CBCS SCHEME

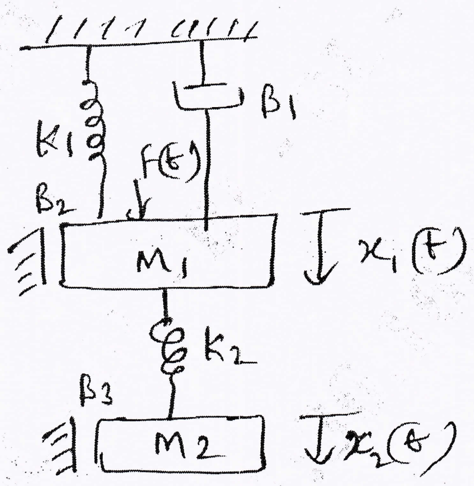
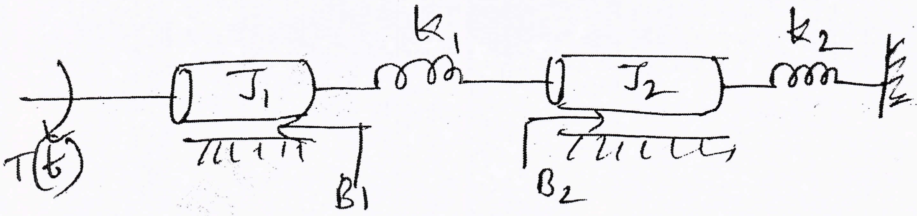
USN | | | | | | | | | |

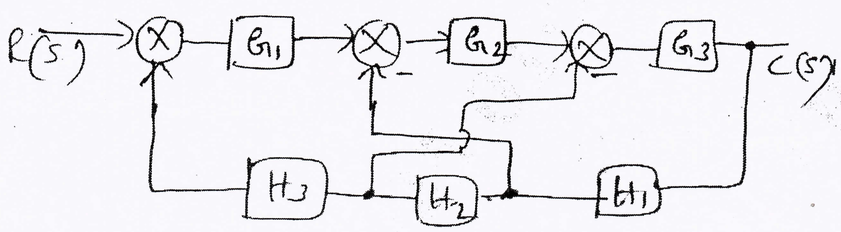
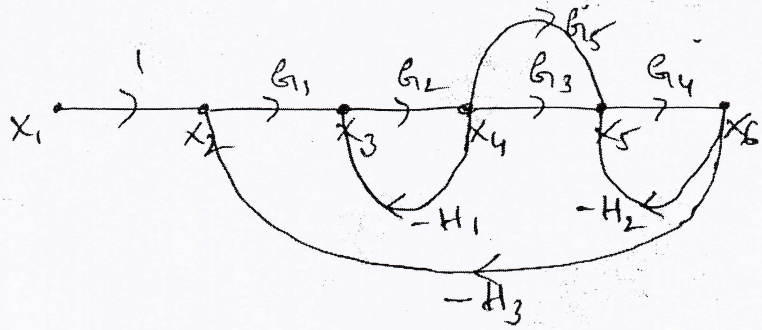
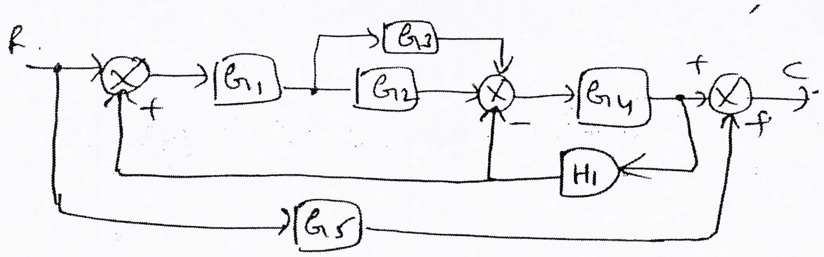
BEE602

Sixth Semester B.E./B.Tech. Degree Examination, June/July 2025
Control Systems

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Construct mathematical model for the mechanical system shown in Fig.Q1(a). Draw electrical equivalent network based on force voltage analogy and force current analogy.  Fig.Q1(a)	12	L3	CO1
	b.	Derive the transfer function of armature controlled DC motor.	8	L4	CO1
		OR			
Q.2	a.	Distinguish between open loop and closed loop systems with examples.	8	L2	CO1
	b.	For the mechanical translation system as shown is Fig.Q2(b). Draw the electrical network based on torque current and torque voltage analogy. Write its performance equations.  Fig.Q2(b)	12	L3	CO1

		Module – 2			
Q.3	**a.**	Obtain $\frac{C(s)}{R(s)}$ using block diagram reduction rule.  Fig.Q3(a)	8	L3	C2O
	b.	Determine transfer function $X_6(s)/X_1(s)$ using Mason's gain formula for the signal flow graph shown in Fig.Q3(b).  Fig.Q3(b)	8	L4	CO2
	c.	Define : i) Source and sink node ii) Loop and forward path.	4	L2	CO2
		OR			
Q.4	**a.**	Explain Mason's gain formula indicating each term.	4	L1	CO2
	b.	Illustrate how to perform the following connection with block diagram reduction technique. i) Shifting summing point after a block ii) Shifting take off point ahead of a block iii) Blocks in parallel.	6	L3	CO2
	c.	Determine the transfer function $\frac{C(s)}{R(s)}$ of a system shown in Fig.Q4(c).  Fig.Q4(c)	10	L2	CO2

		Module – 3			
Q.5	**a.**	Derive an expression for rise time and peak-time for a second order system excited by a step input (under dumped case).	**8**	**L4**	**CO3**
	b.	Check the stability of the given characteristic equation using R – H criterion. $S^5 + 6s^4 + 3x^3 + 2s^2 + s + 1 = 0$.	**6**	**L3**	**CO3**
	c.	A second order system is given by $\frac{C(s)}{R(s)} = \frac{25}{s^2 + 6s + 25}$, find rise time, peak time, peak overshoot and settling time for 2% tolerance.	**6**	**L3**	**CO3**
		OR			
Q.6	**a.**	Explain the difficulties encountered while assessing the R – H criteria and how do you eliminate these difficulties with examples.	**8**	**L1**	**CO3**
	b.	A unity feedback control system has $G(s) = \frac{K(s+4)}{s(s+1)(s+2)}$ using R-H criterion. Find the range of K for which system to be stable and also determine the frequency of oscillations.	**8**	**L3**	**CO3**
	c.	Obtain an expression for time response of the first order system subjected to unit step input.	**4**	**L4**	**CO3**
		Module – 4			
Q.7	**a.**	Explain the terms given below with respect to root locus : i) Break away point ii) Asymptotes iii) Intersection of root locus branches with J^{ω} axis.	**6**	**L2**	**CO4**
	b.	A unity feedback system the open loop transfer function is given by : $G(s) = \frac{K}{s(s+2)(s^2+6s+25)}$ i) Sketch the root locus for $0 \leq K \leq \infty$ ii) At what value of ' K' the system becomes stable iii) At this point of instability determine the frequency of oscillation of system.	**14**	**L3**	**CO3**
		OR			
Q.8	**a.**	A unity FBCS with $G(s) = \frac{80}{s(s+2)(s+20)}$. Find gain and phase margin using bode plot.	**12**	**L2**	**CO4**
	b.	Derive an expression for resonant peak and resonant frequency for a second order system.	**8**	**L4**	**CO4**

		Module – 5			
Q.9	**a.**	Explain PID controller and discuss the effect on the behavior of the system.	**10**	**L1**	**CO5**
	b.	Explain the step by step design procedure of lead compensation network.	**10**	**L2**	**CO5**
		OR			
Q.10	**a.**	Mention the properties of state transition matrix. Given that : $A_1 = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$; $A_2 = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}$; $A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$ Compute e^{AT}.	**10**	**L2**	**CO5**
	b.	Explain the concept of state. Define : i) State variable ii) State vector iii) State space iv) State trajectory.	**10**	**L2**	**CO5**

* * * * *

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Department of Electrical & Electronics Engineering

VTU QP Solution : June/July 2025

Subject : Control System.

Code : BEE 602

Prepared by,
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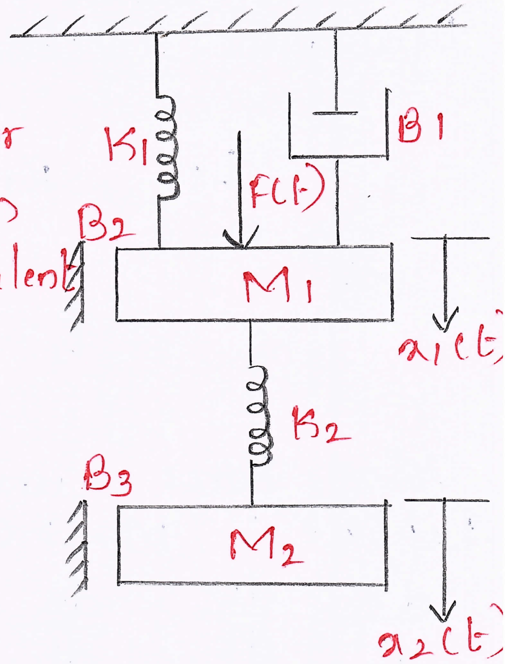
EEE Department

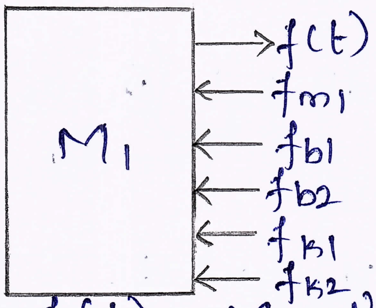
Module - 01

Q1a) Construct mathematical model for the mechanical system shown in Fig. Q1 (a). Draw electrical equivalent network based on force-voltage & force-current analogy.

[12 Marks]



Soln:- Free body diagram for M1.

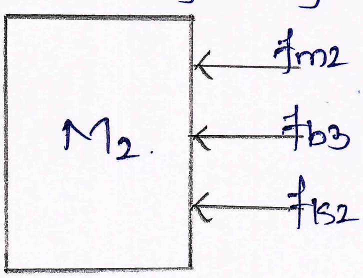


According to Newton's law,

$$f(t) = f_{m1} + f_{b1} + f_{b2} + f_{k1}$$

$$f(t) = M_1 \cdot \frac{d^2x_1(t)}{dt^2} + B_1 \frac{dx_1(t)}{dt} + B_2 \frac{dx_1(t)}{dt} + K_1 x_1(t) + K_2 [x_1(t) - x_2(t)] \quad \text{——} (1)$$

Free body diagram for M2.



$$0 = f_{m2} + f_{b3} + f_{k2}$$

$$0 = M_2 \cdot \frac{d^2x_2(t)}{dt^2} + B_3 \frac{dx_2(t)}{dt} + K_2 [x_2(t) - x_1(t)] \quad \text{——} (2)$$

Using force-voltage analogy,

Replace : $M \rightarrow L$, $B \rightarrow R$, $K = 1/C$.

$$\frac{d^2x}{dt} = \frac{di}{dt} ; \quad \frac{dx}{dt} = i ; \quad x = \int i \cdot dt$$

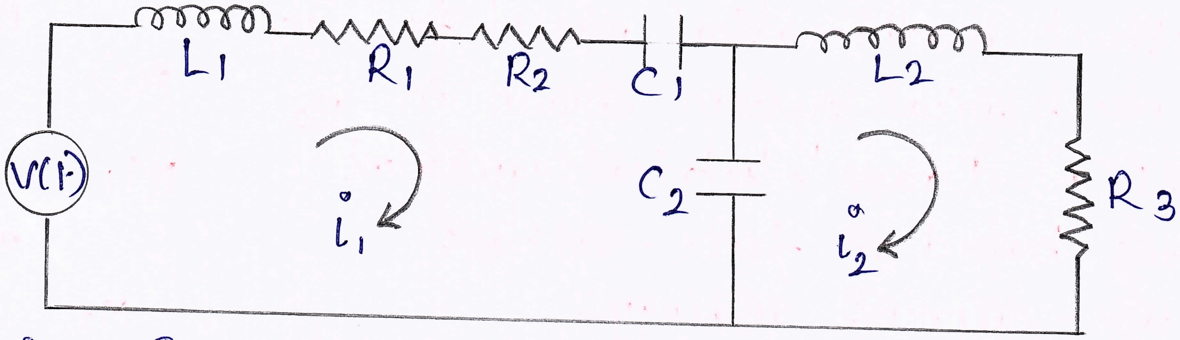
Eqn. (1) can be written as,

$$v(t) = L_1 \cdot \frac{di_1(t)}{dt} + R_1 i_1(t) + R_2 i_1(t) + \frac{1}{C_1} \int i_1 \cdot dt + \frac{1}{C_2} \int (i_1 - i_2) \cdot dt \quad \text{--- (3)}$$

