CBCS SCHEME

USN 4DM25CS034

BPHYS102

First Semester B.E./B.Tech. Degree Examination, Dec.2025/Jan.2026

Quantum Physics and Applications

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks, L: Bloom's level, C: Course outcomes.
3. VTU formula handbook is permitted.

		Module – 1	M	L	C
Q.1	a.	Set up one dimensional time-independent Schrodinger's wave equation.	8	L2	CO1
	b.	Write a brief note on- Physical significance of wave function, Principle of complementarily, Expectation value& Quantum tunneling.	8	L2	CO1
	c.	Calculate the de-Broglie wavelength associated with an electron having a kinetic energy of 100 eV.	4	L3	CO1
		OR			
Q.2	a.	Derive a normalized wave function for a particle inside one dimensional infinite potential well.	8	L2	CO1
	b.	State and explain Heisenberg Uncertainty principle with three relationships. Use the energy-time uncertainty to explain the broadening of spectral lines.	8	L2	CO1
	c.	An electron is bound in a 1-dimensional potential well of width 1 Å & of infinite height. Find its energy values in eV for the ground state & the first two excited states.	4	L3	CO1
		Module – 2			
Q.3	a.	Define Fermi energy and Fermi factor. Discuss the variation of Fermi factor with temperature and energy.	8	L2	CO2
	b.	What is Hall effect? Derive an expression for Hall voltage in terms of Hall coefficient with all necessary diagrams.	8	L2	CO2
	c.	Find the temperature at which there is 1% probability that a state with an energy 0.5 eV above Fermi energy is occupied.	4	L3	CO2
		OR			
Q.4	a.	Explain the failure of Classical free electron theory of metals and list assumptions of quantum free electron theory.	8	L2	CO2
	b.	Derive the expression for Fermi energy in terms of energy gap of intrinsic semiconductor.	8	L2	CO2
	c.	A semiconductor sample 0.5 mm thick carries a current of 5 mA in a magnetic field of 0.2 T. If the Hall voltage is 1 mV, determine the Hall coefficient.	4	L3	CO2

		Module – 3			
Q.5	**a.**	What are phonons? Explain the role of phonons in Cooper pair formation.	**8**	**L2**	**CO3**
	b.	Explain flux quantization with neat diagram. Discuss DC & AC Josephson effect.	**8**	**L2**	**CO3**
	c.	For a superconducting sample with critical temperature 7.2 K and critical field at 0K is 6.5×10^4 Am^{-1}, find the critical field at 4 K.	**4**	**L3**	**CO3**
		OR			
Q.6	**a.**	Explain Silsbee effect and hence derive an expression for the critical current for a superconducting cylindrical wire.	**8**	**L2**	**CO3**
	b.	Define Meissner's effect. Differentiate between Type I and Type II superconductors.	**8**	**L2**	**CO3**
	c.	A long thin superconducting wire has a radius of 0.5 mm and a critical field of 8 kA m^{-1}. Determine its critical current.	**4**	**L3**	**CO3**
		Module – 4			
Q.7	**a.**	Discuss stimulated emission process. Derive energy density of radiation in thermal equilibrium using Einstein coefficients.	**8**	**L2**	**CO4**
	b.	Explain the construction & working of superconducting nano wire single photon detectors (SNSPDs). Write it's any two advantages.	**8**	**L2**	**CO4**
	c.	The attenuation of light in an optical fiber is estimated at 2.2 dB/km. What fractional initial intensity remains after 2 km and after 6 km?	**4**	**L3**	**CO4**
		OR			
Q.8	**a.**	Define acceptance angle for an optical fiber. Hence, derive the expression for its Numerical Aperture (NA)and arrive at the condition for propagation.	**8**	**L2**	**CO4**
	b.	Describe the construction and working of a semiconductor laser based on energy band diagram.	**8**	**L2**	**CO4**
	c.	A fiber has a core refractive index of 1.48 and a cladding index of 1.46. Calculate its numerical aperture (NA) and acceptance angle in air.	**4**	**L3**	**CO4**
		Module – 5			
Q.9	**a.**	State Moore's law. Distinguish between classical and Quantum computing.	**8**	**L2**	**CO5**
	b.	Outline the operation of the CNOT gate and define its standard matrix and logical truth table.	**8**	**L2**	**CO5**
	c.	Given $\lvert\psi\rangle = \begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}$ and $\lvert\phi\rangle = \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}$. Prove that $\langle\psi\vert\phi\rangle = \langle\phi\vert\psi\rangle^*$	**4**	**L3**	**CO5**
		OR			

