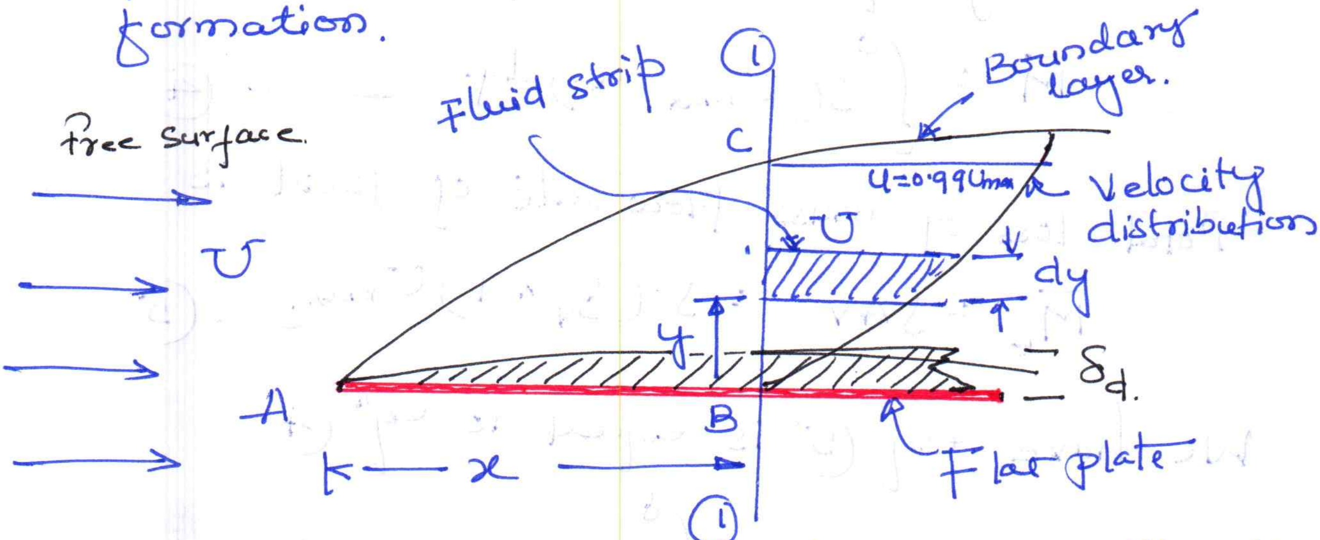
Main Assumption of Boundary layer Theory.

(i) Fluid is steady.
(ii) Fluid is incompressible.
(iii) Viscosity is important only within the boundary layer.
(iv) Boundary layer thickness is very small compared to body dimensions.
(v) Pressure across the boundary layer is constant

Q7 (b)

Displacement thickness (δ^*)

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the direction in flow rate on account of boundary layer formation.



Once the fluid layer moves from one position to another, in the boundary layer region, there arise change in mass flow rate $\overline{M}$ between fluid layer along the boundary layer thickness, in perpendicular distance from the flat plate and this change in mass flow rate is a loss of mass flow rate $\overline{M}_L$ due to effect present in the boundary layer up to a thickness of (δ_d) and hence

$$\overline{M}_L = \overline{M}$$

consider a fluid elementry strip thickness of dimension (dy, 1) at a distance of 'y' from the flat plate.

Change in mass flow rate occurs during the displacement of molecule layers, over entire boundary thickness (δ) be $\overline{M}$ where having $\overline{M}_2$ the mass flow rate occured at the free surface and $\overline{M}_1$ the mass flow rate at the elementary strip.

$$\overline{M}_2 = \int_0^{\delta} (\overline{M}_2 - \overline{M}_1) \quad \text{—— (1)}$$

mass flow rate in the boundary layer at free surface,

$$\overline{M}_2 = \rho(dy * 1)U_{max} \quad (2) \quad \Big| \because \overline{M}_2 = \rho AV.$$

mass flow rate in the boundary layer

$$\overline{M}_1 = \rho(dy * 1)u \quad —— (3)$$

Substituting eq^n (2), (3) in eq^n (1).