Eqn. (2) can be written as,

$$0 = L_2 \cdot \frac{di_2(t)}{dt} + R_3 i_2(t) + \frac{1}{C_2} \int [i_2 - i_1] \, dt \quad \text{--- (4)}$$

Equivalent electrical circuit using eqns. (3) & (4)



Using Force-current analogy,

Replace : $M \rightarrow C$, $B \rightarrow 1/R$, $K \rightarrow 1/L$

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} ; \quad \frac{dx}{dt} = v ; \quad x = \int v \cdot dt$$

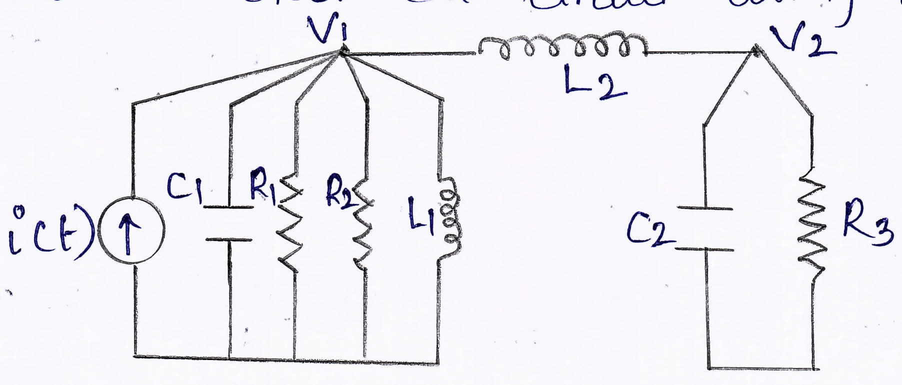
Eqn. (1) can be written as,

$$i(t) = C_1 \cdot \frac{dv_1(t)}{dt} + \frac{v_1}{R_1} + \frac{v_1}{R_2} + \frac{1}{L_1} \int v_1 \cdot dt + \frac{1}{L_2} \int (v_1 - v_2) dt \quad \text{--- (5)}$$

Eqn. (2) can be written as,

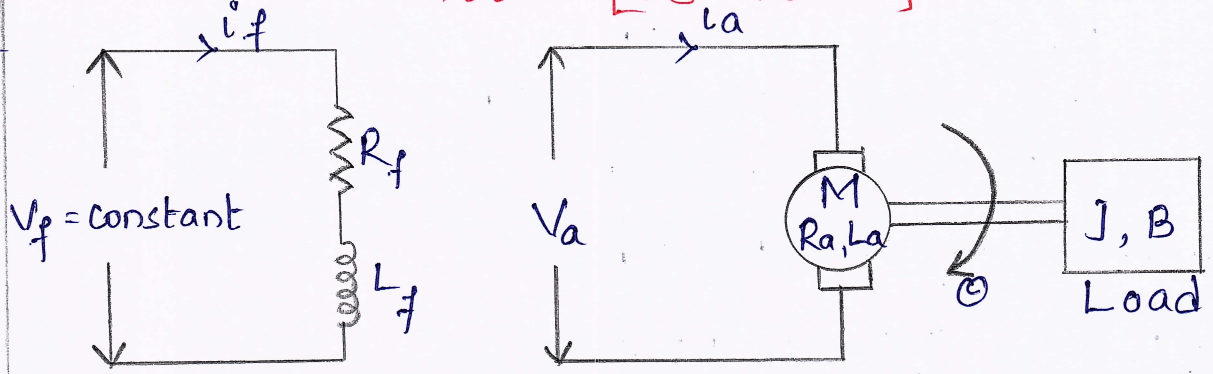
$$0 = C_2 \cdot \frac{dv_2(t)}{dt} + \frac{v_2}{R_3} + \frac{1}{L_2} \int (v_2 - v_1) \cdot dt \quad \text{--- (6)}$$

Equivalent electrical circuit using eqns. (5) & (6)



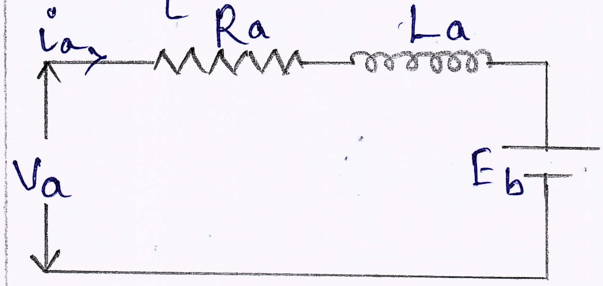
1 b) Derive the transfer function of armature controlled DC motor. [08 marks]

Soln:-



The speed of DC motor is directly proportional to armature voltage & inversly proportional to flux in the field winding. In armature controlled DC motor, the desired speed is obtained by varying the armature voltage where as field is considered constant.

The equivalent circuit of armature is shown below.



By Kirchoff's law we can write,

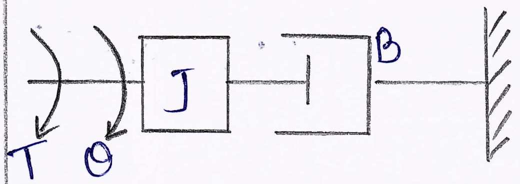
$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \quad \text{--- (1)}$$

Torque of DC motor is proportional to product of flux & current. Since flux is constant torque is proportional to current only.

ie $T \propto i_a \Rightarrow T = K_t \cdot i_a$ --- (2)

where K_t - torque constant.

The mechanical system of the motor is shown below



The differential equation governing mechanical system is,

$$J \cdot \frac{d^2\theta}{dt^2} + B \cdot \frac{d\theta}{dt} = T \quad \text{--- (3)}$$

The back emf of the DC motor is proportional to speed (angular velocity) of shaft.

ie $e_b \propto \frac{d\theta}{dt} \Rightarrow e_b = K_b \cdot \frac{d\theta}{dt}$ --- (4)

where K_b - back emf constant.

On taking Laplace transform of above eqns. 1, 2, 3 & 4

$$I_a(s)R_a + L_a s I_a(s) + E_b(s) = V_a(s) \quad \text{---} \quad (5)$$

$$T(s) = K_t \cdot I_a(s) \quad \text{---} \quad (6)$$

$$Js^2\theta(s) + Bs\,\theta(s) = T(s) \quad \text{---} \quad (7)$$

$$E_b(s) = K_b s\,\theta(s) \quad \text{---} \quad (8)$$

On equating eqns. 6 & 7,

$$K_t I_a(s) = Js^2\theta(s) + Bs\,\theta(s)$$

$$I_a(s) = \frac{[Js^2 + Bs]}{K_t} \cdot \theta(s) \quad \text{---} \quad (9)$$

Eqn. 5 can be written as,

$$[R_a + s \cdot L_a]\, I_a(s) + E_b(s) = V_a(s) \quad \text{---} \quad (10)$$

Substituting eqns. (8) & (9) in eqn. (10)

$$[R_a + s \cdot L_a]\frac{[Js^2 + Bs]}{K_t}\theta(s) + K_b s\,\theta(s) = V_a(s)$$

$$\frac{\theta(s)\left\{[R_a + s \cdot L_a][Js^2 + Bs] + K_b K_t s\right\}}{K_t} = V_a(s)$$

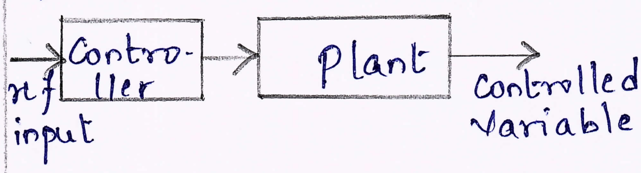
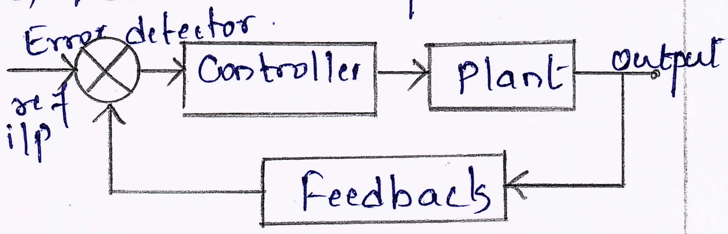
The required Transfer function is $\theta(s)/V_a(s)$

$$\therefore \frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + s \cdot L_a)(Js^2 + Bs) + sK_bK_t}$$

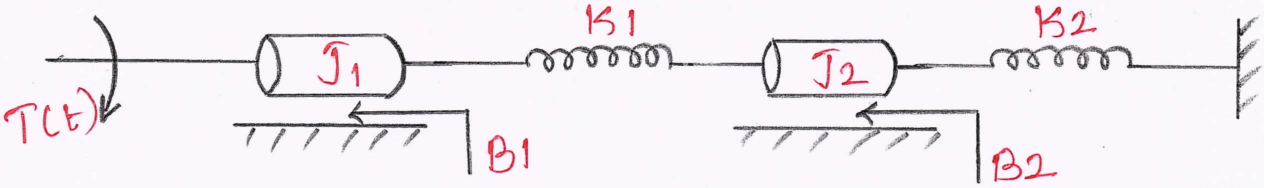
$$\boxed{\frac{\theta(s)}{V_a(s)} = \frac{K_t}{s[JL_a s^2 + (JR_a + BL_a)s + (BR_a + K_bK_t)]}}$$

2a) Distinguish between open-loop & closed loop system with examples. [08 Marks]

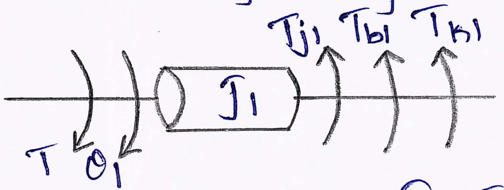
Soln:-

Open-loop	Closed-loop
1) Simple and economical	01) Complex and costly.
2) Inaccurate & unreliable	2) Accurate & reliable.
3) Changes in output due to external disturbances are not corrected automatically.	3) Changes in output due to external disturbances are corrected automatically.
4) Stable operation.	4) Care should be taken while designing to get stable operation.
5) Feedback is absent	5) Feedback is present.
6) 	

2b) For the mechanical translational system as shown in Fig. Q2(b), draw the electrical network based on torque-voltage & torque-current analogy. Write its performance equations. [12 marks]



Soln:- Free body diagram J_1

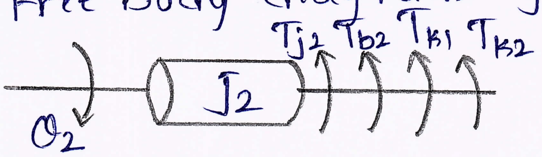


According to Newton's Law.

$$T = T_{j1} + T_{b1} + T_{k1}$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + K_1(\theta_1 - \theta_2) \qquad \text{—— (1)}$$

Free body diagram J_2.



$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_1(\theta_2 - \theta_1) + K_2\theta_2 \qquad \text{—— (2)}$$

Torque - voltage analogy.