Q.10	a.	Define bit and qubit. Mention four properties of a qubit. Explain the representation of qubit using Bloch sphere.	8	L2	CO5
	b.	Mention the Pauli X and Y gate and apply these on the state $\|0>$ and $\|1>$ mention the truth table along with circuit symbol.	8	L2	CO5
	c.	Prove, using matrix algebra, that two consecutive T gates are equivalent to a single S gate in quantum circuit.	4	L3	CO5

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Q 1. a> One Dimensional Time-Independent Schrodinger equation, The time-dependent Schrodinger equation is

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x,t)$$

Assume separation of variables

$$\psi(x,t) = \psi(x)\, e^{-iEt/\hbar}$$

Substituting and simplifying

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

b> 1> Physical Significance of Wave function

* $|\psi|^2 \rightarrow$ Probability density
* Must satisfy normalization $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$

2> Complementarity Principle

Wave-particle duality; particle and wave aspects are complementary and cannot be observed simultaneously.

3> Expectation value

$$\langle A \rangle = \int \psi^* A \psi \, dx$$

4> Quantum Tunneling

Particle can penetrate potential barrier even if $E < V_0$. Transmission probability decreases exponentially.

c) De-Broglie Wavelength (100 eV)

$E = 100\,eV$

$\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda(\mathring{A}) = \frac{12.27}{\sqrt{E}}$

$= \frac{12.27}{\sqrt{100}} = 1.227\mathring{A}$

$\boxed{\lambda = 1.23\mathring{A}}$

Q2)
a) Derive a normalized wave function for a particle inside one dimensional infinite potential well.

Boundary conditions for $0 < x < L$.

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

We know that $\psi = A \sin kx$

Boundary conditions $\psi(0) = 0$, $\psi(L) = 0$

$$k = \frac{n\pi}{L}$$

Normalization: $\int_0^L |\psi|^2 dx = 1$

$$A = \sqrt{\frac{2}{L}}$$

Final normalized wavefunction

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

b) State and explain Heisenberg uncertainty principle

Heisenberg's uncertainty principle states that it is impossible to simultaneously know both the exact position and the exact momentum of a particle

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

$$\Delta L \cdot \Delta \theta \geq \frac{\hbar}{2}$$

Broading of spectral lines $\Delta E = \frac{\hbar}{\Delta t}$

Finite lifetime → Energy uncertainty → Line broadening.

2c) Given: widt $L = 1 \mathring{A} = 10^{-10} m$

Formula: $E_n = \frac{n^2 h^2}{8 m L^2}$

$L = 1\mathring{A} = 10^{-10} m$

$E_1 = 37.6 \, eV$

$E_2 = 4E_1 = 150.4 \, eV$

$E_3 = 9E_1 = 338.4 \, eV$

Module - 2

3.a) Define fermi energy and fermi factor. Discuss the variation of fermi factor with temperature and energy.

Fermi energy is the highest energy level that electrons can occupy at absolute zero temperature, while the fermi factor is the probability that an electron state at a specific energy level is occupied at a given temperature.

Fermi factor $f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$

At 0K, $\Rightarrow E < E_F \Rightarrow f = 1$

$E > E_F \Rightarrow f = 0$

At high T → smooth curve will be obtained

3.b> Hall effect: when a current-carrying conductor or semiconductor is placed in a magnetic field that is perpendicular to the direction of current flow. As a result of the magnetic field, a transverse electric field is developed across the material.

This produces a potential difference known as the Hall voltage.

Consider → Rectangular conducting or semiconductor plate.

