Q 1. a> One Dimensional Time-Independent Schrodinger equation, The time-dependent Schrodinger equation is

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x,t)$$

Assume separation of variables

$$\psi(x,t) = \psi(x)\, e^{-iEt/\hbar}$$

Substituting and simplifying

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

b>. 1> Physical Significance of Wave function

* $|\psi|^2 \rightarrow$ Probability density
* Must satisfy normalization $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

2> Complementarity Principle

Wave-particle duality; particle and wave aspects are complementary and cannot be observed simultaneously.

3> Expectation value

$$\langle A \rangle = \int \psi^* A \psi \, dx$$

4> Quantum Tunneling

Particle can penetrate potential barrier even if $E < V_0$. Transmission probability decreases exponentially.

c) De-Broglie Wavelength (100 eV)

$E = 100\,eV$

$$\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda(\mathring{A}) = \frac{12.27}{\sqrt{E}}$$

$$= \frac{12.27}{\sqrt{100}} = 1.227\,\mathring{A}$$

$$\boxed{\lambda = 1.23\,\mathring{A}}$$

Q2) a) Derive a normalized wave function for a particle inside one dimensional infinite potential well.

Boundary conditions for $0 < x < L$.

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

We know that $\psi = A \sin kx$

Boundary conditions $\psi(0) = 0$, $\psi(L) = 0$

$$k = \frac{n\pi}{L}$$

Normalization: $\int_0^L |\psi|^2 dx = 1$

$$A = \sqrt{\frac{2}{L}}$$

Final normalized wavefunction

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

b) State and explain Heisenberg uncertainty principle.

Heisenberg's uncertainty principle states that it is impossible to simultaneously know both the exact position and the exact momentum of a particle.

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

$$\Delta L \cdot \Delta \theta \geq \frac{\hbar}{2}$$

Broading of spectral lines $\Delta E = \frac{\hbar}{\Delta t}$

Finite lifetime → Energy uncertainty → Line broadening.

2c) Given: widt $L = 1 Å = 10^{-10} m$

Formula: $E_n = \frac{n^2 h^2}{8mL^2}$

$L = 1Å = 10^{-10} m$

$E_1 = 37.6 eV$

$E_2 = 4E_1 = 150.4 eV$

$E_3 = 9E_1 = 338.4 eV$

Module - 2

3. a) Define fermi energy and fermi factor. Discus the variation of fermi factor with temperature and energy.

Fermi energy is the highest energy level that electrons can occupy at absolute zero temperature, while the fermi factor is the probability that an electron state at a specific energy level is occupied at a given temperature.

Fermi factor $f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$

At 0K, : $\Rightarrow E < E_F \Rightarrow f = 1$

$E > E_F \Rightarrow f = 0$

At high T → smooth curve will be obtained

3.b> Hall effect: When a current-carrying conductor or semiconductor is placed in a magnetic field that is perpendicular to the direction of current flow. As a result of the magnetic field, a transverse electric field is developed across the material.

This produces a potential difference known as the Hall voltage.

Consider → Rectangular conducting or semiconductor plate.

→ current flows along the x-axis

→ Magnetic field along the z-axis

→ The Hall voltage develops along the y axis

The current I is given by

Fig

$$I = nqv_d(w \times t)$$

$$v_d = \frac{I}{nqwt}$$

when B is applied ⊥lar to the current,

$$F_B = qv_dB$$

As charge accumulate, an electric field E_H, Hall field develops in the opposite direction.

$$F_E = qE_H$$

at equilibrium $F_B = F_E$

$$qE_H = qv_dB$$

$$E_H = v_dB$$

<u>Hall voltage</u>: $V_H = E_H \times w$

$$V_H = v_d Bw \qquad \because E_H = v_d B$$

$$\boxed{V_H = \frac{IB}{nqt}} \qquad \because v_d = \frac{I}{nqwt}$$

Hall coefficient is defined as

$$R_H = \frac{E_H}{jB}$$

where j is current density $j = \frac{I}{wt}$

$$E_H = \frac{IB}{nqwt} = \frac{jB}{nq}$$

Thus, $R_H = \frac{1}{nq}$

Substitute $R_H = \frac{1}{nq}$ into $V_H = \frac{IB}{nqt}$

$$\boxed{V_H = \frac{BIR_H}{t}}$$

3. e) Solution:

$$f(E) = 0.01$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/K_BT}}$$

$$0.01 = \frac{1}{1 + e^{0.5/KT}}$$

$$e^{0.5/KT} = 99$$

$$\frac{0.5}{KT} = \ln 99 = 4.6$$

$$T = \frac{0.5}{4.6 \times 8.617 \times 10^{-5}}$$

$$\boxed{T \simeq 1260 \text{ K}}$$

4. a) 1) Failure to explain Electronic Specific Heat.

Predications (classical theory)

$$E = \frac{3}{2}kT \quad \text{per electron}$$

electronic specific heat $C_e = \frac{3}{2}Nk$. → This is a large value comparable to lattice specific heat.

Experimental result shows very small specific heat.

2) Failure to explain Temperature Dependence of Conductivity $\sigma = \frac{ne^2\tau}{m}$

3) Failure to Explain Wiedemann-Franz law quantitatively. $\frac{K}{\sigma T} = \text{constant}$

4) Failure to Explain Hall effect Correctly.

$$R_H = -\frac{1}{ne}$$

5) Failure to Explain Magnetic Properties

6) Failure to Explain why some solids are Insulators.

Assumptions of quantum free theory:

1) Conduction electrons move freely inside the metal like particle in a box.

2) Pauli exclusion Principle applies, no two electrons can have the same set of quantum numbers

3) Electrons obeys Fermi-Dirac distribution, no Maxwell-Boltzman statistics.

4) Electrons move independently, electron-electron interaction is neglected.

4b) In an intrinsic semiconductor, $n = p$. — ①

Electron concentration is given $n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

Hole concentration in valence Band $p = N_v \exp\left(-\frac{E_F - E_v}{kT}\right)$

$$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

From ① $N_c \exp\left(-\frac{E_c - E_F}{kT}\right) = N_v \exp\left(-\frac{E_F - E_v}{kT}\right)$

Take log and rearranging. $\frac{E_c - E_F}{kT} - \frac{E_F - E_v}{kT} = \ln \frac{N_c}{N_v}$

$$E_F = \frac{E_c + E_v}{2} - \frac{kT}{2} \ln \frac{N_c}{N_v}$$

Expression interms of Energy gap, $E_g = E_c - E_v$,

$$\frac{N_c}{N_v} = \left(\frac{m_e^*}{m_h^*}\right)^{3/2} \quad E_F = \frac{E_c + E_v}{2} + \frac{3}{2} kT \left(\frac{m_h^*}{m_e^*}\right)$$

4.c) Given: $t = 0.5\,mm = 5 \times 10^{-4}\,m$

$I = 5\,mA = 5 \times 10^{-3}\,A$

$B = 0.2\,T$

$V_H = 1\,mV = 10^{-3}\,V$

$$R_H = \frac{V_H t}{IB} = \frac{(10^{-3})(5 \times 10^{-4})}{(5 \times 10^{-3})(0.2)}$$

$$R_H = 5 \times 10^{-4}\,m^3/C$$

5.a) Phonons are the quatized modes of lattice vibrations in a crystalline Solid.

Energy of a Phonon $E = \hbar\omega$

where $\hbar = \frac{h}{2\pi}$, ω angular frequency of vibration

Rule of phonon in cooper pair formation.

Normally two electrons repel each other due to coulomb force. But in a superconductor at very low temperature electron Distorts the Lattice

⇒ Electron moving through the lattice attracts nearby positive ion.

⇒ This creates a slight local lattice distortion.

⇒ This distortion produces a phonon.

⇒ The lattice distortion increases positive charge density locally.

⇒ Acts like a temperary positive potential well.

⇒ Another electron feels attraction toward this positive region. This indirect ~~a~~ attraction via phonon overcomes coulomb repulsion at low temperature.

⇒ Two electrons form a weakly bound pair, have opposite spins, have opposite momenta and total spin zero. This pair is called cooper pair.

5b) Flux Quantization:

Super conducting electrons form Cooper pair (Charge=2e).