Replace : $J \rightarrow L$; $B \rightarrow R$; $K \rightarrow 1/C$.

$$\frac{d^2\theta}{dt^2} = \frac{di}{dt} \; ; \; \frac{d\theta}{dt} = i \; ; \; \theta = \int i \cdot dt$$

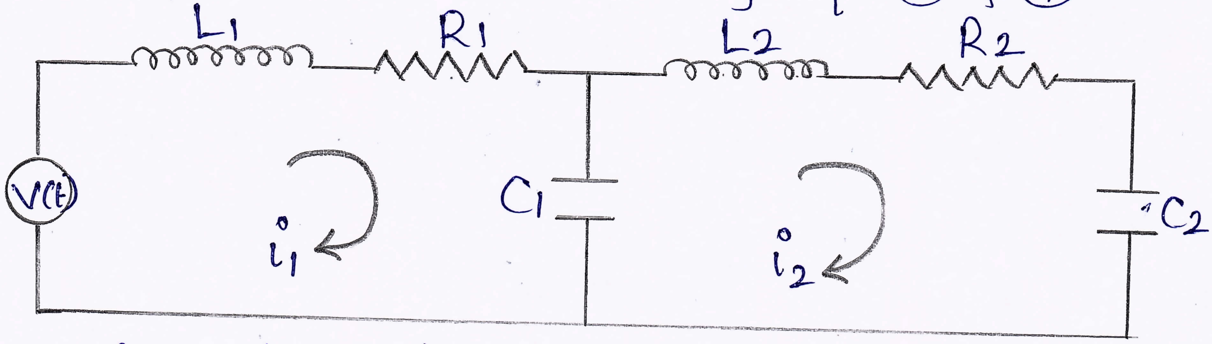
Eqn. 1 can be written as,

$$v(t) = L_1 \cdot \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt \quad \text{--- (3)}$$

Eqn. 2 can be written as,

$$0 = L_2 \cdot \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_1}\int (i_2 - i_1)\,dt + \frac{1}{C_2}\int i_2\,dt \quad \text{--- (4)}$$

Equivalent electrical circuit using eqn 3 & 4



Torque - Current analogy.

Replace : $J \rightarrow C$; $B \rightarrow 1/R$; $K = 1/L$

$$\frac{d^2\theta}{dt^2} = \frac{dv}{dt} \; ; \; \frac{d\theta}{dt} = v \; ; \; \theta = \int v \cdot dt$$

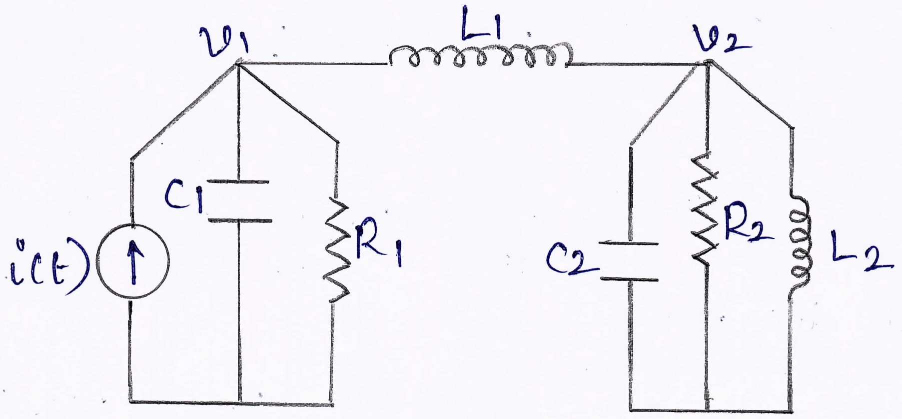
Eqn. 1 can be written as,

$$i(t) = C_1 \cdot \frac{dv_1}{dt} + \frac{v_1}{R_1} + \frac{1}{L_1}\int (v_1 - v_2)\,dt \quad \text{--- (5)}$$

Eqn. 2 can be written as,

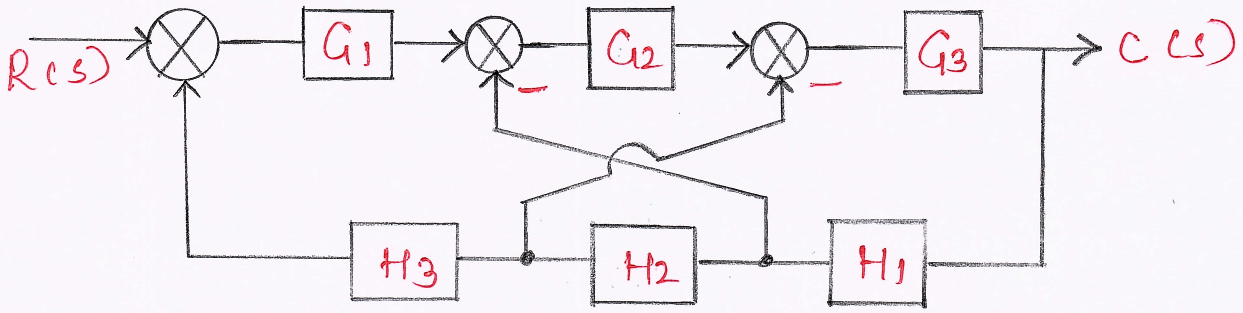
$$0 = C_2 \cdot \frac{dv_2}{dt} + \frac{v_2}{R_2} + \frac{1}{L_1}\int (v_2 - v_1) \cdot dt + \frac{1}{L_2}\int v_2\,dt \quad \text{--- (6)}$$

Equivalent electrical circuit using eqn. 5 & 6

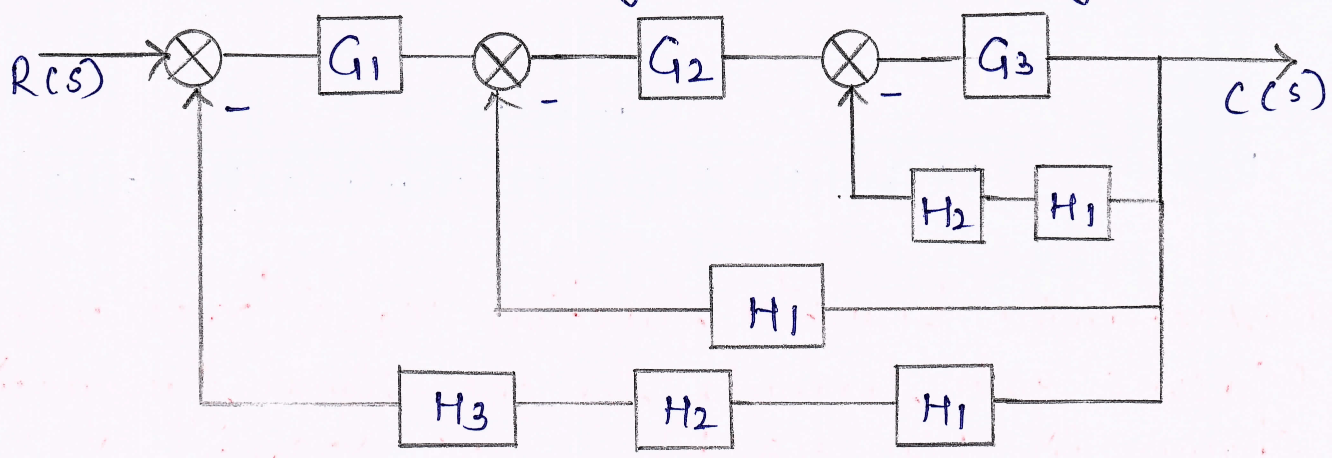


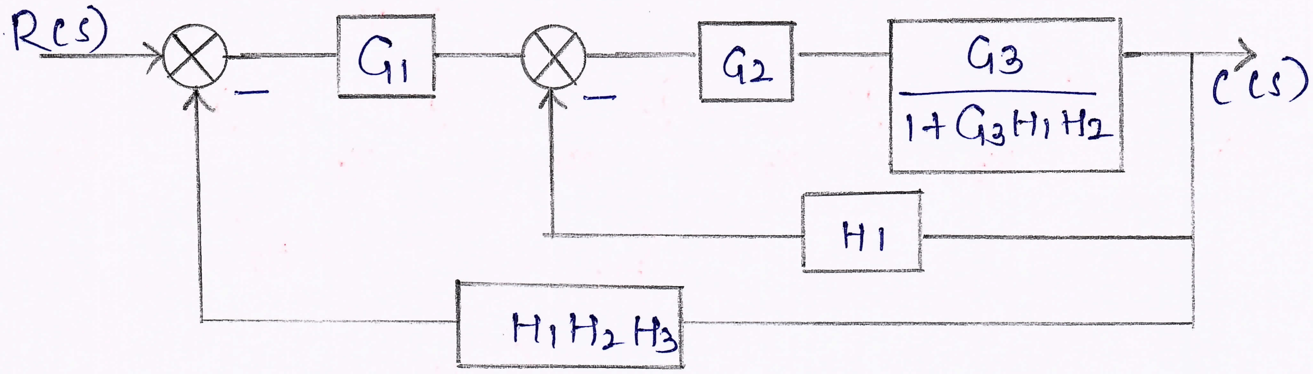
Module-02

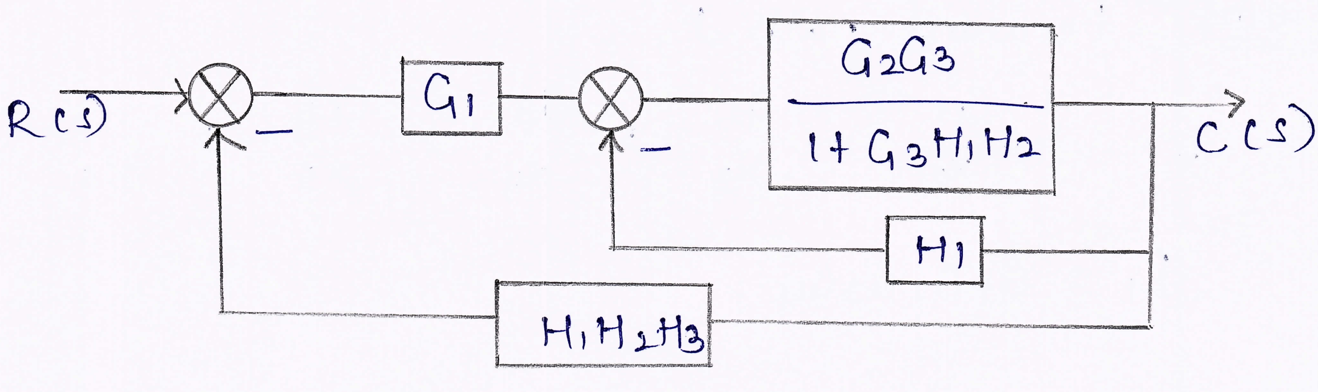
3 a) Obtain $\frac{C(s)}{R(s)}$ using block diagram reduction rule. [08 marks]

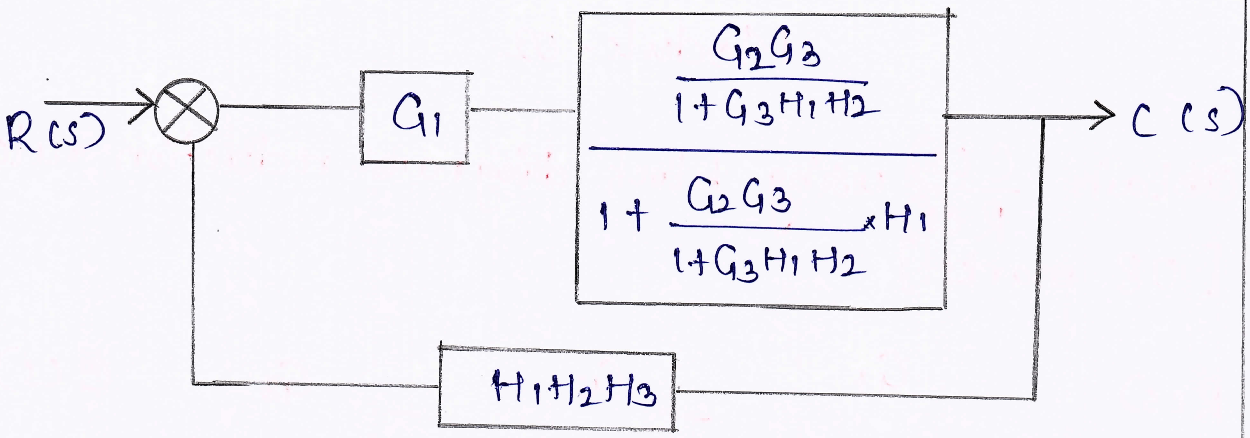


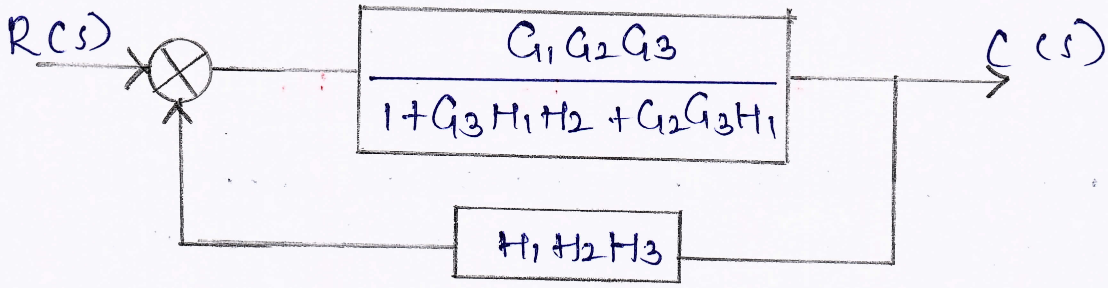
Soln:- Separating out the feedbacks at different summing points, we can rearrange the block diagram as below