$$\overline{M} = \int_0^\delta \rho(U_{max} - u)\,dy \quad —— (4)$$

Total loss of mass flow rate of fluid is

$$\overline{M}_L = \rho AV = \rho(\delta_d * 1)U_{max} \quad —— (5)$$

We have eq (5) is equal to eq^n (4)

$$\Rightarrow \rho\delta_d U_{max} = \int_0^\delta \rho(U_{max} - u)\,dy$$

$$\boxed{\delta_d = \int_0^\delta \left(1 - \frac{u}{U_{max}}\right)dy}$$

Q 7 (e)

Given data :

Free stream velocity $U = 2$ m/s.

thin plate area $A = 2 \times 2$ m^2.

specific gravity as 0·86 — $\rho = 0.86 \times 1000 = 860$

Kinematic viscosity as 10^{-5} m^2/s.

Calculate : (I) Boundary layer thickness.
(II) shear stress at trailing point
(III) Total surface ~~~~ resistance of the plate :

Sol^n: (I) Boundary layer thickness

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$Re_x = \frac{U_{max} * L}{\nu} = \frac{2 * 2}{10^{-5}} = 400{,}000$$

$$\therefore \delta = \frac{5 \times 2}{\sqrt{400{,}000}} = \frac{10}{632.45}$$

$$\boxed{\delta = 0.0158 \text{ m}}$$

(II) shear stress at trailing point is

$$\tau_w = \frac{\rho U_M^2}{2} * C_f$$

$$= \frac{860 \times 4}{2} \times 1.049 \times 10^{-3}$$

$$\tau_w = 1720 \times 1.049 \times 10^{-3}$$

$$\tau_w = 1.788$$

where $C_f = \frac{0.664}{\sqrt{Re_x}}$

According to Blasius eq^n:

$\therefore C_f = 1.049 \times 10^{-3}$

$U_M = 2$ m/s

Q8 (a)

(i) Reynold's number (R_e): Is the ratio of Inertia force and viscous force. And it Indicates the overcome of inertia force over the viscous force and signifies in knowing the type of flow, i.e. Laminar or Turbulent.

$$\therefore R_e = \frac{\text{Inertia force}}{\text{Viscous force}} \Rightarrow R_e = \frac{m * a}{\tau * A}$$

(ii) Froude's Number (F_r): Is the ratio of inertia force and gravity force. It indicates the overcome of inertia force over the gravity force.

$$F_r = \sqrt{\frac{\text{Inertia force}}{\text{Gravity force}}} \Rightarrow F_r = \sqrt{\frac{\rho A V^2}{m * g}}$$

(iii) Euler's Number (E_r): Is the ratio of inertia force and pressure force.

$$E_r = \sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}} \Rightarrow E_r = \frac{V}{\sqrt{P/\rho}}$$

(iv) Weber's Number (W_r): Is the ratio of inertia force and surface tension force.

$$W_r = \sqrt{\frac{\text{Inertia force}}{\text{Surface tension force}}} \Rightarrow W_r = \frac{V}{\sqrt{\sigma/\rho L}}$$

(v) Mach Number (M): Is the ratio of inertia force and elastic/compressive force.

$$M = \sqrt{\frac{\text{Inertia force}}{\text{elastic/compressive force}}}$$

$$M = \frac{V}{a}.$$

Q8 (b)

The functional relationship between the dependent and Independent variables is given by,

$$R = f(l, V, \mu, \rho, K)$$

or $f(R, l, V, \mu, \rho, K) = 0$ or constant.

Dimensions of different variables are.

$$\frac{F}{MLT^{-2}} \quad \frac{l}{[L]} \quad \frac{V}{[LT^{-1}]} \quad \frac{\mu}{[ML^{-1}T^{-1}]} \quad \frac{\rho}{[ML^{-3}]} \quad \frac{K}{[ML^{-1}T^{-2}]}$$

* Total number of variables, $n = 6$
- Number of primary dimensions, $m = 3$.