The wave function of these electrons are single-valued. This leads to quantization of flux.

In a superconducting ring, the magnetic flux is quantized.

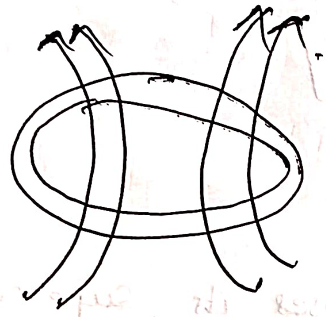
i.e $\phi = n\phi_0$ where $n = 1, 2, 3 \ldots$

$\phi_0 = \frac{h}{2e}$ = flux quantized.

$\phi_0 = 2.07 \times 10^{-15}$ Wb.

$$\boxed{\phi = n\frac{h}{2e}}$$

Diagram of Flux quantization:



DC Josephson Effect

when no voltage is applied across the Josephson junction a supercurrent flows through it.

$$I = I_c \sin\phi$$

where I_c = Critical current ϕ = Phase difference

when, $V = 0$, ⇒ Pure Supercurrent (no resistance), current depends on phase difference. This is called DC Josephson Effect

AC Josephson Effect: when a constant voltage V is applied across the junction, the supercurrent oscillates with time

$$f = \frac{2eV}{h}$$ where f = frequency of oscillation

V - applied voltage

Explanation: Applying voltage causes

$\frac{d\phi}{dt} = \frac{2eV}{h}$, current becomes $I = I_c \sin(\omega t)$

where $\omega = \frac{2eV}{h}$

So current oscillates produce AC current.

5 c). $T_c = 7.2K$ $H_c(0) = 6.5\times10^4\ Am^{-1}$

$T = 4K$ $H_c(4) = ?$

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$H_c(4) = 6.5\times10^4\left[1-\left(\frac{4}{7.2}\right)^2\right] = 6.5\times10^4(1-0.308)$$

$$\boxed{H_c = 4.49\times10^4\ A/m}$$

or

6a). A superconductor loses its superconducting property when the magnetic field produced by the current flowing through it becomes equal to its critical magnetic field.

In other words, superconductivity is destroyed when

$H_{surface} = H_c$.

Consider a long superconducting cylindrical wire of radius r and current flowing I.

Ampere's law can be written

$$\oint \vec{H}\cdot\vec{dl} = I$$

For long cylindrical wire

$$H(2\pi r) = I \Rightarrow H = \frac{I}{2\pi r}$$ this is B at the surface.

According silsbee effect, $H = H_c$

$$\frac{I_c}{2\pi r} = H_c$$

$$\boxed{I_c = 2\pi r H_c}$$

Since H_c depends on temperature

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$I_c(T) = 2\pi r H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

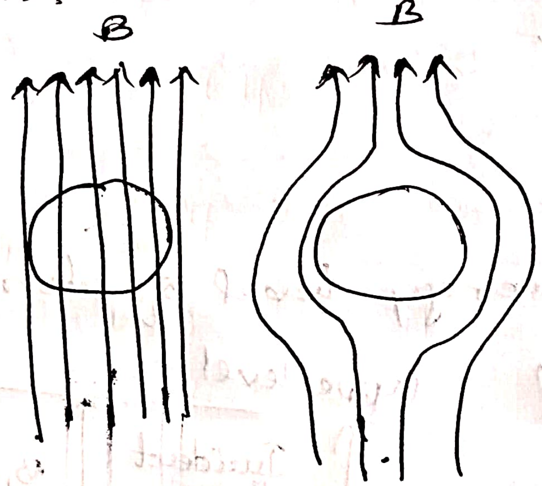
6b) The Meissner effect is the phenomenon in which a superconductor completely expels magnetic flux from its interior when it is cooled below its critical temperature T_c.

→ when a ~~magnetic~~ material is cooled below T_c, it becomes superconducting.

→ If placed in a magnetic field, the magnetic field lines are expelled from the interior. Magnetic induction inside the superconductor becomes $B=0$.

Thus superconductor behaves as a perfect diamagnet.

Diagram.