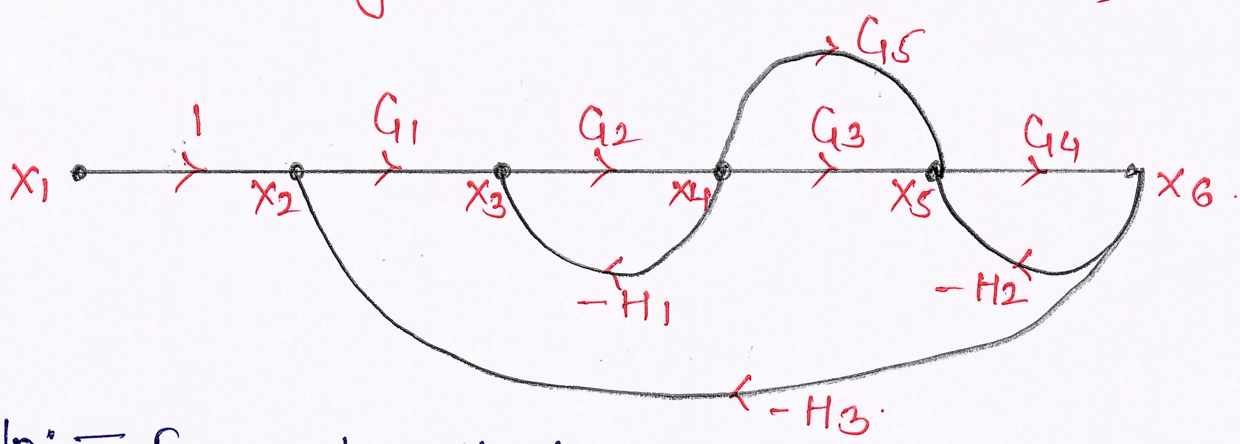




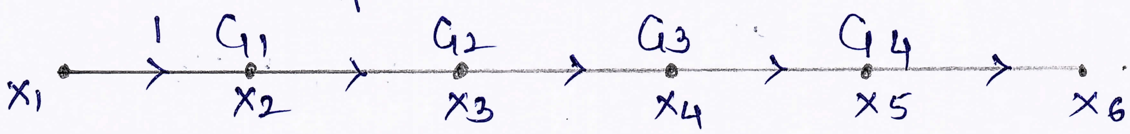


$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_3H_1H_2+G_2G_3H_1+G_1G_2G_3H_1H_2H_3}}$$

3b) Determine transfer function $X_6(s)/X_1(s)$ using Mason's gain formula for the signal flow graph shown in Fig. Q3(b). [08 Marks]

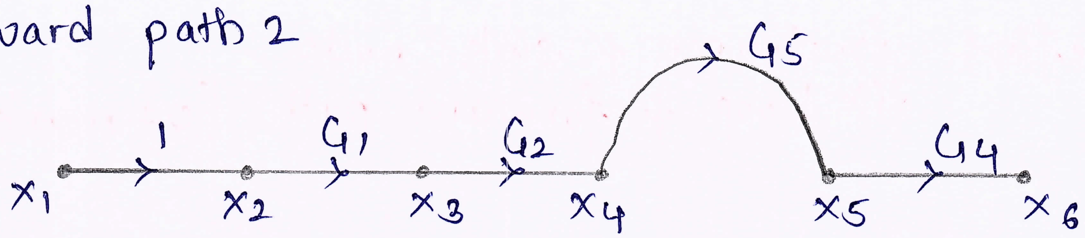


Soln:- Forward path 1.



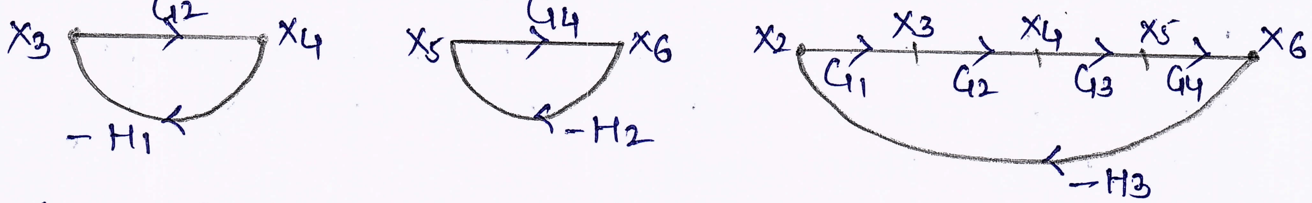
$P_1 = G_1G_2G_3G_4$

Forward path 2

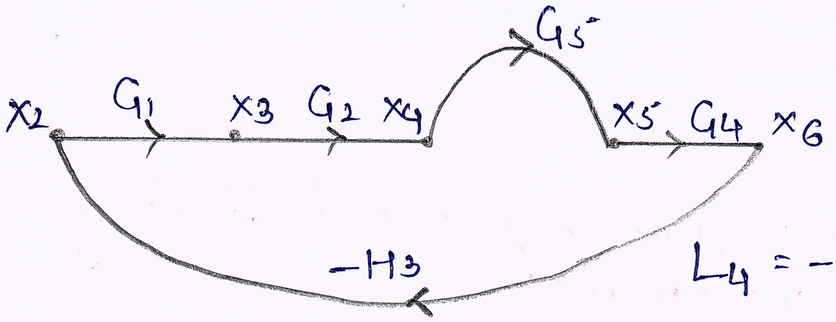


$$P_2 = G_1 G_2 G_4 G_5$$

Individual loop gains.



$L_1 = -G_2 H_1$ $\quad$ $L_2 = -G_4 H_2$ $\quad$ $L_3 = -G_1 G_2 G_3 G_4 H_3$



$$L_4 = -G_1 G_2 G_4 G_5 H_3$$

Two non-touching loops.

$$L_1 L_2 = G_2 G_4 H_1 H_2$$

$$\therefore \Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_2]$$

$$= 1 + G_2 H_1 + G_4 H_2 + G_1 G_2 G_3 G_4 H_3 + G_1 G_2 G_4 G_5 H_3 + G_2 G_4 H_1 H_2$$

$$\Delta_1 = \Delta_2 = 1.$$

Using Mason's gain formula,

$$T(s) = \frac{1}{\Delta} \sum P_k \cdot \Delta_k = \frac{1}{\Delta} [P_1 \Delta_1 + P_2 \Delta_2]$$

$$\boxed{\therefore T(s) = \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_4 G_5}{1 + G_2 H_1 + G_4 H_2 + G_1 G_2 G_3 G_4 H_3 + G_1 G_2 G_4 G_5 H_3 + G_2 G_4 H_1 H_2}}$$

3c) Define i) Source & sink node
ii) Loop & forward path. [04 marks]

Soln:

Source: It is node that has only outgoing branches.

Sink: It is node that has only incoming branches.

Loop: It is a closed path starting from a node & after passing through a certain part of a graph arrives at same node without crossing any node more than once.

Forward path: It is path from an input node to an output node that does not cross any node more than once.

4a) Explain Mason's gain formula indicating each term. [04 marks]

Soln:— Let $R(s)$ = input to the system.
$C(s)$ = output of the system

Transfer function $T(s) = C(s)/R(s)$

Mason's gain formula states the overall gain of the system as follows.

Overall gain $T = \frac{1}{\Delta} \sum_{k} P_k \cdot \Delta_k$

where, $T = T(s) = C(s)/R(s)$ transfer function.

P_k = forward path gain of k^{th} forward path

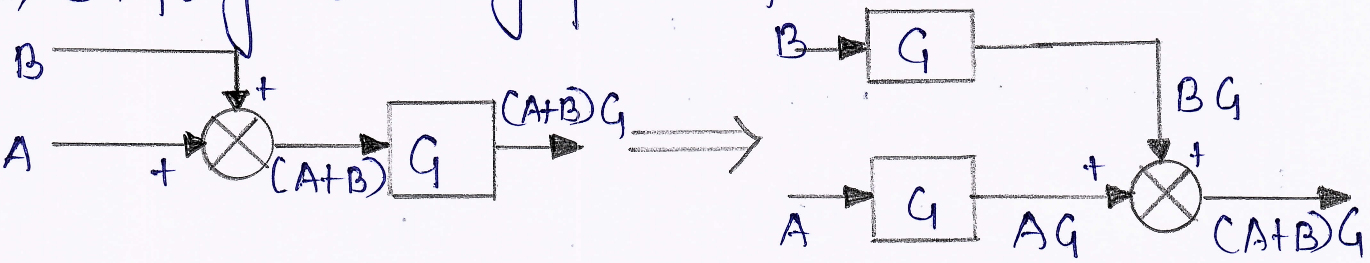
k = no. of forward paths in the signal flowgraph

Δ = 1 − (Sum of individual loop gains) + (Sum of gain products of all possible combinations of two non-touching loops) − (Sum of gain products of all possible combinations of three non-touching loops) + ...

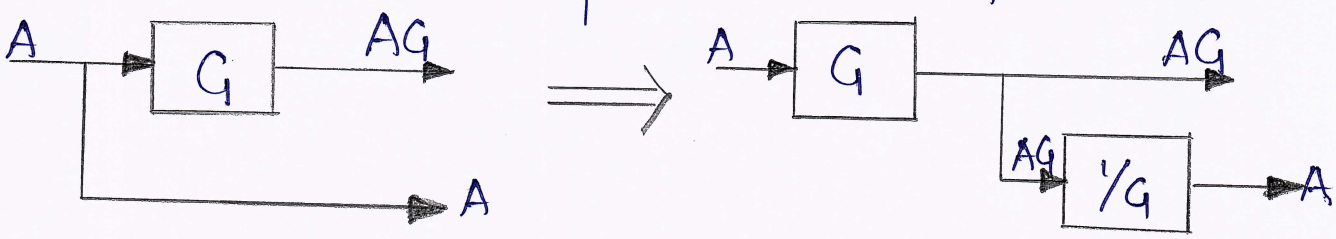
Δ_k = Δ for that part of the graph which is not touching k^{th} forward path.