$\therefore$ Number of π terms $= n - m = 6 - 3 = \underline{\underline{3}}$

$\therefore f(\pi_1, \pi_2, \pi_3) = 0$ or constant. —— a

The repeating variables are selected such that,

i) Geometric property (l)

ii) Flow property (V)

(iii) fluid property (ρ).

Now, as per Buckingham π theorem the π terms can be written as,

$$\pi_1 = l^{x_1} V^{y_1} \rho^{z_1} R \qquad \text{—— b}$$

$$\pi_2 = l^{x_2} V^{y_2} \rho^{z_2} \mu \qquad \text{—— c}$$

$$\pi_3 = l^{x_3} V^{y_3} \rho^{z_3} K. \qquad \text{—— d}$$

1. consider equation (b) and substitute the dimensions.

$$M^0L^0T^0 = [L]^{x_1}[LT^{-1}]^{y_1}[ML^{-3}]^{z_1}[MLT^{-2}]$$

Equating the power of MLT on both sides,

for M : $0 = z_1 + 1$ $\quad\therefore z_1 = -1$

For L : $0 = x_1 + y_1 - 3z_1 + 1$ $\quad\therefore x_1 + y_1 = -4$

for T : $0 = -y_1 - 2$ $\quad\therefore y_1 = -2$

and $x_1 = -2$

Substitute the value of x_1, y_1, z_1 in eq (b).

$$\pi_1 = l^{-2} V^{-2} \rho^{-1} F$$

$$\therefore \boxed{\pi_1 = \frac{F}{l^2 V^2 \rho}}$$

2. considering equation (c) and substitute the dimensions.

$$M^0L^0T^0 = [L]^{x_2}[LT^{-1}]^{y_2}[ML^{-3}]^{z_2}[ML^{-1}T^{-1}]$$

Equating power of MLT on both sides.

for M : $0 = z_2 + 1$ $\quad\therefore z_2 = -1$

for L : $0 = x_2 + y_2 - 3z_2 - 1$ $\quad\therefore x_2 + y_2 = -2$

for T : $0 = -y_2 - 1$ $\quad\therefore y_2 = -1$

and $x_2 = -1$.

substituting values of x_2, y_2, z_2 in eq^n (c)

$$\therefore \pi_2 = l^{-1} V^{-1} \rho^{-1} \mu$$

$$\therefore \boxed{\pi_2 = \frac{\mu}{l V \rho}}$$

3. consider equation (d) and substitute the dimensions,

$$M^0 L^0 T^0 = [L]^{x_3} [LT^{-1}]^{y_3} [ML^{-3}]^{z_3} [ML^{-1}T^{-2}]$$

Equating the power of MLT on both sides.

For M: $0 = z_3 + 1$ $\therefore z_3 = -1$

For L: $0 = x_3 + y_3 - 3z_3 - 1$ $\therefore x_3 + y_3 = -2$

For T: $0 = -y_3 - 2$ $\therefore y_3 = -2$

and $x_3 = 0$.

Substituting the values of x_3, y_3, z_3 in eq^n (d)

$$\therefore \pi_3 = l^0 V^{-2} \rho^{-1} K$$

$$\therefore \boxed{\pi_3 = \frac{K}{V^2 \rho}}$$

Now, put the values of π_1, π_2, π_3 in eq^n (a)

$$\therefore f\left(\frac{R}{l^2 V^2 \rho}, \frac{\mu}{l V \rho}, \frac{K}{V^2 \rho}\right) = 0 \text{ or constant.}$$

$$\text{or} \quad \frac{R}{l^2 V^2 \rho} = \phi\left(\frac{\mu}{l V \rho}, \frac{K}{V^2 \rho}\right)$$