Distinction Between Type I and Type II Superconductors

Property	Type I	Type II
critical field (H_c)	one H_c	Two H_{c_1}, H_{c_2}
Magnetic behavior	complete expulsion	Partial expulsion
Transition	Sudden	Gradual
Magnetic field Strength	Low	Very high
Materials	Pure metals	Alloys & compounds

6 c) $r = 0.5\ mm = 0.5 \times 10^{-3}\ m$

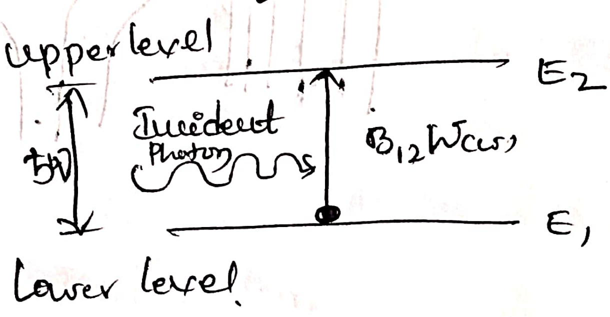
$H_c = 8\ kA m^{-1} = 8 \times 10^{3}\ A m^{-1}$

$I_c = 2\pi r H_c$

$= 2\pi (5 \times 10^{-4})(8 \times 10^{3})$

$$\boxed{I_c = 25\ A}$$

7. a) Consider two energy level of atomic system, lower energy level E_1 and upper energy level E_2. The energy difference between them is $h\nu$.



There are three possible radiation processes:

1). Absorption 2). Spontaneous emission 3) Stimulated emission

When an atom in excited state E_2 is struck by a photon of energy $h\nu$:

$$h\nu = E_2 - E_1$$

The atom is forced to return to ground state E_1 and emits a second photon.

Rate of stimulated Emission:

If $\rho(\nu)$ is the energy density of radiation.

Rate of stimulated emission $= B_{21}\,\rho(\nu)\,N_2$

Derivation: Einstein coefficients

Absorption $\quad R_{abs} = B_{12}\,\rho(\nu)\,N_1$

Spontaneous Emission : $\quad R_{sp} = A_{21}N_2$

Stimulated Emission $\quad R_{stim} = B_{21}\,\rho(\nu)\,N_2$

At thermal equilibrium

Absorption rate = Total emission rate

$$B_{12}\,\rho(\nu)\,N_1 = A_{21}N_2 + B_{21}\,\rho(\nu)\,N_2$$

$$\rho(\nu) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

Using Boltzman distribution $\quad \frac{N_2}{N_1} = e^{-\frac{h\nu}{KT}}$

$$\rho(\nu) = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\nu}{KT}} - 1}$$

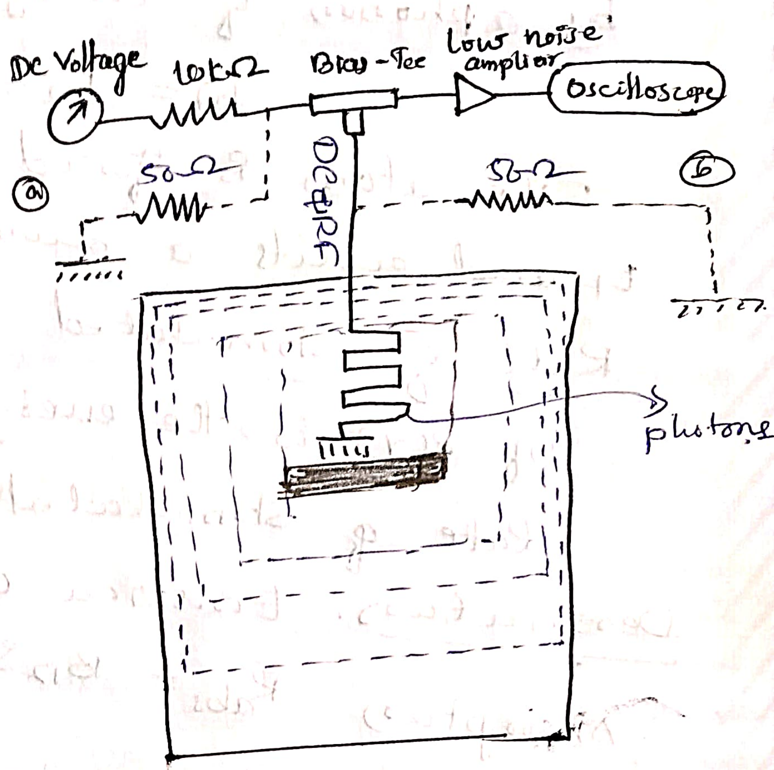
Einstein Relations.