4b) Illustrate how to perform the following connection with block diagram reduction technique.
i) Shifting summing point after a block
ii) Shifting take off point ahead of a block
iii) blocks in parallel. [06 Marks]

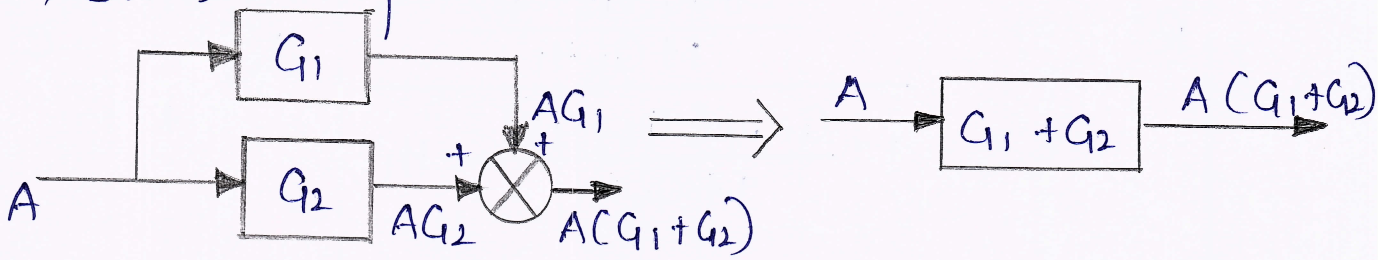
Soln: i) Shifting summing point after a block



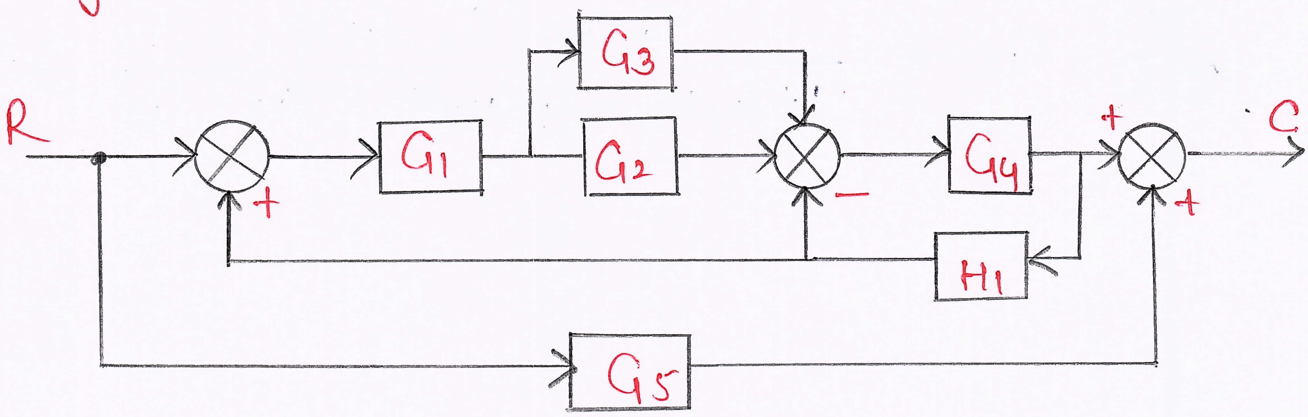
ii) Shifting take-off point ahead of a block.



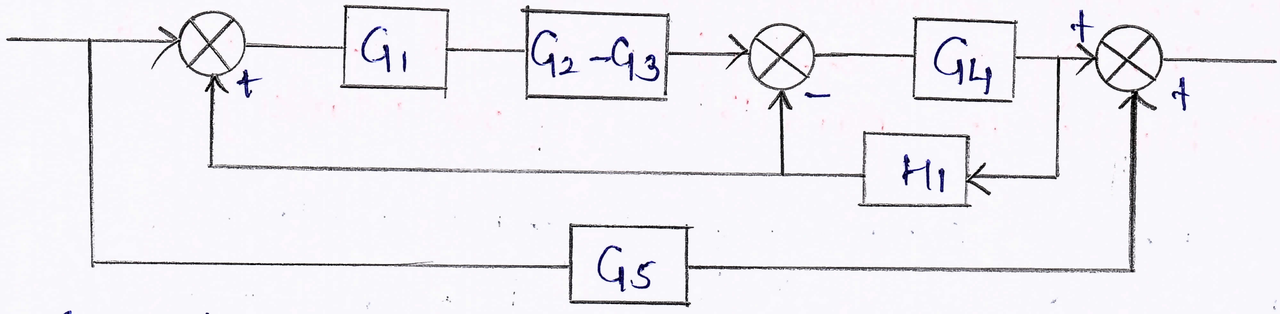
iii) blocks in parallel.



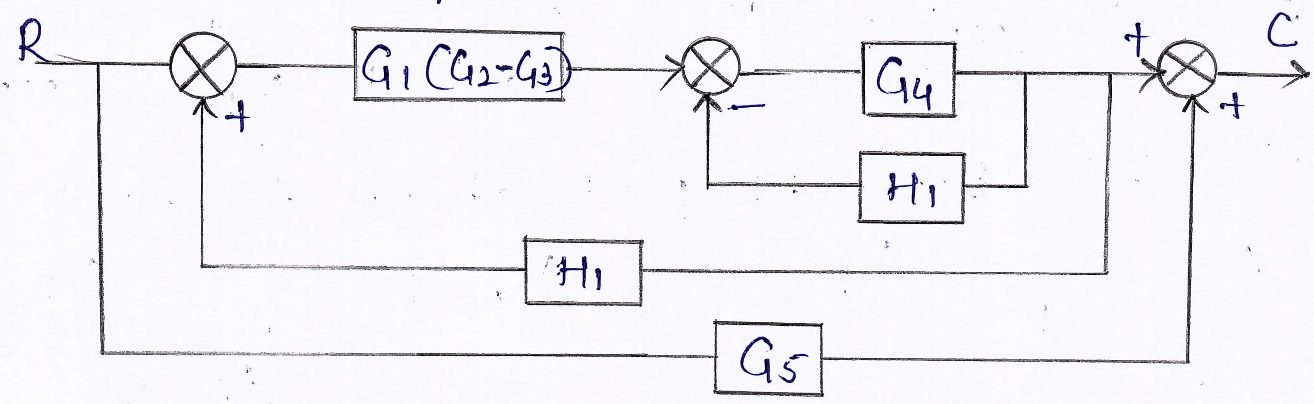
4c) Determine the transfer function C(s)/R(s) of a system shown in Fig. Q4(c). [10 marks]

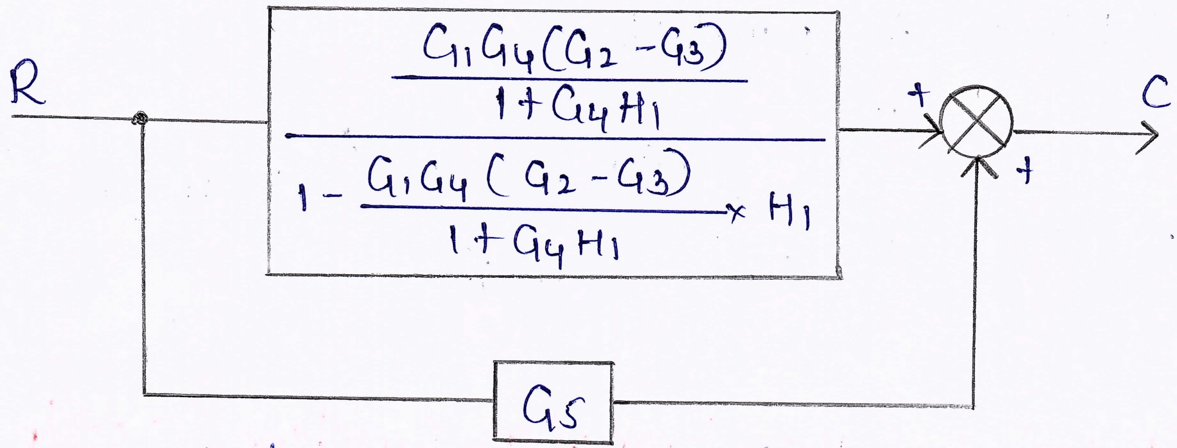


Soln:— The blocks G_2 & G_3 are in parallel, so reducing them,



Separate the paths as shown.





The two blocks are in parallel.

$$\therefore \boxed{\frac{C(s)}{R(s)} = G_5 + \frac{G_1G_4(G_2-G_3)}{1+G_4H_1 - G_1G_4H_1(G_2-G_3)}}$$

Module-03

5 a) Derive an expression for rise-time & peak-time for a second order system excited by a step input (under damped case) [08 Marks]

Soln:- Unit step response of 2nd order system for under damped case is,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin[\omega_d \cdot t + \theta]$$

at $t = t_r$, $c(t) = c(t_r) = 1$.

$$\therefore c(t) = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 1$$

$$-\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t_r + \theta) = 0$$

Since $-e^{-\xi\omega_n t_r} \neq 0$ so $\sin(\omega_d t_r + \theta) = 0$

when $\phi = 0, \pi, 2\pi, 3\pi \ldots\ldots$ $\sin\phi = 0$

$$\therefore \omega_d t_r + \theta = \pi$$

$$\omega_d t_r = \pi - \theta$$

$$\therefore \boxed{\text{Rise time } t_r = \frac{\pi - \theta}{\omega_d} \text{ in sec}}$$

where $\theta = \tan^{-1}\sqrt{1-\xi^2}$ in radians.

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

***Peak-time (t_p)**

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \sin(\omega_d t + \theta)$$

Expression for t_p is obtained by,

$$\left.\frac{d}{dt} c(t)\right|_{t = t_p} = 0$$

$$\therefore \frac{d\,c(t)}{dt} = \frac{-e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot (-\xi\omega_n)\sin(\omega_d t+\theta) + \left[-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \cos(\omega_d t+\theta)\cdot\omega_d\right]$$

Put $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$= \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \xi\omega_n \sin(\omega_d t+\theta) - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \omega_n\sqrt{1-\xi^2}\cos(\omega_d t+\theta)$$

$$= \frac{e^{-\xi\omega_n t}\,\omega_n}{\sqrt{1-\xi^2}}\left[\xi\sin(\omega_d t+\theta) - \sqrt{1-\xi^2}\cdot\cos(\omega_d t+\theta)\right]$$

$$= \frac{\omega_n}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t} \cdot \left[\cos\theta\sin(\omega_d t+\theta) - \sin\theta\cos(\omega_d t+\theta)\right]$$

$$= \frac{\omega_n}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t}\left[\sin(\omega_d t+\theta-\theta)\right]$$

$$= \frac{\omega_n}{\sqrt{1-\xi^2}} \cdot e^{-\xi\omega_n t}\sin(\omega_d t)$$

at $t = t_p$, $\frac{d}{dt}c(t_p) = 0$

$$\therefore \frac{\omega_n}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t_p}\sin(\omega_d t_p) = 0$$

Since $e^{-\xi\omega_n t} \neq 0$ $\quad \sin(\omega_d t_p) = 0$

when $\phi = 0, \pi, 2\pi, 3\pi \ldots\ldots$ $\quad \sin\phi = 0$.

$$\therefore \omega_d t_p = \pi$$

$$t_p = \pi/\omega_d$$

$$\boxed{\text{Peak-time } \; t_p = \pi/\omega_n\sqrt{1-\xi^2}}$$

5b) Check the stability of the given characteristic equation using R-H criterion.

$$S^5 + 6S^4 + 3S^3 + 2S^2 + S + 1 = 0$$ [06 Marks]

Soln:-

S^5	1	3	1
S^4	6	2	1
S^3	2.6	0.83	
S^2	0.08	1	
S^1	-31.67		
S^0	1		

System is unstable as there is sign change in first column. There is 2 times sign changes hence two roots lying in right half of s-plane.

5c) A second order system is given by, $\frac{C(s)}{R(s)} = \frac{25}{s^2+6s+25}$ find rise time, peak time, peak overshoot & settling time for 2% tolerance. [06 Marks]

Soln:- We have, $\omega_n = \sqrt{25} = 5$ rad/sec.

$$2\xi\omega_n = 6$$

$$\therefore \xi = \frac{6}{2\times5} = 0.6$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 5\sqrt{1-(0.6)^2} = 4 \text{ rad/sec.}$$

i) Rise time $t_r = \frac{\pi - \theta}{\omega_d}$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 53.13^\circ = 0.927 \text{ rad.}$$

$$= \frac{\pi - 0.927}{4}$$

$$t_r = 0.553 \text{ sec}$$

ii) peak-time $t_p = \pi/\omega_d = \pi/4 = 0.785$ sec.

iii) % peak overshoot $\%M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$

$\%M_p = 9.47\%$

iv) Settling time $t_s = \dfrac{\ln(\%\text{ error})}{\xi\omega_n} = \dfrac{\ln(0.02)}{0.6 \times 5}$

$t_s = 1.33$ sec.

6 a) Explain the difficulties encountered while assessing the R-H criteria & how do you eliminate these difficulties with examples. [08 marks]

Soln:—

01) The first element in any one row is zero, but others are not.

Remedy: Replace '0' by small number ϵ & continue calculating elements of routh array. Once routh array completes replace ϵ by 0.

OR Substitute $s = 1/z$ in the characteristic equation & form new characteristic equation, for this form Routh array.