→ current flows along the x-axis

→ Magnetic field along the z-axis

→ The Hall voltage develops along the y axis

The current I is given by   Fig

$$I = nqv_d(w \times t)$$

$$v_d = \frac{I}{nqwt}$$

when B is applied ⊥lar to the current,

$$F_B = q v_d B$$

As charge accumulate, an electric field E_H, Hall field develops in the opposite direction.

$$F_E = q E_H$$

at equilibrium $F_B = F_E$

$$qE_H = qV_d B$$

$$E_H = V_d B$$

<u>Hall voltage</u> : $V_H = E_H \times w$

$$V_H = V_d B w \qquad \because E_H = V_d B$$

$$\boxed{V_H = \frac{IB}{nqt}} \qquad \because V_d = \frac{I}{nqwt}$$

Hall coefficient is defined as

$$R_H = \frac{E_H}{jB}$$

where j is current density $j = \frac{I}{wt}$

$$E_H = \frac{IB}{nqwt} = \frac{jB}{nq}$$

Thus, $R_H = \frac{1}{nq}$

Substitute $R_H = \frac{1}{nq}$ into $V_H = \frac{IB}{nqt}$

$$\boxed{V_H = \frac{BIR_H}{t}}$$

3. e) Solution:

$$f(E) = 0.01$$

$$f(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}}$$

$$0.01 = \frac{1}{1 + e^{0.5/kT}}$$

$$e^{0.5/kT} = 99$$

$$\frac{0.5}{kT} = \ln 99 = 4.6$$

$$T = \frac{0.5}{4.6 \times 8.617 \times 10^{-5}}$$

$$\boxed{T \simeq 1260\ K}$$

4. a) 1) Failure to explain Electronic Specific Heat.

Predictions (classical theory)

$$E = \frac{3}{2}kT \quad \text{per electron}$$

electronic specific heat $C_e = \frac{3}{2}Nk$. → This is a large value comparable to lattice specific heat.

Experimental result shows very small specific heat.

2) Failure to explain Temperature Dependence of conductivity $\sigma = \frac{ne^2\tau}{m}$

3) Failure to Explain Wiedemann-Franz law quantitatively. $\frac{K}{\sigma T}$ = constant

4) Failure to Explain Hall effect Correctly.

$$R_H = -\frac{1}{ne}$$

5) Failure to Explain Magnetic Properties

6) Failure to Explain why some solids are Insulators.

Assumptions of quantum free theory:

1) Conduction electrons move freely inside the metal like particle in a box.

2) Pauli exclusion Principle applies, no two electrons can have the same set of quantum numbers.

3) Electrons obeys Fermi-Dirac distribution, no Maxwell-Boltzman statistics.

4) Electrons move Independently, electron-electron interaction is neglected.

4b) In an intrinsic semiconductor, $n = p$. —①

Electron concentration is given $n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

Hole concentration in valence band $p = N_v \exp\left(-\frac{E_F - E_v}{kT}\right)$

$$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

From ① $N_c \exp\left(-\frac{E_c - E_F}{kT}\right) = N_v \exp\left(-\frac{E_F - E_v}{kT}\right)$

Take log and rearranging. $\frac{E_c - E_F}{kT} - \frac{E_F - E_v}{kT} = \ln \frac{N_c}{N_v}$

$$E_F = \frac{E_c + E_v}{2} - \frac{kT}{2} \ln \frac{N_c}{N_v}$$

Expression in terms of Energy gap, $E_g = E_c - E_v$,

$\frac{N_c}{N_v} = \left(\frac{m_e^*}{m_h^*}\right)^{3/2}$ & $E_F = \frac{E_c + E_v}{2} + \frac{3}{2} kT \left(\frac{m_h^*}{m_e^*}\right)$

4. c) Given: $t = 0.5\,m = 5 \times 10^{-4}\,m$

$I = 5\,mA = 5 \times 10^{-3}\,A$

$B = 0.2\,T$

$V_H = 1\,mV = 10^{-3}\,V$

$$R_H = \frac{V_H t}{IB} = \frac{(10^{-3})(5 \times 10^{-4})}{(5 \times 10^{-3})(0.2)}$$

$$R_H = 5 \times 10^{-4}\,m^3/C$$

5.a> Phonons are the quatized modes of lattice vibrations in a crystalline solid.

Energy of a Phonon $E = \hbar\omega$

where $\hbar = \frac{h}{2\pi}$, ω angular frequency of vibration

Rule of phonon in cooper pair formation.