$$B_{12} = B_{21}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

Final expression for Energy Density $\boxed{\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\cdot\frac{1}{e^{\frac{h\nu}{KT}} - 1}}$

7b): Superconducting Nanowire Single-Photon Detector:



Construction of SNSPD:

→ A SNSPD is an ultra-sensitive photon detector made from a thin superconducting nanowire patterned into a meander structure.

Components:

1). Superconducting nanowire:
Material: NbN, NbTiN,
Width: ~50–150nm
Thickness: 5–10nm

2). Usually Silicon substrate.

3) Bias current just below critical current I_c.

4) Cryogenic system.

Working of SNSPD:

⇒ Super conducting State: The nanowire is cooled below its critical temperature T_c. A bias current I_b is applied $I_b \lesssim I_c$. Wire is in superconducting state (zero resistance).

=> Absorption of photon temporarily destroy the superconductivity.

⇒ Formation of Resistance Barrier: The ~~bias~~ bias current is forced to flow around the hotspot. Current density increases. If it exceeds critical current density, the region becomes resistive.

⇒ As hotspot cools quickly, superconductivity is restored. Detector becomes ready for next photon.

7. c) Given: $n_1 = 1.48$ $n_2 = 1.46$

Numerical Aperture $NA = \sqrt{2.1904 - 2.1316}$

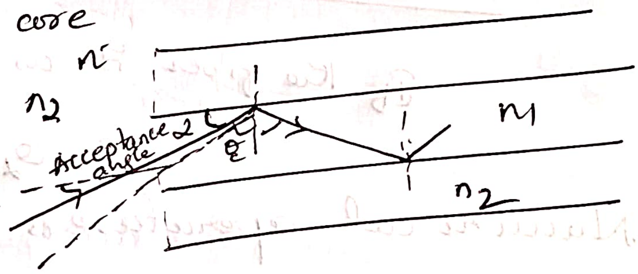
$NA = 0.242$

Acceptance angle $\theta = \sin^{-1}(0.242)$

$\boxed{\theta = 14^\circ}$

8. a) Acceptance Angle and Numerical Aperature of an optical Fiber.

An optical fiber consists of core of R.I n_1 and cladding R.I n_2 for light to propagate inside the fiber $n_1 > n_2$



So that Total internal Reflection (TIR) occurs at the core-cladding interface.