02) All the elements in one row of routh array are zero.

Remedy: Take a row above zero row as auxilary row & form auxilary equation $A(s)$. Find $\dfrac{dA(s)}{ds}$ & use in the place of zero row.

6 b) A unity feedback control system has,

$G(s) = \dfrac{K(s+4)}{s(s+1)(s+2)}$ using R-H criterion, find the range of K for which system to be stable & also determine the frequency of oscillations. [08 Marks]

Soln:- Closed loop T.F. $\dfrac{C(s)}{R(s)} = \dfrac{K(s+4)}{s(s+1)(s+2) + K\cdot(s+4)}$

Characteristic equation is,

$$s(s+1)(s+2) + K(s+4) = 0$$
$$s(s^2+3s+2) + Ks + 4K = 0$$
$$s^3 + 3s^2 + (2+K)s + 4K = 0$$

Routh array,

s^3	1	$2+K$
s^2	3	$4K$
s^1	$\dfrac{6+3K-4K}{3}$	0
s^0	$4K$	

For system to be stable,

$$4K > 0$$
$$K > 0$$
$$6 + 3K - 4K > 0$$
$$6 - K > 0$$
$$\text{or } K < 6$$

So, $0 < K < 6$ for system to be stable.

When $K = 6$; s^1 row becomes zero.

Auxilary equation is, $3s^2 + 4K = 0$

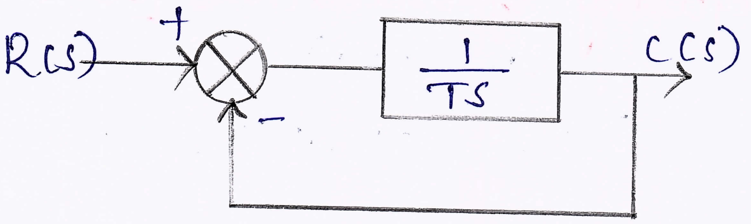
Put $K = 6$; $3s^2 + 24 = 0$

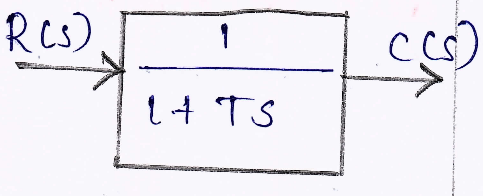
$s = \pm 2.82$

When $K = 6$, system has roots on imaginary ($j\omega$) axis & it oscillates at $\omega = 2.82$ rad/sec.

6 c) Obtain an expression for time response of the first order system subjected to unit step input. [04 marks]

Soln:— Closed loop 1st order system is as shown below.





$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{1+TS}$$

If input is unit step then $r(t) = 1$ & $R(s) = 1/s$

∴ The response in s-domain,

$$C(s) = R(s) \cdot \frac{1}{1+TS} = \frac{1}{s} \cdot \frac{1}{1+TS}$$

$$= \frac{1}{sT(1/T + s)} = \frac{\frac{1}{T}}{s[s + 1/T]}$$

By partial fraction expansion,

$$C(s) = \frac{1/T}{s(s+1/T)} = \frac{A}{s} + \frac{B}{s+1/T}$$

$$A(s + 1/T) + Bs = 1/T$$

Put $s = 0 \Rightarrow A = 1$.

Put $s = -1/T \Rightarrow B = -1$

$$\therefore C(s) = \frac{1}{s} - \frac{1}{s+1/T}$$

Time response is given by,

$$c(t) = \mathcal{L}^{-1}[C(s)] = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{s+1/T}\right]$$

$$= 1 - e^{-t/T}$$

$\therefore c(t) = 1 - e^{-t/T}$ for unit step input

$c(t) = A[1 - e^{-t/T}]$ for step ~~un~~input.

Module - 04

7 a) Explain the terms given below with respect to root locus.
i) Break-away point
ii) Asymptotes.
iii) Intersection of root locus branches with Jω axis.
[06 Marks]

Soln:— **Break-away point:**

The break-away or break-in points either lie on real axis or exist as complex conjugate pairs. If there is root locus on real axis between 2 poles then there exist a breakaway point. If there is a root locus real axis between two zeros, then there exist a break-in point. If there is a root locus on real axis between poles & zero, then there may be or may not be breakaway or break in point.

Characteristic eqn. $B(s) + K \cdot A(s) = 0$

$$\therefore K = -B(s)/A(s)$$

The breakaway & break-in point is given by roots of the equation $dK/ds = 0$. The roots of $dK/ds = 0$ are actual break-away or break-in point provided, for this value of roots, the gain K should be positive & real.

Asymptotes:

n-m root locus branches terminate at zeros at infinity. These n-m root locus branches will go along an asymptotic path & meets the asymptotes at ∞. Hence no. of asymptotes is equal to no. of root locus branches going to infinity.

$$\text{Angle of asymptote} = \frac{\pm 180 (2q+1)}{n-m}, \quad q = 0, 1, 2 \ldots \ldots (n-m)$$

Point of intersection of root-locus with imaginary axis.

Substituting $s = j\omega$ in characteristic eqn. & separate real & imaginary part, solve to get ω & K.
ω - gives the points where the root locus crosses imaginary axis. The value of K gives the value of gain K at there crossing point.

7b) A unity feedback system the open-loop transfer function is given by, $G(s) = \dfrac{K}{s(s+2)(s^2+6s+25)}$

i) Sketch the root locus for $0 \le K \le \infty$.

ii) At what value of 'K' the system becomes stable.

iii) At this point of instability determine the frequency of oscillation of system. [14 marks]

Soln:-

Step 1: Location of poles & zeros.

no. of zeros, $m = 0$.

no. of poles, $n = 4 \Rightarrow 0, -2, -3+j4, -3-j4$

Step 2: Root locus on real axis.

Real axis between 0 to -2 is part of root locus.

Step 3: Angle of asymptotes & centroid.

$$\theta = \frac{\pm(2q+1)\times 180}{n-m}, \quad q = 0, 1, 2, 3, 4.$$

$q = 0 \Rightarrow \theta_1 = \pm 45$

$q = 1 \Rightarrow \theta_2 = \pm 135^\circ$

$q = 2 \Rightarrow \theta_3 = \pm 225^\circ$

$q = 3 \Rightarrow \theta_4 = \pm 315^\circ$

$q = 4 \Rightarrow \theta_5 = \pm 405^\circ = \pm 45^\circ$

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

$$\text{Centroid} = \frac{[0 - 2 - 3 + j4 - 3 - j4] - 0}{4}$$

$$\text{Centroid} = -2$$

Step 4: Break-away point.

Consider Characteristic eqn. as,

$$s(s+2)(s^2+6s+25) + K = 0.$$

$$s(s^3+8s^2+37s+50) + K = 0$$

$$\text{ie} \quad K = -s^4 - 8s^3 - 37s^2 - 50s \quad \text{——} \quad (1)$$

$$\frac{dK}{ds} = -4s^3 - 24s^2 - 74s - 50 = 0$$

$$\text{ie} \quad s^3 + 6s^2 + 18.5s + 12.5 = 0$$

The roots of this cubic equation gives us breakaway point. On solving, $s = -0.898$ is valid breakaway point for which $K = 20.2$ from eqn. (1)

Step 5: Intersection with imaginary axis.

$$s^4 + 8s^3 + 37s^2 + 50s + K = 0.$$

s^4	1	37	K
s^3	8	50	
s^2	30.75	K	
s^1	$\frac{1537.5 - 8K}{30.75}$		
s^0	K		

For K_{mar}, $1537.5 - 8K = 0$.

$\therefore K_{mar} = 192.1875$

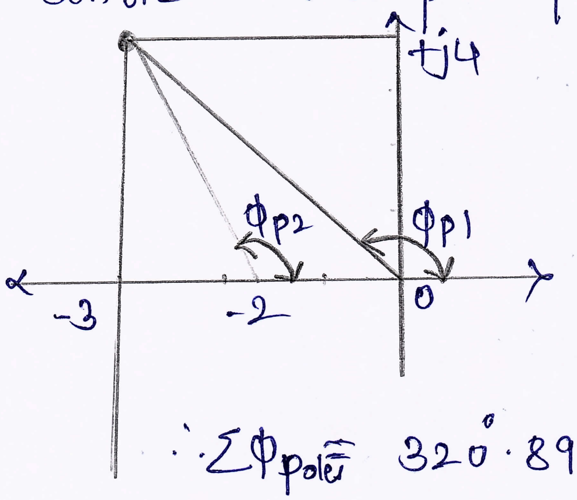
$A(s) = 30.75 s^2 + K = 0$.

$$\therefore s^2 = -\frac{K}{30.75} = \frac{-192.18}{30.75} = -6.25$$

$\therefore s = \pm j2.5$

Step 6: Angle of departure.

Consider a complex pole $-3+j4$



$\phi_{p1} = 180 - \tan^{-1}\frac{4}{3} = \cancel{\frac{4}{3}} = 126.86^\circ$

$\phi_{p2} = 180^\circ - \tan^{-1}\frac{4}{1} = 104.03^\circ$.

$\phi_{p3} = 90^\circ$.

$\therefore \Sigma\phi_{poles} = 320^\circ.89$ and $\Sigma\phi_{zeros} = 0^\circ$.

$\therefore \phi = \Sigma\phi_{poles} - \Sigma\phi_{zeros} = 320^\circ.89^\circ$

$\phi_d = 180 - \phi = -140.89^\circ$ at $-3+j4$

$= +140.89^\circ$ at $-3-j4$

The complete root locus is shown in graph sheet.

SCALE X - 3 unit = 1.
Y - 2 unit = 1.

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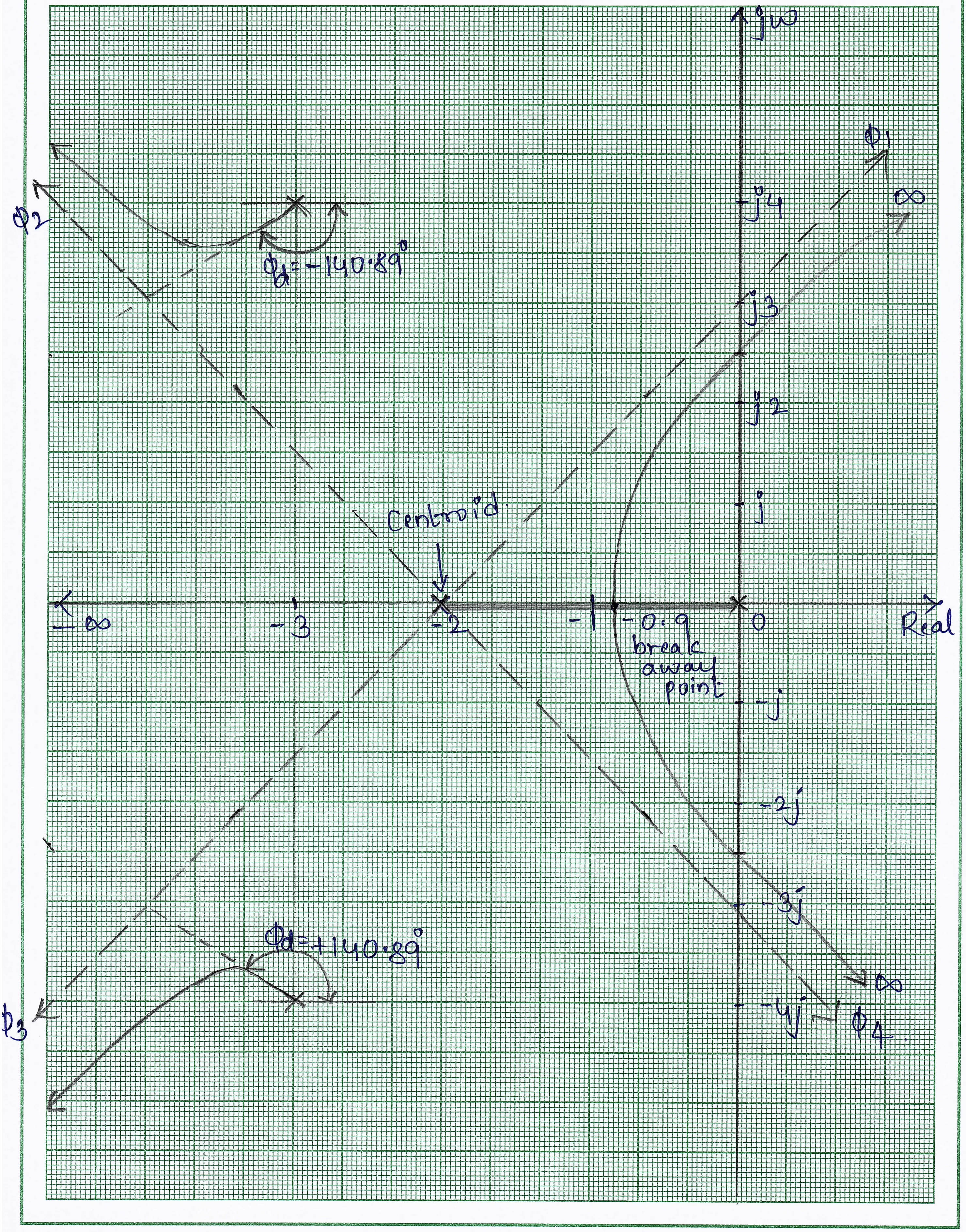


8a) For a unity FBCS with $G(s) = \frac{80}{s(s+2)(s+20)}$. Find gain & phase margin using bode plot. [12 marks]

Soln:- Let $H(s) = 1$

$$\therefore G(s)H(s) = \frac{80}{s(s+2)(s+20)} = \frac{80}{s \times 2[1+0.5s] \times 20[1+0.05s]}$$

Replace s by $j\omega$.

$$G(j\omega) = \frac{2}{j\omega[1+0.5j\omega][1+0.05j\omega]}$$

Magnitude plot.