Normally two electrons repel each other due to coulomb force. But in a superconductor at very low temperture electron Distorts the Lattice

⇒ Electron moving through the lattice attracts nearby positive ion.

⇒ This creates a slight local lattice distortion.

⇒ This distortion produces a phonon.

⇒ The lattice destortion increases positive charge density locally.

⇒ ~~Acts~~ like a temperary positive potential well.

⇒ Another electron feels attraction toward this positive region. This indirect ~~a~~ attraction via phonon overcomes coulomb repulsion at low temperature.

⇒ Two electrons form a weakly bound pair, have opposite spins, have opposite momenta and total spin zero. This pair is called cooper pair.

5b> Flux Quantization:

Super conducting electrons form Cooper pair (charge 2e).

The wave function of these electrons are single-valued.

This leads to quantization of flux.

In a Superconducting ring, the magnetic flux is quantized.

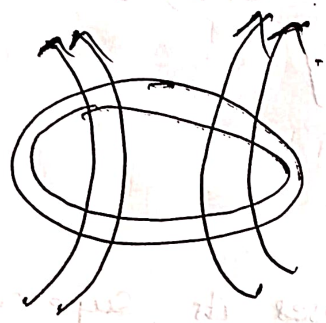
i.e $\phi = n\phi_0$ where $n = 1, 2, 3 \ldots$

$\phi_0 = \frac{h}{2e}$ = flux quantized.

$\phi_0 = 2.07 \times 10^{-15}$ Wb.

$$\boxed{\phi = n\frac{h}{2e}}$$

Diagram of Flux quantization:



DC Josephson Effect

When no voltage is applied across the Josephson junction a supercurrent flows through it.

$I = I_c \sin\phi$

where I_c = Critical current ϕ = Phase difference

When, $V = 0$, $\Rightarrow$ Pure Supercurrent (no resistance), current depends on phase deffrence. This is called DC Josephson Effect

AC Josephson Effect: When a constant voltage V is applied across the junction, the supercurrent oscillates with time

$f = \frac{2eV}{h}$ where f = frequency of oscillation

V - applied voltage

Explanation: Applying voltage causes

$\frac{d\phi}{dt} = \frac{2eV}{h}$, current becomes $I = I_c \sin(\omega t)$

where $\omega = \frac{2eV}{h}$

So current oscillate produce AC current.

5 c). $T_c = 7.2K$ $H_c(0) = 6.5\times10^4\ Am^{-1}$

$T = 4K$ $H_c(4) = ?$

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$H_c(4) = 6.5\times10^4\left[1-\left(\frac{4}{7.2}\right)^2\right] = 6.5\times10^4(1-0.308)$$

$$\boxed{H_c = 4.49\times10^4\ A/m}$$

or

6a). A superconductor loses its superconducting property when the magnetic field produced by the current flowing through it becomes equal to its critical magnetic field.

In otherwords, superconductivity is destroyed when

$H_{surface} = H_c$.

Consider a long superconducting cylindrical wire of radius r and current flowing I.

Ampere's law can be written

$$\oint \vec{H}.\vec{dl} = I$$

For long cylindrical wire

$$H(2\pi r) = I \Rightarrow H = \frac{I}{2\pi r}$$ this is B at the surface.

According silsbee effect, $H = H_c$

$$\frac{I_c}{2\pi r} = H_c$$

$$\boxed{I_c = 2\pi r H_c}$$

Since H_c depends on temperature

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$I_c(T) = 2\pi r H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

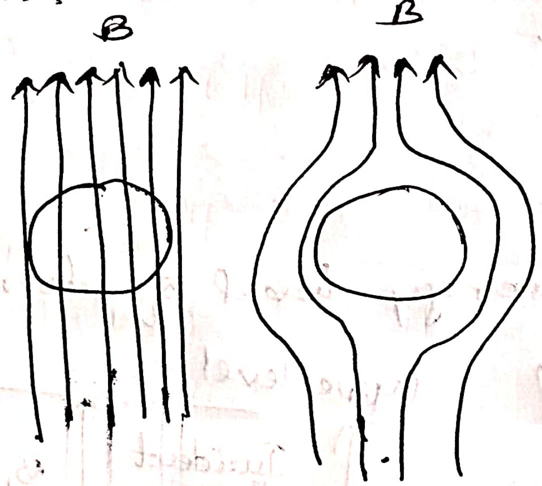
6b) The Meissner effect is the phenomenon in which a superconductor completely expels magnetic flux from its interior when it is cooled below its critical temperature T_c.