Using Snell's law $n_0 \sin\theta_a = n_1 \sin\theta_1$

θ_1 = angle of refraction inside the core

For TIR at core-cladding interface

$\theta_2 \geq \theta_c$

where θ_c = critical angle

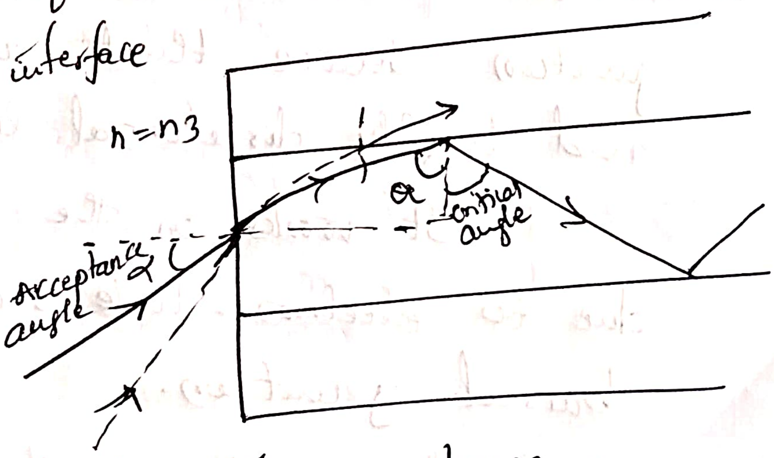
$\theta_2 = 90^\circ - \theta_1$

Critical angle condition

$\sin\theta_c = \frac{n_2}{n_1}$

For limiting case

$\theta_2 = \theta_c$



Ray outside acceptance cone is lost from core

So, $90° - \theta_1 = \theta_c$ $\theta_1 = 90° - \theta_c$

Taking sine

$$\sin\theta_1 = \cos\theta_c$$

$$\sin\theta = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

substitute in Snell's law

$$n_0 \sin\theta_a = n_1 \sin\theta_1$$

$$n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2}$$

Acceptance angle $\sin\theta_a = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

If the fiber is in air ($n_0 = 1$)

$$\sin\theta_a = \sqrt{n_1^2 - n_2^2}$$

Numerical aperature is the light gathering capacity of the fiber

$$NA = n_0 \sin\theta_a$$

$$\boxed{NA = \sqrt{n_1^2 - n_2^2}}$$

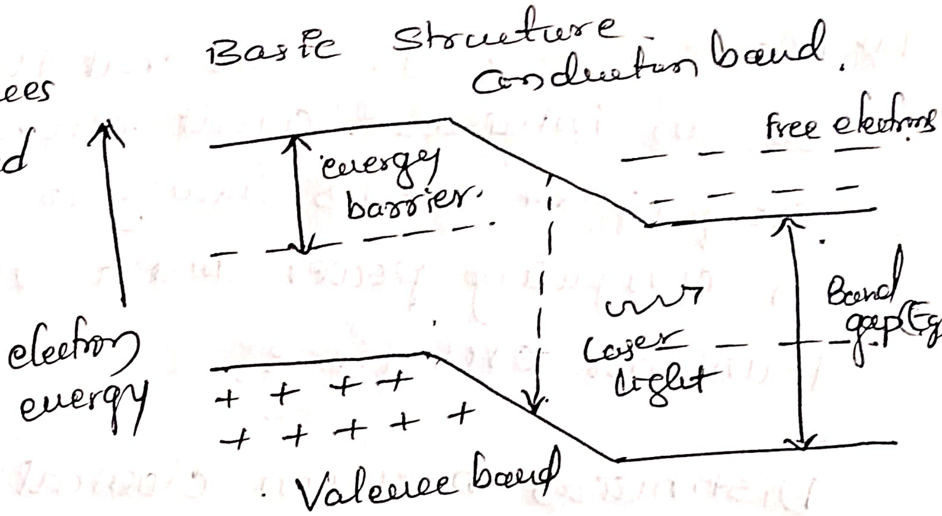
8. b) . A semiconductor laser (laser diode) is a p-n junction device that emit coherent, monochromatic and highly directional light when forwarded baised.

It works on the principle of stimulated emission due to electron - hole recombination in a forward baised junction.

Construction: A heavily doped p-type semiconductor from p-n junction. The device is fabricated from direct band gap materials such as GaAs, InP,

GaAlAs

The two opposite faces of the crystal are cleaved and polished to form parallel reflecting surfaces



9a) Moore's law :- It states that the number of transistors on an integrated circuit doubles approximately every 18-24 months, leading to exponential growth in computing power and a reduction in cost per transistor over time.

Distinction between classical and Quantum computing

Aspect	Classical computing	Quantum computing
Basic unit	Bit	Qubit
State	0 or 1	0, 1, or superposition of both
principle	Boolean logic and deterministic operations	Quantum mechanics principles
Phenomena used	Electrical signals	Superposition and Entanglement
parallelism	Limited	massive parallelism due to superposition
Hardware	CPUs, Microcontrollers	Quantum processors

9b) CNOT Gate :- Two-qubit quantum logic gate widely used in quantum computing

It consists of : * Control qubit

* Target qubit

Operation rule :-

If the control qubit $= |0\rangle$, the target qubit remain unchanged.

If the control qubit $= |1\rangle$ the target qubit is flipped

Thus, the CNOT gate performs a conditional NOT operation.