Term	Corner frequency rad/sec.	Slope db/dec.	Change in slope.
$2/j\omega$	—	-20	—
$1/1+0.5j\omega$	$\omega_{c1} = 2$	-20	$-20-20 = -40$.
$1/1+0.05j\omega$	$\omega_{c2} = 20$	-20	$-40-20 = -60$

Let $\omega_l = 0.1$ rad/sec.

$\omega_h = 100$ rad/sec.

at $\omega = \omega_l = 0.1$ rad/sec.

$$A = 20 \log(2/\omega)$$
$$= 20 \log(2/0.1)$$
$$A = 26.02 \text{ db}.$$

at $\omega = \omega_h = 100$ rad/sec

$$A = 20 \log(2/100)$$
$$= -33.97 \text{ db}.$$

at $\omega = \omega_{c1} = 2$

$$A = 20 \log(2/2)$$
$$= 0 \text{ db}$$

at $\omega = \omega_{c2} = 20$

$$A = 20 \log(2/20)$$
$$= -20 \text{ db}$$

Phase plot

$$\phi = \angle G(j\omega) = -90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.05\omega)$$

$\phi(\omega)$ rad/sec	$\phi(\omega)$ in deg.
0.1	-93.14°
0.2	-96.27°
2	-140.7°
10	$-195^\circ.29^\circ$
20	-219.28°
50	-245.9°
80	-254.53°
100	-257.5°

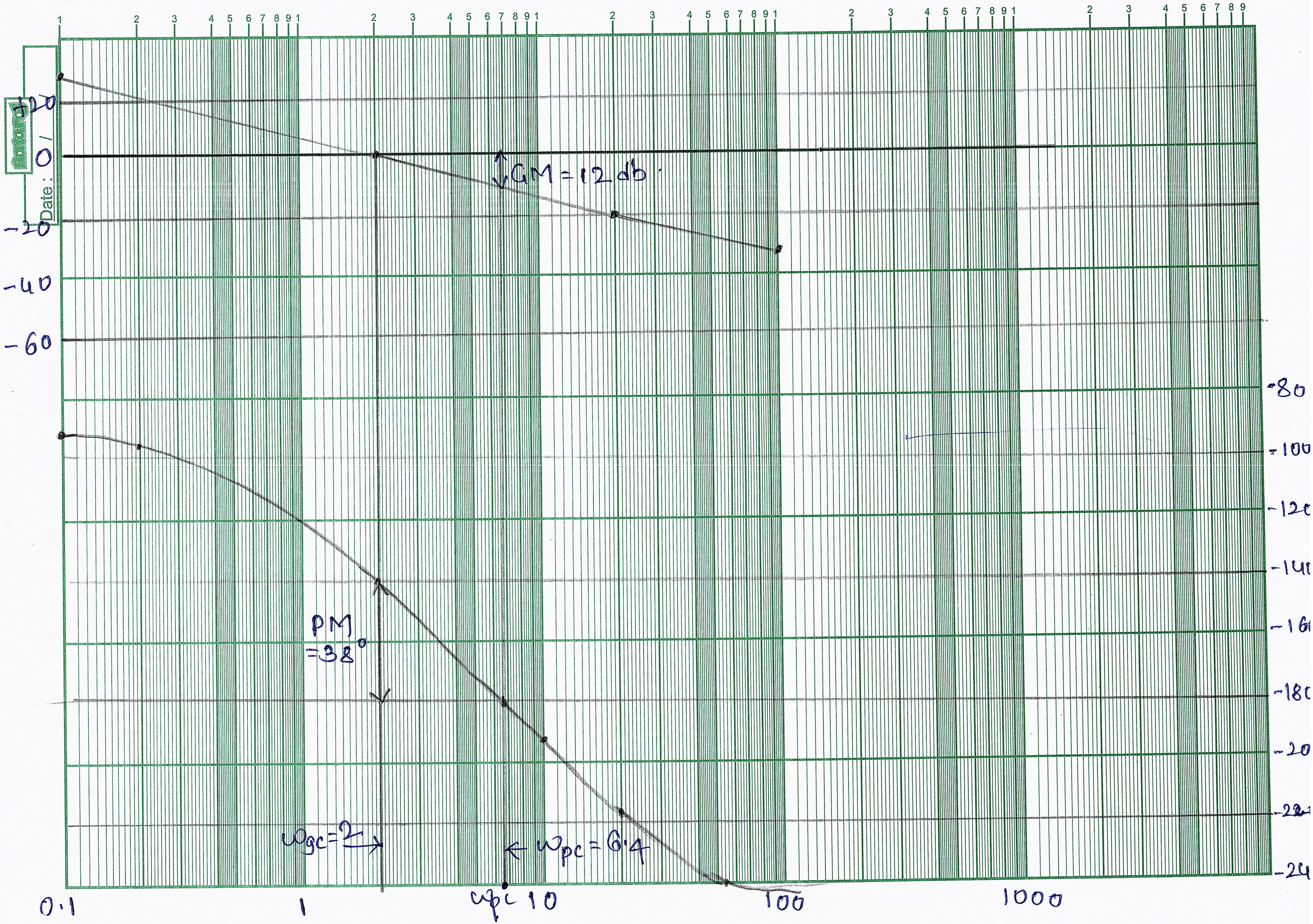
The graph is shown below.
From the bode plot we can have,

$\omega_{gc} = 2$

$\omega_{pc} = 6.4$

G.M. = 12 db

P.M. = 38°.



+20
0
-20
-40
-60
GM = 12 db
-80
-100
-120
-140
-160
-180
-200
-220
-240
PM = 38°
ω_{gc} = 2
ω_{pc} = 6.4
0.1
1
ω_{pc} 10
100
1000
Date : / /

8 b) Derive an expression for resonant peak & resonant frequency for a second order system. [08 Marks]

Soln:- Consider the closed loop transfer function of 2nd order system.

$$\frac{C(s)}{R(s)} = M(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \text{--- (1)}$$

The sinusoidal transfer function $M(j\omega)$ is obtained by letting $s = j\omega$

$$\therefore M(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + j2\xi\omega_n\omega + \omega_n^2}$$

$$= \frac{\omega_n^2}{\omega_n^2\left[\frac{-\omega^2}{\omega_n^2} + j2\xi\frac{\omega}{\omega_n} + 1\right]}$$

$$= \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}}$$

Let normalised frequency, $u = (\omega/\omega_n)$

$$\therefore M(j\omega) = \frac{1}{1 - u^2 + j2\xi u}$$

Let M = Magnitude of closed Loop transfer function.

α = Phase of closed loop transfer function.

$$M = |M(j\omega)| = \left[\frac{1}{(1-u^2)^2 + (2\xi u)^2}\right]^{1/2} = \left[(1-u^2)^2 + 4\xi^2u^2\right]^{-1/2} \quad \text{--- (2)}$$

$$\alpha = \angle M(j\omega) = -\tan^{-1}\frac{2\xi u}{1-u^2}$$

The resonant peak is the maximum value of M. The condition for maximum value of M can be obtained by differentiating the equation of M w.r.t u & letting $dM/du = 0$ when $u = u_r$

where $u_r = \frac{\omega_r}{\omega}$ = normalised resonant frequency.

On differentiating eqn (2) w.r.t. u we get,

$$\frac{dM}{du} = \frac{d}{du}\left[(1-u^2)^2 + 4\xi^2 u^2\right]^{-1/2}$$

$$= -\frac{1}{2}\left[(1-u^2)^2 + 4\xi^2 u^2\right]^{-3/2} \cdot \left[2(1-u^2)(-2u) + 8\xi^2 u\right]$$

$$= -\frac{\left[-4u(1-u^2) + 8\xi^2 u\right]}{2\left[(1-u^2)^2 + 4\xi^2 u^2\right]^{3/2}}$$

$$= \frac{4u(1-u^2) - 8\xi^2 u}{2\left[(1-u^2)^2 + 4\xi^2 u^2\right]^{3/2}} \quad \text{—— (3)}$$

Replace u by u_r in eqn. (3) & equate to zero.

$$\frac{4u_r(1-u_r^2) - 8\xi^2 u_r}{2\left[(1-u_r^2)^2 + 4\xi^2 u_r^2\right]^{3/2}} = 0. \quad \text{—— (4)}$$

The eqn. (4) will become;

$$4u_r(1-u_r^2) - 8\xi^2 u_r = 0.$$

$$4u_r - 4u_r^3 - 8\xi^2 u_r = 0.$$

$$\therefore 4u_r^3 = 4u_r - 8\xi^2 u_r$$

$$\therefore u_r^2 = 1 - 2\xi^2$$

$$u_r = \sqrt{1-2\xi^2}$$

$\therefore$ Resonant peak occurs when $u_r = \sqrt{1-2\xi^2}$

Put this condition in eqn (2) for M & solve for M_r.

$$\therefore M_r = \left.\frac{1}{\left[(1-u^2)^2 + 4\xi^2 u^2\right]^{1/2}}\right|_{u=u_r}$$

$$= \frac{1}{\left[(1-(1-2\xi^2))^2 + 4\xi^2(1-2\xi^2)\right]^{1/2}}$$

$$= \frac{1}{\left[4\xi^4 + 4\xi^2 - 8\xi^4\right]^{1/2}}$$

$$= \frac{1}{\left[4\xi^2 - 4\xi^4\right]^{1/2}} = \frac{1}{\left[4\xi^2(1-\xi^2)\right]^{1/2}} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\therefore \boxed{\text{Resonant peak, } M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$

*Resonant Frequency (ω_r)

Normalised resonant frequency, $u_r = \frac{\omega_r}{\omega_n} = \sqrt{1-2\xi^2}$

$$\therefore \boxed{\text{The resonant frequency, } \omega_r = \omega_n\sqrt{1-2\xi^2}}$$

9a) Explain PID Controller & discuss the effect on the behaviour of the system. [10 Marks]

Soln:- The PID controller produces an output, which is the combination of the outputs of proportional, integral & derivative controllers.