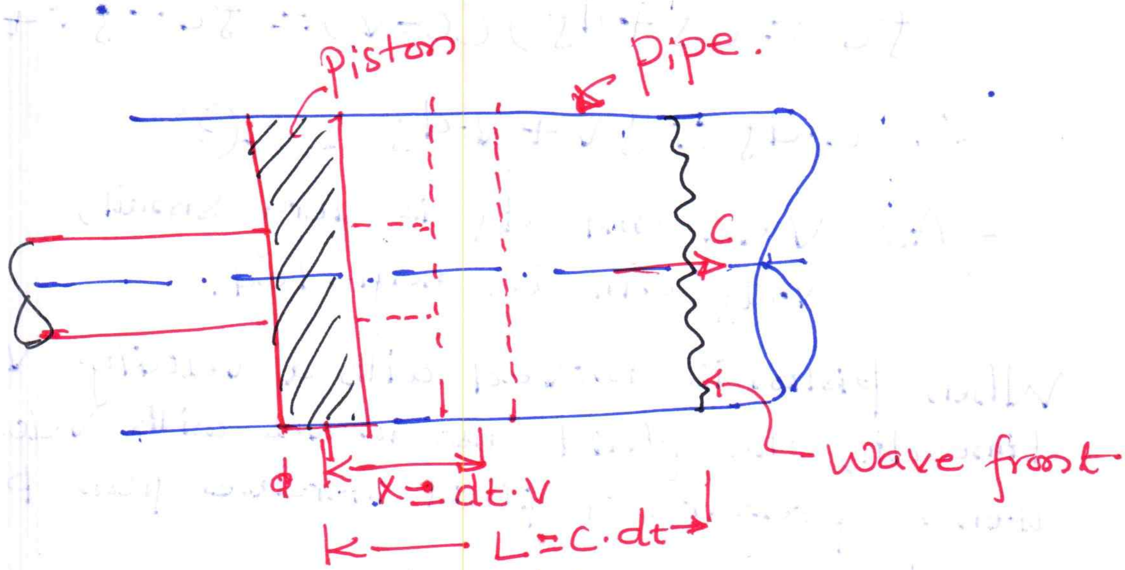
$$\text{But,} \quad \frac{\mu}{l V \rho} = \frac{1}{(l V \rho / \mu)} = \frac{1}{R_e}$$

$$\text{and} \quad \frac{K}{V^2 \rho} = \frac{1}{V^2 \rho / K} = \frac{1}{M^2}$$

$$\therefore \boxed{R = l^2 V^2 \rho \, \phi(R_e, M)}$$

Module : 5

Q9 (a)



Let, V = Velocity of piston

A = Cross-section area of the pipe.

P & ρ = Pressure and density of fluid in pipes before the movement of the piston.

C = Velocity of pressure wave or sound wave.

L ~~dt~~ = Distance travelled by the pressure wave.

dt = Small internal of time in which piston

X = Distance travelled by the piston in time.

$P + dp$ = pressure after compression.

$\rho + d\rho$ = Density after compression.

- The mass of fluid before compression (for L)

m_1 = ρ x volume of fluid upto length L

$\therefore \; m_1 = \rho \times AL = \rho \times A \times C \cdot dt$ — (1)

$|||^{ly}$, The mass of fluid after compression (for L−X)

$m_2 = (\rho + d\rho) \times A \times (L - X)$

$\therefore \; m_2 = (\rho + d\rho) \times A \times (C - V)\,dt.$ — (2)

But, the mass of fluid before ~~expa~~ compression must be equal to the mass of fluid After compression (1) = (2).

$\rho A C\, dt = (\rho + d\rho) A (C - V) dt$

$$\rho c = (\rho + d\rho)(c - v) = \rho c - \rho v + c \cdot d\rho - v \cdot d\rho$$

$$\therefore \; c \cdot d\rho = \rho V + V \cdot d\rho \quad \text{— (3)}$$

As $V < c$ and $d\rho$ is very small, $V \cdot d\rho$ can be neglected.