→ when a ~~magnetic~~ material is cooled below T_c, it becomes superconducting.

→ If placed in a magnetic field, the magnetic field lines are expelled from the interior. Magnetic induction inside the superconductor becomes $B=0$.

Thus superconductor behaves as a perfect diamagnet.

Diagram.



Distinction Between Type I and Type II Superconductors

Property	Type I	Type II
critical field (H_c)	one H_c	Two H_{c_1}, H_{c_2}
Magnetic behavior	complete expulsion	Partial expulsion
Transition	Sudden	Gradual
Magnetic field Strength	Low	Very high
Materials	Pure metals	Alloys & compounds

6 c) $r = 0.5\ mm = 0.5 \times 10^{-3}\ m$

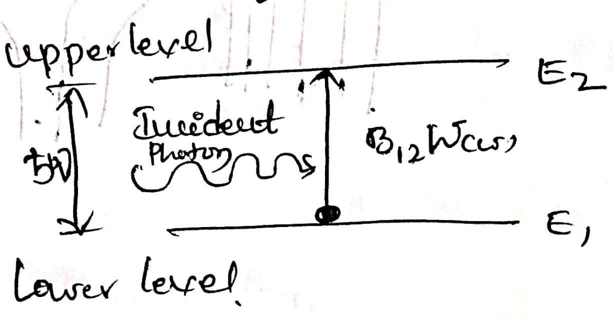
$H_c = 8\ kAm^{-1} = 8 \times 10^{3}\ Am^{-1}$

$I_c = 2\pi r H_c$

$= 2\pi (5 \times 10^{-4})(8 \times 10^{3})$

$\boxed{I_c = 25\ A}$.

7. a) Consider two energy level of atomic system, lower energy level E_1 and upper energy level E_2. The energy difference between them is $h\nu$.



There are three possible radiation processes:

1) Absorption 2) Spontaneous emission 3) Stimulated emission

When an atom in excited state E_2 is struck by a photon of energy $h\nu$:

$$h\nu = E_2 - E_1$$

The atom is forced to return to ground state E_1 and emits a second photon.

Rate of stimulated Emission:

If $\rho(\nu)$ is the energy density of radiation.

Rate of stimulated emission $= B_{21}\rho(\nu)N_2$

Derivation: Einstein coefficients

~~Absorption~~ $R_{abs} = B_{12}\rho(\nu)N_1$

Spontaneous Emission: $R_{sp} = A_{21}N_2$

Stimulated Emission $R_{stim} = B_{21}\rho(\nu)N_2$

At thermal equilibrium

Absorption rate = Total emission rate

$$B_{12}\rho(\nu)N_1 = A_{21}N_2 + B_{21}\rho(\nu)N_2$$

$$\rho(\nu) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

Using Boltzman distribution $\frac{N_2}{N_1} = e^{-\frac{h\nu}{KT}}$

$$\rho(\nu) = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\nu}{KT}} - 1}$$

Einstein Relations.