$$u(t) = K_P \cdot e(t) + K_I \int e(t) \cdot dt + K_D \cdot \frac{de(t)}{dt}$$

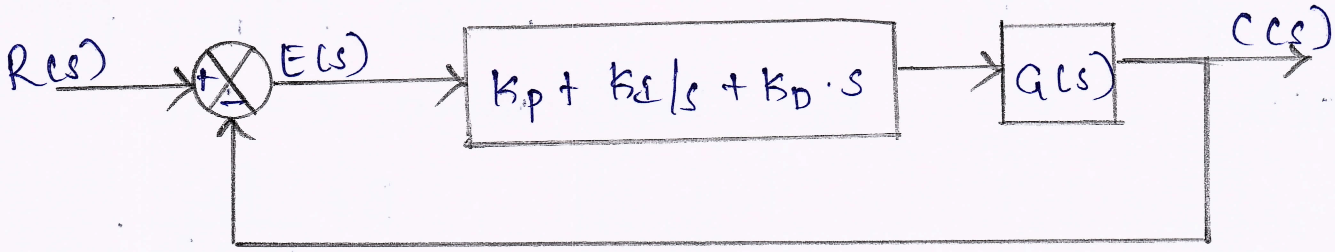
Apply Laplace Transform on both side.

$$U(s) = \left[K_P + \frac{K_I}{s} + K_D \cdot s\right] E(s)$$

$$\frac{U(s)}{E(s)} = \left[K_P + \frac{K_I}{s} + K_D \cdot s\right]$$

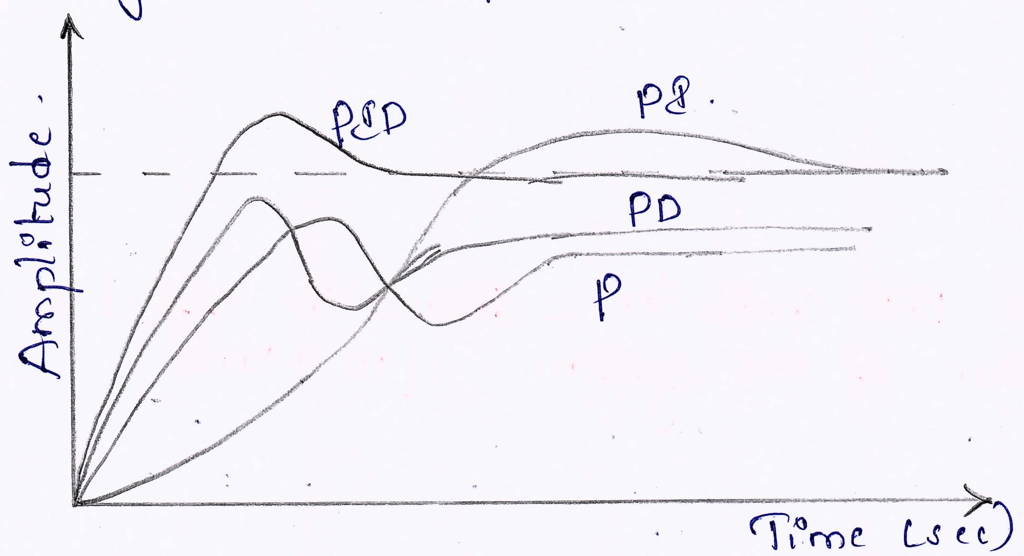
Therefore, the transfer function of the PID Controller is $[K_P + K_I/s + K_D \cdot s]$

The block diagram of the unity -ve feedback closed loop control system along with the PID controller is shown below.



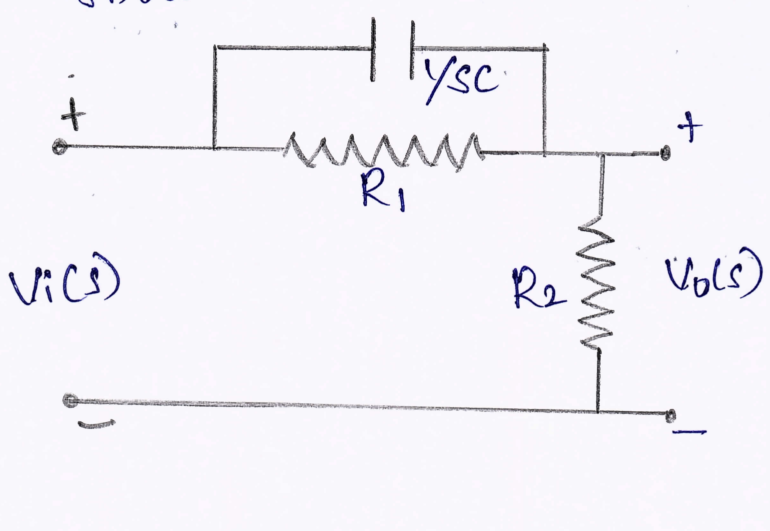
* PID controller has advantages of the modes. The integral mode eliminates the offset error of proportional mode & response is also fast due to derivative mode.

* With PID controller there is no offset, no oscillation with the least settling time. So, there is improvement in both transient as well as steady state response.



9b) Explain the step by step design procedure of lead compensation networks. [10 Marks]

Soln:— The lead compensator is an electrical network which produces a sinusoidal output having phase lead, when sinusoidal input is applied. The lead compensator circuit in 'S' domain is shown below.



The T.F. of the lead Compensator is,

$$\frac{V_o(s)}{V_i(s)} = \beta\left[\frac{s\tau + 1}{\beta s\tau + 1}\right]$$

where $\tau = R_1C$

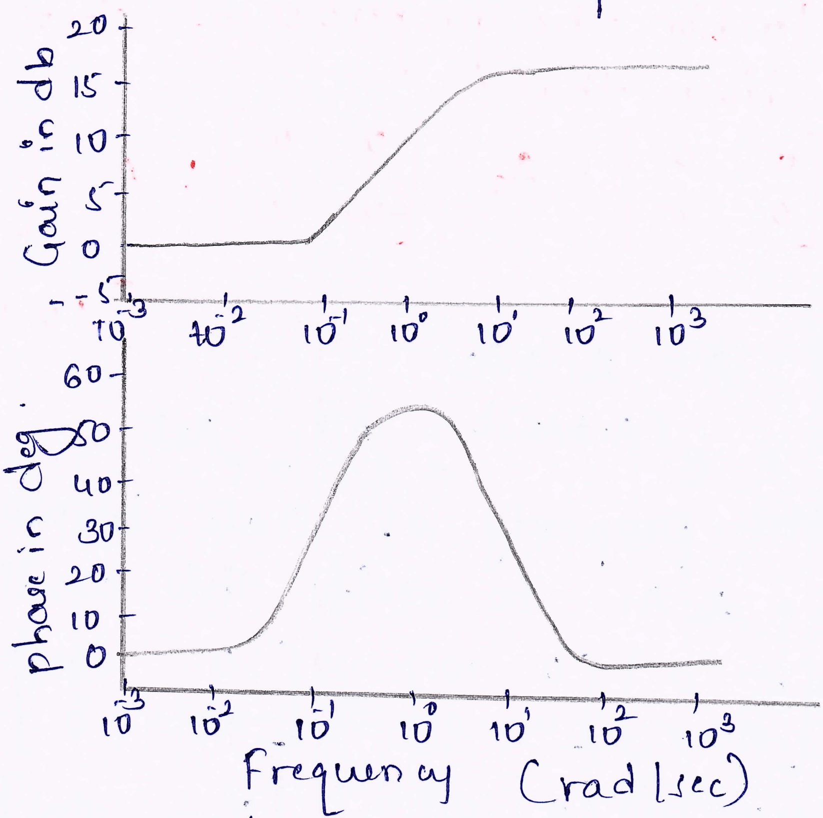
$$\beta = \frac{R_2}{R_1 + R_2}$$

From the transfer function, we can conclude that the lead compensator has pole at $s = -1/\beta$ & zero at $s = -1/\beta\tau$

Substitute $s = j\omega$ in the transfer function,

$$\frac{V_o(j\omega)}{V_i(j\omega)} = \beta \left[\frac{j\omega\tau + 1}{\beta j\omega\tau + 1} \right]$$

Phase angle $\phi = \tan^{-1}\omega\tau - \tan^{-1}\beta\omega\tau$



*Basic design procedure:

1) Determine loop gain, K to satisfy either steady-state error requirement or bandwidth requirement.
   a) Set K to provide the required static error constant or
   b) Set K to place the crossover frequency an octave below desired closed loop bandwidth.

2) Evaluate the phase margine of the uncompensated system using value of K.

3) If necessary, determine the required PM from ξ or overshoot specifications. Evaluate the PM of the uncompensated system & determine the required phase lead at the cross-over frequency

to achieve this PM.

Add $\sim 10°$ additional phase. This is ϕ_{max}

04) Calculate β from ϕ_{max}.

05) Set $\omega_{max} = \omega_{pm}$. calculate T from ω_{max} & β.

06) Simulate & iterate, if necessary.

Q 10 a) Mention the properties of state transition matrix. Given that:

$$A_1 = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}; \quad A_2 = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}; \quad A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

[10 marks]

Soln:- If $A = A_1 + A_2$

then $e^{At} = e^{(A_1 + A_2)t} = e^{A_1 t} \times e^{A_2 t}$

this is the property of state transition matrix if $A_1 A_2 = A_2 A_1$. So verify this condition.

$$A_1 A_2 = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} = \begin{bmatrix} 0 & \sigma\omega \\ -\sigma\omega & 0 \end{bmatrix}$$

$$A_2 A_1 = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} = \begin{bmatrix} 0 & \sigma\omega \\ -\sigma\omega & 0 \end{bmatrix}$$

As $A_1 A_2 = A_2 A_1$, the above property holds good. So calculate $e^{A_1 t}$ & $e^{A_2 t}$ & then multiply to obtain e^{At}.

$$e^{A_1 t} = L^{-1}[sI - A_1]^{-1}$$

$$[sI - A_1] = \begin{bmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{bmatrix}, \quad \text{Adj}[sI - A_2] = \begin{bmatrix} s-\sigma & 0 \\ 0 & s-\sigma \end{bmatrix}$$

$$|sI - A_1| = (s-\sigma)^2$$

$$[sI - A_1]^{-1} = \frac{Adj[sI - A_1]}{|sI - A_1|} = \begin{bmatrix} \frac{1}{s-\sigma} & 0 \\ 0 & \frac{1}{s-\sigma} \end{bmatrix}$$

$$\therefore e^{A_1 t} = L^{-1}[sI - A_1]^{-1} = \begin{bmatrix} e^{\sigma t} & 0 \\ 0 & e^{\sigma t} \end{bmatrix}$$

$$[sI - A_2] = \begin{bmatrix} s & -\omega \\ \omega & s \end{bmatrix}, \quad adj[sI - A_2] = \begin{bmatrix} s & \omega \\ -\omega & s \end{bmatrix}$$

$$|sI - A_2| = s^2 + \omega^2$$

$$[sI - A_2]^{-1} = \frac{Adj[sI - A_2]}{|sI - A_2|} = \begin{bmatrix} \frac{s}{s^2+\omega^2} & \frac{\omega}{s^2+\omega^2} \\ \frac{-\omega}{s^2+\omega^2} & \frac{s}{s^2+\omega^2} \end{bmatrix}$$

$$\therefore e^{A_2 t} = L^{-1}[sI - A_2]^{-1} = \begin{bmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{bmatrix}$$

$$\therefore e^{At} = \begin{bmatrix} e^{\sigma t} & 0 \\ 0 & e^{\sigma t} \end{bmatrix} \times \begin{bmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{\sigma t}\cos\omega t & e^{\sigma t}\sin\omega t \\ -e^{\sigma t}\sin\omega t & e^{\sigma t}\cos\omega t \end{bmatrix}$$

10b) Explain concept of state. Define.

i) State variable.
ii) State vector
iii) State space.
iv) State trajectory [10 Marks]

Soln:- State : The state of a dynamic system is defined as a minimal set of variables such that knowledge of these variables at $t = t_0$ together with the knowledge of the inputs for $t \geq t_0$, completely determines the behaviour of the system for $t > t_0$.

2) State Variables: The variables involved in determining the state of a dynamic system $x(t)$, are called state variables. $x_1(t)$, $x_2(t)$, ... $x_n(t)$ are state variables & are normally energy storing elements contained in the system.

3) State Vector: The 'n' state variables necessary to describe the complete behaviour of the system can be considered as 'n' components of a vector $x(t)$ called state vector at time t. The state vector $x(t)$ is the vector sum of all the state variables.

4) State Space: The space whose co-ordinate axes are nothing but the 'n' state variables with time as the implicit variable is called the state space.

5) State Trajectory: It is the locus of the tips of the state vectors, with time as the implicit variable.

(Rajeshwari N.)