* Matrix Form:

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

* Logic Truth Table of CNOT gate

Control qubit	Target qubit	Target output
0	0	0
0	1	1
1	0	1
1	1	0

9c) Given that $|\psi\rangle = \begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix}\beta_1 \\ \beta_2\end{pmatrix}$

To prove $\langle\psi|\phi\rangle = (\langle\phi|\psi\rangle)^*$

Hermitian conjugate $\langle\psi| = (\alpha_1^* \ \alpha_2^*)$

$\langle\phi| = (\beta_1^* \ \beta_2^*)$

$$\langle\psi|\phi\rangle = (\alpha_1^* \alpha_2^*)\begin{pmatrix}\beta_1 \\ \beta_2\end{pmatrix}$$

$$\langle\psi|\phi\rangle = \alpha_1^* \beta_1 + \alpha_2^* \beta_2$$

$$\langle\phi|\psi\rangle = \begin{pmatrix} \beta_1^* & \beta_2^* \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$= (\beta_1^* \alpha_1 + \beta_2^* \alpha_2)$$

$$(\langle\phi|\psi\rangle)^* = (\beta_1^* \alpha_1 + \beta_2^* \alpha_2)^*$$

$$= \alpha_1^* \beta_1 + \alpha_2^* \beta_2$$

$$\boxed{\therefore \langle\psi|\phi\rangle = (\langle\phi|\psi\rangle)^*}$$

10a) A bit is the fundamental unit of information in classical computing. It can exist in only one of two definite states at a time. 0, or 1.

Qubit :- A qubit is the fundamental unit of information in quantum computing. Unlike a classical bit, a qubit can exist in superposition of both state. $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

where $\alpha, \beta \in \mathbb{C}$

$$|\alpha|^2 + |\beta|^2 = 1$$

Properties of qubits ;

(a) Superposition : A qubit can exist in a combination of $|0\rangle$ and $|1\rangle$ simultaneously.

(b) Probability amplitude:
The coefficients α and β are complex number.

probability of $|0\rangle = |\alpha|^2$

$|1\rangle = |\beta|^2$

(c) Measurement collapse.

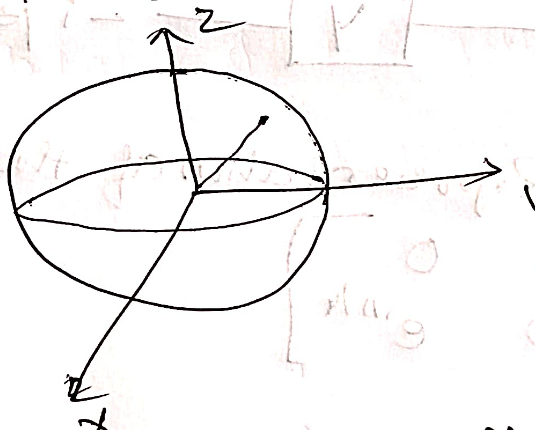
(d) Phase information.

* Representation of Qubit using Bloch sphere.

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$0 \le \theta \le \pi$

$0 \le \phi < 2\pi$



North pole $\rightarrow |0\rangle$, South pole $\rightarrow |1\rangle$.

10 by Pauli - x and Pauli - y Gates

(1) Pauli - x Gate (Quantum NOT Gate)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$

Truth Table

Input	output
1	0
0	1

* Pauli - Y Gate :-

$$|\psi\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$Y|0\rangle = i|1\rangle$

$Y|1\rangle = -i|0\rangle$

Truth Table :

Input	output
$\|0\rangle$	$i\|1\rangle$
$\|1\rangle$	$-i\|0\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$-i\beta\|0\rangle + i\alpha\|1\rangle$

$\alpha|0\rangle + \beta|1\rangle$ —— $\boxed{Y}$ —— $-i\beta|0\rangle + i\alpha|1\rangle$

10 c) Matrix Representation of the T - Gate

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$T^2 = T \cdot T$

$$T^2 = \begin{bmatrix} 1 \cdot 1 & 0 \\ 0 & e^{i\pi/4} \cdot e^{i\pi/4} \end{bmatrix}$$

$e^{i\pi/2} = i$

$$T^2 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

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$$\boxed{T^2 = S}$$