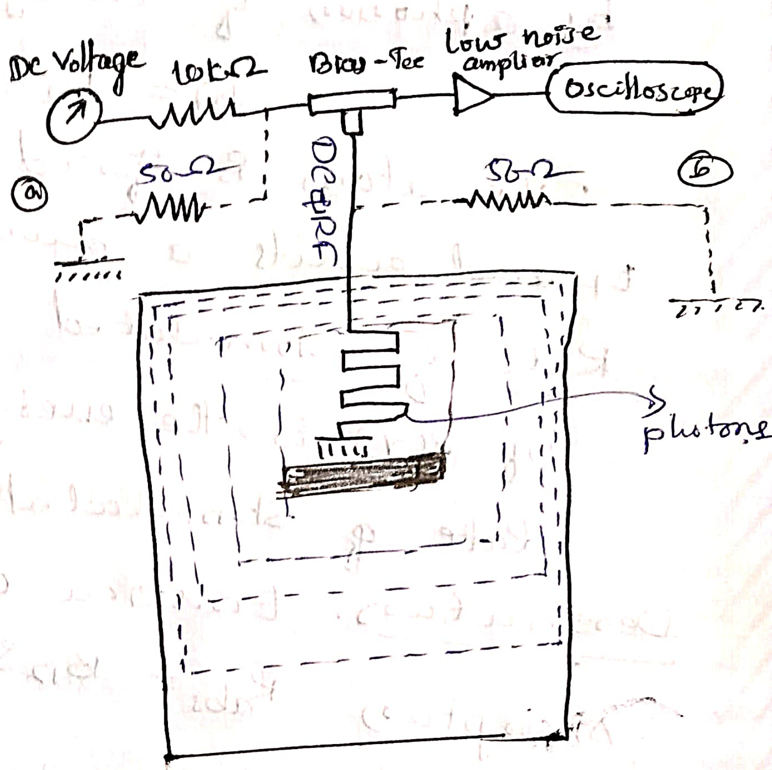
$$B_{12} = B_{21}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

Final expression for Energy Density $\boxed{\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{KT}} - 1}}$

7b): Superconducting Nanowire Single-Photon Detector:

Construction of SNSPD:



→ A SNSPD is an ultra-sensitive photon detector made from a thin superconducting nanowire patterned into a meander structure.

Components:

1). Superconducting nanowire:
Material: NbN, NbTiN,
Width: ~50-150nm
Thickness: 5-10nm

2). Usually Silicon substrate.

3) Bias current just below critical current I_c.

4) Cryogenic system.

Working of SNSPD:

⇒ Super conducting State: The nanowire is cooled below its critical temperature T_c. A bias current I_b is applied $I_b \lesssim I_c$. Wire is in superconducting state (zero resistance).

⇒ Absorption of photon temporarily destroy the superconductivity

⇒ Formation of Resistance Barrier: The ~~bias~~ bias current is forced to flow around the hotspot. Current density increases. If it exceeds critical current density, the region becomes resistive.

⇒ As hotspot cools quickly, superconductivity is restored. Detector becomes ready for next photon.

i.c) Given: $n_1 = 1.48$ $n_2 = 1.46$

Numerical Aperture $NA = \sqrt{2.1904 - 2.1316}$

$NA = 0.242$

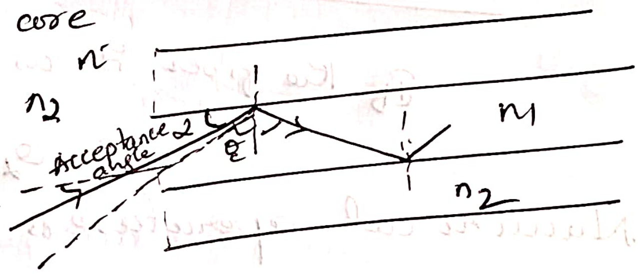
Acceptance angle $\theta = \sin^{-1}(0.242)$

$\boxed{\theta = 14^\circ}$

Q.a) Acceptance Angle and Numerical Aperature of an optical Fiber.

An optical fiber consists of core of R.I n_1 and cladding R.I n_2

For light to propagate inside the fiber $n_1 > n_2$



So that Total Internal Reflection (TIR) occurs at the core-cladding interface.