When piston is moved with a velocity V for time dt, the fluid also moves with velocity V. Hence pressure of fluid increases from p to $p + dp$.

$$\therefore \; (p + dp)A - p \times A = \text{Total mass} \times \frac{\text{Change in velocity}}{\text{Time}}$$

$$\therefore \; p \times A + A \cdot dp - p \times A = \frac{\rho A L}{dt}(V - 0)$$

$$\therefore \; A \cdot dp = \frac{\rho A c \cdot dt}{dt}(V) = \rho A c V$$

$$\therefore \; dp = \rho c V$$

$$\rho V = \frac{dp}{c} \quad \text{— (4)}$$

Substituting eq^n (4) in eq^n (3).

$$\therefore \; c \cdot d\rho = \frac{dp}{c}$$

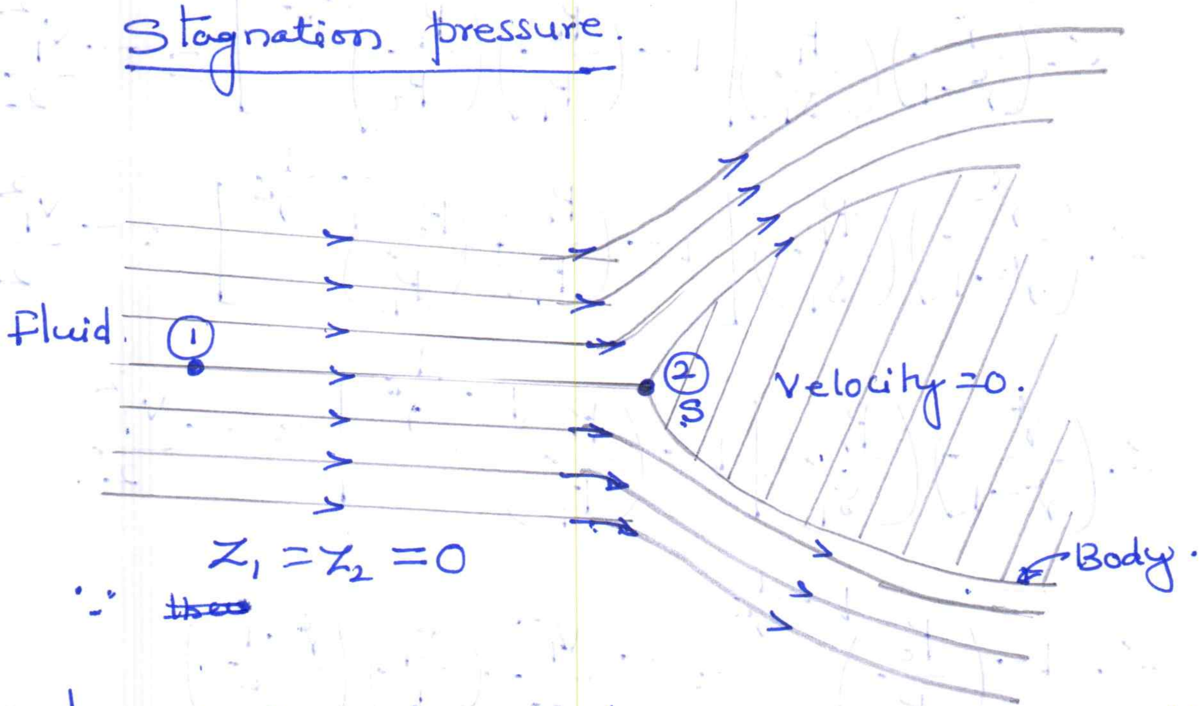
$$\therefore \; c^2 d\rho = dp$$

or $$c^2 = \frac{dp}{d\rho}$$

or $$\boxed{C = \sqrt{\frac{dp}{d\rho}}}$$

Q10

(a) Stagnation pressure.



$Z_1 = Z_2 = 0$

$\therefore$ ~~then~~

Now applying Bernoulli's equation between the points 1 and 2.