Using Snell's law $n_0 \sin\theta_a = n_1 \sin\theta_1$

θ_1 = angle of refraction inside the core

For TIR at core-cladding interface

$\theta_2 \geq \theta_c$

where θ_c = critical angle

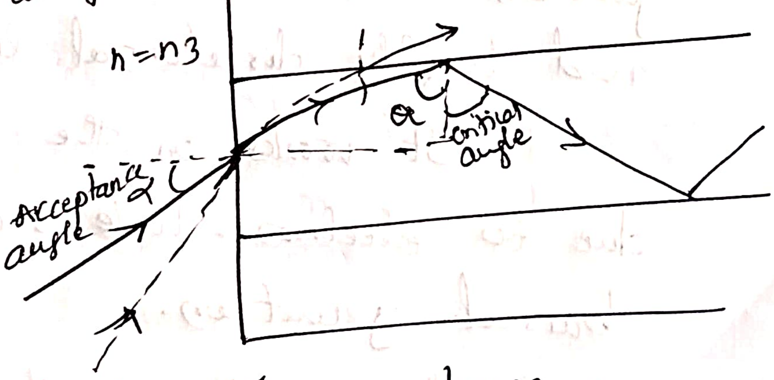
$\theta_2 = 90^\circ - \theta_1$

Critical angle condition

$\sin\theta_c = \frac{n_2}{n_1}$

For limiting case

$\theta_2 = \theta_c$



Ray outside acceptance cone is lost from core

So, $90° - \theta_1 = \theta_c$ $\quad \theta_1 = 90° - \theta_c$

Taking sine

$$\sin\theta_1 = \cos\theta_c$$

$$\sin\theta = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

substitute in Snell's law

$$n_0 \sin\theta_a = n_1 \sin\theta_1$$

$$n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2}$$

Acceptance angle $\sin\theta_a = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$

If the fiber is in air ($n_0 = 1$)

$$\sin\theta_a = \sqrt{n_1^2 - n_2^2}$$

Numerical aperture is the light gathering capacity of the fiber

$$NA = n_0 \sin\theta_a$$

$$\boxed{NA = \sqrt{n_1^2 - n_2^2}}$$

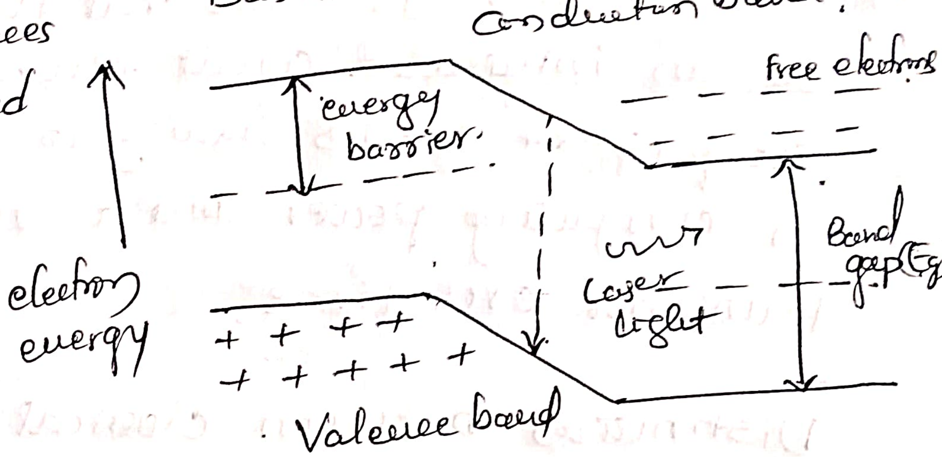
8. b) . A semiconductor laser (laser diode) is a p-n junction device that emit coherent, monochromatic and highly directional light when forwarded baised.

It works on the principle of stimulated emission due to electron - hole recombination in a forward baised junction.

Construction: A heavily doped p-type semiconductor from p-n junction. The device is fabricated from direct band gap materials such as GaAs, InP, GaAlAs

The two opposite faces of the crystal are cleaved and polished to form parallel reflecting surfaces

Basic structure



9a) **Moore's law:-** It states that the number of transistors on an integrated circuit doubles approximately every 18-24 months, leading to exponential growth in computing power and a reduction in cost per transistor over time.

Distinction between classical and Quantum computing

Aspect	Classical computing	Quantum computing
Basic unit	Bit	Qubit
State	0 or 1	0, 1, or Superposition of both
principle	Boolean logic and deterministic operations	Quantum mechanics principles
Phenomena used	Electrical signals	Superposition and Entanglement
parallelism	Limited	massive parallelism due to superposition
Hardware	CPUs, Microcontroller	Quantum processors

9b) **CNOT Gate:-** Two-qubit quantum logic gate widely used in quantum computing

It consists of : * Control qubit,
* Target qubit

Operation rule:-

If the control qubit = $|0\rangle$, the target qubit remain unchanged.

If the control qubit = $|1\rangle$ the target qubit is flipped

Thus, the CNOT gate performs a conditional NOT operation.

* **Matrix Form**:

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

* Logic Truth Table of CNOT gate

Control qubit	Target qubit	Target output
0	0	0
0	1	1
1	0	1
1	1	0

9c) Given that $|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$

To prove $\langle\psi|\phi\rangle = (\langle\phi|\psi\rangle)^*$

Hermitian conjugate $\langle\psi| = (\alpha_1^* \;\; \alpha_2^*)$

$\langle\phi| = (\beta_1^* \;\; \beta_2^*)$

$$\langle\psi|\phi\rangle = (\alpha_1^* \;\; \alpha_2^*) \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$$

$$\langle \psi | \phi \rangle = \alpha_1^* \beta_1 + \alpha_2^* \beta_2$$

$$\langle \phi | \psi \rangle = \begin{pmatrix} \beta_1^* & \beta_2^* \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$= (\beta_1^* \alpha_1 + \beta_2^* \alpha_2)$$

$$(\langle \phi | \psi \rangle)^* = (\beta_1^* \alpha_1 + \beta_2^* \alpha_2)^*$$

$$= \alpha_1^* \beta_1 + \alpha_2^* \beta_2$$

$$\boxed{\therefore \langle \psi | \phi \rangle = (\langle \phi | \psi \rangle)^*}$$

10a) A bit is the fundamental unit of information in classical computing. It can exist in only one of two definite states at a time. 0, or 1.

Qubit :- A qubit is the fundamental unit of information in quantum computing. Unlike a classical bit, a qubit can exist in superposition of both state. $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

where $\alpha, \beta \in C$

$$|\alpha|^2 + |\beta|^2 = 1$$

Properties of qubits :

(a) Superposition : A qubit can exist in a combination of $|0\rangle$ and $|1\rangle$ simultaneously.

(b) Probability amplitude:

The coefficients α and β are complex number.

probability of $|0\rangle = |\alpha|^2$

$|1\rangle = |\beta|^2$

(c) Measurement collapse.

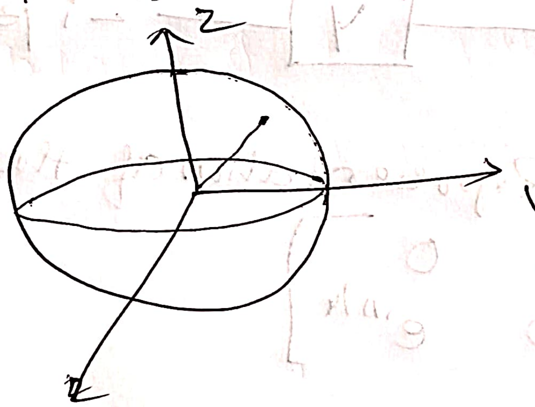
(d) Phase information.

* Representation of Qubit using Bloch sphere.

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$0 \le \theta \le \pi$$

$$0 \le \phi < 2\pi$$



North pole $\rightarrow |0\rangle$, South pole $\rightarrow |1\rangle$.

10 by Pauli-x and Pauli-y Gates

(1) Pauli-x Gate (Quantum NOT Gate)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X|0\rangle = |1\rangle$$

$$X|1\rangle = |0\rangle$$

Truth Table

Input	Output
1	0
0	1

* Pauli - Y Gate :-

$$|\psi\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$Y|0\rangle = i|1\rangle$

$Y|1\rangle = -i|0\rangle$

Truth Table :

Input	output
$\|0\rangle$	$i\|1\rangle$
$\|1\rangle$	$-i\|0\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$-i\beta\|0\rangle + i\alpha\|1\rangle$

$$\alpha|0\rangle + \beta|1\rangle \text{ —— } \boxed{Y} \text{ —— } -i\beta|0\rangle + i\alpha|1\rangle$$

10c) Matrix Representation of the T - Gate

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

$$T^2 = T \cdot T$$

$$T^2 = \begin{bmatrix} 1\cdot 1 & 0 \\ 0 & e^{i\pi/4} \cdot e^{i\pi/4} \end{bmatrix}$$

$$e^{i\pi/2} = i$$

$$T^2 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

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$$\boxed{T^2 = S}$$