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{P_1}{\rho_1 g}\right) + \frac{V_1^2}{2g} + Z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 = Z_2 = 0, \quad V_2 = 0, \quad \rho_2 = \rho_0, \quad P_2 = P_0.$$

$$\therefore \left(\frac{\gamma}{\gamma-1}\right)\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} = \left(\frac{\gamma}{\gamma-1}\right)\frac{P_0}{\rho_0 g}$$

$$\therefore \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{P_1}{\rho_1} - \frac{P_0}{\rho_0}\right) = -\frac{V_1^2}{2}$$

$$\therefore \left(\frac{\gamma}{\gamma-1}\right)\frac{P_1}{\rho_1}\left[1 - \left(\frac{P_0}{P_1}\right)\left(\frac{\rho_1}{\rho_0}\right)\right] = -\frac{V_1^2}{2} \quad \text{——— (1)}$$

For an adiabatic process,

$$\frac{P}{\rho^\gamma} = \text{constant} \qquad \therefore \frac{P_1}{\rho_1^\gamma} = \frac{P_0}{\rho_0^\gamma}$$

$$\therefore \frac{\rho_1}{\rho_0} = \left(\frac{P_1}{P_0}\right)^{1/\gamma} \quad \text{——— (2)}$$

Substituting eq^n (2) in equation (1).

$$\therefore \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{P_1}{\rho_1}\right)\left[1-\left(\frac{P_0}{P_1}\right)\left(\frac{P_1}{P_0}\right)^{\frac{1}{\gamma}}\right] = -\frac{V_1^2}{2}$$

$$\therefore \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{P_1}{\rho_1}\right)\left[1-\left(\frac{P_0}{P_1}\right)\left(\frac{P_0}{P_1}\right)^{-\frac{1}{\gamma}}\right] = -\frac{V_1^2}{2}$$

$$\therefore \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{P_1}{\rho_1}\right)\left[1-\left(\frac{P_0}{P_1}\right)^{1-\frac{1}{\gamma}}\right] = -\frac{V_1^2}{2}.$$

$$\therefore 1-\left(\frac{P_0}{P_1}\right)^{1-\frac{1}{\gamma}} = -\frac{V_1^2}{2}\left(\frac{\rho_1}{P_1}\right)\left(\frac{\gamma-1}{\gamma}\right) \quad \text{--- (3)}$$

For an adiabatic process, the velocity of sound is $C = \sqrt{\gamma R T} = \sqrt{\gamma\left(\frac{P}{\rho}\right)}$ $\quad \| \because RT = \frac{P}{\rho}$

<u>At point 1</u>

$$C_1 = \sqrt{\gamma\left(\frac{P_1}{\rho_1}\right)}, \quad \therefore C_1^2 = \gamma\left(\frac{P_1}{\rho_1}\right)$$

$$\therefore \left(\frac{\rho_1}{P_1}\right) = \frac{\gamma}{C_1^2} \quad \text{--- (4)}$$

Substituting eq^n (4) in equation (3).

$$\therefore 1-\left(\frac{P_0}{P_1}\right)^{1-\frac{1}{\gamma}} = -\frac{V_1^2}{2}\left(\frac{\cancel{\gamma}}{C_1^2}\right)\left(\frac{\gamma-1}{\cancel{\gamma}}\right)$$

$$\left(\frac{P_0}{P_1}\right)^{1-\frac{1}{\gamma}} = 1+\left[\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma}{\gamma-1}}$$

As Mach No. $M = \frac{V}{\ldots}$

$$\therefore \boxed{P_0 = P_1\left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma}{\gamma-1}}} \quad \text{— (5)}$$

Equation (5) is known as equation for stagnation pressure.

Stagnation Temperature

For point 2 ie, stagnation point.

$$P_0 = \rho_0 R T_0$$

$$\therefore T_0 = \frac{P_0}{\rho_0 R} \quad \text{— (6)}$$

wkt.

$$P_0 = P_1\left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma}{\gamma-1}} \quad \text{— (a)}$$

and

$$\rho_0 = \rho_1\left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{1}{\gamma-1}} \quad \text{— (b)}$$

Substituting eq^n (a) and (b) in (6)

$$T_0 = \frac{1}{R}\cdot\frac{P_1\left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{\gamma}{\gamma-1}}}{\rho_1\left[1+\left(\frac{\gamma-1}{2}\right)\cdot M^2\right]^{\frac{1}{\gamma-1}}}$$

$$T_0 = \frac{P_1}{\rho_1 R}\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\cancel{\frac{\gamma-1}{\gamma-1}}}$$

$$\therefore \boxed{T_0 = T_1\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]} \quad \text{— (d)}$$

The above equation (d) is known as stagnation temperature.

Q9 (b)

Stagnation Pressure (P_0): Also called "Total pressure" this is the pressure the fluid would reach if it were brought to rest isentropically (without friction or head loss).

Static Pressure (P): The actual thermodynamic pressure of the fluid. It is measured by an observer moving at the same velocity as the fluid, or by a probe placed parallel to the flow that does not distrub it.

Feature	Static	Stagnation
Motion:	Fluid is in motion (relative to the observer)	Fluid brought to Zero Velocity.
Formula:	P (the base pressure)	$P_0 = P_{static} + P_{dynamic}$
Energy:	Represents only the potential Energy	Includes both potential and kinetic Energy.
Measurement	Measured normal (perpendicular to the flow)	Measured by a pitot tube facing the flow.

Q9 (c)

<u>Given data</u> :

$\alpha = 30^\circ$, $R = 0.28714$ kJ/kg K

$\gamma = 1.4$, $T_{air} = 273 + 15 = 288$ K.

(I) calculate the velocity of bullet fired.

Velocity of sound wave is

$$C = \sqrt{\gamma R T} = \sqrt{1.4 \times 0.28714 \times 288}$$

$$C = 97.84 \text{ m/s}$$

(II) Mach angle is given by,

$$\sin\alpha = \frac{C}{V}$$

$$\therefore \sin 30^\circ = \frac{97.84}{V}$$

$$\therefore V = \frac{97.84}{\sin 30^\circ} = 195.68 \text{ m/s.}$$

$$\boxed{V = 195.68} \text{ m/s.}$$

Q10 (b)

Advantages and Disadvantages of CFD.

Advantages of CFD

i) Relative low cost :
- * Using physical experiments and test to get essential engineering data in design can be expensive.
- * CFD simulation are relatively inexepensive and cost are likely to decrease as computers becomes more powerful.

ii) Speed :
- * CFD simulation can be executed in a short period of time.
- * Quick turnaround means engineering data can be introduced early in the design proeess.

iii) Ability to simulate real conditions:
- * Mainly flow and heat transfer processes can't be (easily) tested. eg: hypersonic flow.

Disadvantages of CFD

i) It requires fair amount of knowledge and experience with both CFD and the phenomenon under investigation.

ii) The cost of tool or software require for CFD is very heigh.

iii) The solution of CFD problem is not fully reliable.

iv) To solve the CFD problem, it requires large number of input data.

Q10 (c)

Steps involved in solving a CFD problem.

(I) During preprocessing

* The geometry (physical bounds) of the problem is defined.
* The volume occupied by the fluid is divided into discrete cells (the mesh). The mesh may be uniform or non-uniform.
* The physical modeling is defined - for example the equation of motions, enthalpy, radiation, etc.,
* Boundary conditions are defined. This involves specifying the fluid behaviour and properties at the boundaries of the problems.

(II) The Simulation is started and the equations are solved iteratively as a steady-state or transient.

(III) Finally a postprocessor is used for the analysis and the results are interpreted in the ~~visual~~ visual form like graphs, images, animations etc.,

The following methods are commonly used:

i) Finite volume method.
ii) Finite element method.
iii) Finite difference method
iv) Spectral element method.
v) Boundary element method.

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